

Question 5

5) x^3 is $O(g(x))$ for each $g(x)$ a) $g(x) = x^2$

$$f(x) = x^3$$

$$g(x) = x^2$$

$$|x^3| \leq C|x^2|$$

$$\text{when } x > c \quad |x^3| > C|x^2|$$

so $f(x) = x^3$ is not $O(x^2)$

$$|f(x)| \leq C|g(x)|$$

$$f(x) > g(x)$$

$$x^3 > x^2$$

b) $g(x) = x^3$

$$f(x) = x^3$$

$$g(x) = x^3$$

$$f(x) = g(x)$$

$$C = 1 \quad k = 0$$

so $f(x) = x^3$ is $O(x^3)$ f) $g(x) = x^3/2$

$$f(x) = x^3$$

$$g(x) = x^3/2$$

$$2(g(x)) = f(x)$$

so when $k=0$ and $C=2$, $f(x) = x^3$ is $O(x^3/2)$ c) $g(x) = x^2 + x^3$

$$f(x) = x^3$$

$$g(x) = x^2 + x^3$$

$$k = 0$$

$$0 + x^3 < x^2 + x^3$$

so $f(x) = x^3$ is $O(x^2 + x^3)$ d) $g(x) = x^2 + x^4$

$$f(x) = x^3$$

$$g(x) = x^2 + x^4$$

$$g(x) = x^2 + x^4 > x^3 = f(x)$$

so $f(x) = x^3$ is $O(x^2 + x^4)$ e) $g(x) = 3^x$

$$f(x) = x^3$$

$$g(x) = 3^x$$

$$k = 3$$

when $x > 3$ $g(x) = 3^x > x^3 = f(x)$ so $f(x) = x^3$ is $O(3^x)$

Questions

5) x^3 is $O(g(x))$ for each $g(x)$ a) $g(x) = x^2$

$$f(x) = x^3$$

$$|f(x)| \leq C|g(x)|$$

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$$f(x) > g(x)$$

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so $f(x) = x^3$ is not $O(x^2)$ b) $g(x) = x^3$

$$f(x) = x^3$$

$$f) g(x) = x^{3/2}$$

$$f(x) = x^3$$

$$g(x) = x^3$$

$$g(x) = x^{3/2}$$

$$f(x) = g(x)$$

$$2(g(x)) = f(x)$$

$$C=1 \quad k=0$$

so when $k=0$ and $C=2$,so $f(x) = x^3$ is $O(x^3)$

$$f(x) = x^3 \text{ is } O(x^{3/2})$$

c) $g(x) = x^2 + x^3$

$$f(x) = x^3$$

$$g(x) = x^2 + x^3$$

$$k=0$$

$$0 + x^3 < x^2 + x^3$$

$$C=1 \quad \text{so } f(x) = x^3 \text{ is } O(x^2 + x^3)$$

d) $g(x) = x^2 + x^4$

$$f(x) = x^3$$

$$g(x) = x^2 + x^4$$

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so $f(x) = x^3$ is $O(3^x)$

Question 3

3) Are the functions $O(x^2)$

a) $f(x) = 17x + 11$

$f(x) = 17x + 11$

$k = 18$

$q(x) = x^2$

$x > 18$

$f(x) = 17x + 11 \leq 18 \leq x^2 + x$

$C = 1$

$\therefore f(x) = O(x^2)$

b) $f(x) = x^2 + 1000$

$k = 100$

$x > 100$

$x^2 > 100^2 \therefore x^2 + 1000 < x^2 + x^2 = 2x^2$

$C = 2$

$\therefore f(x) = O(x^2)$

c) $f(x) = x \log(x)$

$k = 0$

$x \log x \leq x \cdot x = x^2$

$C = 1$

$\therefore f(x) = O(x^2)$

d) $f(x) = x^4/2$

there is no value C such that $x^4/2 \leq C|x|$ that holds for large values of x

Not possible that $f(x) = O(x^2)$

e) $f(x) = 2^x$

$f(x) = 2^x \leq Cx^2$

$C \geq 2^x/x^2$

$\lim_{x \rightarrow \infty} C \geq \lim_{x \rightarrow \infty} 2^x/x^2$

$C \geq \infty \therefore f(x) \neq O(x^2)$

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$$k = 3$$

when $x > 3 \quad g(x) = 3^x > x^3 = f(x)$ So $f(x) = x^3$ is $O(3^x)$

Question 2

2) a) 87 cents

$$\# \text{ of quarters} = q = 3$$

$$\# \text{ of dimes} = d = 1$$

$$\# \text{ of pennies} = n = 2$$

Same as when using nickels

Lowest # of coins possible

c) 99 cents

$$q = 3$$

$$d = 2$$

$$p = 4$$

same as when using nickels

Lowest # of coins possible

b) 49 cents

$$q = 1$$

$$d = 2$$

$$p = 4$$

same as when using nickels

Lowest # of coins possible

d) 83 cents

quarters

$$83 \geq 25; q = 1$$

$$83 - 25 = 58$$

$$58 < 25$$

dimes

$$58 < 10; d = 0$$

pennies

$$58 \geq 1; p = 1$$

$$58 - 1 = 57$$

$$57 \geq 1; p = 2$$

$$57 - 1 = 56$$

$$56 \geq 1; p = 4$$

$$56 - 1 = 55$$

$$55 \geq 1; p = 6$$

$$55 - 1 = 54$$

$$54 \geq 1; p = 8$$

$$54 - 1 = 53$$

$$q = 1; d = 0; p = 8 \rightarrow \# \text{ coins} = 9$$

Not lowest # of coins possible

$$q = 0; d = 3; p = 3 \rightarrow \# \text{ coins} = 6$$

$$6 < 9$$

Question 1

c) 99 cents

quarters

$$99 \geq 25; q=1$$

$$1) 99 - 25 = 74$$

$$74 \geq 25; q=2$$

$$2) 74 - 25 = 49$$

$$49 \geq 25; q=3$$

$$3) 49 - 25 = 24$$

$$24 < 25$$

dimes

$$24 \geq 10; d=1$$

$$1) 24 - 10 = 14$$

$$14 \geq 10; d=2$$

$$2) 14 - 10 = 4$$

$$4 < 10$$

nickels

$$4 < 5; n=0$$

pennies

$$4 \geq 1; p=1$$

$$1) 4 - 1 = 3$$

$$3 \geq 1; p=2$$

$$2) 3 - 1 = 2$$

$$2 \geq 1; p=3$$

$$3) 2 - 1 = 1$$

$$1 \geq 1; p=4$$

$$4) 1 - 1 = 0$$

$$0 < 1$$

$$q=3; d=2; n=0; p=4$$

d) 88 cents

quarters

$$88 \geq 25; q=1$$

$$1) 88 - 25 = 63$$

$$63 \geq 25$$

dimes

$$63 \geq 10; d=10$$

nickels

$$63 \geq 5; n=1$$

$$1) 63 - 5 = 58$$

$$58 < 5$$

pennies

$$58 \geq 1; p=1$$

$$1) 58 - 1 = 57$$

$$2) 57 - 1 = 56$$

$$3) 56 - 1 = 55$$

$$4) 55 - 1 = 54$$

$$5) 54 - 1 = 53$$

$$6) 53 - 1 = 52$$

$$7) 52 - 1 = 51$$

$$8) 51 - 1 = 50$$

$$9) 50 - 1 = 49$$

$$10) 49 - 1 = 48$$

$$11) 48 - 1 = 47$$

$$12) 47 - 1 = 46$$

$$13) 46 - 1 = 45$$

$$14) 45 - 1 = 44$$

$$15) 44 - 1 = 43$$

$$16) 43 - 1 = 42$$

$$17) 42 - 1 = 41$$

$$18) 41 - 1 = 40$$

$$19) 40 - 1 = 39$$

$$20) 39 - 1 = 38$$

$$21) 38 - 1 = 37$$

$$22) 37 - 1 = 36$$

$$23) 36 - 1 = 35$$

$$24) 35 - 1 = 34$$

$$25) 34 - 1 = 33$$

$$26) 33 - 1 = 32$$

$$27) 32 - 1 = 31$$

$$28) 31 - 1 = 30$$

$$29) 30 - 1 = 29$$

$$30) 29 - 1 = 28$$

$$31) 28 - 1 = 27$$

$$32) 27 - 1 = 26$$

$$33) 26 - 1 = 25$$

$$34) 25 - 1 = 24$$

$$35) 24 - 1 = 23$$

$$36) 23 - 1 = 22$$

$$37) 22 - 1 = 21$$

$$38) 21 - 1 = 20$$

$$39) 20 - 1 = 19$$

$$40) 19 - 1 = 18$$

$$41) 18 - 1 = 17$$

$$42) 17 - 1 = 16$$

$$43) 16 - 1 = 15$$

$$44) 15 - 1 = 14$$

$$45) 14 - 1 = 13$$

$$46) 13 - 1 = 12$$

$$47) 12 - 1 = 11$$

$$48) 11 - 1 = 10$$

$$49) 10 - 1 = 9$$

$$50) 9 - 1 = 8$$

$$51) 8 - 1 = 7$$

$$52) 7 - 1 = 6$$

$$53) 6 - 1 = 5$$

$$54) 5 - 1 = 4$$

$$55) 4 - 1 = 3$$

$$56) 3 - 1 = 2$$

$$57) 2 - 1 = 1$$

$$58) 1 - 1 = 0$$

$$59) 0 - 1 = -1$$

$$60) -1 < 0$$

$$q=1; d=0; n=1; p=3$$

Questions

5) x^3 is $O(g(x))$ for each $g(x)$ a) $g(x) = x^2$

$$f(x) = x^3$$

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$$|x^3| \leq C|x^2|$$

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$$f) g(x) = x^3/2$$

$$f(x) = x^3$$

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So when $k=0$ and $C=2$, $f(x) = x^3$ is $O(x^3/2)$ c) $g(x) = x^2 + x^3$

$$f(x) = x^3$$

$$g(x) = x^2 + x^3$$

$$k=0$$

$$0 + x^3 < x^2 + x^3$$

$$C=1 \quad \text{so } f(x) = x^3 \text{ is } O(x^2 + x^3)$$

d) $g(x) = x^2 + x^4$

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So $f(x) = x^3$ is $O(x^2 + x^4)$ e) $g(x) = 3^x$

$$f(x) = x^3$$

$$g(x) = 3^x$$

$$k=3$$

$$\text{when } x > 3 \quad g(x) = 3^x > x^3 = f(x)$$

So $f(x) = x^3$ is $O(3^x)$

Question 3

a) Are the functions $O(x^2)$

a) $f(x) = 17x + 11$

$f(x) = 17x + 11$ $k = 18$

$g(x) = x^2$ $x > 18$

$f(x) = 17x + 11 \leq 18 \leq x^2 + x$

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b) $f(x) = x^2 + 1000$

$k = 100$

$x > 100$

$x^2 > 100^2 \therefore x^2 + 1000 < x^2 + x^2 = 2x^2$

$C = 2$

$\therefore f(x) = O(x^2)$

c) $f(x) = x \log(x)$

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$x \log x \leq x \cdot x = x^2$

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d) $f(x) = x^4/2$

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Not possible that $f(x) = O(x^2)$

e) $f(x) = 2^x$

$f(x) = 2^x \leq Cx^2$

$C \geq 2^x/x^2$

$\lim_{x \rightarrow \infty} C \geq \lim_{x \rightarrow \infty} 2^x/x^2$

$C \geq \infty \therefore f(x) \neq O(x^2)$

Question 1

c) 99 cents

quarters

$$99 \geq 25; q=1$$

$$1) 99 - 25 = 74$$

$$74 \geq 25; q=2$$

$$2) 74 - 25 = 49$$

$$49 \geq 25; q=3$$

$$3) 49 - 25 = 24$$

$$24 < 25$$

dimes

$$24 \geq 10; d=1$$

$$1) 24 - 10 = 14$$

$$14 \geq 10; d=2$$

$$2) 14 - 10 = 4$$

$$4 < 10$$

nickels

$$4 < 5; n=0$$

pennies

$$4 \geq 1; p=1$$

$$1) 4 - 1 = 3$$

$$3 \geq 1; p=2$$

$$2) 3 - 1 = 2$$

$$2 \geq 1; p=3$$

$$3) 2 - 1 = 1$$

$$1 \geq 1; p=4$$

$$4) 1 - 1 = 0$$

$$0 < 1$$

$$q=3; d=2; n=0; p=4$$

d) 83 cents

quarters

$$83 \geq 25; q=1$$

$$1) 83 - 25 = 58$$

$$58 \geq 25$$

dimes

$$58 < 10; d=0$$

nickels

$$58 \geq 5; n=1$$

$$1) 58 - 5 = 53$$

$$53 < 5$$

pennies

$$53 \geq 1; p=1$$

$$1) 53 - 1 = 52$$

$$22 \geq 1; p=2$$

$$2) 22 - 1 = 21$$

$$12 \geq 1; p=3$$

$$3) 12 - 1 = 11$$

$$0 < 1$$

$$q=1; d=0; n=1; p=3$$

Question 2

2) a) 87 cents

$$\# \text{ of quarters} = q = 3$$

$$\# \text{ of dimes} = d = 1$$

$$\# \text{ of pennies} = n = 2$$

Same as when using nickels

Lowest # of coins possible

c) 99 cents

$$q = 3$$

$$d = 2$$

$$p = 4$$

same as when using nickels

Lowest # of coins possible

b) 49 cents

$$q = 1$$

$$d = 2$$

$$p = 4$$

same as when using nickels

Lowest # of coins possible

d) 33 cents

quarters

$$33 \geq 25; q = 1$$

$$33 - 25 = 8$$

$$8 < 25$$

dimes

$$8 < 10; d = 0$$

pennies

$$8 \geq 1; p = 8$$

$$8 - 1 = 7$$

$$7 \geq 1; p = 2$$

$$7 - 1 = 6$$

$$6 \geq 1; p = 4$$

$$6 - 1 = 5$$

$$5 \geq 1; p = 3$$

$$5 - 1 = 4$$

$$4 \geq 1; p = 2$$

$$4 - 1 = 3$$

$$3 \geq 1; p = 1$$

$$3 - 1 = 2$$

$$2 \geq 1; p = 1$$

$$2 - 1 = 1$$

$$1 \geq 1; p = 1$$

$$1 - 1 = 0$$

$$q = 1; d = 0; p = 8 \rightarrow \# \text{ coins} = 9$$

Not lowest # of coins possible

$$q = 0; d = 3; p = 3 \rightarrow \# \text{ coins} = 6$$

$$6 < 9$$

Question 3

3) Are the functions $O(x^2)$

a) $f(x) = 17x + 11$

$$f(x) = 17x + 11 \quad k = 18$$

$$g(x) = x^2 \quad x > 18$$

$$f(x) = 17x + 11 \leq 18 \leq x^2 + x$$

$$C = 1$$

$$\therefore f(x) = O(x^2)$$

b) $f(x) = x^2 + 1000$

$$k = 100$$

$$x > 100$$

$$x^2 > 100^2 \therefore x^2 + 1000 < x^2 + x^2 = 2x^2$$

$$C = 2$$

$$\therefore f(x) = O(x^2)$$

c) $f(x) = x \log(x)$

$$k = 0$$

$$x \log x \leq x \cdot x = x^2$$

$$C = 1$$

$$\therefore f(x) = O(x^2)$$

d) $f(x) = x^4/2$

there is no value C such that $x^4/2 \leq C|x|$ that holds for large values of x

Not possible that $f(x) = O(x^2)$

e) $f(x) = 2^x$

$$f(x) = 2^x \leq Cx^2$$

$$C \geq 2^x/x^2$$

$$\lim_{x \rightarrow \infty} C \geq \lim_{x \rightarrow \infty} 2^x/x^2$$

$$C \geq \infty \therefore f(x) \neq O(x^2)$$

Question 4

4) least integer n such that $f(x)$ is $O(x^n)$

a) $f(x) = 2x^2 + x^2 \log x$

exists c and k

such that $|f(x)| \leq c |g(x)|$

when $x > k$

$\log x \leq x \therefore 2x^2 + x^2 \log x \leq 2x^2 + x^4$

smallest n such that $f(x)$ is $O(x^n)$

$n = 4$

b) $f(x) = (x^4 + x^2 + 1) / (x^4 + 1)$

$f(x) = \frac{x^4 + x^2 + 1}{x^4 + 1} = 1 + \frac{x^2}{x^4 + 1}$

smallest n such that $f(x)$ is $O(x^n)$

$n = 0$

QUESTIONS

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$g(x) = x^2$

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when $x > c$ $|x^3| > |Cx^2|$

so $F(x) = x^3$ is not $O(x^2)$

$|F(x)| \leq C|g(x)|$

$F(x) > g(x)$

$x^3 > x^2$

b) $g(x) = x^3$

$F(x) = x^3$

$g(x) = x^3$

$F(x) = g(x)$

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$g(x) = x^3/2$

$2(g(x)) = F(x)$

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$F(x) = x^3$

$g(x) = x^2 + x^3$

$k = 0$

$0 + x^3 < x^2 + x^3$

 $C = 1$ so $F(x) = x^3$ is $O(x^2 + x^3)$

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when $x > 3$ $g(x) = 3^x > x^3 = F(x)$ so $F(x) = x^3$ is $O(3^x)$

CS 2060 Homework 7 Question 1

i) quarter = 25, dime = 10, nickel = 5, penny = 1

a) 87 cents

quarters: $q = \# \text{ of quarters}$

$$87 \geq 25; q = 1$$

$$1) 87 - 25 = 62$$

$$62 \geq 25; q = 2$$

$$2) 62 - 25 = 37$$

$$37 \geq 25; q = 3$$

$$3) 37 - 25 = 12$$

$$12 < 25$$

dimes: $\# \text{ of dimes} = d$

$$12 \geq 10; d = 1$$

$$1) 12 - 10 = 2$$

nickels: $\# \text{ of nickels} = n$

$$2 < 5; n = 0$$

pennies: $\# \text{ of pennies} = p$

$$2 \geq 1; p = 1$$

$$1) 2 - 1 = 1$$

$$1 \geq 1; p = 2$$

$$2) 1 - 1 = 0$$

$$q = 3; d = 1; n = 0; p = 2$$

b) 49 cents

quarters

$$49 \geq 25; q = 1$$

$$1) 49 - 25 = 24$$

$$24 < 25$$

dimes

$$24 \geq 10; d = 1$$

$$1) 24 - 10 = 14$$

$$14 \geq 10; d = 2$$

$$2) 14 - 10 = 4$$

$$4 < 10$$

nickels

$$4 < 5; n = 0$$

pennies

$$4 \geq 1; p = 1$$

$$1) 4 - 1 = 3$$

$$3 \geq 1; p = 2$$

$$2) 3 - 1 = 2$$

$$2 \geq 1; p = 3$$

$$3) 2 - 1 = 1$$

$$1 \geq 1; p = 4$$

$$4) 1 - 1 = 0$$

$$q = 1; d = 2; n = 0; p = 4$$

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$$33 \geq 25; q = 1$$

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pennies

$$8 \geq 1; p = 1$$

$$8 - 1 = 7$$

$$7 \geq 1; p = 2$$

$$7 - 1 = 6$$

$$6 \geq 1; p = 3$$

$$6 - 1 = 5$$

$$5 \geq 1; p = 4$$

$$5 - 1 = 4$$

$$4 \geq 1; p = 5$$

$$4 - 1 = 3$$

$$3 \geq 1; p = 6$$

$$3 - 1 = 2$$

$$2 \geq 1; p = 7$$

$$2 - 1 = 1$$

$$1 \geq 1; p = 8$$

$$1 - 1 = 0$$

$$q = 1; d = 0; p = 8 \rightarrow \# \text{ coins} = 9$$

Not lowest # of coins possible

$$q = 0; d = 3; p = 3 \rightarrow \# \text{ coins} = 6$$

$$6 < 9$$

Question 3

a) Are the functions $O(x^2)$

a) $f(x) = 17x + 11$

$$f(x) = 17x + 11 \quad k = 18$$

$$g(x) = x^2 \quad x > 18$$

$$f(x) = 17x + 11 \leq 18 \leq x^2 + x$$

$$C = 1$$

$$\therefore f(x) = O(x^2)$$

b) $f(x) = x^2 + 1000$

$$k = 100$$

$$x > 100$$

$$x^2 > 100^2 \therefore x^2 + 1000 \leq x^2 + x^2 = 2x^2$$

$$C = 2$$

$$\therefore f(x) = O(x^2)$$

c) $f(x) = x \log(x)$

$$k = 0$$

$$x \log x \leq x \cdot x = x^2$$

$$C = 1$$

$$\therefore f(x) = O(x^2)$$

d) $f(x) = x^4/2$

there is no value C such that $x^4/2 \leq C|x|$ that holds for large values of x

Not possible that $f(x) = O(x^2)$

e) $f(x) = 2^x$

$$f(x) = 2^x \leq Cx^2$$

$$C \geq 2^x/x^2$$

$$\lim_{x \rightarrow \infty} C \geq \lim_{x \rightarrow \infty} 2^x/x^2$$

$$C \geq \infty \therefore f(x) \neq O(x^2)$$

Question 4

4) least integer n such that $f(x)$ is $O(x^n)$

a) $f(x) = 2x^3 + x^2 \log x$

exists c and k

such that $|f(x)| \leq c|\log(x)|$

when $x > k$

$\log x \leq x \therefore 2x^3 + x^2 \log x \leq 2x^3 + x^4$

Smallest n such that $f(x)$ is $O(x^n)$

$n = 4$

b) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

$f(x) = \frac{x^4 + x^2 + 1}{x^4 + 1} = 1 + \frac{x^2}{x^4 + 1}$

Smallest n such that $f(x)$ is $O(x^n)$

$n = 0$

CS 2060 Homework 7 Question 1

i)

quarter = 25, dime = 10, nickel = 5, penny = 1

a) 87 cents

quarters: $q = \# \text{ of quarters}$

$$87 \geq 25; q = 1$$

$$1) 87 - 25 = 62$$

$$62 \geq 25; q = 2$$

$$2) 62 - 25 = 37$$

$$37 \geq 25; q = 3$$

$$3) 37 - 25 = 12$$

$$12 < 25$$

dimes: $\# \text{ of dimes} = d$

$$12 \geq 10; d = 1$$

$$1) 12 - 10 = 2$$

nickels: $\# \text{ of nickels} = n$

$$2 < 5; n = 0$$

pennies: $\# \text{ of pennies} = p$

$$2 \geq 1; p = 1$$

$$1) 2 - 1 = 1$$

$$1 \geq 1; p = 2$$

$$2) 1 - 1 = 0$$

$$q = 3; d = 1; n = 0; p = 2$$

b) 49 cents

quarters

$$49 \geq 25; q = 1$$

$$1) 49 - 25 = 24$$

$$24 < 25$$

dimes

$$24 \geq 10; d = 1$$

$$1) 24 - 10 = 14$$

$$14 \geq 10; d = 2$$

$$2) 14 - 10 = 4$$

$$4 < 10$$

nickels

$$4 < 5; n = 0$$

pennies

$$4 \geq 1; p = 1$$

$$1) 4 - 1 = 3$$

$$3 \geq 1; p = 2$$

$$2) 3 - 1 = 2$$

$$2 \geq 1; p = 3$$

$$3) 2 - 1 = 1$$

$$1 \geq 1; p = 4$$

$$4) 1 - 1 = 0$$

$$q = 1; d = 2; n = 0; p = 4$$

QUESTIONS

5) x^3 is $O(g(x))$ for each $g(x)$

a) $g(x) = x^2$

$F(x) = x^3$

$g(x) = x^2$

$|x^3| \leq C|x^2|$

when $x > C$ $|x^3| > C|x^2|$

so $F(x) = x^3$ is not $O(x^2)$

$|F(x)| \leq C|g(x)|$

$F(x) > g(x)$

$x^3 > x^2$

b) $g(x) = x^3$

$F(x) = x^3$

$g(x) = x^3$

$F(x) = g(x)$

$C = 1$; $k = 0$

so $F(x) = x^3$ is $O(x^3)$

f) $g(x) = x^3/2$

$F(x) = x^3$

$g(x) = x^3/2$

$2(g(x)) = F(x)$

so when $k=0$ and $C=2$, $F(x) = x^3$ is $O(x^3/2)$

c) $g(x) = x^2 + x^3$

$F(x) = x^3$

$g(x) = x^2 + x^3$

$k = 0$

$0 + x^3 < x^2 + x^3$

 $C = 1$ so $F(x) = x^3$ is $O(x^2 + x^3)$

d) $g(x) = x^2 + x^4$

$F(x) = x^3$

$g(x) = x^2 + x^4$

$g(x) = x^2 + x^4 > x^3 = F(x)$

so $F(x) = x^3$ is $O(x^2 + x^4)$

e) $g(x) = 3^x$

$F(x) = x^3$

$g(x) = 3^x$

$k = 3$

when $x > 3$ $g(x) = 3^x > x^3 = F(x)$ so $F(x) = x^3$ is $O(3^x)$