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Problem Set 11

$$L = \frac{1}{2}\left(\hat{v}(s_t, w) - v_\pi(s_t)\right)^2$$

$v_\pi(s_t) = 4.1$
$x_1(s_t) = 1.3$
$x_2(s_t) = 0.7$

a) $\varepsilon^{(2)} \doteq \frac{\partial L}{\partial \hat{v}}$

$= \partial/\partial\hat{v}\left[\frac{1}{2}\left(\hat{v}(s_t,w) - v_\pi(s_t)\right)^2\right]$

$= 2 \cdot 1/2 \left(\hat{v} - v_\pi(s_t)\right)$

$= \hat{v} - v_\pi(s_t)$

$= 2.276 - 4.1 = \boxed{-1.824}$

Note

$\hat{v} = f(w_0^{(2)} + w_1^{(2)} \mu_1 + w_2^{(2)} \mu_2)$

$= f(0.2 + -0.8\mu_1 + 1.2\mu_2)$

$= (0.2 + 0 + (1.2)(1.73))$

$= \boxed{2.276}$

$\mu_1 = f(w_0^1 + w_1^1 x_1 + w_2^1 x_2)$

$= (-1.2 + 0.1(1.3) + 0.5(0.7))$

$= -0.49 = \boxed{0}$

$\mu_2 = f(w_0^2 + w_1^2 x_1 + w_2^2 x_2)$

$= (0.9 + 0.8(1.3) - 0.3(0.7))$

$= 1.73$

b) $\varepsilon_h^{(1)} \doteq \partial L/\partial u_h \quad (h = 1, 2)$

$\partial L/\partial u_h = \partial L/\partial\hat{v} \cdot \partial\hat{v}/\partial u_h$

$= \cancel{\partial\hat{v}/\partial u_h}\ \cancel{(\hat{v} - v_\pi(s_t))}$

$\partial v/\partial u_h = w_h^{(2)}$

$= \varepsilon^{(2)}\ \partial\hat{v}/\partial u_h = \varepsilon^{(2)} w_h^{(2)}$

$\boxed{= -1.824 \cdot w_h^{(2)}}$

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b) $\varepsilon_h^{(1)} = \frac{\partial L}{\partial u_h} = \varepsilon^{(2)} \omega_h^{(2)}$

$$\boxed{\begin{aligned} \varepsilon_1^{(1)} &= -1.824 \cdot (-0.8) = 1.46 \\ \varepsilon_2^{(1)} &= -1.824 \cdot (1.2) = 2.19 \end{aligned}}$$

c) $\frac{\partial L}{\partial \omega_h^{(2)}}$, $h = 1, 2, \underline{0}$

$$\frac{\partial L}{\partial \omega_h^{(2)}} = \frac{\partial L}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \omega} = \varepsilon^{(2)} \frac{\partial \hat{v}}{\partial \omega} = \varepsilon^{(2)} \frac{\partial}{\partial \omega}\left[f(\omega_0^{(2)} + \omega_1^{(2)} u_1 + \omega_2^{(2)} u_2\right.$$

$$= \underline{\varepsilon^{(2)} u_h}$$

$h = 0$; $\varepsilon^{(2)} \cdot u_0 = -1.824$

$h = 1$; $\varepsilon^{(2)} u_1 = -1.824 \cdot 0 = 0$

$h = 2$; $\varepsilon^{(2)} u_2 = -1.824 \cdot 1.73 = -3.16$

d) $\frac{\partial L}{\partial w_{jh}^{(1)}}$, $j = 0, 1, 2$
$h = 1, 2$

$$\frac{\partial L}{\partial w_{jh}^{(1)}} = \frac{\partial L}{\partial u_h}\frac{\partial u_h}{\partial w_{jh}^{(1)}} = \varepsilon_h^{(1)} \cdot \frac{\partial u_h}{\partial w_{jh}^{(1)}} = \varepsilon_h^{(1)} f\underbrace{(w_0^{(h)} + w_1^{(h)}x_1 + w_2^{(h)}x_2)}_{\cdot\, \partial/\partial w}$$

$$= \boxed{\varepsilon_h^{(1)} x_j \cdot f'(w_j^{(h)} x_j)}$$

$\underline{h=1}$

$\cancel{\varepsilon_1^{(1)} x_0 \leq 2}$

$f'(w_{jh}^{(h)} x_j) = 0$

so $\boxed{\frac{\partial L}{\partial w_{j1}^{(1)}} = 0}$ for $h = 1$ & $j \in \{0, 2\}$

$h = 2$ $\quad f'(w_{j2} x_j) = 1$

so $\varepsilon_2^{(1)} x_0 = \varepsilon_2^{(1)} = -2.188 = \frac{\partial L}{\partial w_{02}}$

$\varepsilon_2^{(1)} x_1 = -2.188 \cdot 1.3 = -2.845 = \frac{\partial L}{\partial w_{12}}$

$\varepsilon_2^{(1)} x_2 = -2.188 \cdot 0.7 = -1.532 = \frac{\partial L}{\partial w_{22}}$

e) ~~SGD~~ SGD Upate using $\alpha = 0.1$

$\underline{w - \alpha \nabla L}$

~~w~~ $\underline{w = 0}$ 1st update ?

$\cancel{-\alpha\left(\frac{\partial L}{\partial w}\right.}$

$$-\alpha\left(\frac{\partial L}{\partial w_{01}^{(1)}}, \frac{\partial L}{\partial w_{11}^{(1)}}, \frac{\partial L}{\partial w_{21}^{(1)}}, \frac{\partial L}{\partial w_{02}}, \frac{\partial L}{\partial w_{12}}, \frac{\partial L}{\partial w_{22}},\right.$$

$$\left.\frac{\partial L}{\partial w_0^{(2)}}, \frac{\partial L}{\partial w_1^{(2)}}, \frac{\partial L}{\partial w_2^{(2)}}\right)$$

$$= -\alpha\left(0, 0, 0, -2.18, -2.845, -1.532, -1.824, 0, -3.16\right)$$

$$\boxed{= (0, 0, 0, -.22, -.29, -.15, -.18, 0, -.32)}$$