

Thank you, next: demographic consequences of partner diversity and turnover in a multi-species ant-plant mutualism

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Abstract

The diversity of partners in a multi-species mutualism causes varied demographic effects on the population of the focal mutualist which can be explained by several mechanisms: portfolio effect, complementarity, and sampling effect. *Cylindropuntia imbricata* (tree cholla) produce extrafloral nectar and various ant partners provide defense from herbivores and seed predators. We used plant demographic censuses to parameterize a series of Bayesian hierarchical generalized linear vital rate models to determine the impacts of different partners on the focal mutualists. We constructed an Integral Projection Model in which we simulate different combinations of ant partners that don't occur in the wild. The hierarchical models revealed that different ant partners had different impacts on the cholla vital rates. Specifically, *C. opuntiae* tended plants had advantages in both growth and survival when small, and large *L. apiculatum* tended plants had floral viability advantages. The IPM results revealed that scenario which included *L. apiculatum* only resulted in the highest possible fitness for the tree cholla. This suggests that there are no benefits of diversity in this system. This study highlights that partner diversity does not always increase the benefits a focal mutualist receives.

Introduction

Mutualisms are species interactions where all participants receive net benefits, leading to higher individual fitness and increased population growth rates. **They are widespread species interactions [8, 18, 23, 36, 46] but can deteriorate into commensalism or parasitism under conditions that elevate costs or dampen benefits [9, 48, 65, 69, 74].**¹ Mutualisms are considered more context dependent than other species interactions [23, 36], meaning the magnitude and sign of interaction strength are often determined by environmental conditions and species' identities [46, 59].

Mutualism is defined at the level of a species pair (+/+) but these interactions are embedded within multi-species communities, and growing evidence suggests that pairwise interactions are poor predictors of the net effects of multi-species mutualism [4, 12, 25, 63]. A focal mutualist may interact with multiple guilds of partner types (e.g., plants that interact with pollinators, seed dispersers, soil microbes, and ant defenders) or with multiple partner species within the same guild (e.g., plants visited by multiple pollinator species). Within a mutualist guild, partner species often differ in the amount or type of goods or services they provide, making partner identity an important source of contingency in mutualism [72]. Whether and how partner diversity modifies the demographic effects of mutualistic interactions remain open questions within relevance in applied settings [35, 63, 66, 73].

There are multiple mechanisms by which partner diversity can influence the net benefits accrued by a focal mutualist, mirroring the mechanisms by which, at a larger scale of organization, biodiversity can influence ecosystem function [10, 79, 83]. First, when there is a hierarchy of fitness effects (a consistent ranking of best to worst mutualists), a more diverse sample of the partner community may be more likely to include the best partner [36]. This can lead to an apparent benefit of diversity driven by a sampling effect [13] – but there is no benefit of diversity *per se*, only better and worse partners. If partner associations are mutually exclusive (a focal

¹*I suggest condensing the two sets of citations into one at the end of the sentence. The statement that they are widespread species interactions is so broad and vague that it is not really worth citing.*

mutualist may engage with only one partner at a time), then partner diversity may impose opportunity costs, leading to negative effects of a diverse mutualist assemblage relative to exclusive association with the single best partner [52]. Second, even within a single mutualist guild, the benefits conferred by alternative partner species can vary in type and not just degree [20, 70, 72]. This can lead to a positive effect of partner diversity through complementarity of alternative functions [13]. Interference or synergies between partners can make their combined effect different than the expected from the sum of complementary functions [4]. Third, partner species can have species-specific responses to environmental variation, either spatially [62] or temporally [6]. Multiple partners can therefore act as a 'portfolio' that stabilizes fitness benefits across spatial or temporal heterogeneity, leading to positive effects of partner diversity through the portfolio effect [13, 42, 45].

Partner diversity can have different effects depending on whether partners are present simultaneously or sequentially (partner turnover) [10, 21, 25, 27, 57, 79]. Sequential associations are likely when alternative partners engage in interference competition for access to a shared mutualist [13, 44, 78, 82]. Turnover can happen at different timescales, from minutes to years [41, 61]. The frequency of partner turnover can impact the level of benefits received by the focal mutualist, particularly if the benefits continue to accumulate with successive turnover (e.g., when sequential partners provide complementary functions) or if they saturate over time [31, 68]. Directionality of turnover can also influence effects of partner diversity if partner identity changes consistently across ontogeny of a focal mutualist [26, 32, 59]. For example, plant susceptibility to enemies can change across life stages [11, 17], so the benefits of a diverse guild of defensive mutualists are greatest when more defensive partner species align with more vulnerable life stages [26, 27].

Defensive ant-plant mutualisms – where plants provide food and/or housing to ants that in turn defend them from enemies – are widespread interactions that offer valuable model systems for the ecology and evolution of mutualism [19, 20]. Extrafloral nectar (EFN) -bearing plants can serve as dietary resources that promote ant abundance and colony size [22, 28, 57, 58]. In turn, presence of defensive ant partners is often linked to reductions in herbivory [67, 76] and demo-

graphic advantages for the plant partner [16]. Defensive ant-plant mutualisms are commonly multi-species, where a guild of ant partner species share, and often compete for, a plant mutualist [5, 14, 19, 76]. Ant partners can vary in their ability to deter herbivores [21], and visitation by low quality ant partners can prevent visitation by higher quality partners, consequently causing a reduction in fitness through missed opportunity costs [34, 35]. Susceptibility to herbivory can also vary significantly throughout the life stages of the plant [17], suggesting that the order and timing of successive partners is important to the fitness impacts of the combined partner guild [11, 17, 32]. Finally, herbivore identity and pressure can vary inter-annually [80], much like mutualist identity and presence, meaning the threat plants face can vary just as much as the protection they receive due to temporal stochasticity. Previous studies have investigated how ant partner diversity affects plant fitness [4, 25, 31, 37, 47, 63] However, little is known about the combined effects of partner identity, directional partner turnover, and temporal stochasticity, particularly because the necessary long-term data are rarely available.

This study examined the consequences of partner diversity in a food-for-protection mutualism between the tree cholla cactus (*Cylindriopuntia imbricata*), a long-lived EFN-bearing plant, and multiple species of ant partners. Previous studies have shown that herbivory by specialized insect herbivores negatively affects cactus fitness [54], and that ant defense reduces herbivore damage [52]. Tree cholla are tended by two common ant species (*Liometopum apiculatum* and *Crematogaster opuntiae*) and several additional rarer species, all of which collect EFN during foraging visits but their colonies are ground-nesting and not housed by the plants. These ant species locally co-occur but individual plants are typically tended by only one species that patrols the plant around-the-clock and maintains control of the plant's nectar resources, usually for an entire growing season [28, 60]. Switches between partner species, or between vacancy and ant occupancy, commonly occur from one growing season to the next [52]. Prior experiments suggested a hierarchy of mutualist quality, with *L. apiculatum* providing strong anti-herbivore defense and *C. opuntiae* having net negative effects because herbivore deterrence is outweighed by deterrence of pollinators [52, 60]. However, all of our previous studies in this system have focused on single

life stages (adult plants) or vital rates (seed production) and did not integrate the demographic effects of ant defense across the life cycle, which may be essential for understanding net fitness effects of a diverse partner guild [e.g., 63]. To our knowledge no previous study has incorporated inter-annual stochasticity into models of ant-plant dynamics, which limits our understanding of diversity benefits that may arise through the portfolio effect.

Here we used a unique long-term data set that allows us to explore mutualistic associations with multiple partner species, longitudinal turnover in partner identity, and how the demographic effects of alternative partners varies across plant size structure and nearly 20 years of inter-annual fluctuations. We used this observational data set of plant demography and ant-plant associations, contextualized by previous ant exclusion experiments, to investigate whether and through which mechanism(s) partner diversity affects the fitness benefits of ant visitation. Specifically, we asked:

1. What are the demographic effects of association with alternative partners and how do these effects fluctuate across years?
2. What are the frequency and direction of partner turnover across the plant life cycle?
3. What is the net effect of partner diversity on plant fitness, and what mechanism(s) explain(s) this effect?

To answer these questions we used a hierarchical Bayesian statistical approach to estimate demographic vital rates for hosts in different states of ant occupancy and to quantify state-dependent partner turnover from the long-term data. We then used a stochastic, multi-state integral projection model (IPM) that combines diverse effects on vital rates and pathways of partner turnover to quantify effects of partner diversity on plant fitness.

Methods

Study System

117

This study was conducted in the Los Piños mountains, a small mountain chain located on the Sevilleta National Wildlife Refuge, a Long-term Ecological Research site (SEV-LTER) in central New Mexico, USA. This is an area characterized by steep, rocky slopes, and perennial vegetation including grasses (*Bouteloua eriopoda* and *B. gracilis*), yuccas, cacti, and junipers. Tree cholla cacti are common in high Chihuahuan desert habitats of the southwestern USA [15]. These areborescent plants produce cylindrical segments with large spines. In the growing season (May to August in New Mexico), the plants initiate new vegetative segments and flower buds at the ends of existing segments. While most plants produce new segments every season, only those which are reproductively mature produce flower buds. Like other EFN-bearing cacti, tree cholla secrete nectar from specialized glands on young vegetative segments and flower buds [57, 61]. Flower buds produce more and higher-quality EFN than vegetative segments, making reproductive cholla valuable mutualist partners [53]. Smaller, non-flowering cholla produce little to no EFN and are commonly vacant (no ant visitation at the time of our census) [53].

Tree cholla EFN is harvested by various ground-nesting ant species. At SEV-LTER, cholla are visited primarily by two species, *Crematogaster opuntiae* and *Liometopum apiculatum*, as well as other rarer species, including *Forelius pruinosus* and unidentified species in the genera *Aphaenogaster* and *Camponotus*. *L. apiculatum* are the most frequent visitors with 25% – 60% of tree cholla tended by these ants in a given year, followed by *C. opuntiae* (5% – 20%) [28]. Between 30% – 80% of cacti are vacant in any given year. Workers of different species rarely co-occur on individual plants, likely due to interspecific competition. For example, staged introductions of *C. opuntiae* to *L. apiculatum*-tended plants, and vice versa, provoke aggressive responses by residents (A. Campbell, personal observation).

Several insect herbivores and seed predators specialize on tree cholla [49], and defense against

141 these enemies is the main pathway by which ant visitation affects plant demography. The Cer-
ambycid beetle *Moneilema appressum* and an unidentified weevil (Coleoptera: Curculionidae) of
the genus *Gerstaekeria* feed on vegetative and reproductive structures as adults and their larvae
144 feed internally. Two species of cactus bugs, *Narnia pallidicornis* and *Chelinidea vittiger* (Hemiptera:
Coreidae), feed on all cholla parts with a preference for flower buds; their damage can induce
floral abortion [56]. A seed predator, *Cahela ponderosella* (Lepidoptera: Pyralidae), oviposits in
147 open flowers and larvae eat seeds in developing fruits. These consumers can have significant
negative impacts on plant fitness and depress population growth [54]. Prior experiments showed
that ant-tended tree cholla experience less herbivory and seed predation than plants from which
150 ants were excluded [52, 60].

Data Collection

This study is based on long-term demographic data spanning 2004 to 2023 at SEV-LTER. From
153 2004 to 2008, we censused 134 plants distributed across three spatial blocks. This initial census
group was discontinued in 2009, when we established six 30 × 30-meter plots and tagged all tree
cholla within those plots. Two additional 30 × 30-meter plots were added in 2011, and this group
156 of eight plots has since been censused annually through 2023 (with the exception of 2020 due to
the pandemic shutdown). For all plants, in May or early June of each year we recorded plant
survival since the last survey and, for survivors, we recorded height (cm), maximum crown
159 width (cm), and crown width perpendicular to the maximum (cm). Size measurements were
used to calculate plant volume (cm^3) based on the volume of an elliptical cone. We measured
reproduction by counting flowerbuds, and we distinguished between flowerbuds that were viable
and aborted. We recorded the ant species present (or vacancy if no ants present). Occurrences
162 of more than one ant species on one plant were rare (less than 5% of observations), and for the
purpose of this analysis we classified the plant as being occupied by the more abundant species.
165 Plots were searched for new recruits each year, and these were added to the census. These data
allowed us to link each plant's demographic fate (survival, growth, and reproduction) to its

state of ant visitation. In total, the data set includes a total of 9,787 observations of 1141 unique
168 individuals across 19² complete transition years (spanning May/June of year t to May/June of
year $t + 1$).³

We used additional, smaller data sets from previously published studies to estimate seed and
171 seed bank parameters. Ohm et al. 2014 provide data on the number of seeds per fruit for plants
tended by *L. apiculatum*, *C. opuntiae*, or no ants (experimental exclusion), accounting for their
effects on pollinator visitation. Elderd and Miller 2016 provide data on seed entry to the seed
174 bank and seedling germination and survival rates.

Multi-state Integral Projection Model

The demographic data were used to parameterize a multi-state Integral Projection Model (IPM).
177 IPMs describe population dynamics in discrete time with functions that relate vital rates to con-
tinuous state variables, typically size [30]. While IPMs are a natural choice for populations with
continuous size structure, they can also be modified to accommodate a combination of contin-
180 uous and discrete state variables, as we do here. We constructed a stochastic, multi-state IPM
that stitches together population structure associated with plant size and ant state, allowing us to
determine the individual fitness effects of each ant species and the composite effects of multiple
183 partners, with ant transition dynamics and inter-annual variability modeled explicitly.

Given the low frequency of ant occupancy states other than *L. apiculatum* and *C. opuntiae* (<8%
of observations) we combined all other ants into an “other” category, such that our multi-state
186 IPM included four ant states: vacant, *L. apiculatum*, *C. opuntiae*, and Other. The “Other” cate-
gory was made up of *Forelius pruinus* (3.5% of observations), unidentified species belonging to
the genera *Camponotus* (0.9%), *Aphaenogaster* (0.4%), *Myrmecocystus* (0.08%), *Tetramorium* (0.02%),
189 *Brachymyrmex* (0.02%), and additional ants not identified to genus or species (2.8%).

²There are actually 15 complete transition years.

³You should cite the data package here. The published package only goes up to 2018, so we should talk about updating that
package with you as lead author, which should be easy to do as long as the data are QAQC'd.

Ant state is included as a predictor variable in IPM sub-models where there are biologically realistic pathways through which ants could impact the outcome of that process. For example, ant partners defend cacti from herbivores, and prior experimental work indicates that ant tending can reduce vegetative tissue loss and floral abortion. Therefore, ant state was included in sub-models for survival, growth, flowerbud viability, and seed number per flowerbud. In contrast, we have no reason to expect that ant tending can directly influence the probability of flowering or flowerbud production independently of its influence on plant size, so these sub-models do not include ant state as a predictor variable.

We modeled the tree cholla life cycle using continuously size-structured plants where number of plants of size x and ant state a in year t ($n(x, a)_t$) predicts the number of plants of size x' and ant state a' in year $t + 1$ ($n(x', a')_{t+1}$) based on a size- and ant-specific vital rates. The models also includes two discrete seed banks (B_t^1 and B_t^2) corresponding to 1 and 2-year old seeds. Seed bank dynamics are given by:

$$B_{t+1}^1 = \delta \sum_{a=1}^A \int_L^U \kappa(a') P(x; \boldsymbol{\tau}^P) F(x; \boldsymbol{\tau}^F) V(a; \boldsymbol{\tau}_a^V) n(x, a)_t dx$$

$$B_{t+1}^2 = (1 - \gamma_1) B_t^1$$

Functions $P(x; \boldsymbol{\tau}^P)$ and $F(x; \boldsymbol{\tau}^F)$ give the probability of flowering in year t and the number of flowerbuds produced in year t , respectively, by plants of size x in year t . The proportion of flowerbuds that remain viable through fruit set ($V(a; \boldsymbol{\tau}_a^V)$) and the number of seeds per fruit ($\kappa(a')$) is **dependent on ant state a'** ⁴. The vectors $\boldsymbol{\tau}$ give year-specific deviates (mean zero) and appear in functions for which we can estimate temporal stochasticity from the long-term data; superscripts indicate the corresponding vital rate and, when present, the a subscript indicates that deviates are specific to plants in ant state a . For example, temporal deviates $\boldsymbol{\tau}_a^V$ describe better- and worse-than-average years for flowerbud viability and plants in different ant states can fluctuate independently (good years for *L. apiculatum* -occupied plants may not be good

⁴The text says ant state a but the notation here and in the equation shows ant state a' . Which is it? (Should be a .)

years for *C. opuntiae*-occupied plants, for example). Seed production is integrated over the size distribution, from the lower L to upper U size limits, and summed over all possible initial ant states ($A = 4$) giving total seed production. Seeds are multiplied by the probability of escaping post-dispersal seed predation (δ) to give the number of seeds that enter the one-year old seed bank. Plants can recruit out of the year-one seed bank with probability γ_1 or transition to the two-year seed bank with a probability of $1 - \gamma_1$. Seeds in the two-year seed bank are assumed to either germinate with probability γ_2 or die.

For the above-ground part of the life cycle, the number of plants of size x' and ant state a' in year $t + 1$ is given by survival/growth transitions from size x and ant state a in year t , plus germination out of the seed banks:

$$n(x', a')_{t+1} = (\gamma_1 B_t^1 + \gamma_2 B_t^2) \eta(x') \omega \rho_0(a') + \sum_{a=1}^A \int_L^U S(x, a; \tau_a^S) G(x', x, a; \tau_a^G) \rho(x, a, a'; \tau^e) n(x, a)_t dx$$

The first term in Eq. ⁵ estimates the number of individuals recruiting from a one or two-year seed bank to a plant of size x' and ant state a' based on the recruit size distribution $\eta(x')$ and the probability of over-winter seedling survival (ω) from germination (late summer) to the census (May). This term is multiplied by $\rho_0(a')$, which gives the probability that a new recruit has ant state a' ($\sum \rho_0(a') = 1$). The second term represents all possible transitions from size x and ant a to size x' and ant a' , conditioned on survival. Survival ($S(x, a; \tau_a^S)$) and growth from size x to x' ($G(x', x, a; \tau_a^G)$) are both dependent on initial size and ant state. As above, these functions include inter-annual variability through year-specific deviates that can vary by ant state (τ_a). Finally, ant transition function $\rho(a', a, x; \tau^e)$ gives the probability that an individual transitions from ant state a to a' in the next census, conditional on initial size x . This function includes inter-annual variability through year-specific intercepts which are consistent across initial ant states (τ^e).

⁵We should label equations. I am not sure why the equation label is not working here and I did not try to figure it out. It is probably something with the `linenomath` formatting.

Statistical modeling and parameter estimation

We parameterized the IPM using a series of generalized linear mixed models in a hierarchical Bayesian framework. Vital rate models included spatial and temporal random effects associated with plot and year variation, respectively (only year variation is used in the IPM), and included plant size (the natural logarithm of volume, $\log(\text{cm}^3)$; x, x'), ant partner state (a, a'), or both as fixed-effect predictor variables. As in the IPM, our statistical modeling assumed that demographic effects of ant occupancy are limited to survival, growth, and flowerbud viability.

Growth. We fit the growth sub-model ($G(x', x, a; \tau_a^G)$) to data on size in year $t + 1$ (y^G) using the skewed normal distribution to account for left-skewed size transitions (at some initial sizes, transitions below the expected future size were more common than transitions above it). The skew-normal has three parameters corresponding to location (\hat{G}), shape (σ), and scale (α):

$$y_i^G \sim \text{SkewedNormal}(\hat{G}_i, \sigma_i, \alpha_i)$$

$$\hat{G}_i = \beta_{a[i]}^0 + \beta_{a[i]}^1 x_i + \beta_{a[i]}^2 x_i^2 + u_{\text{year}[i], a[i]} + w_{\text{plot}[i]}$$

$$\log(\sigma_i) = \beta^3 + \beta^4 x_i$$

$$\alpha_i = \beta^5 + \beta^6 x_i$$

Here, the location parameter for the i th observation \hat{G}_i is defined as a second-order polynomial with ant-size interactions because preliminary analysis found this was an improvement over a linear relationship (note that the location parameter of the skew-normal is not the mean, but the mean can be derived as $\hat{G} + \frac{\sigma\alpha}{\sqrt{1+\alpha^2}} \sqrt{\frac{2}{\pi}}$). The year- and ant-specific random effect u (which parameterizes the τ_a^G vectors) and plot-specific random effect w are normally distributed with variances σ_{year}^2 and σ_{plot}^2 , respectively. Parameters σ_i and α_i control residual variance and skewness, respectively, and were defined as linear functions of initial size x_i (σ_i is strictly positive and was modeled with a log link function). We assume growth variance and skewness were not dependent on ant occupancy state.

Survival. The survival sub-model ($S(a, x; \tau_a^S)$) estimates the probability of survival from year t to year $t + 1$, with fixed effects of size x and ant partner a in year t . We fit this model to the survival data (alive or dead) using a Bernoulli distribution with a similar linear predictor for the probability of survival as in the growth model but with a logit link function and without the second-order influence of size.

Reproduction. The flowering sub-model ($P(x; \tau^P)$) estimates the probability of reproducing in year t , with fixed effects size x in year t and random effects of plot and year. We fit this model to the reproductive status data (vegetative or flowering) using a Bernoulli distribution and a logit link function, similar to the survival model above but with no ant effects. The flower bud function $F(x; \tau^F)$ estimates the total flowers produced by a reproducing plant in year t , with fixed effects of size x in year t . We fit this model to flowerbud count data (sum of viable and aborted buds) using a zero-truncated negative binomial distribution with a log link and normally distributed year and plot random effects.

The flowerbud viability sub-model ($V(a; \tau_a^V)$) estimates the probability that flowerbuds produced in year t remain viable (not aborted), with fixed effects of ant partner a in year t . We fit this model to floral viability data using a binomial distribution where trials and successes are given by the total number of **flower buds**⁶ and the number that are viable, respectively. This model used a logit link function and included random effects for plot and year following the same structure as the growth and survival models, with ant-specific year random effects.

Additional reproductive parameters for the number of seeds per fruit, probability of entry to the seed bank, germination rates, and recruit size were estimated following methods described in Appendix XX. Estimates for the number of seeds per fruit were ant specific, but the data (cite tom's data) only included *L. apiculatum* and *C. opuntiae* ants. **We used the vacancy estimates for plants tended by other ants.**⁷ **The rest of these parameters were not ant specific.**⁸

⁶*Sometimes this is flowerbuds and sometimes it is flower buds. I would tighten this up.*

⁷*This is an assumption that needs to be explained or justified. Reviewers will ask if results are sensitive to this assumption.*

⁸*Because the seed number model is ant-specific, I suggest moving it from the appendix to the main text.*

Ant Transitions. The ant transition model ($\epsilon(x, a, a'; \tau^\epsilon)$) estimates the probability of a cactus being occupied by ant partner a' in year $t + 1$ given that it was occupied by initial ant partner a in year t , with fixed effect of initial size x . We fit this model to ant partner data using a multinomial distribution with a logit link function and random effects for plot and year.

Parameter estimation. We fit models using Stan run through version 4.0.2 of R [64, 71]. We used vague priors for all parameters. For each model, we obtained three chains of 10,000 iterations, discarding the first 1,500 iterations. We visually assessed parameter convergence between and within chains (Figures 3 – 9 b) and assessed overall model fit with posterior predictive checks to examine how well the fitted model can generate simulated data similar to the real data (Figures 3 – 9 a).⁹

IPM Analysis

Analyzing an IPM requires that we discretize the continuous IPM kernel into an approximating matrix. Size variable x was discretized into b bins, resulting in a $b \times b$ matrix. In our model there is additional complexity in the form of transitions between A ant states and two additional discrete states (year one and year two seed banks), leading to a matrix size of $A(b + 2) \times A(b + 2)$. We used $b = 200$ bins¹⁰, which we found to be sufficient for numerically stable outputs, and extended the integration limits beyond the minimum (L) and maximum (U) observed sizes to avoid unintentional eviction using the “floor-and-ceiling” method [81].

For stochastic analyses, we estimated the approximating matrix corresponding to each t to $t + 1$ transition year. To estimate population mean fitness in a stochastic environment (λ_S) we simulated population dynamics for 500 years by randomly sampling among the 17 annual transition matrices¹¹, discarding the first 100 years of the simulation to minimize the influence of

⁹These figures need to be labeled so that it is clear that they are in the Appendix. Am Nat’s template should allow you to do this automatically.

¹⁰In your script `matsize` is 500.

¹¹Unless you add 2024 I think this is 16.

initial conditions. Sampling observed transition matrices (rather than independently sampling regression coefficients) produces demographic time series that realistically capture inter-annual variation by preserving correlations between vital rates [51]. We tallied the total population size at each time step as $N_t = B_t^1 + B_t^2 + \sum_{a=1}^A \int n(x, a)_t dx$ and calculated the stochastic growth rate as $\log(\lambda_S) = E[\log(\frac{N_{t+1}}{N_t})]$ [50]. We propagated uncertainty from the vital rate models using 1000 draws¹² from the joint posterior distribution of model parameters, resulting in a posterior distribution of λ_S and other derived quantities.

Partner diversity experiments

Using the fully parameterized multi-state IPM, we conducted simulation experiments to quantify how diversity and identity of ant partners influenced plant fitness in a stochastic environment (λ_S). From the full version of the model (described above) corresponding to the observed assemblage of partners and observed frequencies of partner transition, we created treatments corresponding to all eight “counter-factual” scenarios of diversity and composition: no ant partners (complete vacancy); one ant partner (*C. opuntiae* only, *L. apiculatum* only, Other only); two partners (all pairwise combinations of *C. opuntiae*, *L. apiculatum*, and Other); and three partners (observed scenario of all ant states). These simulation experiments were made possible by extrapolating ant-specific demographic performance across the size distribution, even for combinations of size and ant occupancy that were rarely observed. For example, the no-partner scenario modeled a hypothetically ant-free cactus population, even though no such population exists to our knowledge, by applying the statistical knowledge gleaned from vacant plants (which were mostly small and non-reproductive) across the size distribution.

In all scenarios that included any ant partners, we preserved the observed pattern of size-dependent vacancy/occupancy (estimated through the ant transition sub-model) and manipulated partner identity conditional on occupancy. This means, for example, that the *C. opuntiae*-

¹²I did not realize you were doing this many. I think you could use as few as 100, which would make your numerical work much faster.

only scenario included two possible states, vacancy and occupied by *C. opuntiae*. While our
339 statistical models allow us to extrapolate the demographic performance of ant-tended plants to
small sizes that are typically vacant, the natural history of this system tells us that this is not
biologically sensible. Small, non-reproductive plants are typically vacant because they do not
342 produce extrafloral nectar, and once plants begin producing nectar they are nearly always ant-
tended [53]. Our simulation experiments preserved this basic biology, avoiding tiny ant-occupied
plants that do not and could not occur in nature.

345 The partner diversity treatment scenarios required additional assumptions about the mecha-
nisms that give rise to observed occupancy patterns. Based on evidence that EFN-bearing cacti
are nearly always ant-occupied [53], we assume that ant partners competitively exclude one an-
348 other from EFN-bearing cacti and that competition is zero-sum. This means that, in scenarios
that remove species from the partner community, remaining species gain access to plants that the
removed species would have tended. In Appendix XX, we present results under an alternative
351 assumption, that ant visitation is limited by factors other than availability of cactus EFN (e.g.,
nesting sites or off-plant dietary resources), such that when a species is removed from the partner
community, the plants it would have occupied remain vacant.

354 *Temporal stochasticity experiments*

Under the portfolio effect hypothesis, partner diversity may confer a fitness advantage when the
benefits of alternative partners are not perfectly synchronized across temporal environmental
357 variation, yielding an advantage of a diverse “portfolio” of partners when the environment fluc-
tuates. Our statistical estimation of ant-specific year random effects in the vital rates allows for
this possibility. We constructed two versions of the stochastic, multi-state IPMs that allowed us
360 to test this hypothesis by exploring two scenarios of environmental variation. First, we evaluated
the model using empirical estimates for the τ_a vectors that describe ant-specific year deviates. In
this scenario, good years and bad years can differ between ant states, according to the empirical
363 parameter estimates. We also quantified from the fitted random effects how tightly inter-annual

variation was correlated between ant states.¹³ Second, we re-fit the ant-dependent vital rates (survival, growth, flowerbud viability) without ant-specific year random effects, thus assuming
366 that plants in all ant states fluctuated synchronously in response to temporal environmental variation. We evaluated a second, “synchronized” version of the model that effectively turns off any portfolio effect, holding all else equal. Both scenarios of temporal stochasticity, non-synchronized
369 and synchronized, were run for all eight ant partner scenarios described above.

Statistical inference on fitness consequences of partner identity and diversity

The range of models we created generated many outputs; we focus our inference on the following
372 specific contrasts. First, to determine whether ant occupancy and partner diversity are beneficial, we calculated a posterior distribution of λ_S for each of four partner richness levels (zero, one, two, three), averaging over composition scenarios within each level. If cactus fitness increases
375 with partner richness, this would be interpreted as evidence for benefits of partner diversity. Second, to determine whether each partner, in isolation, confers a fitness advantage and to rank alternative partners, we contrasted the fitness of each single partner scenario (*C. opuntiae* only,
378 *L. apiculatum* only, Other only) against vacancy (zero partners). Third, to determine whether any benefits of diversity are due to the sampling effect or complementarity, we contrasted the fitness of multi-partner scenarios against the single best partner scenario. If the best multi-partner
381 scenario exceeds the fitness associated with the best single partner, this would be interpreted as evidence of complementarity, a true benefit of diversity *per se*. Alternatively, the sampling effect hypothesis predicts that no multi-partner scenario yields higher plant fitness than the best single
384 partner. It is also possible that multi-partner scenarios yield lower fitness than the single best partner, which would be consistent with an opportunity cost of diversity. Fourth, to quantify any contribution of the portfolio effect, we contrasted λ_S of the full (four-state) scenario to vacancy
387 for synchronized and non-synchronized responses to temporal stochasticity (as a measure of how much benefit is recieved from all partners being present). If the portfolio effect confers a benefit

¹³Worth doing!

of diversity, the fitness advantage of having all vs. no partners should be greater when temporal
390 fluctuations are not synchronized across ant states.

We base our statistical inferences on the posterior probability distributions of the contrasts described above. For example, the contrast of *C. opuntiae* with vacancy yields a posterior distribution of the difference in λ_S ($\Delta\lambda_{C-V}$). We can quantify from this distribution our certainty in
393 the mutualistic effect of *C. opuntiae*, given the data, as $Pr(\Delta\lambda_{C-V} > 0)$. We apply similar logic to other contrasts described above.

396 Results

*What are the demographic effects of association with alternative parnters and
how do these effects fluctuate across years?*

399 Over the 20-year data set, we found that ant partners influenced demographic performance of cactus hosts, and different ant partners had contrasting demographic effects across host vital rates. Plants tended by *C. opuntiae* had a growth advantage, particularly for small plants, while
402 plants in states of *L. apiculatum*, Other ants, and vacancy had indistinguishable growth trajectories (Figure 1). For all ant states, growth was left-skewed, with transitions to sizes below the mean were more common than sizes above the mean. Similarly, ant visitation enhances cactus
405 survival, and ant partner identity has a significant impact on survival for smaller plants (Figure 2). Mean survival rates ranged from 7.7% to 99.9%, with the smallest plants the most vulnerable to mortality. *C. opuntiae*-occupied plants had a survival advantage over other ant states, particu-
408 larly at smaller sizes, consistent with the positive effects on growth. At larger sizes, plants in any state of ant occupancy had a survival advantage over vacant plants.

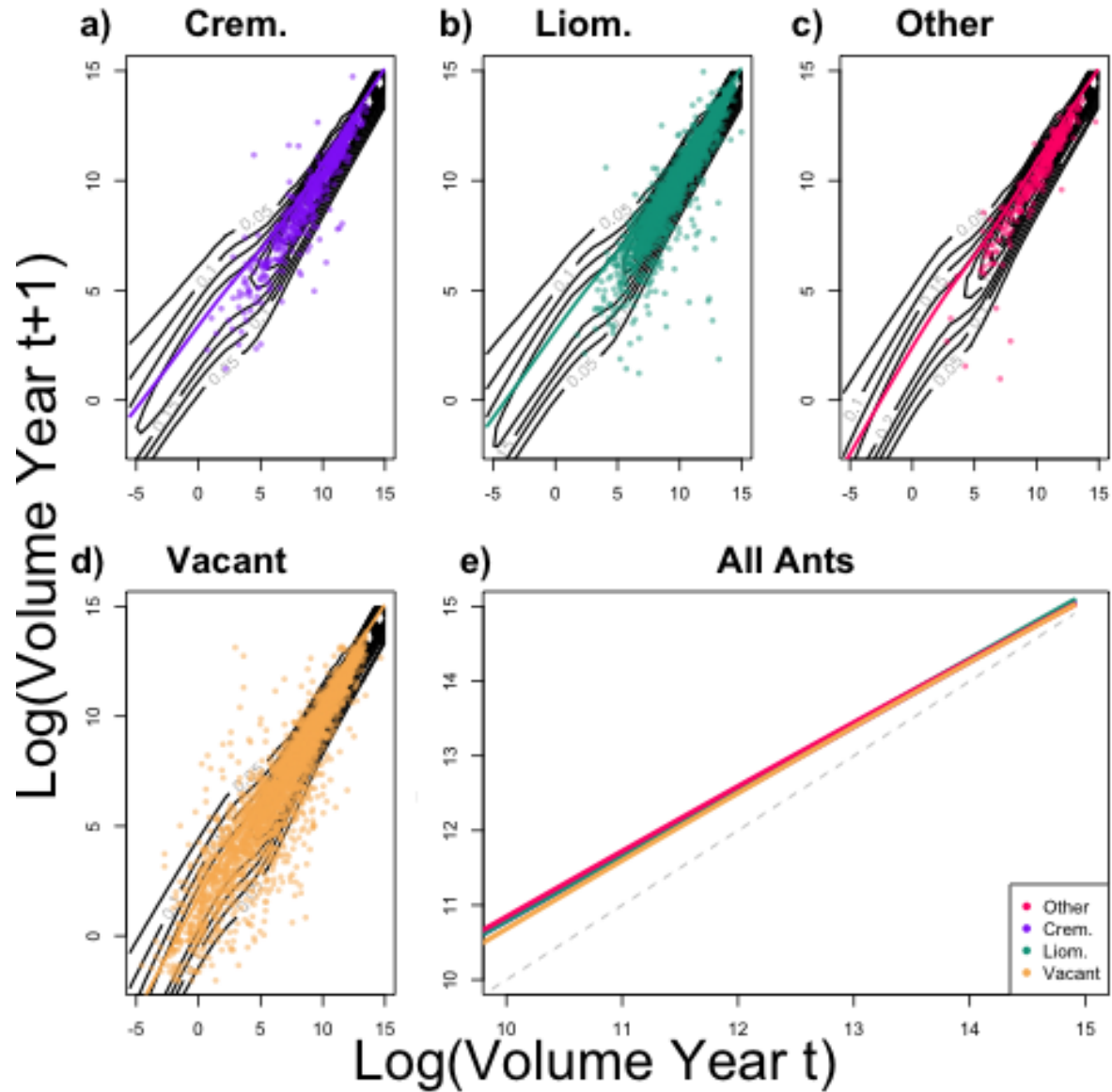


Figure 1: This figure shows the next predicted size of cholla based on previous size with each individual ant partner. The solid colored lines (seen in all panels) are the next mean predicted size of cholla. The points (seen in panels a-d) are the observed data which informs these estimates. The black countour lines (seen in a-d) appear at 5% increments showing where 5%, 10%, etc. of the data is expected to fall. They grey dashed line (in panel e only) shows the line where the next predicted size is the same as the previous (aka there is no growth on this line and below this line is shrinkage).

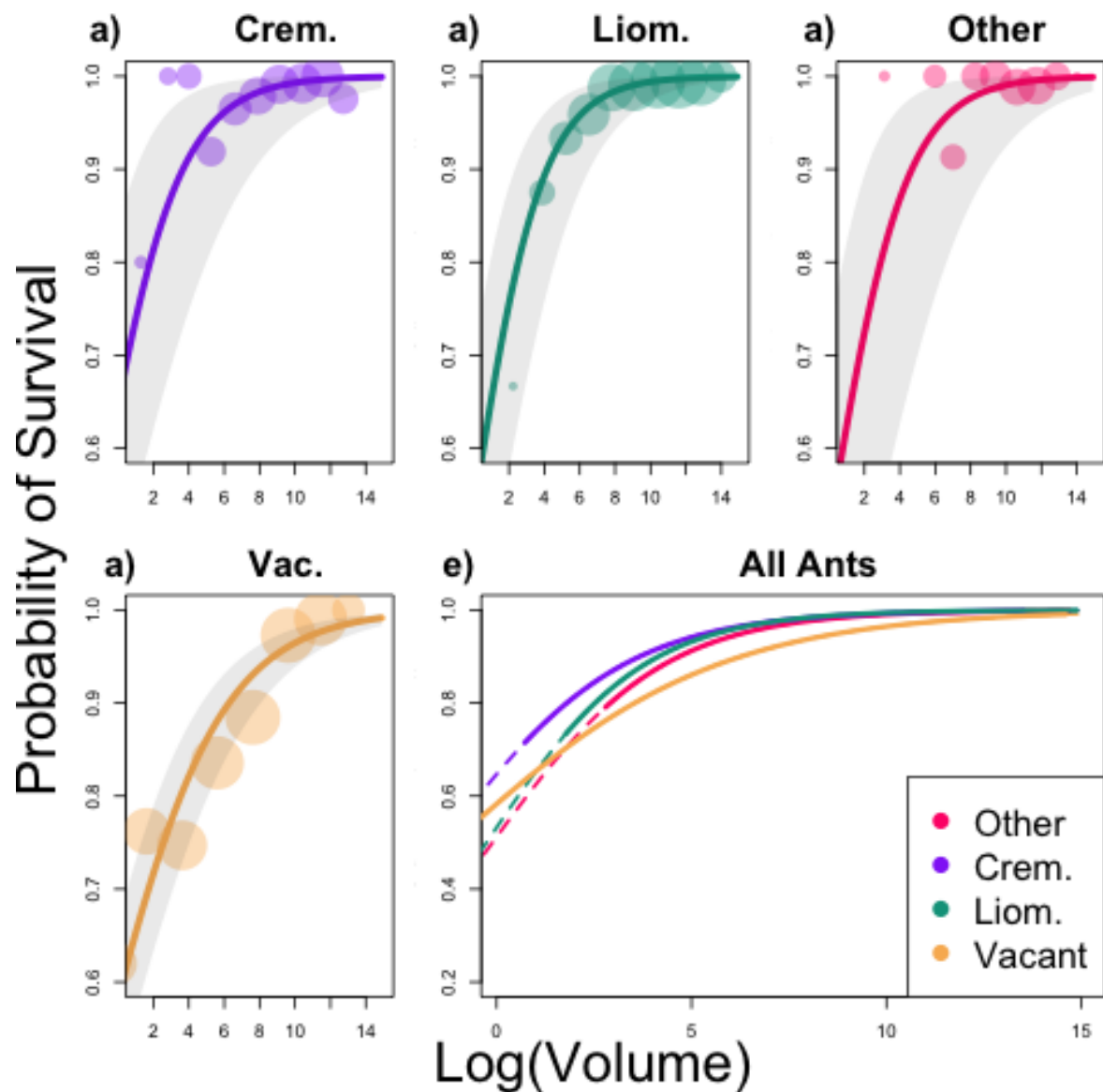


Figure 2: This figure shows the estimated survival rates based on the size of the cactus with each individual ant partner. The solid colored lines (shown on all panels) indicate the mean estimated survival rates. The dashed lines (shown in panel e) indicate extrapolations beyond existing data (where we estimated survival for plants tended by ants where we had never seen a tended cactus of that size). The grey area around the solid lines (shown in panels a-d) show the 90% confidence interval for the estimates. The colored dots are the real data binned by size to show how our estimates align with real survival observations. A larger circle means we had more data on survival of plants of this size with this partner.

We found evidence that ant visitation leads to increased floral viability rates and that ant
411 identity can influence the strength of viability benefits. We observed mean viability rates of flow-
ers between 39% and 96% (Figure ??). Ant partners influence the mean viability rate of flowers,
with *L. apiculatum*-tended plants experiencing the highest mean viability rate (86%, Figure 3b),
414 followed by *C. opuntiae* and Other tended plants (at 74-75%, Figure 3b,c), and vacant plants had
the lowest floral viability rate (71%, Figure 3d). Plants tended by *L. apiculatum* had floral viability
advantages, while plants in states of *C. opuntiae*, Other ants, and vacancy had very similar floral
417 viability rates.

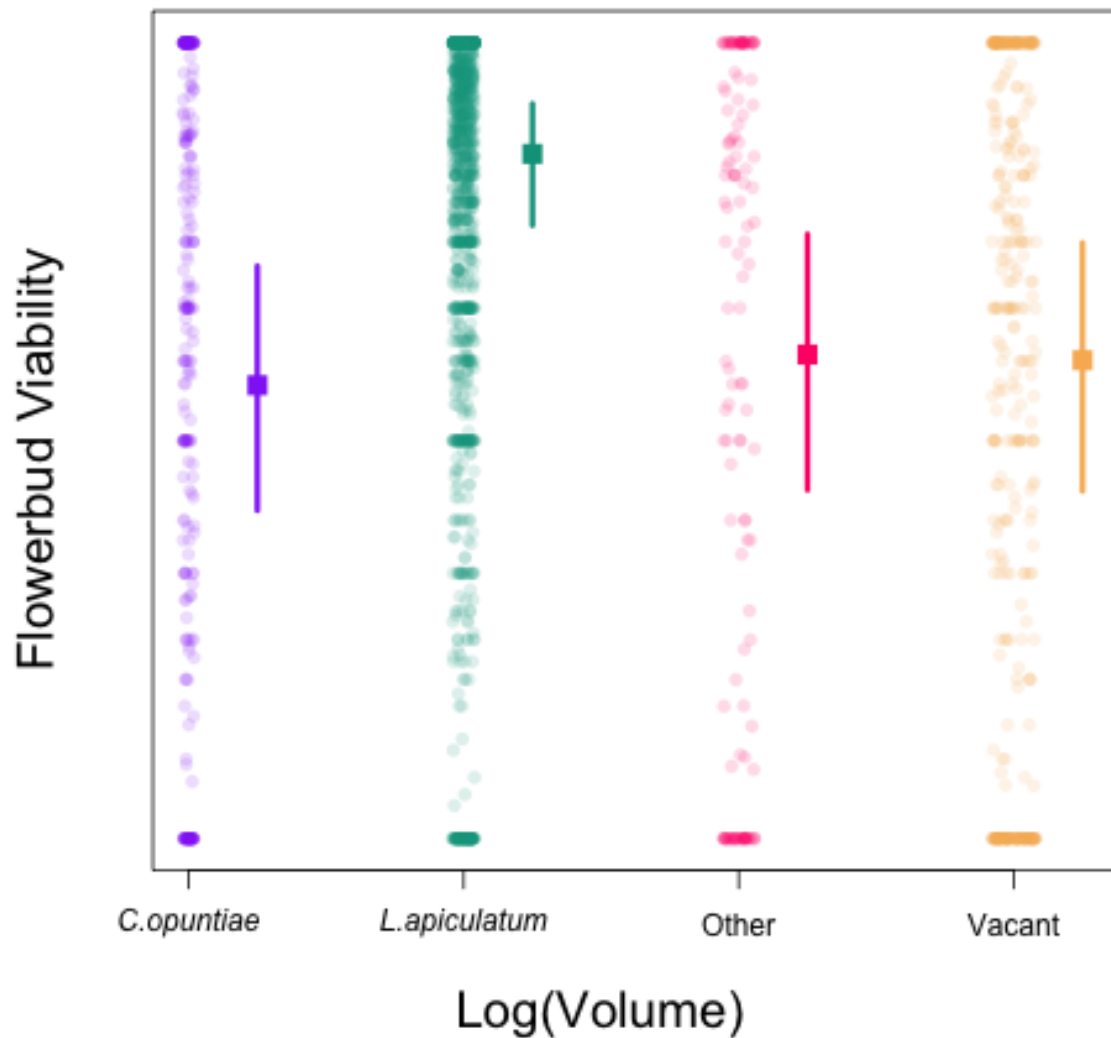


Figure 3: This figure shows the estimated distributions of floral viability rates compared to observed distributions of floral viability rates of cholla based on ant partner identity. The solid lines indicate the estimated viability distribution. The colored histograms represent the observed viability rates of plants with that partner.

We analyzed all other vital rates mentioned in the methods as well. The results of each of these as well as the posterior predictive checks are all included in the Appendix A.

420 We analyzed the correlation coefficients of all models which included ant state as a predictor
and found that the annually varying effects of each ant on survival were the least correlated,
and that the effects on growth were the most correlated. Low correlation indicates synchronicity,
423 which is necessary for portfolio effect to occur because a central thesis of this effect is that partners
must react differently to temporal environmental stochasticity. Across growth, survival, and
viability models, the annually varying random effects of Other ants were the most asynchronous
426 (the least correlated with the effects of other partners), while vacant random effects were the
most synchronous. This variety of synchronicity across ant states and vital rates indicates there
is potential for portfolio effect as many of the ants effects revealed low synchronicity, particularly
429 in the survival model.

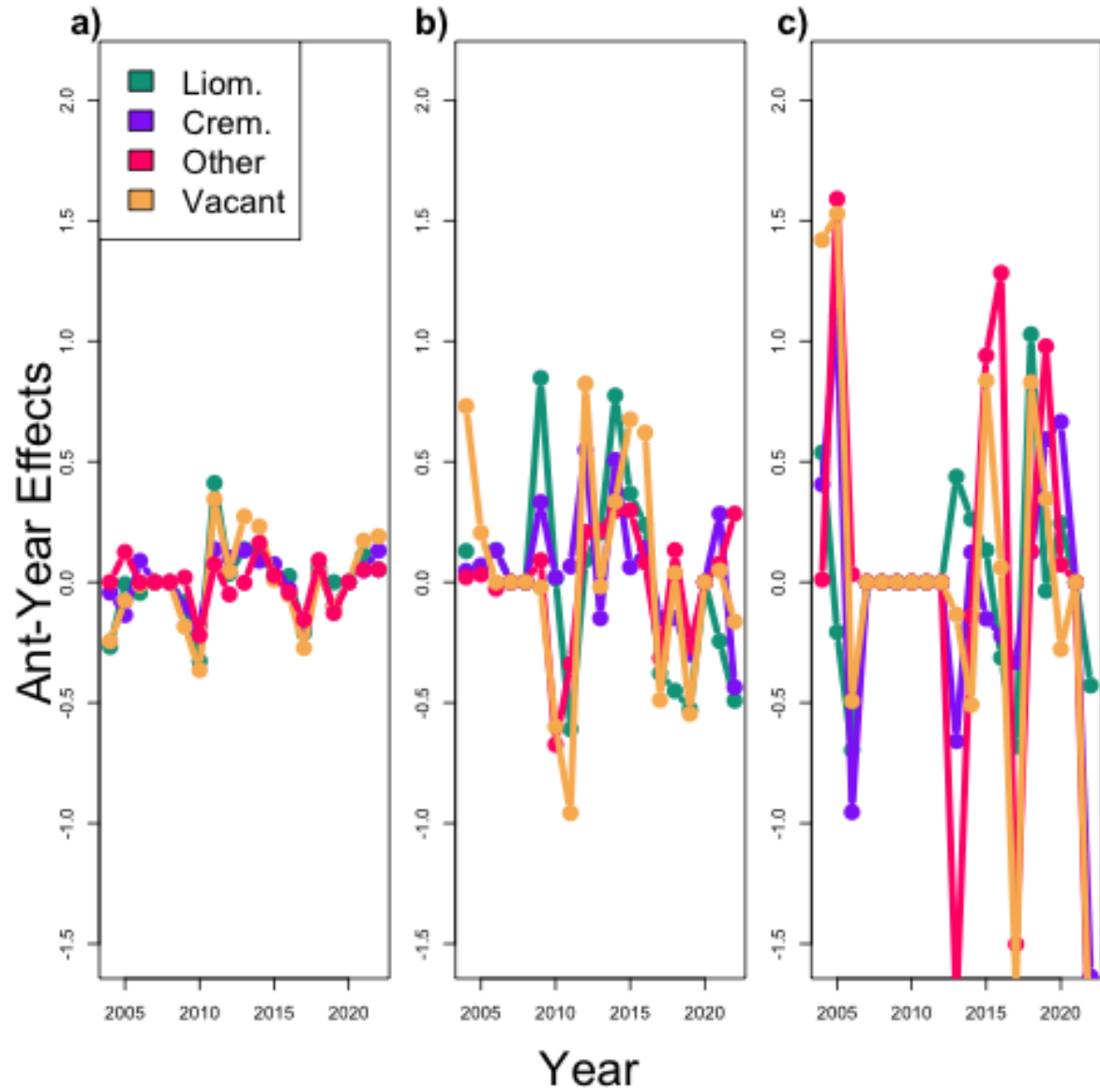


Figure 4: This figure shows the mean affect of each ant partner on a) the estimated next size, b) the estimated survival, and c) the floral viability of cacti across every year of our study. These values are estimated from the fitted random effects of ant and year in our models. Each point represents the mean of the random effect of the identified model, ant, and year (e.g. the lowest dot in panel b) represents the mean effect of vacancy on survival rates in year 2011).

What are the frequency and direction of partner turnover across the plant life cycle?

We found that 40% of plants experienced an ant state transition on average, with very distinct size-dependence and directional patterns (Figure 5). Vacancy is the most likely ant state of small plants ($\leq 10 \log(\text{cm}^3)$). Even when small plants are ant-tended at the start of the transition year, they are most likely to transition back to vacancy (Figure 5b-d). The probability of becoming ant-tended increases with size, though it is not equally likely to be tended by all partners. For large plants that are initially vacant or tended by *L. apiculatum* or Other ants, *L. apiculatum* is the most likely next partner, suggesting that this partner species is able to colonize plants that were previously vacant or occupied by Other ants, and effectively retain plants that it previously occupied. *C. opuntiae* were also able to retain plants they previously occupied, but not as well as *L. apiculatum*: for plants that begin the transition year with *C. opuntiae*, the probability that those plants remain occupied by *C. opuntiae* at the end of the transition year is only slightly greater than the probability of take-over by *L. apiculatum*, while take-over in the other direction is extremely rare. It is also notable that transitions away from the initial state of *L. apiculatum* were almost always transitions to vacancy (Figure 5d), while transitions away from the initial states of *C. opuntiae* and Other were often transitions to other ants. This suggests a competitive hierarchy whereby *L. apiculatum* may abandon low-value plants with little nectar production but is almost never displaced from high-value plants.

¹⁴Honestly not sure if I should include an image for this one or just report some values? We should discuss.

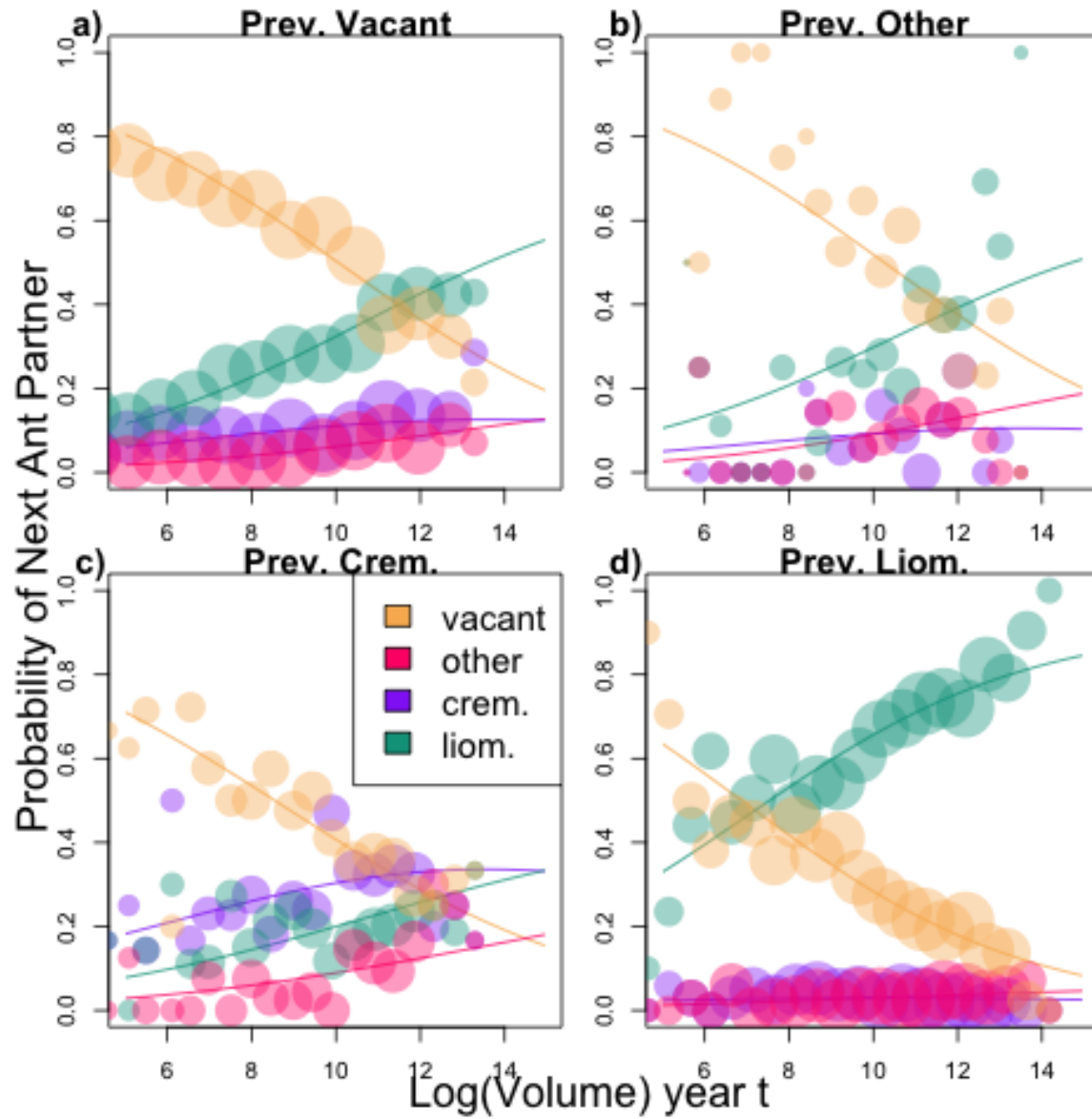


Figure 5: This figure shows the probability of being tended by each ant partner or vacant based on the size of the plant. Each panel shows these probabilities for a different previous ant state. The solid lines represent the mean probability of being tended by a specific partner. The colored points are the real data binned by size to show how our estimates align with real visitation observations. A larger circle means we had more data on visitation of plants of this size with this previous partner.

450 *What is the net effect of partner diversity on plant fitness, and what*
mechanism(s) explain(s) this effect?

By integrating vital rate results and ant transition dynamics into the multi-state IPM we can
453 evaluate the fitness implications of different scenarios of partner diversity and identity. First,
there was strong evidence that ant visitation had mutualistic fitness effects on plant partners.
The lowest mean fitness was $\lambda_{S,Vacant}$, the fitness of the cholla with no partners (Figure 6b).
456 Across all 1+ partner scenarios, we are *****(double check after final runs)82–100% confident that
any scenario of ant visitation elevates fitness. Furthermore, we find an apparent positive effect of
partner diversity, with cactus fitness positive related to partner richness, averaging over partner
459 identities (Figure 6b).

We originally believed the benefits of partner diversity are heavily driven by the presence of
a single best partner rather than overall synergy. However, we found that there was little benefit
462 of any specific partner over another. The identity and number of partners does not appear
to significantly affect the fitness of the cacti. This indicates that while each of these partners are
beneficial, there does not appear to be a significant benefit of partner diversity within this system.

465 After further simulations, we found that this lack of diversity benefits are not driven by
frequency of a single ant. Using the simulations where all ants had equal frequencies across
sizes (further explained and analyzed in Appendix C), we found the same fitness patterns as in
468 the competitive exclusion model described above. Equal probability for transitioning into any ant
state meant that the numbers of *C. opuntiae* and Other ants were boosted significantly. Despite
this, the fitness of scenarios including these less frequent ants were not increased meaningfully.

471 We found no evidence of portfolio effect, meaning the presence of multiple partners did not
buffer against the potentially negative effects of annual fluctuations. The effect of all ant partners
can be measured as $\lambda_{All} - \lambda_{Vacant}$ (Figure 7). We are *****(double check after final runs)94%
474 confident that when all ants are present the cholla experience higher fitness than when no ants
are present according to both the synchronized and non-synchronized model scenarios. When

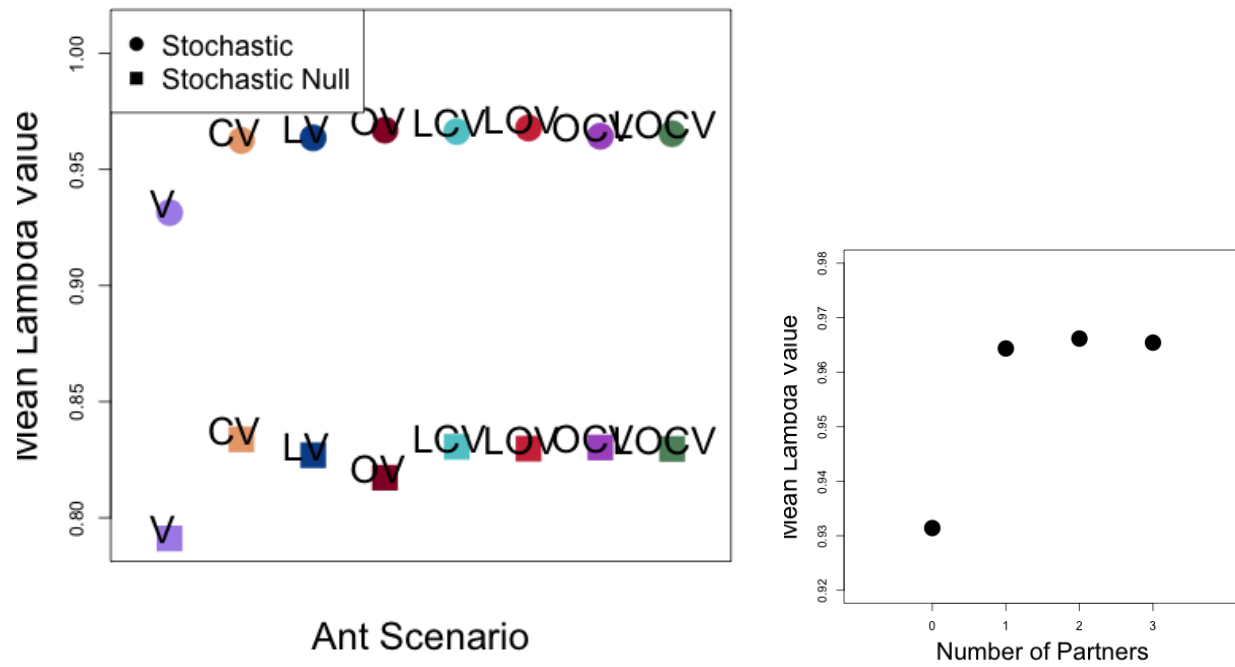


Figure 6: Panel a) shows the mean values of the estimated λ_S (filled in circles) and λ_{SN} (empty circles with an X) for each simulated combination of ant partners. The letters above the points correspond to what ant partners are present (V = Vacant, C = *C. opuntiae*, L = *L. apiculatum*, O = other). Panel b) shows the mean values of the estimated λ_S for

subtracting these two resulting vectors from each other $((\lambda_{S,All} - \lambda_{S,Vacant}) - (\lambda_{SN,All} - \lambda_{SN,Vacant}))$,

we found that we are only *****(double check after final runs) 52% confident that partners offer higher benefits when able to respond uniquely to a fluctuating environment. There is no real difference between the two scenarios, meaning we have no evidence of portfolio effect.

Discussion

¹⁵ Mutualisms commonly involve multiple partners but the ecological consequences of partner diversity remain poorly understood. Here we show that partner identity can play an important role in determining both vital rates and population fitness. The results of our heirarchical models revealed that different ant partners had different effects on vital rates, with *C. opuniae* tended plants experiencing advantages in growth and survival when small, and *L. apiculatum* tended plants experiencing floral viability advantages. The results of our stochastic IPM revealed that all diversity scenarios which included partners resulted in the highest possible fitness for tree cholla, suggesting that partner diversity may not be beneficial or negative in this system. The results of our stochastic null IPM revealed that there is no evidence of portfolio effect in our system. These results highlight that partner presence can increase the overall benefits of a focal mutualism without synergistic benefits and the importance of a mechanistic understanding to explain the benefits of diversity across systems. ¹⁶

Similar studies have reported complementarity [1, 2, 3, 40, 63] while we found that neither complementarity nor sampling effect explains the benefits of diversity in our system. In our first round of results, focused on individual ant effects on vital rates, we found that different ant partners affected different vital rates uniquely (Figures 1,2,3). This indicated to us that we may be dealing with complementarity, and that each ant partner offered a unique form of herbivore protection across ontogeny and or across different processes. However, our IPM results showed

¹⁵*I am struggling reframing this part. I think I may need to start with a new outline, because a lot of what we had doesn't apply to our new conclusions that mutualism is good but we don't see a benefit of partner diversity?*

¹⁶*This may not be great, but I am struggling a bit with this framing*

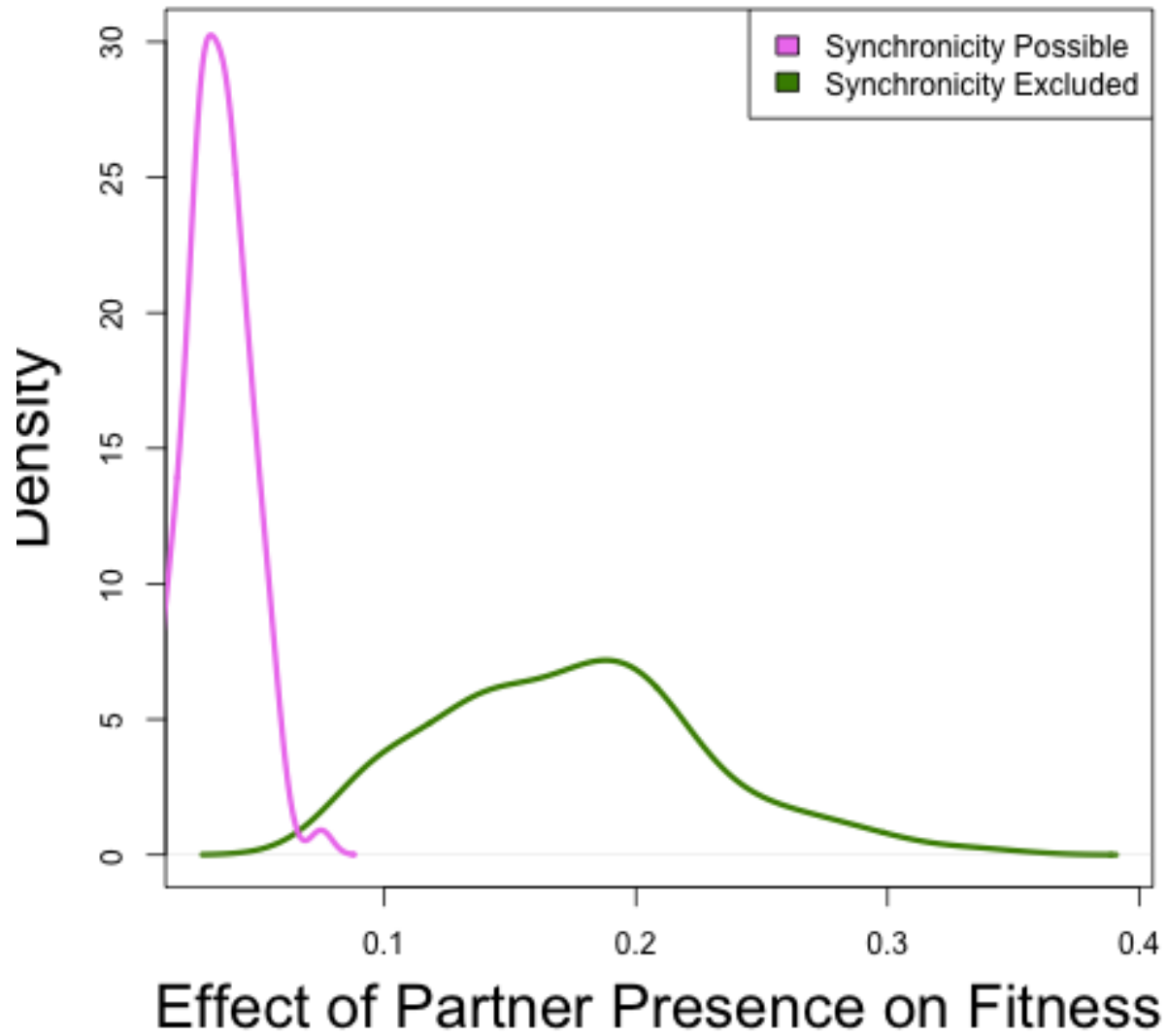


Figure 7: This figure shows the distribution of $\lambda_{S,All} - \lambda_{S,Vacant}$ in pink and $\lambda_{SN,All} - \lambda_{SN,Vacant}$ in green. The vertical dashed line shows where the effect of partners on the fitness of the cholla is 0 (to the left the partners have a negative effect, to the right the partners are beneficial).

that we were actually observing a lack of beneficial diversity, leaving us with questions about how these results could fit together.

Partner turnover is likely a significant driver behind the fitness we see within cholla populations, however the processes which drive the actual turnover frequency and directions remain a mystery in this system. In the literature, it is clear that the frequency of partner turnover can have big effects on the fitness of the focal mutualist [31, 41, 61, 68]. In some systems high frequency of turnover is necessary for resiliency and leads to higher fitness benefits [77], while in other systems loyalty is the most beneficial [13]. We found that some ants had higher frequency of turnover than others, specifically the other category appeared to have high frequency of turnover, rarely returning to the same plant, while *C. opuntiae* and *L. apiculatum* often returned. The direction of partner turnover is also important when the identity of partners impacts the quality of benefits recieved [7, 26, 32, 59]. In our study we found that there are distinct patterns to the direction of partner turnover. Specifically, most plants are tended by *L. apiculatum* ants at some point, because vacant plants, other tended plants, and *L. apiculatum* tended plants are all most likely to be tended by *L. apiculatum* ants. This indicates that while *L. apiculatum* ants are loyal to their own plants and return multiple years in a row to the same ones, they are also strong colonizers.

What now remains is to determine the underlying processes that lead to the emergence of these characterized patterns. Throughout the literature, it is reported that non-symbiotic ants and non-mutualist ants are highly attracted to monosaccharide heavy EFN [38, 43] and that plants often produce monosaccharide EFN to avoid exploitation [39]. It has been previously reported that when reproducing, the nectar of stem segments has a greater disaccharide vs monosaccharide ratio, while flowerbuds have a lower ratio [55]. This could potentially have a significant impact on the partners who tend a plant in a given season, though determining the impacts of nectar on directional turnover requires further study. We conducted alternative simulations where the driving mechanism for ant occupation was not competitive exclusion, these results are included in Apendix XX.

Our work explicitly incorporated temporal environmental stochasticity, which raises the op-

portunity for portfolio effect as a mechanism of diversity benefits. Yet, we find no evidence of portfolio effect within our system. When partners exhibit different reactions to varying environments, interacting with multiple partners can lead to more consistent benefits across time [13]. In our system, we do not directly measure the reactions of ant partners to temporal environmental stochasticity, rather we measure the effects of each partner on the cholla across time, allowing us to statistically quantify how the benefits recieved from each partner vary across time. We found that each ant partner had different effects on different vital rates across time, indicating they exhibit different reactions to varying environments, leading us to believe that portfolio effect could be possible in this system. We compared the fitness of cholla under simulations where the variation in benefits across time reflected what we found in nature to siulations where each ant was forced to react the same way to temporal variation (excluding the possibility of portfolio effect). We found the benefits were not stronger across time when the ants were allowed to vary naturally than when they were forced to react synchronously, indicating that portfolio effect does not explain the benefits of diversity in our system. Other portfolio effect studies [45, 75] found One study [24] found that while portfolio effect was easy to show in theoretical models, it is often very weak or nonexistent in empirical data across many systems. This indicates that it may be very difficult to detect, disguised by different mechanisms, or uncommon in real systems.

This paper shows the importance of long-term datasets in investigating species interactions and calls for further use of long-term data. Previously studies have analyzed how partner identity and partner turnover impact focal mutualist fitness [10, 21, 26, 32, 59, 77]. Separate studies have analyzed how inter-annual variability impacts focal mutualists [6, 7, 42, 45, 62]. The long term dataset we used gave us the unique ability to consider the combined effects of partner identity, partner turnover, and temporal stochasticity.

First, we were able to look at many years of data to reveal the complex dynamics of partner turnover and identity, revealing that the frequency of *L. apiculatum* may be primarily responsible for the fitness of the cholla rather than any other ant partner observed. Fewer years of data would not have been able to show the distinct patterns of *L. apiculatum* taking over cacti that

were previously tended by any other partners with as much clarity. Second, our long-term data set allowed us to test the portfolio effect. This data is required, because we must show that the partners react asynchronously to temporal environmental stochasticity for portfolio effect to even be possible. This is a great example of how long-term data may offer more power to test underlying mechanisms in similar systems. Finally, previous studies by [52] hypothesized that *C. opuntiae* may be a parasite based on the effects on the cholla cacti. Using more years of consecutive data in this system, we were able to show that *C. opuntiae*, while the worst individual partner in terms of the overall cholla fitness (Figure 6), increases the growth and survival rates of cacti prior to reproduction. The length of our data set has allowed us to dive into individual vital rates as well as overall fitness with more clarity.

Herbivory is an important driver of the fitness of cholla in this system, as herbivores directly impact the growth, survival, and reproductive efforts of the cacti. Despite this, we do not directly account for herbivory in our system. Rather, we assume that herbivory can be captured in the effects they have on growth, survival, and reproductive effort observations. In the future, further studies on the direct impacts of herbivory would bolster the results reported here and expand our knowledge within this system. This is particularly true in light of our consideration of portfolio effect. In the same way that different species of ants likely react to temporal environmental variation differently, the same can be said for herbivores [73]. Herbivores have variations in their population reflective of the environment changing temporally, meaning there are "good" and "bad" years for herbivores just like for ant defenders [13], so the strength of their negative effect varies. The fluctuations in herbivore populations in conjunction with ant-specific defenses against herbivores provide a solid foundation to study portfolio effect in this system. We found no evidence of this mechanism (Figure 7), but a more herbivore explicit model in the future may reveal more about driving mechanisms.

As with any study, there are limitations to consider when interpreting the results reported here. These results are based on observational data regarding ant effects on plant demography rather than experimental data, meaning we are able to determine correlations but not causation.

We are able to report evidence that plants experience higher growth when associating with a particular partner, but we are not able to determine if the presence of the ant is the cause of the higher growth observed. It is possible, for example, that plants which attract *L. apiculatum* through osition have a different pattern of resource allocation to growth and survival and this is what we are observing rather than a direct affect of *L. apiculatum* on the plant. This is a typical limitation of observational studies, but does not reduce the power or importance of our results, just the interpretation. However, we believe that the ants likely have direct impacts on the plant demography based on other studies which find the presence of ants can lead to direct increases in survival [14], reproductive efforts [33], growth [35], etc. In the future, it would be interesting to verify the causation of our results through experimental manipulations. One further expansion could include nectar analysis of these cacti in conjuntion with ant interactions to look not only at the ant parnter impacts on the plant demography, but also at the ability of plants to attract specific partners through nectar composition shifts.

Another limitation of our study is that we are simulating populations through an IPM to estimate fitness, meaning we have to extrapolate beyond observed interaction. In our study, small plants are rarely seen with partners, yet we report on the expected effects of each ant on the growth and survival of plants from the size of new recruits to the largest plants. While we are confident in our model, there remains the small possibility of interactions which have never been observed occurring (such as a seedling tended by *L. apiculatum*). This means that our fitness estimations are based on extrapolations of our observed data, and while we have done what we can to be confident in our model, it is possible that our assumptions that observed patterns would continue beyond observed interactions could be incorrect. Another example of this includes our single partner simulations. We have no data on what the fitness of cholla would look like if the only ant partner possible was *C. opuntiae*, yet we offer an estimation based on the effects of this partner in a diverse system. The many years of data and 10467 unique plant-year observations included in our analysis allow us to have relatively high confidence in our outputs despite extrapolations.

Acknowledgments

This should be drafted.

Data and Code Availability

This should be drafted.

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Tables

Figure legends

Appendix A: Additional Methods and Parameters

In addition to the models described in the body of the paper, we fit several other simpler models using data from previous studies. These models are described below.

Seeds Per Fruit. With data[56], we fit a model for the number of seeds produced by every fruit on a cholla ($\kappa(a')$) in year $t + 1$ based on the ant partner a' in year $t + 1$. We fit this model to seed data y^κ using a Negative Binomial distribution and the log link function:

$$y^\kappa \sim \text{NegativeBinomial}(\hat{\kappa}, \hat{\phi})$$

$$\hat{\phi} = \beta_0^\phi$$

The data used for this model did not include data on ants in the “other” category, so we used the data from vacant plants to parameterize seeds per flower for plants with “other” ants in the IPM.

Recruit Size Distribution. We fit this model to recruit size data y^η using a Normal distribution with the identity link function:

$$y^\eta \sim N(\hat{\eta}, \hat{\sigma})$$

where $\hat{\sigma}$ is estimated with a non-informative prior.

Germination. With germination data [52], we fit two models for the probability of germinating from the first year seedbank (γ_1) or the second year seedbank (γ_2) in year $t + 1$, with no fixed or random effects. These models were fit to germination data $y^{\gamma_1}, y^{\gamma_2}$ using the binomial distribution with logit link functions:

$$y^{\gamma_1} \sim \text{Binomial}(\hat{\gamma}_1)$$

Parameter	Median (95% CI)	Prior Distribution
growth xi intercept vacant β_{01}^g	-5.210899(-5.686865, -5.491787)	sDE
growth xi intercept other β_{02}^g	-5.8288(-5.956217, 1.766021)	asdf
growth xi intercept <i>C. opuntiae</i> β_{03}^g	-4.529523(-6.0770390, 0.1222112)	asdf
growth xi intercept <i>L. apiculatum</i> β_{04}^g	-5.106802(-5.4499944, 0.5453901)	asdf
growth xi size dependent vacant β_{11}^g	asdf	asdf
growth xi size dependent other β_{12}^g	asdf	asdf
growth xi size dependent <i>C. opuntiae</i> β_{13}^g	asdf	asdf
growth xi size dependent <i>L. apiculatum</i> β_{14}^g	sadf	asdf
growth omega intercept ω_0^g		
growth omega size dependent ω_1^g		
growth alpha intercept α_0^g		
growth alpha size dependent α_1^g		
1-year germination intercept α^{γ_1}		
2-year germination intercept α^{γ_2}		
survival intercept vacant β_{01}^s		
survival intercept other β_{02}^s		
survival intercept <i>C. opuntiae</i> β_{03}^s		
survival intercept <i>L. apiculatum</i> β_{04}^s		
survival size dependent vacant β_{11}^s		
survival size dependent other β_{12}^s		
survival size dependent <i>C. opuntiae</i> β_{13}^s		
survival size dependent <i>L. apiculatum</i> β_{14}^s		
flowering intercept β_0^f		
flowering size dependent β_1^f		
viability intercept vacant β_{01}^v		
viability intercept other β_{02}^v		
viability intercept <i>C. opuntiae</i> β_{03}^v		
viability intercept <i>L. apiculatum</i> β_{04}^v	45	

Table 1: This table includes the median estimates, the 95% confidence intervals, and the prior distribution for each parameter in each model.

$$y^{\gamma_2} \sim \text{Binomial}(\hat{\gamma}_2)$$

Pre-Census Survival. With recruit census data [56], we fit a model for the probability of a seedling (which germinates in early Fall) surviving to when we census in May (δ) of year $t + 1$ (accounting for missed mortality events), with fixed effects of the previous size x and random effects of the transect m . We fit this model to pre-census survival data y^δ using a Bernoulli distribution with a logit link function:

$$y^\delta \sim \text{Bern}(\hat{\delta})$$

where $m \sim N(0, \sigma_{\text{transect}}^2)$ is the random effect of transect where the recruited individual was analyzed for survival.

Appendix B: Observed Herbivory Data

One type of data we took every year is herbivory data, which involved looking at the plant for evidence of recent herbivory (fresh bites) and noting the type and quantity of herbivores observed.

Appendix C: Alternative Ant Transition Simulations

In addition to the competitive exclusion model defined and analyzed in the body of the paper, we simulated results from several other potential models. We chose to include competitive exclusion as our primary results in the paper because we believe it to be the most biologically realistic. However, in building and testing of alternative models we found that the method of ants occupying plants significantly impacts the fitness of the population. We tested two alternative transition models, one called the frequency based model and one called the equal likelihood model.

Frequency Based Model. The first alternative hypothesis we tested was what we called the frequency based model. In this model rather than the proportion of vacant cacti being maintained,

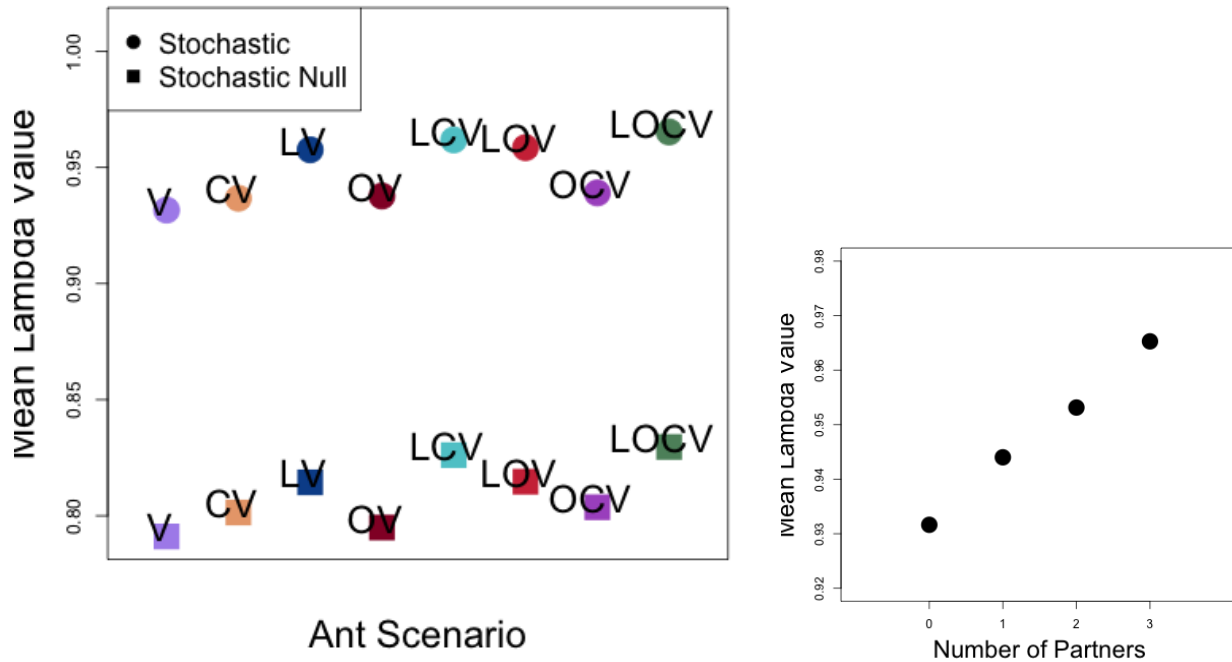


Figure 1: Panel a) shows the mean values of the estimated λ_S (filled in circles) for each simulated combination of ant partners. The letters above the points correspond to what ant partners are present (V = Vacant, C = *C. opuntiae*, L = *L. apiculatum*, O = other). Panel b) shows the mean values of the estimated λ_S for

the proportion of cacti occupied by each species is maintained and when one is removed it is replaced with vacancy. This version of the model assumes that the frequency of each ant we see is reflective of the real frequency of populations rather than some other mechanism. With this model we found very clear evidence of Sampling Effect in the system. When only *C. opuntiae*, Other ants, or both ants are present, there is very little difference in the fitness of the cacti from when no partners are present. Only when *L. apiculatum* ants are present do we see an increase in the fitness of the focal mutualist (Figure 1a). In this simulation, the more partners that are present the higher the fitness of the focal mutualist is, confirming that partner diversity would be beneficial through sampling effect if this transition model were correct. (Figure 1b).

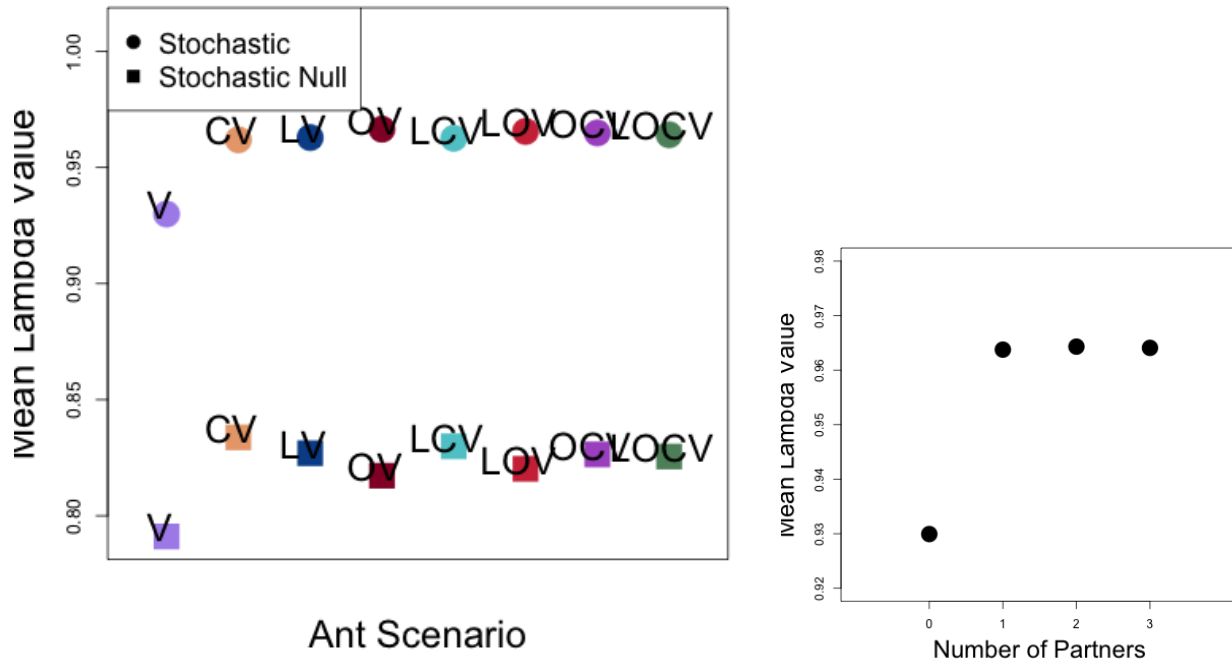


Figure 2: Panel a) shows the mean values of the estimated λ_S (filled in circles) for each simulated combination of ant partners. The letters above the points correspond to what ant partners are present (V = Vacant, C = *C. opuntiae*, L = *L. apiculatum*, O = other). Panel b) shows the mean values of the estimated λ_S for

Equal Likelihood Model. The second alternative hypothesis we tested was what we called the equal likelihood model. In this model we preserved the observed pattern of size-dependent vacancy/occupancy, but occupancy was manipulated to be equally likely for all partner identities. This was designed to remove the effect overwhelming numbers of *L. apiculatum* ants may have. Despite very different proportions, we found very similar outcomes to the competitive exclusion model analyzed in the paper. All ants are beneficial, but having more than one is not necessarily any better than having an individual species as a partner (Figure 2a). Partner presence is beneficial, but neither identity nor number of partners appears to be important (Figure 2b).

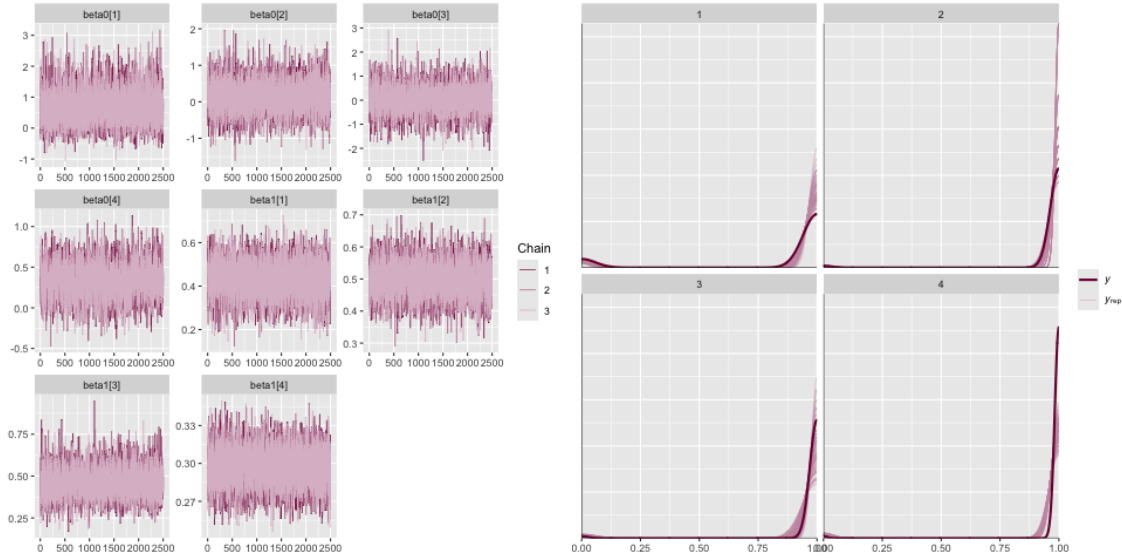


Figure 3: The a) posterior convergence of the parameters estimated by the survival model and the b) posterior distribution of survival estimates (pink lines) for each ant species (1 = *C. opuntiae*, 2 = *L. apiculatum*, 3 = other, 4 = vacant) compared to the mean survival distribution (black line) of the real data.

Appendix D: Posterior Checks and Model Validation

For each model fitted, we conducted two tests to determine if the fit was acceptable to use in our IPM. First, we checked the convergence of each parameter. Below we show the convergence of all β terms listed in the Statistical Modeling subsection of Methods. Second, we checked the posterior fit, comparing the estimated values of each model to the y values of the actual data. We show these posterior checks below, split by ant partner where relevant.

Statistical Models – Results

Below are the results reported of all statistical models not described in the main body of the text.

Reproduction Model. The probability of a plant reproducing in a given year is highly size dependent. The mean probability of reproducing remains at about 0% until the plant reaches a

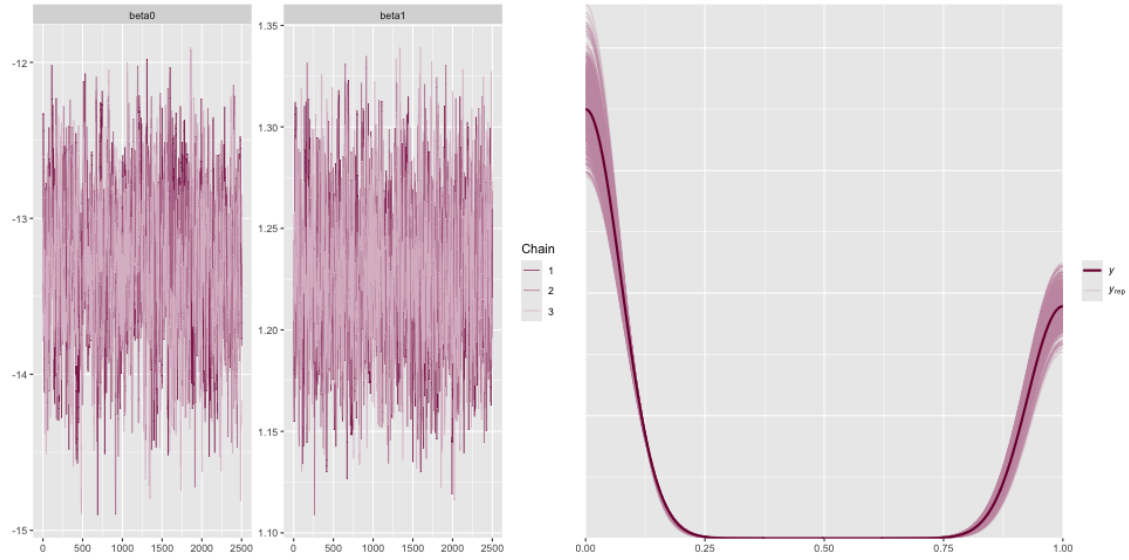


Figure 4: The a) posterior convergence of the parameters estimated by the reproduction model and the b) posterior distribution of reproductive status estimates (pink lines) for each ant species (1 = *C. opuntiae*, 2 = *L. apiculatum*, 3 = other, 4 = vacant) compared to the mean reproductive status distribution (black line) of the real data.

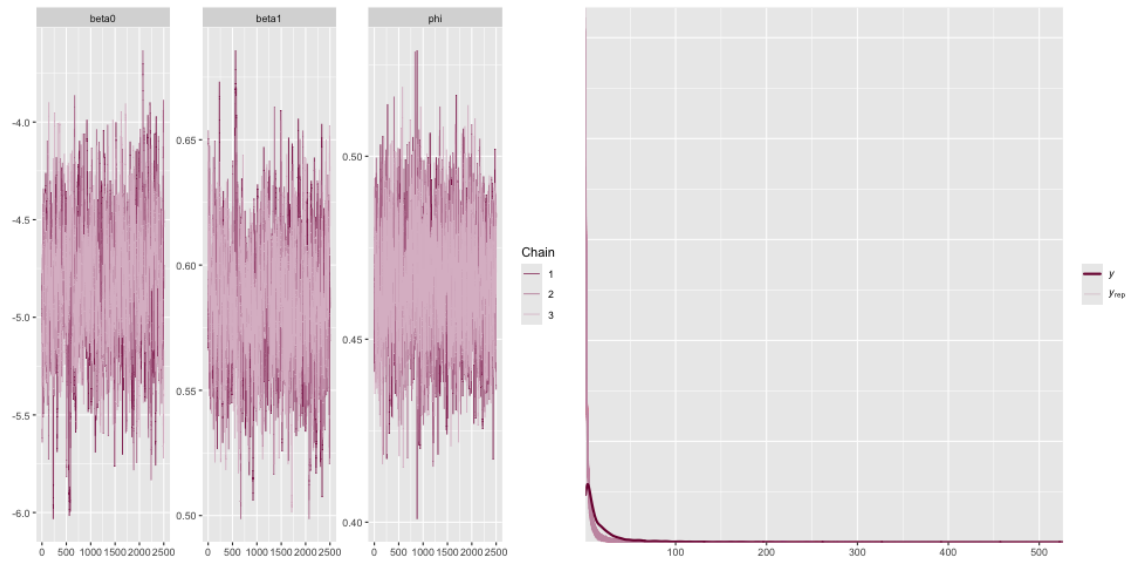


Figure 5: The a) posterior convergence of the parameters estimated by the number of flowers model and the b) posterior distribution of the number of flowers estimated (pink lines) compared to the mean distribution of observed flowers (black line).

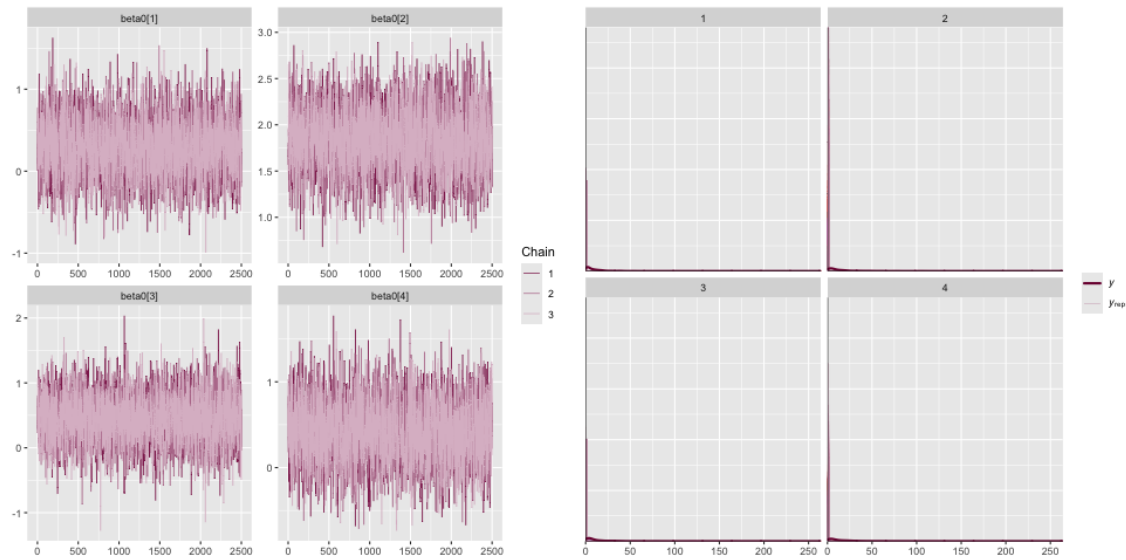


Figure 6: The a) posterior convergence of the parameters estimated by the viability model and the b) posterior distributions of floral viability estimates (pink lines) for each ant species (1 = *C. opuntiae*, 2 = *L. apiculatum*, 3 = other, 4 = vacant) compared to the mean floral viability distribution (black line) of the real data.

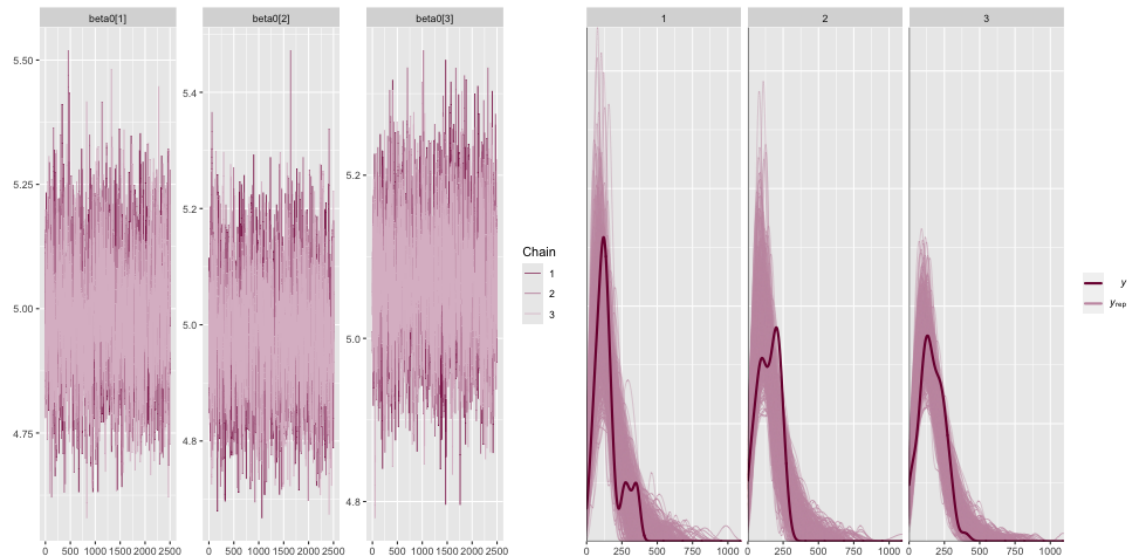


Figure 7: The a) posterior convergence of the parameters estimated by the seeds per fruit model and the b) posterior distributions of seeds per fruit estimates (pink lines) for each ant species (1 = *C. opuntiae*, 2 = *L. apiculatum*, 3 = vacant) compared to the mean seeds per fruit distribution (black line) of the real data.

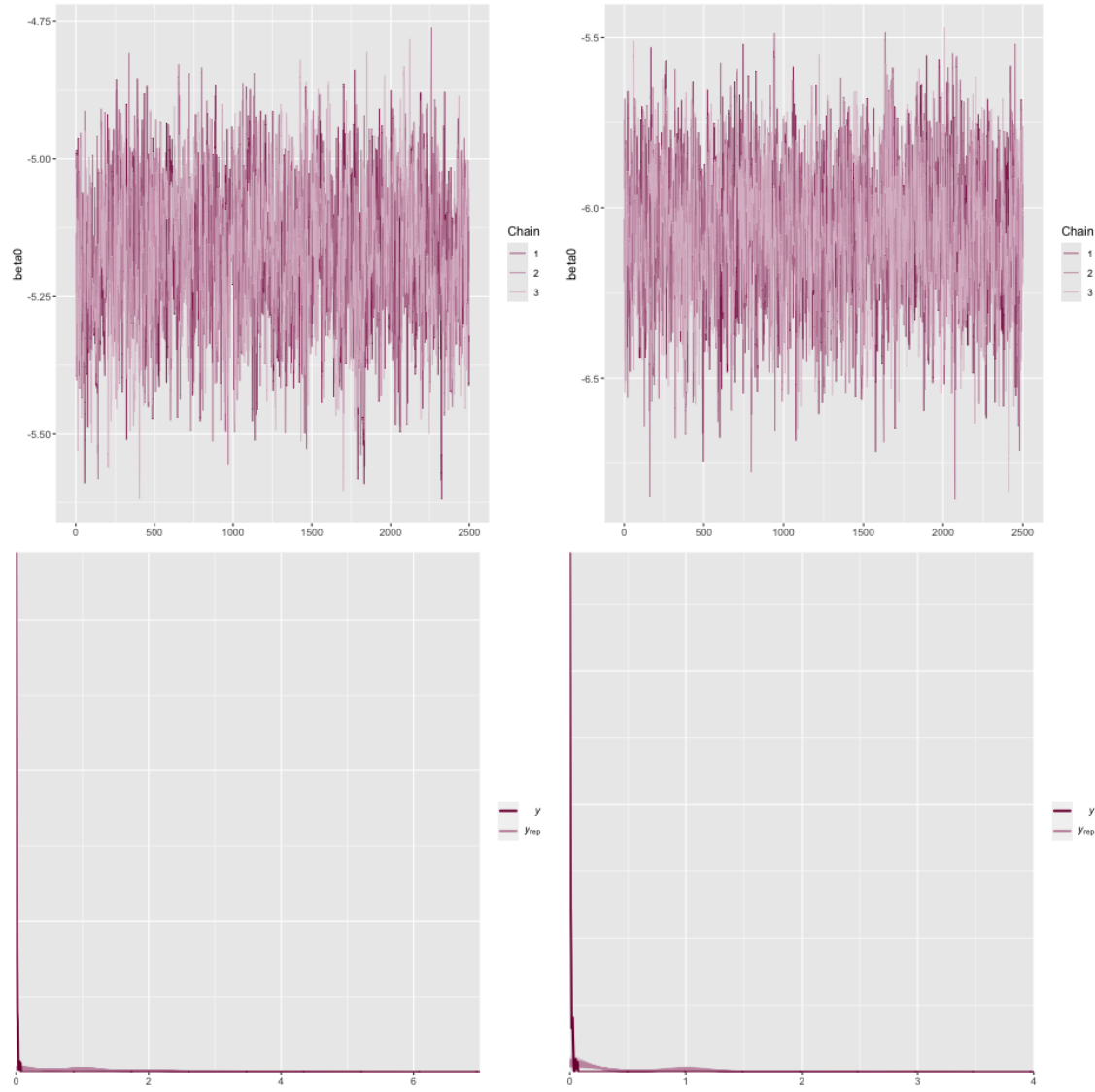


Figure 8: The a-b) posterior convergence of the parameters estimated by the germination from year one seedbank and germination from year two seedbank models respectively. The c-d) posterior distributions of floral viability estimates (pink lines) compared to the mean germination distribution (black line) of the real data for first year germinants and second year germinants respectively.

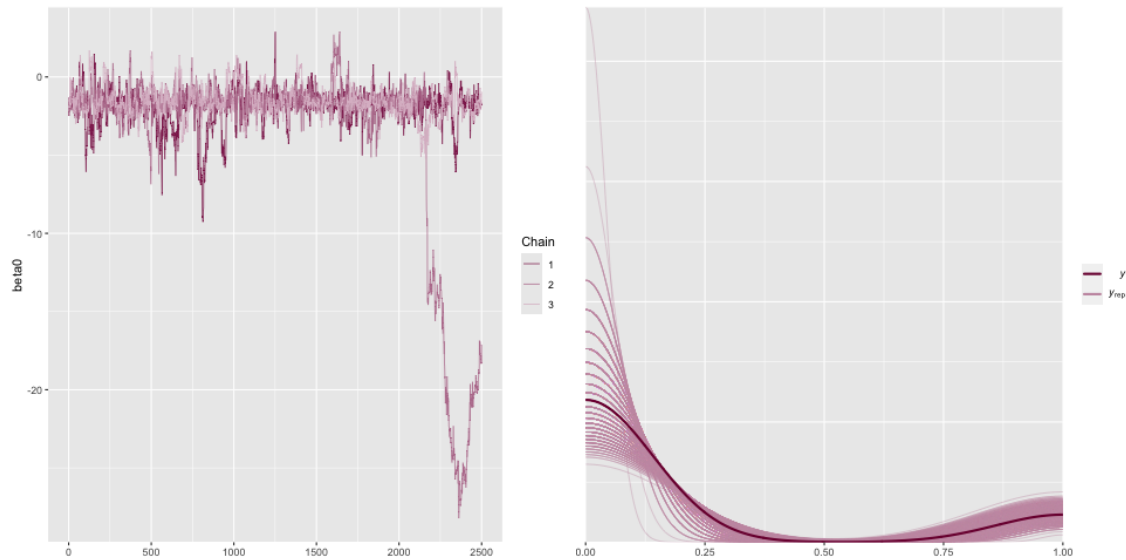


Figure 9: The a) posterior convergence of the parameters estimated by the pre-census survival model and the b) posterior distribution of the pre-census survival estimated (pink lines) compared to the mean distribution of observed pre-census survival (black line).

medium size, after which the mean probability of reproducing increases steadily before reaching about 100% at large sizes.

Seeds Per Flower Model. Each viable flower on a plant produces between 97 and 257 seeds. This number is affected by the ant partner present. *C. opuntiae* tended plants produce a mean of 148 seeds per flower. *L. apiculatum* tended plants produce a mean of 149 seeds per flower.¹⁷ Vacant plants produce a mean of 160 seeds per flower. We are 73% and 78% confident that vacant plants produce more seeds per flower on average than plants tended by *C. opuntiae* and *L. apiculatum* ants respectively.

Precensus Survival Model. Pre-census seed survival rates fall between 0% and 95% with the mean pre-census seed survival at 18%.

¹⁷These results are not consistent with Ohm and Miller, where *Crem* had lower seeds than *Liom*. I would check this. This section should also reference that paper because these are not new results.

861 *Germination Model.* Seeds have a significantly higher probability of germinating in year one than
in year two. Seeds in year one experience germination rates between 50% and 100% with a mean
of 62% germination. Seeds in year two experience germination rates between 50% and 98% with
864 a mean of 58% germination.

New recruits are expected to be between the sizes of 0.11 cm^3 and 0.38 cm^3 with a mean size
of 0.20 cm^3 .¹⁸

¹⁸*Move to an appendix. These results are not relevant for the questions at hand.*