Learning Objectives

1. Quantum Mechanics-Operators and Wavefunctions

2. Few Numerical

3. Free particle- its wavefunction and Hamiltonian

4. Compton Scattering

2. Commutation

$$[A, B] = AB - BA$$

if A and B commute, $AB - BA = 0$

A and B are operators here

Quantity	Operator	Symbol
Position	Multiplication by x	X
Linear momentum (x-component)	$-i\hbar \frac{d}{dx}$	P_x
Kinetic energy	$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$	K
Potential energy	Multiplication by $V(x)$	V
Total mechanical energy	$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$	H

$$[A, B] = AB - BA$$

if A and B commute, $AB - BA = 0$

Ques. $[x, p_x]$ $\qquad p_x = -i\hbar \frac{d}{dx}$

$$[x, p_x] f(x) = x p_x f(x) - p_x x f(x)$$

$$= -x i\hbar \frac{d}{dx} f(x) + i\hbar \frac{d}{dx} (x f(x))$$

$$= \cancel{-i\hbar x \frac{d}{dx} f(x)} + \cancel{i\hbar x \frac{d}{dx} f(x)} + i\hbar f(x)$$

$$= i\hbar f(x)$$

$$\boxed{[x, p_x] = i\hbar}$$

e.g. Let $\psi(x) = Ae^{-x^2/a^2}$ find $\langle \psi | P_x | \psi \rangle$.

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$$P_x \psi(x) = -i\hbar \frac{d}{dx}\left(A e^{-\frac{x^2}{a^2}}\right)$$

$$= -i\hbar A \left(\frac{-2x}{a^2}\right) e^{-x^2/a^2}$$

$$= \frac{2i\hbar}{a^2} x A e^{-x^2/a^2}$$

$$\langle \psi | P_x | \psi \rangle = \int_{-\infty}^{\infty} \psi^*(x)\, P_x\, \psi(x)\, dx$$

$$= \int_{-\infty}^{\infty} A e^{-x^2/a^2} \frac{2i\hbar}{a^2} x A e^{-x^2/a^2} dx$$

y

e.g. Let $\psi(x) = Ae^{-x^2/a^2}$ find $\langle\psi|P_x|\psi\rangle$.

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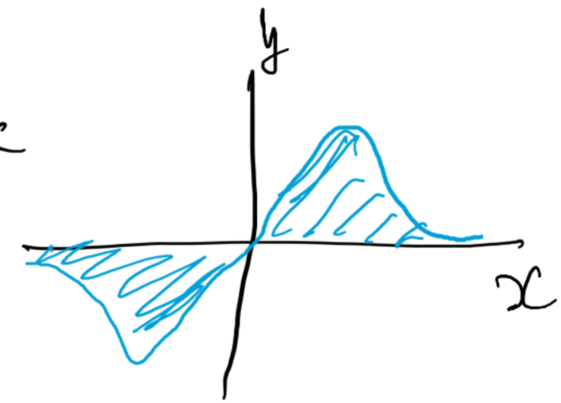
$$\langle\psi|P_x|\psi\rangle = \int_{-\infty}^{\infty}\psi^*(x)\,P_x\,\psi(x)\,dx$$

$$= \int_{-\infty}^{\infty}Ae^{-x^2/a^2}\,\frac{2i\hbar}{a^2}\,x\,Ae^{-x^2/a^2}\,dx$$

$$= \frac{2i\hbar A}{a^2}\int_{-\infty}^{\infty}x\,e^{-2x^2/a^2}\,dx$$

$$= 0$$



$$H = K + V = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \qquad \{1 \text{ dimensional system}\}$$

$$= -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] + V(\vec{r}) \qquad \{3 \text{ dimensional system}\}$$

$$= -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})$$

Time dependent Schrodinger's equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\bar{r},t) + V(\bar{r})\psi(\bar{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\bar{r},t)$$

Example: A free particle in one dimension

$$\psi(x,t) = Ae^{i(kx-\omega t)}$$

$$= Ae^{\frac{i}{\hbar}(px-Et)}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar}$$

$$\omega = 2\pi\nu = \frac{2\pi E}{h} = \frac{E}{\hbar}$$

Wavefunction is given

$$H = K + V = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \quad \{1 \text{ dimensional system}\}$$

$$= -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] + V(\vec{r}) \quad \{3 \text{ dimensional system}\}$$

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for a free particle, no interaction with other objects and hence no potential energy exists. $V = 0$

$$E = \frac{p^2}{2m}$$

$$\psi(x,t) = Ae^{\frac{i}{\hbar}\left(px - \frac{p^2}{2m}t\right)}$$

$$\frac{\partial \psi}{\partial x} = \left(\frac{i}{\hbar}p\right) A e^{\frac{i}{\hbar}\left(px - \frac{p^2}{2m}t\right)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \left(\frac{i}{\hbar}p\right)\left(\frac{i}{\hbar}p\right) A e^{\frac{i}{\hbar}\left(px - \frac{p^2}{2m}t\right)} = -\frac{p^2}{\hbar^2} A e^{\frac{i}{\hbar}\left(px - \frac{p^2}{2m}t\right)}$$

$$\frac{\partial \psi}{\partial t} = \left(-\frac{i p^2}{2m\hbar}\right) A e^{\frac{i}{\hbar}\left(px - \frac{p^2}{2m}t\right)}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t}$$

$$\boxed{H\psi = i\hbar\frac{\partial \psi}{\partial t}}$$

Schrödinger's equation

e.g. a free particle in one dimension, $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

Time independent

$$-\frac{\hbar^2}{2m}\frac{d^2\phi(x)}{dx^2} = E\phi(x)$$ at time $t=0$

$$\frac{d^2\phi(x)}{dx^2} + \frac{2mE}{\hbar^2}\phi(x) = 0$$

Schrodinger's equation is given

Ref. Introduction to Quantum Mechanics - David J. Griffiths e.g. a free particle in one dimension, $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

Time independent

$$-\frac{\hbar^2}{2m}\frac{d^2\phi(x)}{dx^2} = E\phi(x)$$ at time $t=0$

$$\frac{d^2\phi(x)}{dx^2} + \frac{2mE}{\hbar^2}\phi(x) = 0$$

$$\phi(x,p) = \frac{1}{\sqrt{2\pi}} e^{\frac{i}{\hbar}px}, \quad E(p) = \frac{p^2}{2m}$$

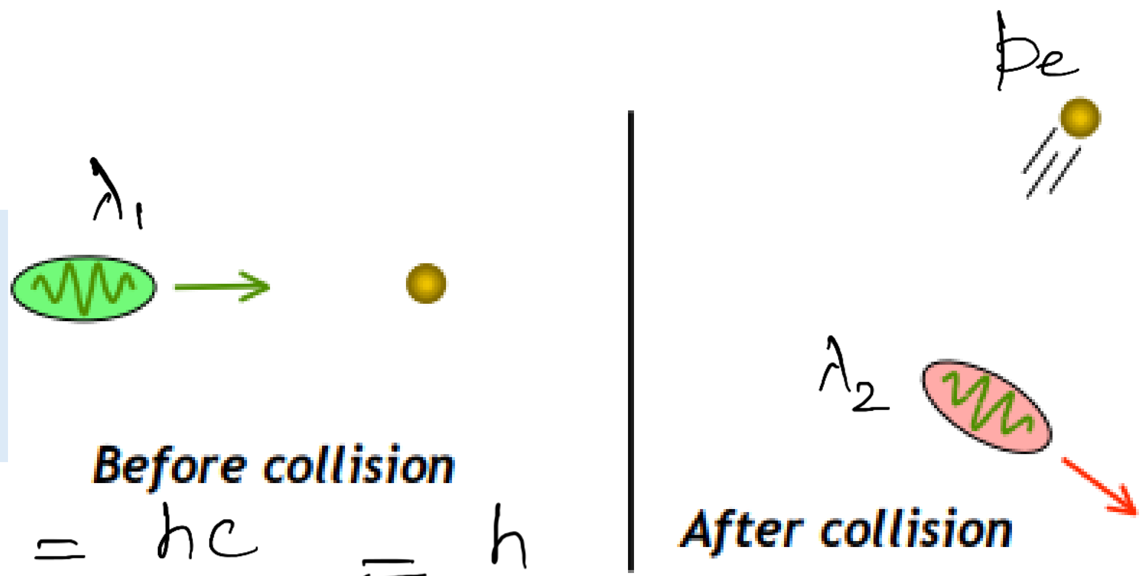
eigenfunction — eigenvalue.

Schrodinger's equation is given

Ref. Introduction to Quantum Mechanics - David J. Griffiths

Quantum Mechanics: Compton Scattering

- In 1923, Compton reported that when X-rays are scattered from materials, the scattered X-rays have larger wavelengths than the incident X-rays.
- This observation is explained by treating scattering of X-ray from material as collision b/w photon of energy (E) and electron of the material which is free and at rest.

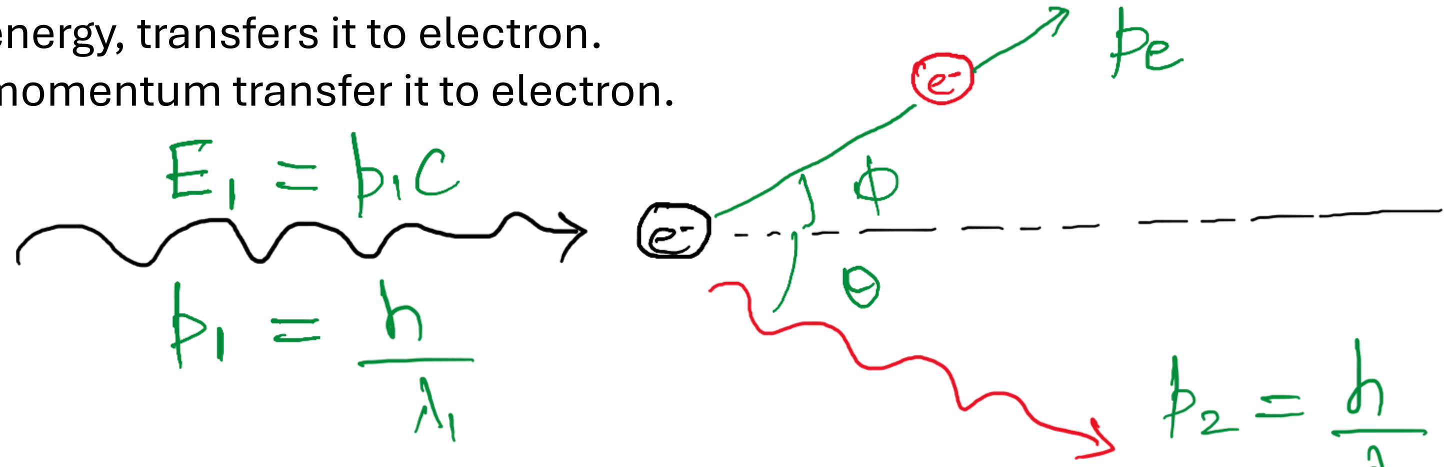


Photon energy E= hv
Photon mass=0
Photon momentum p=E/c

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda}$$

Quantum Mechanics: Compton Scattering

- Photon loses energy, transfers it to electron.
- Photon loses momentum transfer it to electron.



$$E_1 = p_1 c$$

$$p_1 = \frac{h}{\lambda_1}$$

$$p_2 = \frac{h}{\lambda_2}$$

$$E_2 = p_2 c$$

from relativity

rest mass energy

$$E^2 = E_0^2 + (pc)^2$$

before collision		after collision
$E_1 = p_1 c$	Photon	$E_2 = p_2 c$
$E_e^i = E_0$	Electron	$E_e^f = (E_0^2 + p_e^2)^{1/2}$

Quantum Mechanics: Compton Scattering

mom. cons. $\vec{p_1} = \vec{p_2} + \vec{p_e}$

$$\vec{p_e} = \vec{p_1} - \vec{p_2}\ , \quad p_e^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta \quad \text{---}(1)$$

energy cons. $p_1c + E_0 = p_2c + (E_0^2 + p_e^2c^2)^{1/2}$ ——(2)

$$E_0 + c(p_1 - p_2) = (E_0^2 + p_e^2c^2)^{1/2}$$

multiply by $\dfrac{hc}{p_1p_2E_0}$

$$\frac{h}{p_1p_2}(p_1 - p_2) = \frac{hc}{E_0}(1-\cos\theta)$$

$$p_1 = \frac{h}{\lambda_1} \qquad p_2 = \frac{h}{\lambda_2}$$

$$E_0 = m_0c^2$$

$$\boxed{\lambda_2 - \lambda_1 = \frac{h}{m_0c}(1-\cos\theta)}$$

compton wavelength

$$\frac{h}{p_1 p_2}(p_1 - p_2) = \frac{hc}{E_0}(1-\cos\theta)$$

$$\frac{\cancel{h}\,\cancel{\lambda_1\lambda_2}}{\cancel{h^2}}\,\cancel{h}\left(\frac{\lambda_2-\lambda_1}{\cancel{\lambda_1\lambda_2}}\right) = \frac{h\cancel{c}}{m_0 c^{\cancel{2}}}(1-\cos\theta)$$

$$\boxed{\lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1-\cos\theta)}$$

Quantum Mechanics: Compton Scattering

mom. cons. $\vec{p_1} = \vec{p_2} + \vec{p_e}$

$$\vec{p_e} = \vec{p_1} - \vec{p_2} \quad , \quad p_e^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta \quad \text{—(1)}$$

energy cons. $p_1c + E_0 = p_2c + (E_0^2 + p_e^2c^2)^{1/2}$ —(2)

$$E_0 + c(p_1 - p_2) = (E_0^2 + p_e^2c^2)^{1/2}$$

$$\cancel{E_0^2} + c^2(p_1-p_2)^2 + 2E_0c(p_1-p_2) = \cancel{E_0^2} + p_e^2c^2$$

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2 + \frac{2E_0}{c}(p_1-p_2) \quad \text{—(3)}$$

(1) and (3) $\cancel{p_1^2} + \cancel{p_2^2} - 2p_1p_2\cos\theta = \cancel{p_1^2} + \cancel{p_2^2} - 2p_1p_2 + \frac{2E_0}{c}(p_1-p_2)$

$$\frac{E_0}{c}(p_1-p_2) = p_1p_2(1-\cos\theta)$$

multiply by $\frac{hc}{p_1p_2E_0}$

$$\frac{h}{p_1p_2}(p_1-p_2) = \frac{hc}{E_0}(1-\cos\theta)$$

$p_1 = \frac{h}{\lambda_1}$ $\quad p_2 = \frac{h}{\lambda_2}$

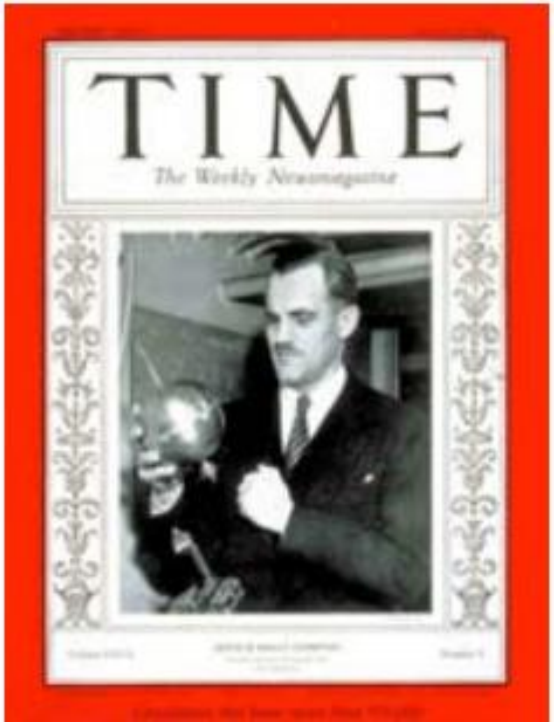
$E_0 = m_0c^2$

$$\boxed{\lambda_2 - \lambda_1 = \frac{h}{m_0c}(1-\cos\theta)}$$

compton wavelength

What have we learnt so far?

- Photons can transfer energy to beam of electrons.
- Determined by conservation of momentum, energy.
- Compton awarded 1927 Nobel prize for showing that this occurs just as two balls colliding.



Arthur Compton
Jan 13, 1936

Quantum Mechanics: What have we learnt so far??

1. Heisenberg Uncertainty principle
2. Wave-particle duality
3. Significance of Heisenberg Uncertainty principle and Wave-particle duality
4. Schrodinger's equation
5. Operators in Quantum mechanics, their properties and use
6. Compton Scattering

Always be happy and grateful