Questions

5) x^3 is $O(g(x))$ for each $g(x)$

a) $g(x) = x^2$

$f(x) = x^3$

$g(x) = x^2$

$|x^3| \le C|x^2|$

when $x > C$ $|x^3| > |Cx^2|$

so $f(x) = x^3$ is not $O(x^2)$

$|f(x)| \le C|g(x)|$

$f(x) > g(x)$

$x^3 > x^2$

b) $g(x) = x^3$

$f(x) = x^3$

$g(x) = x^3$

$f(x) = g(x)$

$C = 1$; $k = 0$

so $f(x) = x^3$ is $O(x^3)$

c) $g(x) = x^2 + x^3$

$f(x) = x^3$

$g(x) = x^2 + x^3$

$k = 0$

$0 + x^3 < x^2 + x^3$

$C = 1$ so $f(x) = x^3$ is $O(x^2 + x^3)$

d) $g(x) = x^2 + x^4$

$f(x) = x^3$

$g(x) = x^2 + x^4$

$g(x) = x^2 + x^4 > x^3 = f(x)$

so $f(x) = x^3$ is $O(x^2 + x^4)$

e) $g(x) = 3^x$

$f(x) = x^3$

$g(x) = 3^x$

$k = 3$

when $x > 3$ $g(x) = 3^x > x^3 = f(x)$

so $f(x) = x^3$ is $O(3^x)$

f) $g(x) = x^3/2$

$f(x) = x^3$

$g(x) = x^3/2$

$2(g(x)) = f(x)$

so when $k = 0$ and $C = 2$,

$f(x) = x^3$ is $O(x^3/2)$

Questions

5) x^3 is $O(g(x))$ for each $g(x)$

a) $g(x)=x^2$

$f(x)=x^3$ $\quad |f(x)| \le C|g(x)|$

$g(x)=x^2$ $\quad f(x) > g(x)$

$|x^3| \le C|x^2|$ $\quad x^3 > x^2$

when $x>C$ $\quad |x^3| > |Cx^2|$

so $f(x)=x^3$ is not $O(x^2)$

b) $g(x)=x^3$

$f(x)=x^3$

$g(x)=x^3$

$f(x)=g(x)$

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so $f(x)=x^3$ is $O(x^3)$

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$f(x)=x^3$

$g(x)=x^2+x^3$

$k=0$

$0+x^3 < x^2+x^3$

$C=1$ so $f(x)=x^3$ is $O(x^2+x^3)$

d) $g(x)=x^2+x^4$

$f(x)=x^3$

$g(x)=x^2+x^4$

$g(x)=x^2+x^4 > x^3=f(x)$

so $f(x)=x^3$ is $O(x^2+x^4)$

e) $g(x)=3^x$

$f(x)=x^3$

$g(x)=3^x$

$k=3$

when $x>3$ $\quad g(x)=3^x > x^3 = f(x)$

so $f(x)=x^3$ is $O(3^x)$

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$f(x)=x^3$

$g(x)=x^3/2$

$2(g(x))=f(x)$

so when $k=0$ and $C=2$,

$f(x)=x^3$ is $O(x^3/2)$

Question 3

3) Are the functions $O(x^2)$

a) $f(x) = 17x+11$

$f(x) = 17x+11$ $\quad k = 18$

$g(x) = x^2$ $\quad x \geq 18$

$f(x) = 17x+11 \leq 18 \leq x^2+x$

$C = 1$

$\therefore f(x) = O(x^2)$

b) $f(x) = x^2+1000$

$k = 100$

$x > 100$

$x^2 > 100^2 \quad \therefore x^2+1000 < x^2+x^2 = 2x^2$

$C = 2$

$\therefore f(x) = O(x^2)$

c) $f(x) = x\log(x)$

$k = 0$

$x\log x \leq x \cdot x = x^2$

$C = 1$

$\therefore f(x) = O(x^2)$

d) $f(x) = x^4/2$

there is no value C such that $x^4/2 \leq C|x|$ that holds for large values of x

Not possible that $f(x) = O(x^2)$

e) $f(x) = 2^x$

$f(x) = 2^x \leq Cx^2$

$C \geq 2^x/x^2$

$\lim_{x \to \infty} C \geq \lim_{x \to \infty} 2^x/x^2$

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$f(x) = x^3$

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$f(x) = g(x)$

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$g(x) = x^2 + x^3$

$k = 0$

$0 + x^3 < x^2 + x^3$

$C = 1$ so $f(x) = x^3$ is $O(x^2 + x^3)$

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$2(g(x)) = f(x)$

so when $k = 0$ and $C = 2$,

$f(x) = x^3$ is $O(x^3/2)$

Question 2

2) a) 87cents

#of quarters = $q = 3$

#of dimes = $d = 1$

#of pennies = $n = 2$

Same as when using nickels

Lowest # of coins possible

c) 99 cents

$q = 3$

$d = 2$

$p = 4$

same as when using nickels

Lowest # of coins possible

b) 49 cents

$q = 1$

$d = 2$

$p = 4$

same as when using nickels

Lowest # of coins possible

d) 33 cents

quarters

$33 \geq 25$; $q = 1$

$33 - 25 = 8$

$8 < 25$

dimes

$8 < 10$; $d = 0$

pennies

$8 \geq 1$; $p = 1$

$8 - 1 = 7$

$7 \geq 1$; $p = 2$

$7 - 1 = 6$

$6 \geq 1$; $p = 3$

$6 - 1 = 5$

$5 \geq 1$; $p = 4$

$5 - 1 = 4$

$4 \geq 1$; $p = 5$

$4 - 1 = 3$

$3 \geq 1$; $p = 6$

$3 - 1 = 2$

$2 \geq 1$; $p = 7$

$2 - 1 = 1$

$1 \geq 1$; $p = 8$

$1 - 1 = 0$

$q = 1$; $d = 0$; $p = 8$ → #coins = 9

Not lowest # of coins possible

$q = 0$; $d = 3$; $p = 3$ → #coins = 6

$6 < 9$

Question 1

c) 99 cents

quarters

99 ≥ 25 ; q = 1

1) 99 - 25 = 74

74 ≥ 25 ; q = 2

2) 74 - 25 = 49

49 ≥ 25 ; q = 3

3) 49 - 25 = 24

24 < 25

dimes

24 ≥ 10 ; d = 1

1) 24 - 10 = 14

14 ≥ 10 ; d = 2

2) 14 - 10 = 4

4 < 10

nickels

4 < 5 ; n = 0

pennies

4 ≥ 1 ; p = 1

1) 4 - 1 = 3

3 ≥ 1 ; p = 2

2) 3 - 1 = 2

2 ≥ 1 ; p = 3

3) 2 - 1 = 1

1 ≥ 1 ; p = 4

4) 1 - 1 = 0

0 < 1

q = 3 ; d = 2 ; n = 0 ; p = 4

d) 33 cents

quarters

33 ≥ 25 ; q = 1

1) 33 - 25 = 8

8 < 25

dimes

8 < 10 ; d = 0

nickels

8 ≥ 5 ; n = 1

1) 8 - 5 = 3

3 < 5

pennies

3 ≥ 1 ; p = 1

1) 3 - 1 = 2

2 ≥ 1 ; p = 2

2) 2 - 1 = 1

1 ≥ 1 ; p = 3

3) 1 - 1 = 0

0 < 1

q = 1 ; d = 0 ; n = 1 ; p = 3

Questions

5) x^3 is $O(g(x))$ for each $g(x)$

a) $g(x) = x^2$

$f(x) = x^3$ $\qquad |f(x)| \le C|g(x)|$

$g(x) = x^2$ $\qquad f(x) > g(x)$

$|x^3| \le C|x^2|$ $\qquad x^3 > x^2$

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$g(x) = 3^x$

$k = 3$

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so $f(x) = x^3$ is $O(3^x)$

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$2(g(x)) = f(x)$

so when $k = 0$ and $C = 2$,

$f(x) = x^3$ is $O(x^3/2)$

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3) Are the functions $O(x^2)$

a) $f(x) = 17x+11$

$f(x) = 17x+11$ $\quad k=18$

$g(x) = x^2$ $\quad x > 18$

$f(x) = 17x+11 \leq 18 \leq x^2+x$

$c=1$

$\therefore f(x) = O(x^2)$

b) $f(x) = x^2+1000$

$k=100$

$x > 100$

$x^2 > 100^2 \quad \therefore \ x^2+1000 < x^2+x^2 = 2x^2$

$c=2$

$\therefore f(x) = O(x^2)$

c) $f(x) = x\log(x)$

$k=0$

$x\log x \leq x \cdot x = x^2$

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Not possible that $f(x) = O(x^2)$

e) $f(x) = 2^x$

$f(x) = 2^x \leq cx^2$

$c \geq 2^x/x^2$

$\lim\limits_{x\to\infty} c \geq \lim\limits_{x\to\infty} 2^x/x^2$

$c \geq \infty \quad \therefore f(x) \neq O(x^2)$

Question 1

c) 99 cents

quarters

$99 \geq 25$; $q=1$

1) $99-25=74$

$74 \geq 25$; $q=2$

2) $74-25=49$

$49 \geq 25$; $q=3$

3) $49-25=24$

$24 < 25$

dimes

$24 \geq 10$; $d=1$

1) $24-10=14$

$14 \geq 10$; $d=2$

2) $14-10=4$

$4 < 10$

nickels

$4 < 5$; $n=0$

pennies

$4 \geq 1$; $p=1$

1) $4-1=3$

$3 \geq 1$; $p=2$

2) $3-1=2$

$2 \geq 1$; $p=3$

3) $2-1=1$

$1 \geq 1$; $p=4$

4) $1-1=0$

$0 < 1$

$q=3$; $d=2$; $n=0$; $p=4$

d) 33 cents

quarters

$33 \geq 25$; $q=1$

1) $33-25=8$

$8 < 25$

dimes

$8 < 10$; $d=0$

nickels

$8 \geq 5$; $n=1$

1) $8-5=3$

$3 < 5$

pennies

$3 \geq 1$; $p=1$

1) $3-1=2$

$2 \geq 1$; $p=2$

2) $2-1=1$

$1 \geq 1$; $p=3$

3) $1-1=0$

$0 < 1$

$q=1$; $d=0$; $n=1$; $p=3$

Question 2

2) a) 87 cents

#of quarters = $q = 3$

#of dimes = $d = 1$

#of pennies = $n = 2$

Same as when using nickels

Lowest # of coins possible

b) 49 cents

$q = 1$

$d = 2$

$p = 4$

Same as when using nickels

Lowest # of coins possible

c) 99 cents

$q = 3$

$d = 2$

$p = 4$

same as when using nickels

Lowest # of coins possible

d) 33 cents

quarters

$33 \geq 25$; $q = 1$

$33 - 25 = 8$

$8 < 25$

dimes

$8 < 10$; $d = 0$

pennies

$8 \geq 1$; $p = 1$

$8 - 1 = 7$

$7 \geq 1$; $p = 2$

$7 - 1 = 6$

$6 \geq 1$; $p = 3$

$6 - 1 = 5$

$5 \geq 1$; $p = 4$

$5 - 1 = 4$

$4 \geq 1$; $p = 5$

$4 - 1 = 3$

$3 \geq 1$; $p = 6$

$3 - 1 = 2$

$2 \geq 1$; $p = 7$

$2 - 1 = 1$

$1 \geq 1$; $p = 8$

$1 - 1 = 0$

$q = 1$; $d = 0$; $p = 8$ → #coins = 9

Not lowest # of coins possible

$q = 0$; $d = 3$; $p = 3$ → #coins = 6

$6 < 9$

Question 3

3) Are the functions $O(x^2)$

a) $f(x) = 17x + 11$

$f(x) = 17x + 11$ $\quad k = 18$

$g(x) = x^2$ $\quad x > 18$

$f(x) = 17x + 11 \leq 18 \leq x^2 + x$

$c = 1$

$\therefore f(x) = O(x^2)$

b) $f(x) = x^2 + 1000$

$k = 100$

$x > 100$

$x^2 > 100^2 \quad \therefore x^2 + 1000 < x^2 + x^2 = 2x^2$

$c = 2$

$\therefore f(x) = O(x^2)$

c) $f(x) = x \log(x)$

$k = 0$

$x \log x \leq x \cdot x = x^2$

$c = 1$

$\therefore f(x) = O(x^2)$

d) $f(x) = x^4/2$

there is no value c such that $x^4/2 \leq c|x|$ that holds for large values of x

Not possible that $f(x) = O(x^2)$

e) $f(x) = 2^x$

$f(x) = 2^x \leq cx^2$

$c \geq 2^x/x^2$

$\lim_{x \to \infty} c \geq \lim_{x \to \infty} 2^x/x^2$

$c \geq \infty \quad \therefore f(x) \neq O(x^2)$

Question 4

4) least integer n such that $f(x)$ is $O(x^n)$

a) $f(x) = 2x^3 + x^3 \log x$

exists C and k

such that $|f(x)| \leq C|g(x)|$

when $x > k$

$\log x \leq x \quad \therefore \quad 2x^3 + x^3 \log x \leq 2x^3 + x^4$

smallest n such that $f(x)$ is $O(x^n)$

$n = 4$

b) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

$f(x) = \dfrac{x^4 + x^2 + 1}{x^4 + 1} = 1 + \dfrac{x^2}{x^4 + 1}$

smallest n such that $f(x)$ is $O(x^n)$

$n = 0$

Questions

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$f(x) = x^3$ $\qquad |f(x)| \leq C|g(x)|$

$g(x) = x^2$ $\qquad f(x) > g(x)$

$|x^3| \leq C|x^2|$ $\qquad x^3 > x^2$

when $x > C$ $\quad |x^3| > |Cx^2|$

so $f(x) = x^3$ is not $O(x^2)$

b) $g(x) = x^3$

$f(x) = x^3$

$g(x) = x^3$

$f(x) = g(x)$

$C = 1$; $k = 0$

so $f(x) = x^3$ is $O(x^3)$

c) $g(x) = x^2 + x^3$

$f(x) = x^3$

$g(x) = x^2 + x^3$

$k = 0$

$0 + x^3 < x^2 + x^3$

$C = 1$ so $f(x) = x^3$ is $O(x^2 + x^3)$

d) $g(x) = x^2 + x^4$

$f(x) = x^3$

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f) $g(x) = x^3/2$

$f(x) = x^3$

$g(x) = x^3/2$

$2(g(x)) = f(x)$

so when $k = 0$ and $C = 2$,

$f(x) = x^3$ is $O(x^3/2)$

CS 2050 Homework 7 Question 1

1) quarter = 25, dime = 10, nickel = 5, penny = 1

a) 87 cents

quarters: q = # of quarters

$87 \geq 25$; $q = 1$

1) $87 - 25 = 62$

$62 \geq 25$; $q = 2$

2) $62 - 25 = 37$

$37 \geq 25$; $q = 3$

3) $37 - 25 = 12$

$12 < 25$

dimes: # of dimes = d

$12 \geq 10$ $d = 1$

1) $12 - 10 = 2$

nickels: # of nickels = n

$2 < 5$ $n = 0$

pennies: # of pennies = p

$2 \geq 1$ $p = 1$

1) $2 - 1 = 1$

$1 \geq 1$ $p = 2$

2) $1 - 1 = 0$

$q = 3$; $d = 1$; $n = 0$; $p = 2$

b) 49 cents

quarters

$49 \geq 25$; $q = 1$

1) $49 - 25 = 24$

$24 < 25$

dimes

$24 \geq 10$; $d = 1$

1) $24 - 10 = 14$

$14 \geq 10$; $d = 2$

2) $14 - 10 = 4$

$4 < 10$

nickels

$4 < 5$; $n = 0$

pennies

$4 \geq 1$; $p = 1$

1) $4 - 1 = 3$

$3 \geq 1$; $p = 2$

2) $3 - 1 = 2$

$2 \geq 1$; $p = 3$

3) $2 - 1 = 1$

$1 \geq 1$; $p = 4$

4) $1 - 1 = 0$

$q = 1$; $d = 2$; $n = 0$; $p = 4$

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$x > 100$

$x^2 > 100^2 \quad \therefore x^2+1000 < x^2+x^2 = 2x^2$

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$f(x) = 2^x \leq Cx^2$

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$\lim_{x\to\infty} C \geq \lim_{x\to\infty} 2^x/x^2$

$C \geq \infty \quad \therefore f(x) \neq O(x^2)$

Question 2

2) a) 87 cents

\# of quarters = q = 3

\# of dimes = d = 1

\# of pennies = n = 2

Same as when using nickels

Lowest # of coins possible

b) 49 cents

q = 1

d = 2

p = 4

Same as when using nickels

Lowest # of coins possible

c) 99 cents

q = 3

d = 2

p = 4

same as when using nickels

Lowest # of coins possible

d) 33 cents

quarters

$33 \geq 25$; $q = 1$

$33 - 25 = 8$

$8 < 25$

dimes

$8 < 10$; $d = 0$

pennies

$8 \geq 1$; $p = 1$

$8 - 1 = 7$

$7 \geq 1$; $p = 2$

$7 - 1 = 6$

$6 \geq 1$; $p = 3$

$6 - 1 = 5$

$5 \geq 1$; $p = 4$

$5 - 1 = 4$

$4 \geq 1$; $p = 5$

$4 - 1 = 3$

$3 \geq 1$; $p = 6$

$3 - 1 = 2$

$2 \geq 1$; $p = 7$

$2 - 1 = 1$

$1 \geq 1$; $p = 8$

$1 - 1 = 0$

$q = 1$; $d = 0$; $p = 8$ → # coins = 9

Not lowest # of coins possible

$q = 0$; $d = 3$; $p = 3$ → # coins = 6

$6 < 9$

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Question 3

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e) $f(x) = 2^x$

$f(x) = 2^x \leq cx^2$

$c \geq 2^x/x^2$

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Question 4

4) least integer n such that $f(x)$ is $O(x^n)$

a) $f(x) = 2x^2 + x^3 \log x$

exists C and k

such that $|f(x)| \leq C|g(x)|$

when $x > k$

$\log x \leq x \quad \therefore \quad 2x^2 + x^3 \log x \leq 2x^2 + x^4$

Smallest n such that $f(x)$ is $O(x^n)$

$n = 4$

b) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

$f(x) = \frac{x^4 + x^2 + 1}{x^4 + 1} = 1 + \frac{x^2}{x^4 + 1}$

smallest n such that $f(x)$ is $O(x^n)$

$n = 0$

CS 2050 Homework 7 Question 1

1) quarter = 25, dime = 10, nickel = 5, penny = 1

a) 87 cents

quarters: q = # of quarters

$87 \geq 25$; q = 1

1) 87 - 25 = 62

$62 \geq 25$; q = 2

2) 62 - 25 = 37

$37 \geq 25$; q = 3

3) 37 - 25 = 12

$12 < 25$

dimes : # of dimes = d

$12 \geq 10$ d = 1

1) 12 - 10 = 2

nickels : # of nickels = n

$2 < 5$ n = 0

pennies : # of pennies = p

$2 \geq 1$ p = 1

1) 2 - 1 = 1

$1 \geq 1$ p = 2

2) 1 - 1 = 0

q = 3 ; d = 1 ; n = 0 ; p = 2

b) 49 cents

quarters

$49 \geq 25$; q = 1

1) 49 - 25 = 24

$24 < 25$

dimes

$24 \geq 10$; d = 1

1) 24 - 10 = 14

$14 \geq 10$; d = 2

2) 14 - 10 = 4

$4 < 10$

nickels

$4 < 5$; n = 0

pennies

$4 \geq 1$; p = 1

1) 4 - 1 = 3

$3 \geq 1$; p = 2

2) 3 - 1 = 2

$2 \geq 1$; p = 3

3) 2 - 1 = 1

$1 \geq 1$; p = 4

4) 1 - 1 = 0

q = 1 ; d = 2 ; n = 0 ; p = 4

Questions

5) x^3 is $O(g(x))$ for each $g(x)$

a) $g(x) = x^2$

$f(x) = x^3$ $\quad |f(x)| \le C|g(x)|$

$g(x) = x^2$ $\quad f(x) > g(x)$

$|x^3| \le C|x^2|$ $\quad x^3 > x^2$

when $x > c$ $\quad |x^3| > |Cx^2|$

so $f(x) = x^3$ is not $O(x^2)$

b) $g(x) = x^3$

$f(x) = x^3$

$g(x) = x^3$

$f(x) = g(x)$

$C = 1$; $k = 0$

so $f(x) = x^3$ is $O(x^3)$

c) $g(x) = x^2 + x^3$

$f(x) = x^3$

$g(x) = x^2 + x^3$

$k = 0$

$0 + x^3 < x^2 + x^3$

$C = 1$ so $f(x) = x^3$ is $O(x^2 + x^3)$

d) $g(x) = x^2 + x^4$

$f(x) = x^3$

$g(x) = x^2 + x^4$

$g(x) = x^2 + x^4 > x^3 = f(x)$

so $f(x) = x^3$ is $O(x^2 + x^4)$

e) $g(x) = 3^x$

$f(x) = x^3$

$g(x) = 3^x$

$k = 3$

when $x > 3$ $\quad g(x) = 3^x > x^3 = f(x)$

so $f(x) = x^3$ is $O(3^x)$

f) $g(x) = x^3/2$

$f(x) = x^3$

$g(x) = x^3/2$

$2(g(x)) = f(x)$

so when $k = 0$ and $C = 2$,

$f(x) = x^3$ is $O(x^3/2)$