Learning Objectives

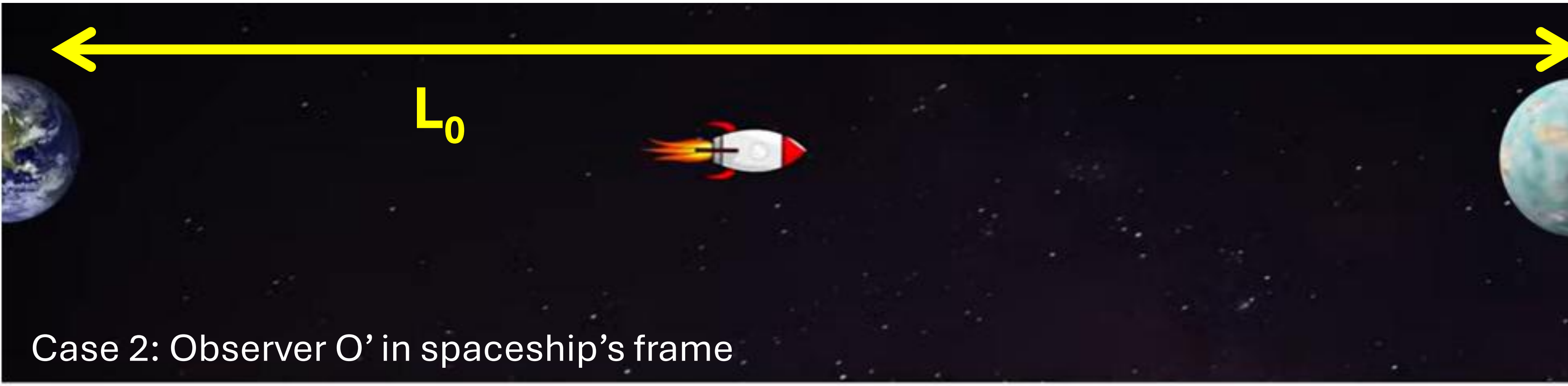
- Length contraction
- Time dilation and length contraction together
- Muon decay problem
- Other formulas related to STR

Ref. Modern Physics by Arthur Beiser
Feynman Lectures on Physics, volume I, chapter 15.

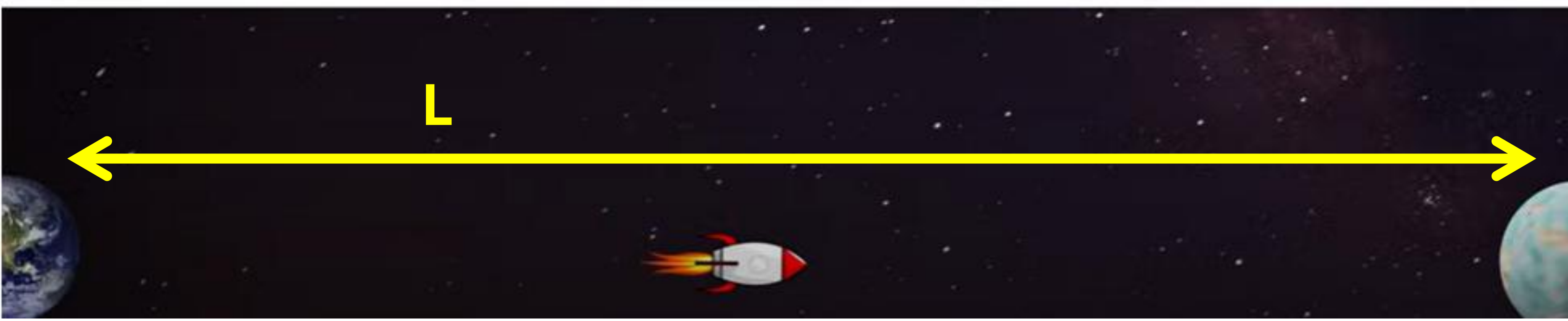
Length contraction

The ‘proper length’ L_0 is the length measured in a frame at rest with respect to objects.

Case 1: Observer O in earth’s frame



Case 2: Observer O’ in spaceship’s frame



Length contraction

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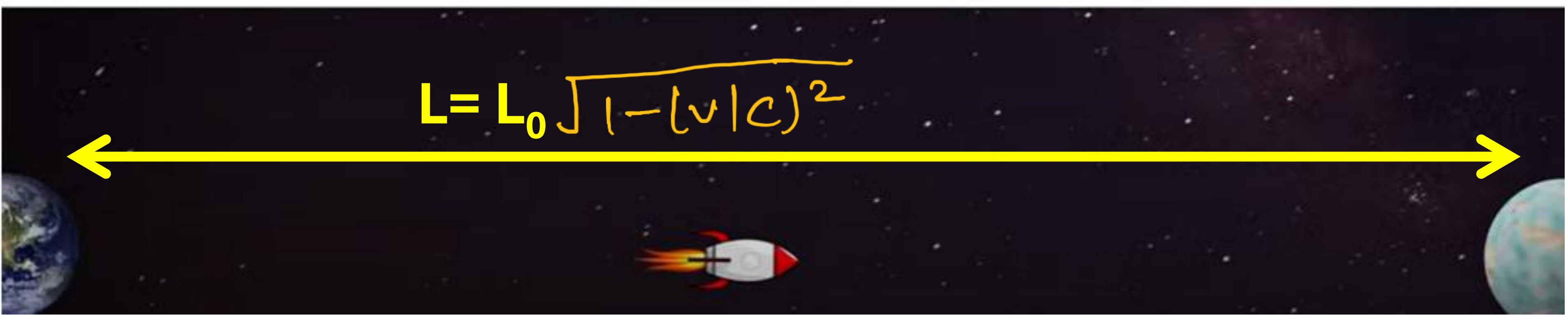
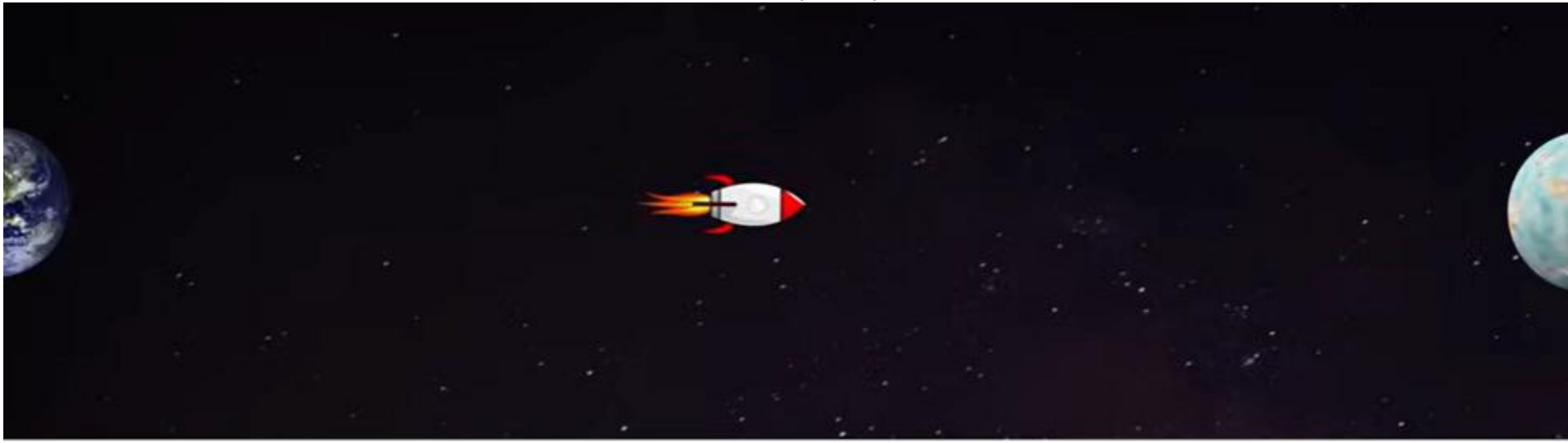
Case 1: Observer O in earth's frame. Earth is at rest w.r.t. start and end points.

Case 2: Observer O' in spaceship's frame . Spaceship is in motion w.r.t. start and end points.



Observer on earth measures dilated time for an event happened inside spaceship

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$



Observer on spaceship measures proper time for an event happened inside spaceship

t_o

Length contraction: Consequences

Muon decay

Muon travels at a speed of 0.998 c and have average lifetime of 2.2 us.

$v = 0.998\ c$

$t_0 = 2.2\ us$

Distance travelled = vt

= 0.998c X 2.2 us

= 0.66 Km

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in time t, muon will cover distance d

$$d = v \times t = 0.998\,c \times 34.8\,\mu s = \underline{10.4\,km}$$

in earth's frame, muon travels distance d

$$d = 10.4 \text{ km}$$

$$L_0 = d$$

in muon's frame of reference, this distance is

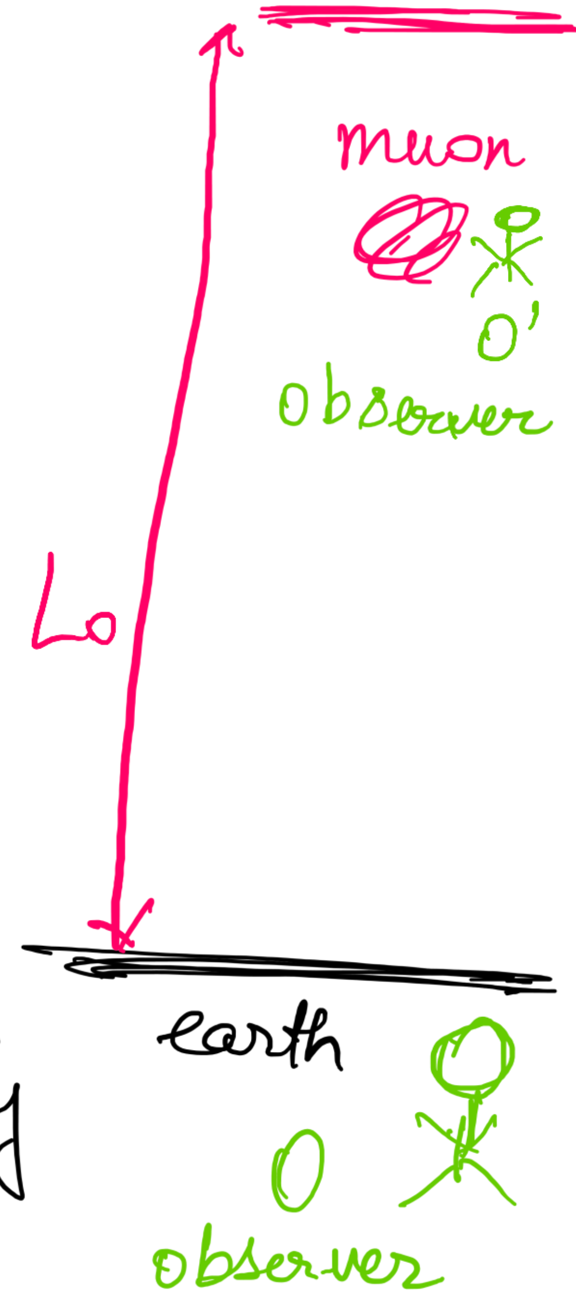
$$L = L_0 \sqrt{1 - v^2/c^2}$$

$$= 10.4 \sqrt{1 - (0.998)^2}$$

$$\boxed{L = 0.66 \text{ km}}$$

length contraction

Relativistic shortening of distance is an example of general contraction of length in the direction of motion.



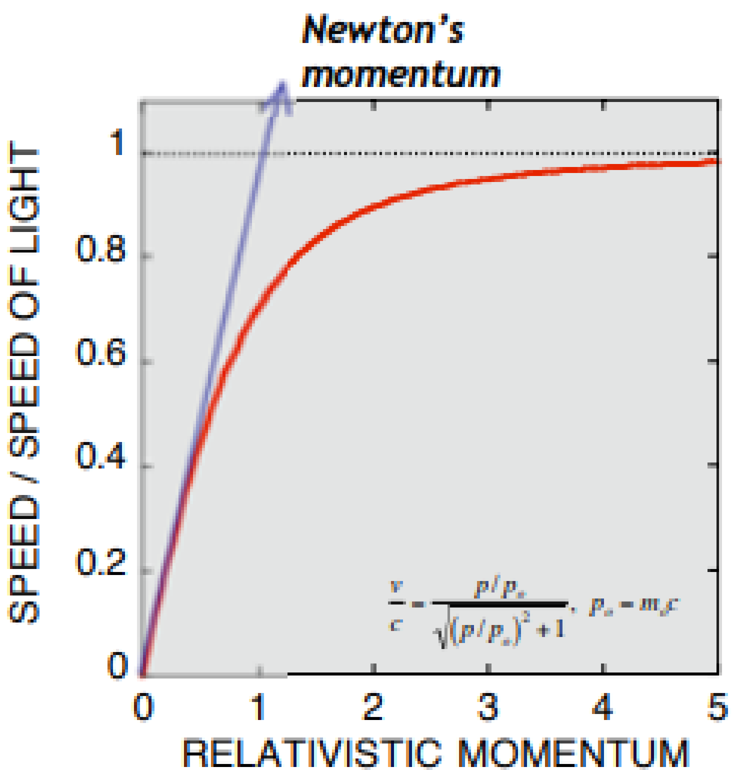
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Relativistic momentum $\quad p = \gamma m_0 v = \dfrac{m_0 v}{\sqrt{1-(v/c)^2}}$

Relativistic mass $\quad m = \gamma \underbrace{m_0}_{\text{rest mass}} = \dfrac{m_0}{\sqrt{1-(v/c)^2}}$

Relativistic second law $\quad F = \dfrac{dp}{dt} = \dfrac{d}{dt}(\gamma m_0 v)$

$$= \frac{m_0 a}{(1-v^2/c^2)^{3/2}}$$



Relativistic momentum for different speeds.

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Relativistic momentum $p = \gamma m_0 v = \frac{m_0 v}{\sqrt{1-(v/c)^2}}$

Relativistic second law $F = \frac{dp}{dt} = \frac{d}{dt}(\gamma m_0 v)$

$$= m_0 \frac{d}{dt}\left(\frac{v}{\sqrt{1-(v/c)^2}}\right)$$

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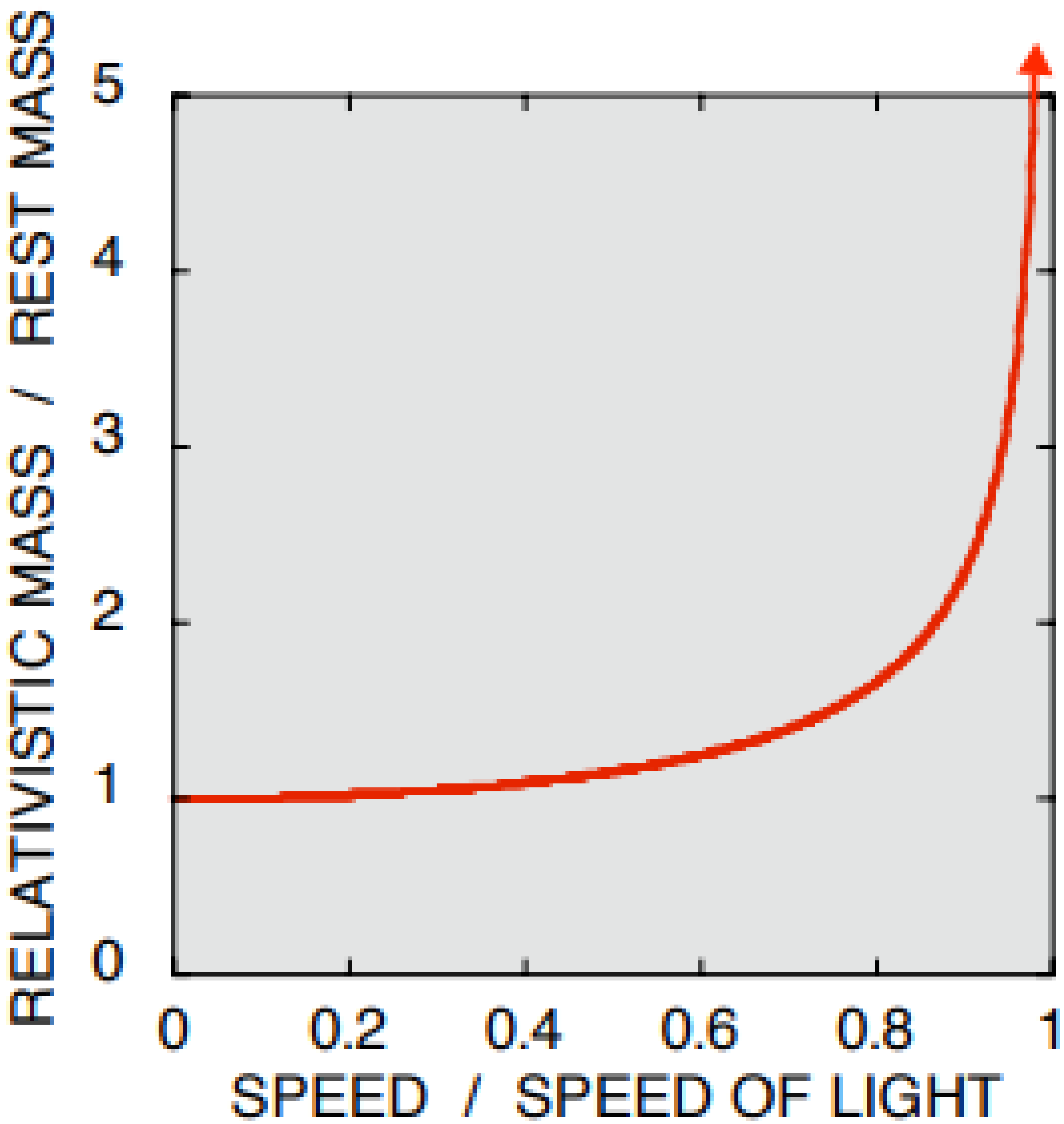
Relativistic second law $\quad F = \dfrac{dp}{dt} = \dfrac{d}{dt}(\gamma m_0 v)$

$$= m_0 \frac{d}{dt}\left(\frac{v}{\sqrt{1-(v/c)^2}}\right)$$

$$= m_0 \left[\frac{1}{\sqrt{1-(v/c)^2}} + \frac{v^2/c^2}{(1-v^2/c^2)^{3/2}}\right]\frac{dv}{dt}$$

$$= \frac{m_0 a}{(1-v^2/c^2)^{3/2}}$$

Relativistic mass $m = \underbrace{\gamma m_0}_{m_0 \to \text{rest mass}} = \dfrac{m_0}{\sqrt{1-(v/c)^2}}$



Relativistic kinetic energy

$$K.E. = \int_0^s F\,ds = \int_0^s \frac{d}{dt}(\gamma m_0 v)\,ds = \int_0^v d(\gamma m_0 v)\,v$$

$$= \int_0^v v\,d(\gamma m_0 v) = \int_0^v v\,d\left(\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right)$$

$$\int x\,dy = xy - \int y\,dx$$

$$K.E. = \frac{m_0 v^2}{\sqrt{1-(v/c)^2}} - m_0\int_0^v \frac{v\,dv}{\sqrt{1-(v/c)^2}}$$

Relativistic kinetic energy

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$$K.E. = \frac{m_0 v^2}{\sqrt{1-(v/c)^2}} - m_0 \int_0^v \frac{v\,dv}{\sqrt{1-(v/c)^2}}$$

$$K.E. = \frac{m_0 c^2}{\sqrt{1-(v/c)^2}} - m_0 c^2$$

$$K.E. = (\gamma - 1) m_0 c^2$$

Total energy = rest mass energy + K.E.

$$= m_0 c^2 + (\gamma - 1) m_0 c^2$$

$$\boxed{E = \gamma m_0 c^2}$$

Was Newton wrong?

Kinetic energy at low speed

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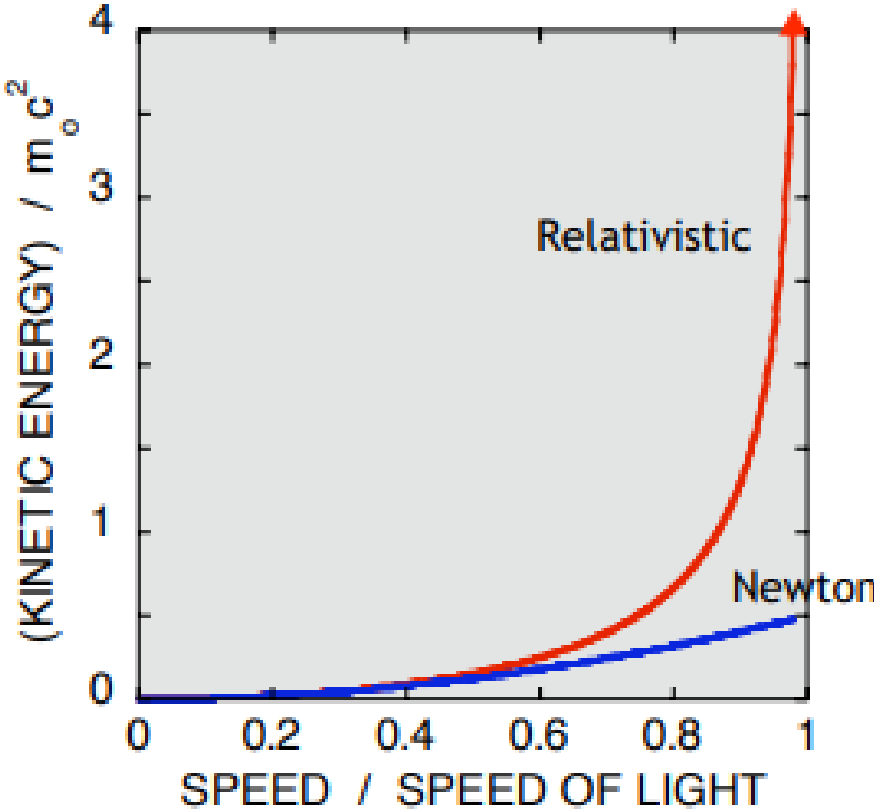
$$= \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2$$



Kinetic energy at low speed

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$$= \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2$$

$$\frac{1}{\sqrt{1 - v^2/c^2}} \simeq 1 + \frac{1}{2}\frac{v^2}{c^2} \quad , v << c$$

$$(1 + x)^n \simeq 1 + nx, \quad |x| \ll 1$$

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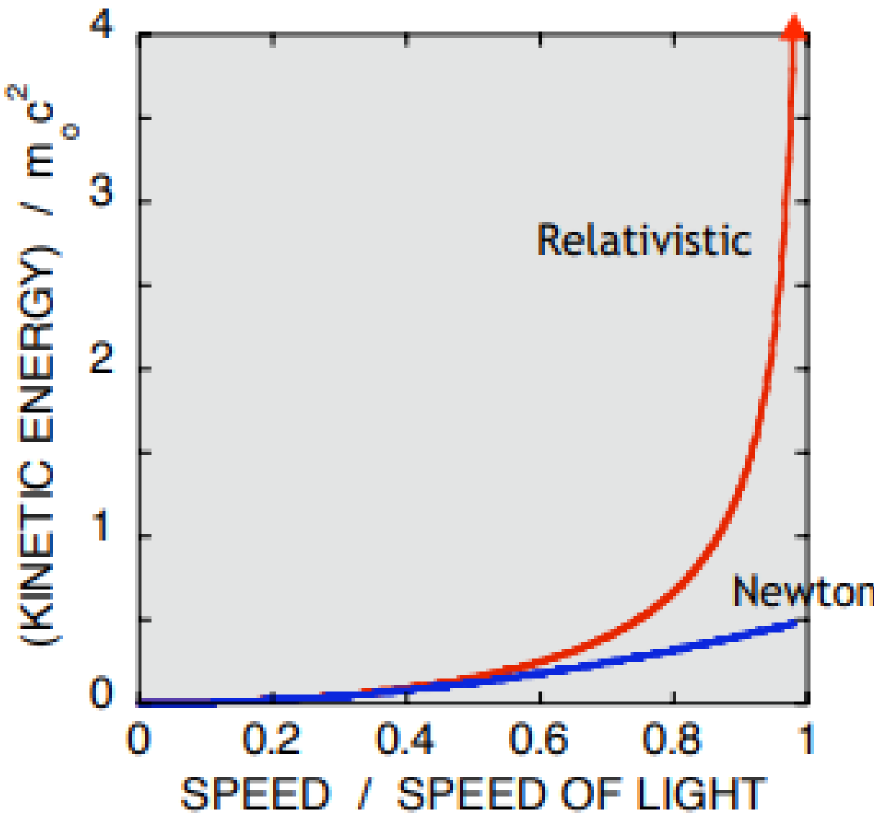
$$(1+x)^n \simeq 1 + nx, \quad |x| \ll 1$$

$$K.E. = m_0 c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) - m_0 c^2$$

$$= \cancel{m_0 c^2} + \frac{1}{2} m_0 v^2 - \cancel{m_0 c^2}$$

$$\boxed{K.E. = \frac{1}{2} m_0 v^2}$$



Energy and momentum

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 V = \frac{m_0 V}{\sqrt{1-(v/c)^2}}$$

$$E^2 = \frac{m_0^2 c^4}{(1-v^2/c^2)}$$

$$p^2 = \frac{m_0^2 v^2}{(1-v^2/c^2)}$$

$$p^2 c^2 = \frac{m_0^2 v^2 c^2}{1-v^2/c^2}$$

$$E^2 - p^2 c^2 = \frac{m_0^2 c^4 - m_0^2 v^2 c^2}{1-v^2/c^2}$$

$$\boxed{E^2 - p^2 c^2 = (m_0 c^2)^2}$$

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$$p^2 = \frac{m_0^2 v^2}{(1-v^2/c^2)}$$

$$p^2c^2 = \frac{m_0^2 v^2 c^2}{1-v^2/c^2}$$

$$E^2 - p^2c^2 = \frac{m_0^2 c^4 - m_0^2 v^2 c^2}{1-v^2/c^2}$$

$$= \frac{m_0^2 c^4 \left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)}$$

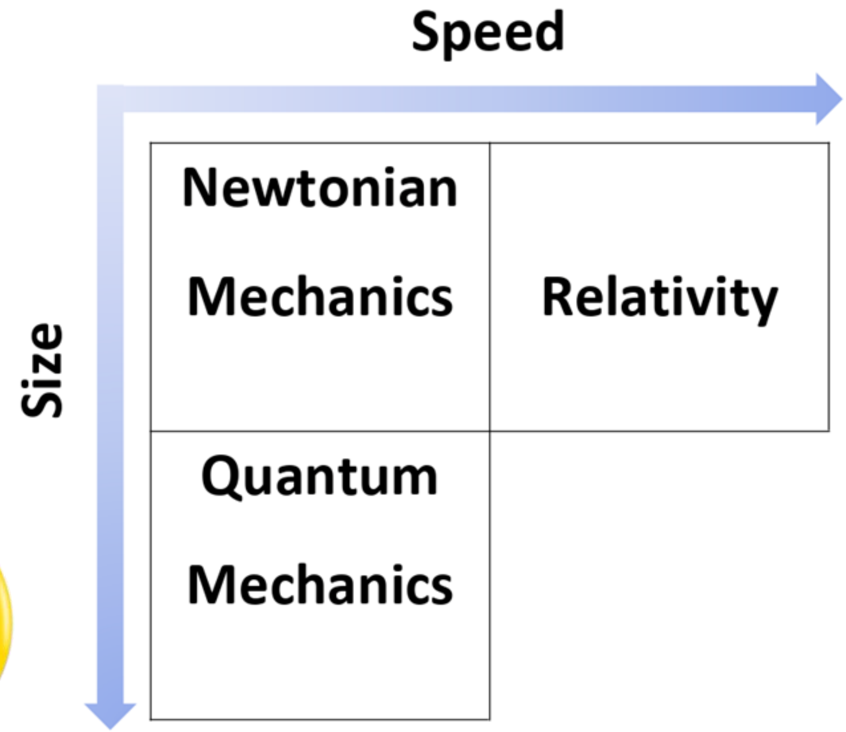
$$E^2 - p^2c^2 = m_0^2 c^4$$

$$E^2 = (m_0 c)^2 + (pc)^2$$

$$\boxed{E^2 = E_0^2 + p^2c^2}$$

What have we learnt so far?

- Importance of frame of reference
- Michelson Morley experiment
- Postulates of Special theory of relativity
- Time dilation
- Length contraction
- Related formulas to STR



Always be happy and grateful

Tutorial 4

1. Suppose observer on train (at rest with respect to laser and mirror) measures round trip time to be one second. What time Observer O on ground is moving at 0.5c with respect to laser/mirror measures?
2. A ship travelling to alpha centauri at 0.95c takes 4.5 years to get there as measured on earth. How long does it seem to passengers?
3. How fast must a spacecraft travel relative to the earth for each day on the spacecraft to correspond to 2 d on the earth?