1. True or False

1. $5/10$: True
2. $9/3$: False
3. $7 \nmid 0$: False

2. Show that if a, b, c, d are integers and $a \neq 0, b \neq 0$ and $a|c$ and $b|d$, then $ab|dc$

p: $a|c$ and $b|d$

q: $ab|cd$

If $a|c$ then there is an integer x such that $c = ax$

If $b|d$ then there is an integer y such that $d = by$

Then $cd = (ax)(by)$

$cd = (ab)(xy)$

and since xy will be an integer such that ab will multiply to equal cd

Then $ab | cd$

3. Find $a \bmod m$

1. $a = 55$, $m = 8$

   $55 = 8(6) + 7$

   $55 \bmod (8) = 7$

2. $a = 1331$, $m = 2050$

   $2050 = 1331(1) + 719$

   $2050 \bmod (1331) = 719$

3. $a = -15$, $m = 4$

   $4 = -15(-1) + -11$

   $4 \bmod (-15) = -11$

4. $a \equiv 11 \bmod (19)$ and $b \equiv 3 \bmod (19)$. Find c when $0 \leq c \leq 18$ such that

1. $c \equiv 8(b) \bmod 19$

   $\equiv 8(3 \bmod 19) \bmod 19$

   $\equiv 8(1) \bmod 19$

   $\equiv \bar{3}$

2. $c \equiv a - b \bmod (19)$

   $\equiv 11 \bmod (19) - (3 \bmod 19) \bmod 19$

   $\equiv 8 - 1 \bmod 19$

   $\equiv 8 - 0 \equiv \bar{8}$

5. Find a such that

1. $17 \pmod{29} \equiv a$

   $a \equiv \bar{12}$

2. $a \equiv 31 \pmod{10}$

   $a \equiv \bar{1}$

3. $a \equiv 24 \bmod (17)$

   $a \equiv \bar{7}$

6. Show that if $a \equiv b \bmod m$ and $c \equiv d \bmod m$ where $a, b, c, d, m \in \mathbb{Z}$ with $m \geq 2$ then $a - c \equiv b - d \pmod m$

p: $a \equiv b \bmod m$ and $c \equiv d (\bmod m)$

q: $a - c \equiv b - d \bmod m$

since m divides $a - b$

then there is a x such that $a - b = m(x)$

since m divides $c - d$

then there is a y such that $c - d = m(y)$

$\therefore (a-b) - (c-d) = m(x-y)$

$\therefore$ there is an integer such that m divides $(a-b)-(c-d)$ and $(a-c) \equiv b - d \bmod m$

7. Use the Euclidean Algorithm to find

1. $\gcd(125, 75)$

   $125 = 75(1) + 50$

   $75 = 50(1) + 25$

$50 = 25(2) + 0$

$\therefore \gcd(125, 75) = 25$

2. $\gcd(123, 277)$

$277 = 123(2) + 31$

$123 = 31(3) + 30$

$31 = 30(1) + 1$

$30 = 1(30) + 0$

$\therefore \gcd(123, 277) = 1$