Tutorial 6
Electromagnetism: Basics

Q1

1. Suppose $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ are two vectors, then find the value of the dot product of these two vectors.

Q2

Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Q3

5. Find the value of λ for which the two vectors $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$ are perpendicular.

Q4

Calculate the cross product between $\mathbf{a} = (3, -3, 1)$ and $\mathbf{b} = (4, 9, 2)$.

Q5

Find the cross product of two vectors $\vec{A} = 3i + 2j - 4k$ **and** $\vec{B} = 2i - 3j - 6k$.

Q6

Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

Q7

1. If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi$ (or grad ϕ) at the point $(1, -2, -1)$.

Q8

15. If $\mathbf{A} = x^2z\,\mathbf{i} - 2y^3z^2\,\mathbf{j} + xy^2z\,\mathbf{k}$, find $\nabla \cdot \mathbf{A}$ (or div $\mathbf{A}$) at the point $(1,-1,1)$.

Q9

23. If $\mathbf{A} = xz^3\,\mathbf{i} - 2x^2yz\,\mathbf{j} + 2yz^4\,\mathbf{k}$, find $\nabla \times \mathbf{A}$ (or curl $\mathbf{A}$) at the point $(1,-1,1)$.

Q10

24. If $\mathbf{A} = x^2y\,\mathbf{i} - 2xz\,\mathbf{j} + 2yz\,\mathbf{k}$, find curl curl $\mathbf{A}$.

Q11

Find the angle between the face diagonals of a cube.

Q12

Example 1.3. Find the gradient of $r = \sqrt{x^2 + y^2 + z^2}$ (the magnitude of the position vector).

Q13

Problem 1.11 Find the gradients of the following functions:

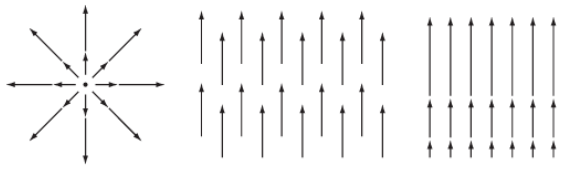
(a) $f(x, y, z) = x^2 + y^3 + z^4$.

(b) $f(x, y, z) = x^2y^3z^4$.

(c) $f(x, y, z) = e^x \sin(y) \ln(z)$.

Q14

Example 1.4. Suppose the functions in Fig. 1.18 are $\mathbf{v}_a = \mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$, $\mathbf{v}_b = \hat{\mathbf{z}}$, and $\mathbf{v}_c = z\,\hat{\mathbf{z}}$. Calculate their divergences.



Q15

Example 1.5. Suppose the function sketched in Fig. 1.19a is $\mathbf{v}_a = -y\hat{\mathbf{x}} + x\hat{\mathbf{y}}$, and that in Fig. 1.19b is $\mathbf{v}_b = x\hat{\mathbf{y}}$. Calculate their curls.

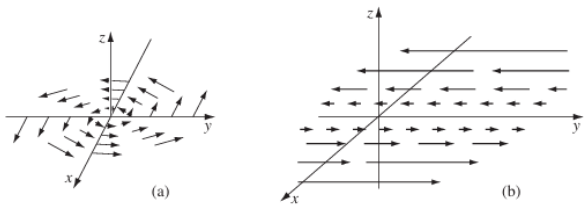


FIGURE 1.19