

1. Fish: 5 entrees  
 meat: 12 entrees  
 vegan: 4 entrees  
 21 options  
 $5 + 12 + 4 = 21$

2. shirt: 12 colors  
 versions: 2  
 sizes: 3 for each version  
 $12 \cdot 2 \cdot 3 = 72$  types

3. # bit strings of length 8  
 $2^8 = 256$

4. 5 person councils, 400 individuals  
 a. How many formed if there are 5 roles?  

$$P(400, 5) = \frac{400!}{(400-5)!} = \frac{400!}{295!} = 5! = 120$$
  
 b. Every member has equal roles  

$$C(400, 5) = \frac{400!}{5!(400-5)!} = \frac{400!}{5!395!} = \frac{400!}{400!} = 1$$

5.  $26^4 - 25^4 = 66,351$  strings

6. integers less than 1000  
 a. divisible by 7 but not by 11  
 # divisible by 7:  $\frac{999}{7} \approx 142$     # divisible by 77:  $\frac{999}{77} \approx 12$   
 # divisible by 11:  $\frac{999}{11} \approx 90$   
 $142 - 12 = 130$   
 b. divisible by either 7 or 11 (inclusive)  
 $142 + 90 - 12 = 220$

7. CS majors: 38 (including joint)  
 math majors: 23 (including joint)  
 joint majors: 7  
 how many students are there?  
 $38 - 7 + 23 - 7 = 47$

8. a.  $C(5, 1)$   
 $= \frac{5!}{1!4!} = 5$   
 b.  $C(5, 2)$   
 $= \frac{5!}{2!3!} = 10$   
 c.  $C(8, 8)$   
 $= \frac{8!}{8!0!} = 1$

9. how many ways of choosing 2 positive integers less than 100?  
 $C(99, 2) =$

$$\frac{99!}{2!97!} = 4851 \text{ ways}$$

10. bit strings of length 10

a. exactly 3 0s

$$C(10, 3) = \frac{10!}{3!7!} = 120$$

b. at least 1 1s

$$C(10, 7) + C(10, 8) + C(10, 9) + C(10, 10) \\ 120 + 45 + 10 + 1 = 176$$

11. letters A-H, contains...

a. string: CDE

$$P(6, 6) = \frac{6!}{(6-6)!} = 6! = 720$$

b. strings: AB, DE, GH

$$P(5, 5) = \frac{5!}{(5-5)!} = 5! = 120$$

c. strings: BCA, ABF

zero strings since B will repeat

12. faculty: 7 women, 9 men

a. ways to select 5 person committee with at least 1 woman on the committee

$$C(16, 5) - C(9, 5) \quad // \text{all possibilities} - \text{committees of only men}$$

$$= \frac{16!}{5!11!} - \frac{9!}{5!4!} = 4368 - 126 = 4242$$

b. ways to select 5 person committee with at least 1 woman and 1 man on the committee

$$\text{only women: } C(7, 5) = \frac{7!}{5!2!} = 21$$

$$C(16, 5) - C(9, 5) - C(7, 5) = 4368 - 126 - 21 = 4221$$

13. select 5 unordered elements from a set w/ 3 elements when repetition is allowed

$$C(3+5-1, 5) = C(7, 5) = \frac{7!}{5!2!} = 21$$

14. # of strings made from AARDVARK, using all letters and 3 A's must be consecutive

6 letters:

$$\text{AAA} \quad \frac{6!}{3!} = 240$$

$$\text{R, R} \quad \frac{1!2!1!1!1!}{1!1!1!1!1!} = 120$$

$$\text{D} \quad 1!1!1!1!1! = 120$$

$$\text{V} \quad 1!1!1!1!1! = 120$$

$$\text{K} \quad 1!1!1!1!1! = 120$$

15. different strings formed with six 1s and eight 0s

$$C(14, 6) = \frac{14!}{6!8!} = 3003$$