Exercise 2.

Try visualizing for $1, 2, 3$ pegs, and you will get that $T_0 = 0, T_1 = 2, T_2 = 8$, and that the winning strategy is to transfer $n-1$ pegs to peg B via peg C, which takes T_{n-1} moves. Now, transfer the biggest peg to C, which takes one move, and then trasnfer the $n-1$ pegs in B to A, which takes T_{n-1} moves. Transfer biggest from C to B, taking one move, and transfer the other pegs to B, taking again T_{n-1} moves, that is

$$\begin{aligned} T_0 &= 0 \\ T_n &= 3T_{n-1} + 2 \end{aligned}$$

To solve this recurrence, we sum 1 to each side (the idea here is that we want to factor T_{n+1}, and summing 1 will make everything a factor of 3), so we have

$$\begin{aligned} T_0 + 1 &= 1 \\ T_n + 1 &= 3T_{n-1} + 3 \end{aligned}$$

and letting $U_n = T_n + 1$, we get

$$\begin{aligned} U_0 &= 1 \\ U_n &= 3U_{n-1} \end{aligned}$$

so clearly $U_n = 3^n$ and $T_n = 3^n - 1$. To prove that this is indeed the case, let's now use inducction. For $n = 0$, $T_0 = 3^0 - 1 = 0$, and assume $T_{n-1} = 3^{n-1} - 1$, so

$$T_n = 3T_{n-1} + 2 = 3(3^{n-1} - 1) + 2 = 3^n - 3 + 2 = 3^n - 1$$