I - Calculs algébriques

À Savoir

Soit a, b, c des réels.

$$a(b+c) = ab + ac.$$

Exemple 1

En utilisant les règles précédentes,

$$\begin{aligned} 3(2\,\mathrm{e}+4) &= 3\times 2\,\mathrm{e}+3\times 4\\ &= 6\,\mathrm{e}+12.\\ 15+175 &= 5\times 3+5\times 25\\ &= 5\times(3+25).\\ 3+75 &= 5\times\frac{3}{5}+5\times 15\\ &= 5\times\left(\frac{3}{5}+15\right). \end{aligned}$$

Solution de l'exercice 1.

1.

$$\left(1-\mathrm{e}^2\right)-1 = 1-\mathrm{e}^2-1 = -\mathrm{e}^2$$

2.

$$\left(1-\mathrm{e}^2\right)(-1) = \left(1-\mathrm{e}^2\right)\times(-1) = -1-\mathrm{e}^2\times(-1) = -1+\mathrm{e}^2$$

3.

$$2^n\left(3-\frac{5}{3}\,\mathrm{e}\right) = 3\times 2^n - \frac{5}{3}2^n\,\mathrm{e}\,.$$

4.

$$(-2^n)\left(3-\frac{5}{3}\,\mathrm{e}\right) = 3\times(-2)^n - \frac{5}{3}(-2)^n\,\mathrm{e}\,.$$

5.

$$-2^n\left(3-\frac{5}{3}\,\mathrm{e}\right) = -3\times 2^n + \frac{5}{3}2^n\,\mathrm{e}\,.$$

6. Lorsque $n=2$,

$$\begin{aligned} 3\times 2^n - \frac{5}{3}2^n\,\mathrm{e} &= 3\times 4 - \frac{5}{3}4\,\mathrm{e} = 12-\frac{20}{3}\,\mathrm{e}\,.\\ 3\times(-2)^n - \frac{5}{3}(-2)^n\,\mathrm{e} &= 3\times 4 - \frac{5}{3}4\,\mathrm{e} = 12-\frac{20}{3}\,\mathrm{e}\,.\\ -3\times 2^n + \frac{5}{3}2^n\,\mathrm{e} &= -3\times 4 + \frac{5}{3}4\,\mathrm{e} = -12+\frac{20}{3}\,\mathrm{e}\,. \end{aligned}$$

□

À Savoir

Soit a, b, c, d des réels. On suppose que les dénominateurs des fractions suivantes sont toujours non nuls.

$$\begin{aligned} -\frac{a}{b} &= \frac{-a}{b} = \frac{a}{-b}\\ \frac{a}{b}+\frac{c}{d} &= \frac{a\times d + c\times b}{b\times d}.\\ \frac{a}{b}\times\frac{c}{d} &= \frac{a\times c}{b\times d}.\\ \frac{a}{\frac{b}{c}} &= a\times\frac{c}{b}. \end{aligned}$$

Solution de l'exercice 2.

1.

$$\frac{\mathrm{e}^3-1}{-1} = -\left(\mathrm{e}^3-1\right) = -\mathrm{e}^3+1.$$

2.

$$\frac{4}{5}\left(5-\frac{1}{\mathrm{e}}\right) = \frac{4}{5}\times 5 - \frac{4}{5}\times\frac{1}{\mathrm{e}} = 4-\frac{4}{5\,\mathrm{e}}.$$

3.

$$\begin{aligned}\frac{-4}{5}\left(2+\mathrm{e}^3\right) &= -\frac{4}{5}\times 2 - \frac{4}{5}\times \mathrm{e}^3\\ &= -\frac{8}{5}-\frac{4\,\mathrm{e}^3}{5}.\end{aligned}$$

4.

$$\begin{aligned}\frac{4}{-5}\left(2-\mathrm{e}^3\right) &= -\frac{4}{5}\times\left(2-\mathrm{e}^3\right)\\ &= -\frac{4}{5}\times 2 + \frac{4}{5}\times \mathrm{e}^3\\ &= -\frac{8}{5}+\frac{4\,\mathrm{e}^3}{5}.\end{aligned}$$

5.

$$\begin{aligned}\frac{4}{5}+\left(5-\frac{1}{\mathrm{e}}\right) &= \frac{4}{5}+5-\frac{1}{\mathrm{e}}\\ &= \frac{4}{5}+\frac{25}{5}-\frac{1}{\mathrm{e}}\\ &= \frac{29}{5}-\frac{1}{\mathrm{e}}.\end{aligned}$$

□

Solution de l'exercice 3.

1.

$$\begin{aligned}\frac{\frac{3}{2}}{1-\frac{4}{5}} &= \frac{\frac{3}{2}}{\frac{5-4}{5}}\\ &= \frac{\frac{3}{2}}{\frac{1}{5}}\\ &= \frac{3}{2}\times 5 \qquad\qquad = \frac{15}{2}.\end{aligned}$$

2.

$$\begin{aligned}\frac{3}{2}+\frac{4}{5} &= \frac{3\times 5}{2\times 5}+\frac{4\times 2}{5\times 2}\\ &= \frac{15+8}{10}\\ &= \frac{23}{10}.\end{aligned}$$

3.

$$\begin{aligned}\frac{3}{2}\times\frac{4}{5} &= \frac{3\times 4}{5\times 2}\\ &= \frac{12}{10} = \frac{6}{5}.\end{aligned}$$

4.

$$\begin{aligned}\frac{\frac{2}{34}}{\frac{63}{40}\times\frac{16}{27}} &= \frac{\frac{1}{17}}{\frac{9\times 7}{8\times 5}\times\frac{8\times 2}{9\times 3}}\\ &= \frac{\frac{1}{17}}{\frac{14}{15}}\\ &= \frac{1}{17}\times\frac{15}{14}\\ &= \frac{15}{238}.\end{aligned}$$

5.

$$\begin{aligned}\frac{3}{2}\left(\frac{\frac{4}{5}\times\frac{10}{3}}{3-\frac{1}{2}}\right) &= \frac{3}{2}\frac{\frac{4}{5}\times\frac{5\times 2}{3}}{\frac{6}{2}-\frac{1}{2}}\\ &= \frac{3}{2}\frac{\frac{8}{3}}{\frac{5}{2}}\\ &= \frac{3}{2}\times\frac{8}{3}\times\frac{2}{5}\\ &= \frac{8}{5}.\end{aligned}$$

6.

$$\begin{aligned}\frac{7}{18}-\frac{13}{60} &= \frac{7}{3\times 6}-\frac{13}{10\times 6}\\ &= \frac{7\times 10-13\times 3}{3\times 10\times 6}\\ &= \frac{70-39}{180}\\ &= \frac{31}{180}.\end{aligned}$$

7.

$$\begin{aligned}\frac{7}{12}-\frac{2}{3}+\frac{2}{9} &= \frac{7\times 3}{9\times 4}-\frac{2\times 12}{9\times 4}+\frac{2\times 4}{9\times 4}\\ &= \frac{21-24+8}{36}\\ &= \frac{5}{36}.\end{aligned}$$

8.

$$\begin{aligned}\left(\frac{3}{2}-\frac{5}{4}\right)\times\left(\frac{9}{4}+\frac{21}{6}\right) &= \frac{3\times 2-5}{4}\times\left(\frac{9}{4}+\frac{7}{2}\right)\\ &= \frac{1}{4}\times\frac{9+2\times 7}{4}\\ &= \frac{23}{16}.\end{aligned}$$

9.

$$\begin{aligned}\frac{\frac{29}{18}\times\frac{-45}{7}}{\frac{39}{14}} &= -\frac{29}{9\times 2}\times\frac{9\times 5}{7}\times\frac{7\times 2}{3\times 13}\\ &= -\frac{29\times 5}{3\times 13}\\ &= -\frac{145}{39}.\end{aligned}$$

10.

$$\begin{aligned}\frac{\frac{3}{2}}{\frac{4}{5}-\frac{-3}{2}\times\frac{7}{15}} &= \frac{\frac{3}{2}}{\frac{4}{5}+\frac{3}{2}\times\frac{7}{5\times 3}}\\ &= \frac{\frac{3}{2}}{\frac{4}{5}+\frac{7}{10}}\\ &= \frac{\frac{3}{2}}{\frac{15}{10}}\\ &= \frac{\frac{3}{2}}{\frac{3}{2}}\\ &= \frac{3}{2}\times\frac{2}{3}\\ &= 1.\end{aligned}$$

11.

$$\begin{aligned}\frac{\frac{3}{2}}{\frac{4}{5}-\frac{-3}{2}\times\frac{7}{-15}} &= \frac{\frac{3}{2}}{\frac{4}{5}-\frac{3}{2}\times\frac{7}{5\times 3}}\\ &= \frac{\frac{3}{2}}{\frac{4}{5}-\frac{7}{10}}\\ &= \frac{\frac{3}{2}}{\frac{1}{10}}\\ &= \frac{3}{2}\times 10\\ &= 15.\end{aligned}$$

12.

$$\begin{aligned}\frac{1-\frac{1}{3}}{\left(\frac{1}{3}\right)^2} &= \frac{\frac{2}{3}}{\frac{1}{3\times 3}}\\ &= \frac{2}{3}\times 3\times 3\\ &= 6.\end{aligned}$$

13.

$$\begin{aligned}\frac{1-\frac{4}{5}}{\left(\frac{4}{5}\right)^2} &= \frac{\frac{1}{5}}{\frac{4\times 4}{5\times 5}}\\ &= \frac{1}{5}\times\frac{5\times 5}{4\times 4}\\ &= \frac{5}{16}.\end{aligned}$$

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