(3)

$$V_\pi(s^L) = \frac{\alpha_0 + \alpha_1 V_\pi(s^R)}{1-\alpha_2} = \frac{\alpha_0}{1-\alpha_2} + \left(\frac{\alpha_1}{1-\alpha_2}\right) V_\pi(s^R)$$

$$= \frac{.7C_2 + .3C_1}{(1-.3\gamma)} + \frac{.7\gamma}{1-.3\gamma}\left(5.2C_1 + 4.9C_2\right)$$

$$= 1.4\left(.7C_2 + .3C_1\right) + .9\left(5.2C_1 + 4.9C_2\right)$$

$(.98 \sim 1.0)$

$$= C_2 + .4C_1 + 4.7C_1 + 4.4C_2$$

$$\boxed{\begin{aligned} V_\pi(s^L) &= 5.1C_1 + 5.4C_2 \\ V_\pi(s^R) &= 5.2C_1 + 4.9C_2 \end{aligned}}$$

<u>Note</u>: I would have expected $V_\pi(s^L)$ to look like $V_\pi(s^R)$ such that

$$V_\pi(s^L) = L_1 C_1 + L_2 C_2$$

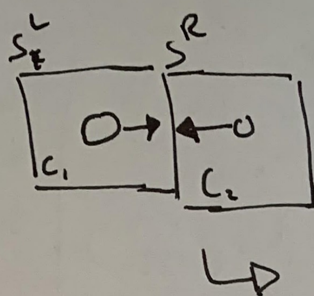
& $V_\pi(s^R) = R_1 C_1 + R_2 C_2$

where $R_1 \simeq L_2$ & $L_1 \simeq R_2$

Andrew Gaide
Assignment 3

$S = \{s^L, s^R\}$

$A = \{0, 1\}$



	$A_t = 0$	$A_t = 1$
s^L	1	2
s^R	3	4

Assignment 3, part 1

a) 1 $P(s', r \mid s^L, 0) = \begin{cases} 1 & \text{if } s' = s^L \ (R = C_1) \\ 0 & \text{else} \end{cases}$

2 $P(s', r \mid s^L, 1) = \begin{cases} 0.7 & \text{if } s' = s^R \ (R = C_2) \\ 0.3 & \text{else} \end{cases}$

3 $P(s', r \mid s^R, 0) = \begin{cases} 1 & \text{if } s' = s^R \ (R = C_2) \\ 0 & \text{else} \end{cases}$

4 $P(s', r \mid s^R, 1) = \begin{cases} 0.7 & \text{if } s' = s^L \ (R = C_1) \\ 0.3 & \text{else} \end{cases}$

$\pi(1 \mid s) = 1$

$\pi(0 \mid s) = 0$

(1) Andrew Caide

Assignment 3

Part 1, b

Bellman Eq: $V_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} P(s',r|s,a) \left[r + \gamma V_\pi(s')\right]$

$\pi_*(1|s) = 1$

$\pi_*(0|s) = 0$

$V_\pi(s^L) = \sum_a \pi(a|s) \cdot \sum_{s',r} P(s',r|s,a) \cdot \left[r + \gamma V_\pi(s')\right]$

note: $\sum_a \pi(a|s^L) = \begin{cases} 1 & \text{if } a = 1 \\ 0 & \text{else} \end{cases}$

$\ldots = 1 \cdot \sum_{s',r} P(s',r|s^L,1) \cdot \left[r + \gamma V_\pi(s')\right]$

note: $P(s',r|s^L,1) = \begin{cases} 0.7 & \text{if } s' = s^L \quad (R = c_2) \\ 0.3 & \text{if } s' = s^R \quad (R = c_1) \end{cases}$

$\ldots = 0.7\left[c_2 + \gamma V_\pi(s^R)\right] + 0.3\left[c_1 + \gamma V_\pi(s^L)\right]$

$= \underbrace{0.7 c_2 + 0.3 c_1}_{\alpha_0} + \gamma \{\underbrace{0.7}_{\alpha_1} V_\pi(s^R) + \underbrace{0.3}_{\alpha_2} V_\pi(s^L)\}$

$$\boxed{\begin{aligned} V_\pi(s^L) &= \alpha_0 + \alpha_1 V_\pi(s^R) + \alpha_2 V_\pi(s^L) \\ \text{where } \alpha_0 &= 0.7 c_2 + 0.3 c_1 \\ \alpha_1 &= 0.7\gamma \\ \alpha_2 &= 0.3\gamma \end{aligned}}$$

Conversely

$V_\pi(s^R) = \beta_0 + \beta_1 V_\pi(s^L) + \beta_2 V_\pi(s^R)$ where

$\beta_0 = 0.7 c_1 + 0.3 c_2$

$\beta_1 = 0.7\gamma, \ \beta_2 = 0.3\gamma$

3

2

1 $V_\pi(s^L) = \alpha_0 + \alpha_1 V_\pi(s^R) + \alpha_2 V_\pi(s^L)$

$V_\pi(s^L)(1-\alpha_2) = \alpha_0 + \alpha_1 V_\pi(s^R)$

$$\boxed{V_\pi(s^L) = \frac{\alpha_0 + \alpha_1 V_\pi(s^R)}{1-\alpha_2}}$$

when

$\alpha_0 = .7C_2 + .3C_1$

$\alpha_1 = .7\gamma$

$\alpha_2 = .3\gamma$

$\beta_0 = .7C_1 + .3C_2$

$* \ \beta_1 = \alpha_1 = .7\gamma$

$* \ \alpha_2 = \beta_2 = .3\gamma$

2 $V_\pi(s^R) = \beta_0 + \beta_1 V_\pi(s^L) + \beta_2 V_\pi(s^R)$

$$= \beta_0 + \beta_1\left(\frac{\alpha_0 + \alpha_1 V_\pi(s^R)}{1-\alpha_2}\right) + \beta_2 V_\pi(s^R)$$

(using $**$)

$$= \beta_0 + \beta_1\left(\frac{\alpha_0 + \beta_1 V(s^R)}{1-\beta_2}\right) + \beta_2 V_\pi(s^R)$$

$$= \beta_0 + \frac{\alpha_0\beta_1 + \beta_1^2 V(s^R)}{(1-\beta_2)} + \beta_2 V_\pi(s^R)$$

$$= \beta_0 + \left(\frac{\alpha_0\beta_1}{1-\beta_2}\right) + \left(\frac{\beta_1^2}{1-\beta_2}\right) V(s^R) + \beta_2 V_\pi(s^R)$$

$$V_\pi(s^R)\left(1 - \frac{\beta_1^2}{1-\beta_2} - \beta_2\right) = \beta_0 + \frac{\alpha_0\beta_1}{1-\beta_2}$$

$$V_\pi(s^R) = \left(\frac{1}{1 - \frac{\beta_1^2}{1-\beta_2} - \beta_2}\right)\left(\beta_0 + \frac{\alpha_0\beta_1}{1-\beta_2}\right)$$

$$= 5.4\left(.7C_1 + .3C_2 + (.863)(.7C_2 + .3C_1)\right)$$

$$\boxed{V_\pi(s^R) = 5.2C_1 + 4.9C_2}$$