HA-3

1. If $Z \sim N(0,1)$, find 'a' such that
   a. $P(Z < a) = 0.90$
   b. $P(Z > a) = 0.25$

2. Diameter (X) of a car component follows the normal distribution with $X \sim N(10;3)$. If we randomly select one of these components, find the probability that its diameter will be larger than 13.4 mm.

3. Customer care of a company receives numerous phone calls throughout the day from customers reporting service problems. Many of these callers are put on hold until an operator is free to help them. The company has determined that the length of time a caller is on hold is normally distributed with a mean of 3.1 minutes and a standard deviation 0.9 minutes. Experts have decided that if as many as 5% of the callers are put on hold for 4.8 minutes or longer, more operators should be hired.

   a. What proportion of the callers is put on hold for more than 4.8 minutes?
   b. Should the company hire more operators?
   c. At another company (same waiting time distribution), 2.5% of the callers are put on hold for longer than 'x' minutes. Find the value of 'x'.

4. Parts for a machine are acceptable within the 'tolerance' limits of 20.5 to 20.6 mm. From previous tests it is known that the manufacturer produces parts to N (20.56, 0. 02). Out of a batch of 1000 parts how many would be expected to be rejected?

5. Buoyancy aids in water sports are tested by adding increasing weights until they sink. A club has two sets of buoyancy aids. One set is two years old, and should support weights according to N (6.0, 0.64) kg; the other set is five years old and should support weights of N (4.5, 1.0) kg. All the aids are tested and any which are unable to support at least 5 kg are thrown out.
   a. If there are 24 two-year-old aids, how many are still usable?
   b. If there are 32 five-year-old aids how many are still usable?

6. A light bulb manufacturer finds that 5% of his bulbs last more than 500 hours. An improvement in the process meant that the mean lifetime was increased by 50 hours. In a new test, 20% of bulbs now lasted longer than 500 hours. Find the mean and standard deviation of the original process.

7. A student is doing a project on the hire of DVDs from a local shop. He finds that the daily demand for DVDs is approximately normal, with mean 50 and SD 10.
   a. What is the probability of more than 65 DVDs will be hired on a particular day?
   b. The shop is considering stopping the hire as it is uneconomical and decides that if demand is less than 40 on more than 3 days out of the next 7 it will stop this service. How likely is this to happen?
   c. The student reckons that with a wider range of DVDs, demand would increase by 25% on average with no effect on the SD. What is the probability of more than 65 videos being hired if this happens?

8. The weights of pieces of a product are normally distributed with mean 34g & standard deviation 5g.
   a. What is the probability that a piece selected at random weighs more than 40g?

b. For some purposes it is necessary to grade the pieces as small, medium or large. It is decided to grade all pieces weighing over 40g as large and to grade the heavier half of the remainder as medium. The rest will be graded as small. What is the upper limit of the small grade?

9. A company has two machines cutting cylindrical corks for wine bottles. The diameters of corks produced by each machine are normally distributed. The specification requires corks with diameters between 2.91 cm and 3.12 cm. Corks cut on Machine A have diameters with a mean 3.03 cm and standard deviation 0.05 cm Calculate the percentage of corks cut on this machine that
   a. Are rejected as undersize.
   b. Meet the specification.

   Machine B cuts corks with a mean diameter of 3.01 cm of which 1.7% are rejected as oversize. Calculate the standard deviation of the diameters of corks cut on Machine B. Which machine, if either, do you consider to be the better? Explain.