

I - Calculs algébriques

À Savoir

Soit a, b, c des réels.

$$a(b + c) = ab + ac.$$

Exemple 1

En utilisant les règles précédentes,

$$\begin{aligned} 3(2e + 4) &= 3 \times 2e + 3 \times 4 \\ &= 6e + 12. \end{aligned}$$

$$15 + 175 = 5 \times 3 + 5 \times 25 = 5 \times (3 + 25).$$

$$\begin{aligned} 3 + 75 &= 5 \times \frac{3}{5} + 5 \times 15 \\ &= 5 \times \left(\frac{3}{5} + 15 \right). \end{aligned}$$

Solution de l'exercice 1.

1.

$$(1 - e^2) - 1 = 1 - e^2 - 1 = -e^2$$

2.

$$(1 - e^2)(-1) = (1 - e^2) \times (-1) = -1 - e^2 \times (-1) = -1 + e^2$$

3.

$$2^n \left(3 - \frac{5}{3}e \right) = 3 \times 2^n - \frac{5}{3}2^n e.$$

4.

$$(-2^n) \left(3 - \frac{5}{3}e \right) = 3 \times (-2)^n - \frac{5}{3}(-2)^n e.$$

5.

$$-2^n \left(3 - \frac{5}{3}e \right) = -3 \times 2^n + \frac{5}{3}2^n e.$$

6. Lorsque $n = 2$,

$$3 \times 2^n - \frac{5}{3}2^n e = 3 \times 4 - \frac{5}{3}4e = 12 - \frac{20}{3}e.$$

$$3 \times (-2)^n - \frac{5}{3}(-2)^n e = 3 \times 4 - \frac{5}{3}4e = 12 - \frac{20}{3}e.$$

$$-3 \times 2^n + \frac{5}{3}2^n e = -3 \times 4 + \frac{5}{3}4e = -12 + \frac{20}{3}e.$$

□

À Savoir

Soit a, b, c, d des réels. On suppose que les dénominateurs des fractions suivantes sont toujours non nuls.

$$\begin{aligned} -\frac{a}{b} &= \frac{-a}{b} = \frac{a}{-b} \\ \frac{a}{b} + \frac{c}{d} &= \frac{a \times d + c \times b}{b \times d}. \\ \frac{a}{b} \times \frac{c}{d} &= \frac{a \times c}{b \times d}. \\ \frac{a}{\frac{b}{c}} &= a \times \frac{c}{b}. \end{aligned}$$

Solution de l'exercice 2.

1.

$$\frac{e^3 - 1}{-1} = -(e^3 - 1) = -e^3 + 1.$$

2.

$$\frac{4}{5} \left(5 - \frac{1}{e} \right) = \frac{4}{5} \times 5 - \frac{4}{5} \times \frac{1}{e} = 4 - \frac{4}{5e}.$$

3.

$$\begin{aligned} \frac{-4}{5} (2 + e^3) \\ &= -\frac{4}{5} \times 2 - \frac{4}{5} \times e^3 \\ &= -\frac{8}{5} - \frac{4e^3}{5}. \end{aligned}$$

4.

$$\begin{aligned} \frac{4}{-5} (2 - e^3) &= -\frac{4}{5} \times (2 - e^3) \\ &= -\frac{4}{5} \times 2 + \frac{4}{5} \times e^3 \\ &= -\frac{8}{5} + \frac{4e^3}{5}. \end{aligned}$$

5.

$$\begin{aligned} \frac{4}{5} + \left(5 - \frac{1}{e} \right) &= \frac{4}{5} + 5 - \frac{1}{e} \\ &= \frac{4}{5} + \frac{25}{5} - \frac{1}{e} \\ &= \frac{29}{5} - \frac{1}{e}. \end{aligned}$$

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Solution de l'exercice 3.**1.**

$$\begin{aligned} \frac{\frac{3}{2}}{1 - \frac{4}{5}} &= \frac{\frac{3}{2}}{\frac{5-4}{5}} \\ &= \frac{\frac{3}{2}}{\frac{1}{5}} \\ &= \frac{3}{2} \times 5 = \frac{15}{2}. \end{aligned}$$

2.

$$\begin{aligned} \frac{3}{2} + \frac{4}{5} &= \frac{3 \times 5}{2 \times 5} + \frac{4 \times 2}{5 \times 2} \\ &= \frac{15 + 8}{10} \\ &= \frac{23}{10}. \end{aligned}$$

3.

$$\begin{aligned} \frac{3}{2} \times \frac{4}{5} &= \frac{3 \times 4}{5 \times 2} \\ &= \frac{12}{10} = \frac{6}{5}. \end{aligned}$$

4.

$$\begin{aligned} \frac{\frac{2}{34}}{\frac{63}{40} \times \frac{16}{27}} &= \frac{\frac{1}{17}}{\frac{9 \times 7}{8 \times 5} \times \frac{8 \times 2}{9 \times 3}} \\ &= \frac{\frac{1}{17}}{\frac{14}{15}} \\ &= \frac{1}{17} \times \frac{15}{14} \\ &= \frac{15}{238}. \end{aligned}$$

5.

$$\begin{aligned}
 \frac{3}{2} \left(\frac{\frac{4}{5} \times \frac{10}{3}}{3 - \frac{1}{2}} \right) &= \frac{3}{2} \frac{\frac{4}{5} \times \frac{5 \times 2}{3}}{\frac{6}{2} - \frac{1}{2}} \\
 &= \frac{3}{2} \frac{\frac{8}{3}}{\frac{5}{2}} \\
 &= \frac{3}{2} \times \frac{8}{3} \times \frac{2}{5} \\
 &= \frac{8}{5}.
 \end{aligned}$$

6.

$$\begin{aligned}
 \frac{7}{18} - \frac{13}{60} &= \frac{7}{3 \times 6} - \frac{13}{10 \times 6} \\
 &= \frac{7 \times 10 - 13 \times 3}{3 \times 10 \times 6} \\
 &= \frac{70 - 39}{180} \\
 &= \frac{31}{180}.
 \end{aligned}$$

7.

$$\begin{aligned}
 \frac{7}{12} - \frac{2}{3} + \frac{2}{9} &= \frac{7 \times 3}{9 \times 4} - \frac{2 \times 12}{9 \times 4} + \frac{2 \times 4}{9 \times 4} \\
 &= \frac{21 - 24 + 8}{36} \\
 &= \frac{5}{36}.
 \end{aligned}$$

8.

$$\begin{aligned}
 \left(\frac{3}{2} - \frac{5}{4} \right) \times \left(\frac{9}{4} + \frac{21}{6} \right) &= \frac{3 \times 2 - 5}{4} \times \left(\frac{9}{4} + \frac{7}{2} \right) \\
 &= \frac{1}{4} \times \frac{9 + 2 \times 7}{4} \\
 &= \frac{23}{16}.
 \end{aligned}$$

9.

$$\begin{aligned}
 \frac{\frac{29}{18} \times \frac{-45}{7}}{\frac{39}{14}} &= -\frac{29}{9 \times 2} \times \frac{9 \times 5}{7} \times \frac{7 \times 2}{3 \times 13} \\
 &= -\frac{29 \times 5}{3 \times 13} \\
 &= -\frac{145}{39}.
 \end{aligned}$$

10.

$$\begin{aligned}
 \frac{\frac{4}{5} - \frac{-3}{2} \times \frac{7}{15}}{\frac{3}{2}} &= \frac{\frac{4}{5} + \frac{3}{2} \times \frac{7}{5 \times 3}}{\frac{3}{2}} \\
 &= \frac{\frac{4}{5} + \frac{7}{10}}{\frac{3}{2}} \\
 &= \frac{\frac{8 + 7}{10}}{\frac{3}{2}} \\
 &= \frac{\frac{15}{10}}{\frac{3}{2}} \\
 &= \frac{3}{2} \times \frac{2}{3} \\
 &= 1.
 \end{aligned}$$

11.

$$\begin{aligned}
 \frac{\frac{4}{5} - \frac{-3}{2} \times \frac{7}{-15}}{\frac{3}{2}} &= \frac{\frac{4}{5} - \frac{3}{2} \times \frac{7}{5 \times 3}}{\frac{3}{2}} \\
 &= \frac{\frac{4}{5} - \frac{7}{10}}{\frac{3}{2}} \\
 &= \frac{\frac{8 - 7}{10}}{\frac{3}{2}} \\
 &= \frac{\frac{1}{10}}{\frac{3}{2}} \\
 &= \frac{3}{2} \times \frac{1}{10} \\
 &= \frac{3}{20}.
 \end{aligned}$$

12.

$$\begin{aligned}\frac{1 - \frac{1}{3}}{\left(\frac{1}{3}\right)^2} &= \frac{\frac{2}{3}}{\frac{1}{3 \times 3}} \\ &= \frac{2}{3} \times 3 \times 3 \\ &= 6.\end{aligned}$$

13.

$$\begin{aligned}\frac{1 - \frac{4}{5}}{\left(\frac{4}{5}\right)^2} &= \frac{\frac{1}{5}}{\frac{4 \times 4}{5 \times 5}} \\ &= \frac{1}{5} \times \frac{5 \times 5}{4 \times 4} \\ &= \frac{5}{16}.\end{aligned}$$

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