Part I of Assignment 9

Please consider a Markov Decision Process (MDP) with $S = \{s^A, s^B, s^C\}$.

Given a particular state $s \in \mathcal{S}$, the agent is allowed to either try staying there or switching to any of the other states. Let's denote an intention to move to state s^A by a^A , to state s^B by a^B , and to state s^C by a^C . The agent does not know transition probabilities, including the distributions of rewards. There is, however, some evidence that the agent gets rewards only at the entrance to s^C ; and transition MDP probabilities to/from s^A appear to be same (or nearly same) as to/from s^B .

Suppose the agent chooses policy $\pi(a^A|s) = 0.05$, $\pi(a^B|s) = 0.05$, $\pi(a^C|s) = 0.90$ for all $s \in \{s^A, s^B, s^C\}$. Because of the apparent symmetry between s^A and s^B , it makes sense to assume that $v_{\pi}(s^A) \approx v_{\pi}(s^B)$ and approximate the state-values as follows:

$$v_{\pi}(s) \approx \hat{v}(s, \mathbf{w}) = w_1 \cdot \mathbb{1}_{(s=s_A)} + w_1 \cdot \mathbb{1}_{(s=s_B)} + w_2 \cdot \mathbb{1}_{(s=s_C)}.$$

Please notice that $\hat{v}(s^A, \mathbf{w}) = \hat{v}(s^B, \mathbf{w})$ for any choice of weights.

Assume that the agent runs the following algorithm with $\alpha = 0.1$ and m = 2 for estimating v_{π} :

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \alpha \sum_{t=mk}^{m(k+1)-1} \left[R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}_k) - \hat{v}(S_t, \mathbf{w}_k) \right] \nabla \hat{v}(S_t, \mathbf{w}_k), \ k = 0, 1, 2, \dots$$

This algorithm is a modification of the Semi-gradient 1-step Temporal-Difference (TD) with the model now being trained in mini-batches of size m. Please use $\gamma = 0.9$ and zero weights for k = 0.

If the agent observes the following sequence of states, actions, and rewards:

$$S_0 = s^A, A_0 = a^C, R_1 = 20,$$

$$S_1 = s^C, A_1 = a^B, R_2 = 0,$$

$$S_2 = s^B, A_2 = a^C, R_3 = 20,$$

$$S_3 = s^C, A_3 = a^C, R_4 = 20,$$

$$S_4 = s^C, A_4 = a^B, R_5 = 20,$$

$$S_5 = s^C, A_5 = a^C, R_6 = 0,$$

$$S_6 = s^B,$$

find (a) weights \mathbf{w}_k and (b) corresponding approximations $\hat{v}(s, \mathbf{w}_k)$ for iteration step k = 1, 2, 3. Specifically, please fill the tables in below:

SOLUTION:

(a) weights
$$\mathbf{w}_k = (w_{1,k}, w_{2,k})^T$$
:

	k = 0	k = 1	k=2	k=3
$w_{1,k}$	0			
$w_{2,k}$	0			

(b) approximations $\hat{v}(s, \mathbf{w}_k)$:

	k = 0	k = 1	k = 2	k = 3
$\hat{v}(s^A, \mathbf{w}_k)$	0			
$\hat{v}(s^B, \mathbf{w}_k)$	0			
$\hat{v}(s^C, \mathbf{w}_k)$	0			