Physics for Engineers

PHL101

Electromagnetism

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Why charge produce Electric field?
Why moving charge produce magnetic field?

https://www.youtube.com/watch?v=SRjyzvx_J_4

Electromagnetism - Maxwell equation

Reference Book: Introduction to Electrodynamics, (Fourth Edition).
Author: David J. Griffiths

Section 3 Maxwell's Equations (Page 336 -348)

- 3.1 Electrodynamics Before Maxwell
- 3.2 How Maxwell Fixed Ampere's law
- 3.3 Maxwell's equation
- 3.5 Maxwell's equation in Matter

Maxwell's equations in free space

Integral form

1. $$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}}$$
2. $$\oint \mathbf{B} \cdot d\mathbf{a} = 0$$
3. $$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$
4. $$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \int \left(\frac{\partial \mathbf{E}}{\partial t} \right) \cdot d\mathbf{a}$$

Differential form

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}}$$

$$\oint \mathbf{B} \cdot d\mathbf{a} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

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Gauss law of electrostatics:
Charges produce electric field and field lines begin and end on charges

Gauss law on magnetostatics:
Magnetic monopole do not exist

Faraday law:
A changing magnetic field induces an electric field

Ampere's law:
Electric current and a changing electric field induces a magnetic field

What is electric charge?

Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.

https://www.quora.com/Can-static-electricity-kill-me-if-not-make-me-blind-or-do-physical-damage-create-fire-or-damage-electronics
https://www.britannica.com/science/static-electricity
https://study.com/academy/lesson/static-electricity-lesson-for-kids-definition-facts.htm
https://www.sciencelearn.org.nz/resources/2746-static-electricity-and-electrical-charge

- Both Balloon and hair are neutral

- Balloon is rubbed on hair
- Electrons move from the hair to balloon.
- Electrons ‘stick’ to the balloon where the balloon was rubbed

- Balloon becomes negatively charged
- Person’s hair becomes positively charged

What is electric force?

Electric force is the attractive or repulsive force between charged objects or point charges.

2.1.2 ■ Coulomb's Law

What is the force on a test charge Q due to a single point charge q, that is at *rest* a distance $\mathscr{r}$ away? The answer (based on experiments) is given by **Coulomb's law**:

$$\boxed{\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}.} \tag{2.1}$$

https://commons.wikimedia.org/wiki/File:Attractive-and-repulsive-electric-force-demonstrations-with-cha

What is electric field?

The electric field is an area where charged particle experience electric force. It is defined as electric force per unit charge.

2.1.3 ■ The Electric Field

If we have *several* point charges $q_1, q_2, \ldots, q_n$, at distances $\mathscr{r}_1, \mathscr{r}_2, \ldots, \mathscr{r}_n$ from Q, the total force on Q is evidently

$$\boxed{\mathbf{F} = Q\mathbf{E},} \tag{2.3}$$

https://letstalkscience.ca/educational-resources/backgrounders/introduction-static
https://www.sciencefacts.net/electric-field.html

What is magnetic monopole?

A hypothetical elementary particle with only one magnetic pole

Electric monopoles exist in the form of particles with positive or negative electric charge.

A **magnetic monopole** would have a net north or south "magnetic charge".

There is no known experimental or observational evidence that magnetic monopoles exist.

https://en.wikipedia.org/wiki/Magnetic_monopole

What is Magnetic force?

Attractive or repulsive force that is exerted between the poles of a magnet and electrically charged moving particles

5.1.2 ■ Magnetic Forces

$$\boxed{\mathbf{F}_{\mathrm{mag}} = Q(\mathbf{v} \times \mathbf{B}).} \tag{5.1}$$

https://physics.aps.org/articles/v9/91

https://www.sciencephoto.com/media/423696/view/magnetic-field

https://www.youtube.com/watch?v=RwQbID53f5o&ab_channel=ProfessorHafner

What is Magnetic field?

a vector field that describes the magnetic influence on moving electric charges, electric currents and magnetic materials.

5.1.1 ■ Magnetic Fields

$$\boxed{\mathbf{F}_{\text{mag}} = Q(\mathbf{v} \times \mathbf{B}).} \tag{5.1}$$

$$\boxed{\mathbf{F}_{\text{mag}} = \int I(d\mathbf{l} \times \mathbf{B}).} \tag{5.16}$$

https://www.alamy.com/stock-photo/magnet-iron-filings.html?sortBy=relevant
https://www.researchgate.net/figure/The-magnetic-field-lines-of-a-carrying-loop-of-wire-in-the-experiment-on-the-left-and_fig3_336783844

Vector operations

dot product

cross product

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

https://www.youtube.com/watch?v=L4aZYLOXFDc&ab_channel=RonHugo

1. Suppose **a** = -2**i** + 3**j** + 5**k** and **b** = **i** + 2**j** + 3**k** are two vectors, then find the value of the dot product of these two vectors.

Solution:

Given,

a = -2**i** + 3**j** + 5**k** and **b** = **i** + 2**j** + 3**k**

As we know, the dot product of two vectors **a =** a_1**i** + a_2**j** + a_3**k** and **b** = b_1**i** + b_2**j** + b_3**k** is
a.b = $a_1b_1 + a_2b_2 + a_3b_3$.

Thus, a.b = (-2)(1) + (3)(2) + (5)(3)

= -2 + 6 + 15

= 19

Find the angle between the vectors **i** – 2**j** + 3**k** and 3**i** – 2**j** + **k**.

Solution:

Let the given vectors be:

a = **i** – 2**j** + 3**k** and **b** = 3**i** – 2**j** + **k**

$|\mathbf{a}| = \sqrt{[1^2 + (-2)^2 + 3^2]} = \sqrt{(1 + 4 + 9)} = \sqrt{14}$

And

$|\mathbf{b}| = \sqrt{[3^2 + (-2)^2 + 1^2]} = \sqrt{(9 + 4 + 1)} = \sqrt{14}$

Now, a.b = (**i** – 2**j** + 3**k**) (3**i** – 2**j** + **k**)

= (1)(3) + (-2)(-2) + (3)(1)

= 3 + 4 + 3

= 10

As we know, **a.b** = |**a**| |**b**| cos θ

cos θ = (**a.b**)/ |**a**| |**b**|

$= 10/ (\sqrt{14}\sqrt{14})$

= 10/14

= 5/7

$\Rightarrow \theta = \cos^{-1}(5/7)$

5. Find the value of λ for which the two vectors $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$ are perpendicular.

Solution:

Let the given vectors be:

$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$

As we know, if two vectors are perpendicular to each other, then their dot or scalar product is equal to 0.

So, $\mathbf{a} \cdot \mathbf{b} = 0$

$(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (3\mathbf{i} + \lambda\mathbf{j} + \mathbf{k}) = 0$

$(2)(3) + (-1)(\lambda) + (2)(1) = 0$

$6 - \lambda + 2 = 0$

$8 - \lambda = 0$

$\lambda = 8$

Calculate the cross product between $\mathbf{a} = (3, -3, 1)$ and $\mathbf{b} = (4, 9, 2)$.

Solution: The cross product is

$$\begin{aligned}
\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 1 \\ 4 & 9 & 2 \end{vmatrix} \\
&= \mathbf{i}(-3 \cdot 2 - 1 \cdot 9) - \mathbf{j}(3 \cdot 2 - 1 \cdot 4) + \mathbf{k}(3 \cdot 9 + 3 \cdot 4) \\
&= -15\mathbf{i} - 2\mathbf{j} + 39\mathbf{k}
\end{aligned}$$

Find the cross product of two vectors $\vec{A} = 3i + 2j - 4k$**and** $\vec{B} = 2i - 3j - 6k$ **.**

$$\vec{A} \times \vec{B} = -24i + 10j - 13k.$$

Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and

$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

Given $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} = \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21) = 19(\hat{j} + \hat{k}) \implies |\vec{a} \times \vec{b}| =$

$19\sqrt{2}$

The unit vector perpendicular to both the vectors $\vec{a}, \vec{b}$ is $\dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \dfrac{19(\hat{j} + \hat{k})}{19\sqrt{2}} =$

$\dfrac{\hat{j} + \hat{k}}{\sqrt{2}}$

del operator

$$\vec{\nabla} \equiv \boldsymbol{\nabla} \equiv \nabla \equiv \hat{\imath}\,\frac{\partial}{\partial x} + \hat{\jmath}\,\frac{\partial}{\partial y} + \hat{k}\,\frac{\partial}{\partial z}$$

Used in Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

del operator = differential operator

Vector A	$A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}$
Gradient $\nabla\phi$	$\frac{\partial\phi}{\partial x}\boldsymbol{i} + \frac{\partial\phi}{\partial y}\boldsymbol{j} + \frac{\partial\phi}{\partial z}\boldsymbol{k}$
Divergence $\nabla \cdot A$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
Curl $\nabla \times A$	$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

$\vec{\phi}$ is scalar field
$\vec{A}$ is vector field

Scalar fields

Scalar field is a function that gives single Value of some variable for every point in space

Example: temperature, voltage, energy

Vector fields

A vector field has both magnitude and direction in space.

Example: Velocity, momentum, acceleration, force, electric field, magnetic field.

Gradient indicates change in magnitude of scalar field

Divergence indicates source of vector field

Curl indicates rotation of vector field

https://www.e-magnetica.pl/doku.php/vector_calculus

Gradient indicates change in magnitude of scalar field

Gradient = grad (scalar field) = scalar	
in a Cartesian system of coordinates	
(simplified notation)	$\text{gradient}(F) \equiv \text{grad}(F) \equiv \vec{\nabla} F \equiv \boldsymbol{\nabla} F \equiv \nabla F$
(full notation)	$\nabla F \equiv \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \right) F \equiv \hat{\imath}\frac{\partial F}{\partial x} + \hat{\jmath}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$

1. If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi$ (or grad ϕ) at the point $(1,-2,-1)$.

1. If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi$ (or grad ϕ) at the point $(1,-2,-1)$.

$$
\begin{aligned}
\nabla\phi &= \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)(3x^2y - y^3z^2) \\
&= \mathbf{i}\,\frac{\partial}{\partial x}(3x^2y - y^3z^2) \;+\; \mathbf{j}\,\frac{\partial}{\partial y}(3x^2y - y^3z^2) \;+\; \mathbf{k}\,\frac{\partial}{\partial z}(3x^2y - y^3z^2) \\
&= 6xy\,\mathbf{i} \;+\; (3x^2 - 3y^2z^2)\,\mathbf{j} \;-\; 2y^3z\,\mathbf{k} \\
&= 6(1)(-2)\,\mathbf{i} \;+\; \{3(1)^2 - 3(-2)^2(-1)^2\}\,\mathbf{j} \;-\; 2(-2)^3(-1)\,\mathbf{k} \\
&= -12\,\mathbf{i} \;-\; 9\,\mathbf{j} \;-\; 16\,\mathbf{k}
\end{aligned}
$$

Divergence indicates source of vector field

Divergence = div (vector field) = scalar	
in a Cartesian system of coordinates	
(simplified notation)	$\text{divergence}(\vec{F}) \equiv \text{div}(\vec{F}) \equiv \vec{\nabla} \cdot \vec{F} \equiv \boldsymbol{\nabla} \cdot \mathbf{F} \equiv \nabla \cdot \mathbf{F}$
(full notation)	$\nabla \cdot \mathbf{F} \equiv \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right)$

$$\text{divergence}(\vec{F}) \equiv \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \right) \cdot \left(\hat{\imath}F_x + \hat{\jmath}F_y + \hat{k}F_z \right) \equiv \text{scalar}$$

15. If $\mathbf{A} = x^2z\,\mathbf{i} - 2y^3z^2\,\mathbf{j} + xy^2z\,\mathbf{k}$, find $\nabla\cdot\mathbf{A}$ (or div $\mathbf{A}$) at the point $(1,-1,1)$.

15. If $\mathbf{A} = x^2z\,\mathbf{i} - 2y^3z^2\,\mathbf{j} + xy^2z\,\mathbf{k}$, find $\nabla\cdot\mathbf{A}$ (or div $\mathbf{A}$) at the point $(1,-1,1)$.

$$\nabla\cdot\mathbf{A} = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)\cdot(x^2z\,\mathbf{i} - 2y^3z^2\,\mathbf{j} + xy^2z\,\mathbf{k})$$

$$= \frac{\partial}{\partial x}(x^2z) + \frac{\partial}{\partial y}(-2y^3z^2) + \frac{\partial}{\partial z}(xy^2z)$$

$$= 2xz - 6y^2z^2 + xy^2 = 2(1)(1) - 6(-1)^2(1)^2 + (1)(-1)^2 = -3 \quad \text{at } (1,-1,1).$$

Curl indicates rotation of vector field

Curl = curl (vector field) = vector	
in a Cartesian system of coordinates	
(simplified notation)	$\operatorname{curl}(\vec{F}) \equiv \vec{\nabla} \times \vec{F} \equiv \boldsymbol{\nabla} \times \mathbf{F} \equiv \nabla \times \mathbf{F}$
(full notation)	$\nabla \times \mathbf{F} \equiv \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$

$$\operatorname{curl}(\vec{F}) \equiv \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(\hat{\imath}F_x + \hat{\jmath}F_y + \hat{k}F_z\right) \equiv \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{pmatrix}$$

23. If $\mathbf{A} = xz^3\,\mathbf{i} - 2x^2yz\,\mathbf{j} + 2yz^4\,\mathbf{k}$, find $\nabla \times \mathbf{A}$ (or curl $\mathbf{A}$) at the point $(1,-1,1)$.

24. If $\mathbf{A} = x^2y\,\mathbf{i} - 2xz\,\mathbf{j} + 2yz\,\mathbf{k}$, find curl curl $\mathbf{A}$.

23. If $\mathbf{A} = xz^3\,\mathbf{i} - 2x^2yz\,\mathbf{j} + 2yz^4\,\mathbf{k}$, find $\nabla \times \mathbf{A}$ (or curl $\mathbf{A}$) at the point $(1,-1,1)$.

$$\nabla \times \mathbf{A} = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) \times (xz^3\,\mathbf{i} - 2x^2yz\,\mathbf{j} + 2yz^4\,\mathbf{k})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xz^3 & -2x^2yz & 2yz^4 \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(2yz^4) - \frac{\partial}{\partial z}(-2x^2yz)\right]\mathbf{i} + \left[\frac{\partial}{\partial z}(xz^3) - \frac{\partial}{\partial x}(2yz^4)\right]\mathbf{j} + \left[\frac{\partial}{\partial x}(-2x^2yz) - \frac{\partial}{\partial y}(xz^3)\right]\mathbf{k}$$

$$= (2z^4 + 2x^2y)\,\mathbf{i} + 3xz^2\,\mathbf{j} - 4xyz\,\mathbf{k} = 3\mathbf{j} + 4\mathbf{k} \quad \text{at } (1,-1,1).$$

24. If $\mathbf{A} = x^2y\,\mathbf{i} - 2xz\,\mathbf{j} + 2yz\,\mathbf{k}$, find curl curl $\mathbf{A}$.

$$\text{curl curl } \mathbf{A} = \nabla \times (\nabla \times \mathbf{A})$$

$$= \nabla \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2y & -2xz & 2yz \end{vmatrix} = \nabla \times \left[(2x+2z)\,\mathbf{i} - (x^2+2z)\,\mathbf{k}\right]$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2x+2z & 0 & -x^2-2z \end{vmatrix} = (2x+2)\,\mathbf{j}$$

James Clerk Maxwell (1831-1879)

James Clerk Maxwell, the great physicist and mathematician,was born in Edinburgh,Scotland, on November 13, 1831

is most notable achievement was to formulate the classical theory of electromagnetic radiation, bringing together for the first time electricity, magnetism, and light as manifestations of the same phenomenon. Maxwell's equations for electromagnetism have been called the "second great unification in physics" after the first one realised by Isaac Newton.

Maxwell's equations in free space

In differential form

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Gauss law of electrostatics:
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How Maxwell fixed Ampere's law?

	Before Maxwell	**After Maxwell's correction**
Gauss law of electrostatics:	$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$	$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$
Gauss law on Magnetostatics:	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
Faraday law:	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampere's law:	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

https://www.e-magnetica.pl/doku.php/vector_calculus

Current Density (**J**)

measure of current flowing into a volume

Charge Density (ρ)

measure of amount of charge within the volume

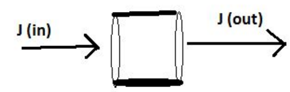
divergence of J

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_V}{\partial t}$$

how much charge is accumulating or leaving in a volume

Continuity equation

…also indicates charge conservation



divergence of **J** is
- positive → if more
- zero → if same
- negative → if less

→ current leaves the volume than enters

Remember two Theorems:

Theorem 1: **Curl of a gradient is the zero**

$$\nabla \times (\nabla f) = \mathbf{0}$$

Theorem 2: **Divergence of a curl is zero**

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

Faraday law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Taking divergence both sides…….

$$\nabla \cdot (\nabla \times \mathbf{E}) = \nabla \cdot \left(-\frac{\partial \mathbf{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \cdot \mathbf{B})$$

zero　　　　　　　　zero

LHS = RHS

Ampere's law (before maxwells):

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Taking divergence both sides…….

$$\nabla \cdot (\nabla \times \mathbf{B}) = \mu_0 (\nabla \cdot \mathbf{J})$$

zero　　　　　　　　May not be zero

Conflict arises!!

Based on purely theoretical arguments
Maxwell substituted Gauss law in Continuity equation

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\epsilon_0 \nabla \cdot \mathbf{E}) = -\nabla \cdot \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

He added this term in Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \xrightarrow{\text{Maxwell's modification}} \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \underbrace{\frac{\partial \mathbf{E}}{\partial t}}$$

He called his
displacement current

Maxwell's equations inside matter

Maxwell's equations

	In free space	Inside Matter
Gauss law of electrostatics:	$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$	(i) $\nabla \cdot \mathbf{D} = \rho_f$,
Gauss law on Magnetostatics:	$\nabla \cdot \mathbf{B} = 0$	(ii) $\nabla \cdot \mathbf{B} = 0$,
Faraday law:	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$,
Ampere's law:	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	(iv) $\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$.

in free space

$$\rho$$

Charge density

Gauss law of electrostatics: $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$

Gauss law on Magnetostatics: $\nabla \cdot \mathbf{B} = 0$

Faraday law: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

Ampere's law: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

Current density

$$\mathbf{J}$$

inside matter

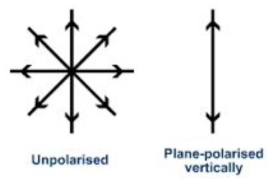
$$\rho = \rho_f + \rho_b$$

If matter is polarized, there is accumulation of "bound" charge and current

$$\mathbf{J} = \mathbf{J}_f + \mathbf{J}_b + \mathbf{J}_p$$

Can light be polarized?

- by polarizer
- by reflection
- by double refraction
- by scattering



Can matter be polarized?

Can matter be polarized?

Yes, by applying electric field

- Comes from charge separation
- acquires electric dipole moment throughout its volume
- Electrically polarized

Yes, by applying magnetic field

- comes from the electron spin
- acquires magnetic dipole moment throughout its volume
- Magnetically polarized

Gauss Law inside matter

What happens to atoms & molecules when placed in an Electric field?

Conductors Insulators Dielectrics

Conductors	Insulators	Dielectrics
High conductivity Unlimited supply of charges. Charges are free to move about through the material Many electrons are <u>not associated to any particular nucleus.</u> Ex: Copper, iron	Low conductivity No free charge <u>Cannot be polarized</u> Ex: Glass, plastic, rubber	Poor conductivity. Charges are attached to specific atoms or molecules Charges move at bit within atoms or molecules. Their cumulative effect account for characteristics behaviour of dielectrics. Electric field can distort the charge distribution of dielectric atom or molecules. <u>Hence can be polarized</u>. Ex: Mica, air, ceramic Used in fabricating capacitors

Electric dipole

A pair of equal and opposite point charges, separated by small distance.

Dipole Moment = Charge * distance of separation

https://www.isis.stfc.ac.uk/Pages/SH20_HelicalOrdering_Electric.aspx

Dielectrics

Dielectric is an electrical insulator that can be polarized to conduct by placing it in an electric field.

- When a dielectric is placed in an electric field, electric charges do not flow through the material as they do in a conductor, but only slightly shift from their average equilibrium positions causing dielectric polarization.
- Because of dielectric polarization, positive charges are displaced toward the field and negative charges shift in the opposite direction.
- This creates an internal electric field that reduces the overall field within the dielectric itself.
- These materials find applications in capacitors, radios, and transmission lines for radio frequency.

Polarization of dielectric material placed in an electric field

Page 172, Section 1.4, Polarization

<u>In neutral atoms (or nonpolar molecules):</u>
tiny **induced dipole moment**, pointing along the field direction

<u>In polar molecules,</u>
Each **permanent dipole** will experience a torque, tending to line it up along the field direction

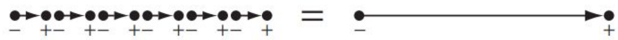
In both case, the material becomes polarized

$$\mathbf{P} \equiv \textit{ dipole moment per unit volume,}$$

Electric Polarization ($\vec{P}$) = density of permanent and induced dipole moment in material

$$P = \varepsilon_o\, \chi_e E$$

Polarization leads to the charge distributions



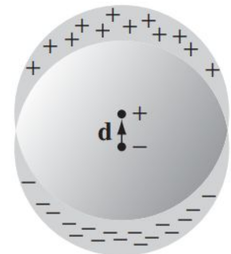


FIGURE 4.15

These accumulations of charge distributions is “**Bound charges**”

$$\rho_b = -\nabla \cdot \mathbf{P}$$

What happens to atoms & molecules when placed in an Electric field?

Formation & alignment of electric dipoles

we call it "Electrically polarized"

Bound charge is created: $\rho_b = -\nabla \cdot \mathbf{P}$

Electric dipoles line up,

- parallel to field: Dielectric

Charge density:

$$\rho = \rho_f + \rho_b$$

free charge density

bound charge density $-\nabla \cdot \mathbf{P}$

Gauss's law:

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}(\rho_f - \nabla \cdot \mathbf{P})$$

$$\nabla \cdot \mathbf{D} = \rho_f$$

where D is electric displacement

$$\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P}$$

Relation between E & D

Electric displacement

$$\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\left\{ \begin{aligned} \mathbf{P} &= \epsilon_0 \chi_e \mathbf{E} \\ \epsilon &\equiv \epsilon_0 (1 + \chi_e) \end{aligned} \right.$$

$$\mathbf{D} = \epsilon \mathbf{E}$$

Ampere’s Law inside matter

What happens to atoms & molecules when placed in an magnetic field?

Magnetic dipole

A magnetic north and south pole separated by a small distance.

What happens to atoms & molecules when placed in a Magnetic field?

Formation & alignment of magnetic dipoles

We call it "Magnetically polarized (or Magnetized)"

Bound current is created: $\mathbf{J}_b = \nabla \times \mathbf{M}$

Magnetic dipoles line up,
- if parallel to field: paramagnets
- if opposite direction of field: diamagnets

*If magnetization retains even after field: ferromagnets

- **Magnetization $\vec{M}$** : It is a process when external $\vec{B}$ aligns the magnetic moments (m) in matter.

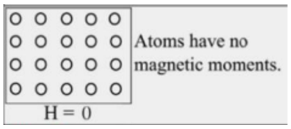
- **Bound currents**: Microscopic current loops inside magnetized matter. It is caused by spin and orbital motion of electrons

$$\mathbf{J}_b = \nabla \times \mathbf{M}$$

Dimagnets

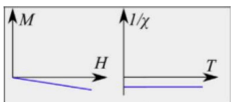
slightly repelled by a magnetic field

Tendency to move from **stronger to weaker** part of external non-uniform magnetic field



No-unpaired electrons

weakly magnetized in opposite direction of external magnetic field

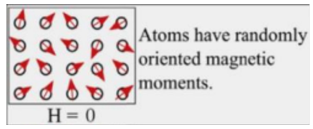


Ex: Copper, Gold

paramagnets

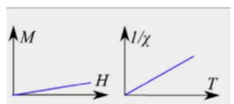
slightly attracted by a magnetic field

Tendency to move from **weaker to stronger** region of external non-uniform magnetic field



Unpaired electrons

weakly magnetized in same direction of external magnetic field

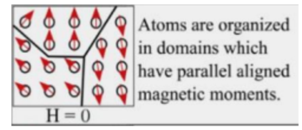


Ex: Aluminum, platinum

ferromagnets

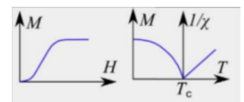
strong attraction to magnetic fields

Tendency to move from **weaker to stronger** region of external non-uniform magnetic field



Unpaired electrons

strongly magnetized in same direction of external magnetic field



Ex: Iron, Cobalt, Nickel

https://www.youtube.com/watch?v=xMsBkyk81Q4

Ampere’s law inside matter

in free space

$$\rho$$

Charge density

Gauss law of electrostatics: $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$

Gauss law on Magnetostatics: $\nabla \cdot \mathbf{B} = 0$

Faraday law: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

Ampere's law: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

Current density

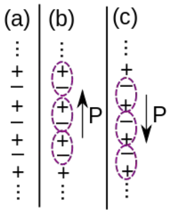
$$\mathbf{J}$$

inside matter

$$\rho = \rho_f + \rho_b$$

If matter is polarized, there is accumulation of "bound" charge and current

$$\mathbf{J} = \mathbf{J}_f + \mathbf{J}_b + \mathbf{J}_p$$

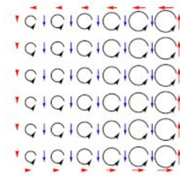


Electric polarization P
results in
bound charge density

$$\rho_b = -\nabla \cdot \mathbf{P}$$

comes from the charge separation

Magnetic polarization (magnetization) M
results in
bound current density



$$\mathbf{J}_b = \nabla \times \mathbf{M}$$

Comes from electron spin and orbital motion

change in electric polarization
results in
polarization current density

P: Polarization:
dipole moment
per unit volume.

$$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t}$$

M: Magnetisation:
magnetic dipole
moment per unit
volume.

comes from linear motion of charge when the electric polarization changes

https://en.wikipedia.org/wiki/Polarization_density

Charge density: $\rho = \rho_f + \rho_b$

free charge density (ρ_f)

bound charge density (ρ_b): $-\nabla \cdot \mathbf{P}$

Current density: $\mathbf{J} = \mathbf{J}_f + \mathbf{J}_b + \mathbf{J}_p$

free current density ($\mathbf{J}_f$)

bound current density ($\mathbf{J}_b$): $\nabla \times \mathbf{M}$

Polarization current density ($\mathbf{J}_p$): $\frac{\partial \mathbf{P}}{\partial t}$

Gauss's law:

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}(\rho_f - \nabla \cdot \mathbf{P})$$

$$\nabla \cdot \mathbf{D} = \rho_f$$

where D is electric displacement

$$\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P}$$

Ampere's law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J}_f + \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t} \right) + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

where H is magnetic field strength

$$\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

Maxwells equation in Matter

$$\text{(i)}\quad \nabla \cdot \mathbf{D} = \rho_f, \qquad \text{(iii)}\quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)}\quad \nabla \cdot \mathbf{B} = 0, \qquad \text{(iv)}\quad \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}.$$

Relation between E & D

Electric displacement

$$\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P}$$

Substitute these

$$\left\{\begin{aligned} &\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \\ &\epsilon \equiv \epsilon_0 (1 + \chi_e) \end{aligned}\right.$$

$$\mathbf{D} = \epsilon \mathbf{E}$$

Relation between B & H

Magnetic field strength

$$\mathbf{H} \equiv \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

Substitute these

$$\left\{\begin{aligned} &\mathbf{M} = \chi_m \mathbf{H} \\ &\mu \equiv \mu_0 (1 + \chi_m) \end{aligned}\right.$$

$$\mathbf{H} = \frac{1}{\mu} \mathbf{B}$$

Page 185, section 4.4.1

Electric Susceptibility, Permittivity, Dielectric Constant

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

$$\epsilon \equiv \epsilon_0 (1 + \chi_e)$$

- Electric susceptibility
- Permittivity of free space
- Permittivity of the material

Relative permittivity, or dielectric constant, of the material

$$\epsilon_r \equiv 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$$

Page 284, section 4.4.1

Magnetic Susceptibility, Permeability

$$\mathbf{M} = \chi_m \mathbf{H}$$

$$\mu \equiv \mu_0 (1 + \chi_m)$$

- Magnetic susceptibility
- Permeability of free space
- Permeability of the material

Relative permeability

$$\mu_r \equiv 1 + \chi_m = \frac{\mu}{\mu_0}$$

Thanks