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Article *in* International Research Journal of Finance and Economics · May 2013

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International Research Journal of Finance and Economics
ISSN 1450-2887 Issue 109 May, 2013
http://www.internationalresearchjournaloffinanceandeconomics.com

Portfolio Risk Management with Value at Risk: A Monte-Carlo Simulation on ISE-100

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Abstract

Value at Risk (VaR) is a common statistical method that has been used recently to measure market risk. In other word, it is a risk measure which can predict the maximum loss over the portfolio at a certain level of confidence. Value at risk, in general, is used by the banks during the calculation process to determine the minimum capital amount against market risks. Furthermore, it can also be exploited to calculate the maximum loss at investment portfolios designated for stock markets. The purpose of this study is to compare the VaR and Markowitz efficient frontier approach in terms of portfolio risks. Along with this angle, we have calculated the optimal portfolio by Portfolio Optimization method based on average variance calculated from the daily closing prices of the ninety-one stocks traded under the Ulusal-100 index of the Istanbul Stock Exchange in 2011. Then, for each of designated portfolios, Monte-Carlo Simulation Method was run for thousand times to calculate the VaR. Finally, we concluded that there is a parallel relationship between the calculated optimum portfolio risks and VaR values of the portfolios.

Keywords: Value at risk, Monte-Carlo Simulation, Portfolio Optimization
JEL Classification Codes: G11, G32, C15.

1. Introduction

In our contemporary world, financial markets have attracted many investors and researchers with its numerous dimensions. Main purposes of these analyses can be either about modeling a financial market and to estimate its behavior, or about generating a solution for the problem caused by unpredictable future. Along with these purposes, there have been many model and theories within the finance and economy literature in the past. Since the incoherency within the human nature has always caused a margin of error in developed theories, this fact prevents that these theories to become de facto mechanisms working in the economy. Hence, this causes investors to face several risks based on uncertainty arise during decision making to allocate their investments.

In early 20th century, modeling of financial markets was studied by many economists. In his study conducted in a period in which conventional economic theories were used to be recognized, Markowitz (1952) proved that the concept of risk is related not only with diversification, but also with direction and degree of the relationship between securities in the market. His theory called Portfolio Optimization based on average variance is being perceived as a foundation of the modern theory and shed light for many successor researchers.

In recent years, another instrument developed to be a guide for investors and researchers as a risk measure was Value at Risk (VaR) approach. VaR, basically, represents the statistical summary of the loss of a portfolio in the market (Linsmeier and Pearson, 2000). The method measuring amount of loss within a certain confidence interval is referred as “portfolio loss distribution” which is calculated on a sampling (Artzner et al., 1998; Bozkus 2005). Calculated VaR value presents the possible amount of maximum loss at previously determined risk level at next day, month or year. In general, it is being used in calculations to determine the amount the capital which should be hold by banks against market risks. Additionally, there have been many studies about its application in financial markets.

The purpose of the study is to compare the VaR and Markowitz efficient frontier approach with regards to portfolio risks. In case the investors evaluate the VaR and optimum rate of portfolio risk together, it is investigated that whether the uncertainty in their investments decrease or not. In our study, we use data composed of annual closing prices of 91 stocks traded in Istanbul Stock Exchange (ISE) included in Ulusal-100 index in 2001. By comparing the risk rates and VaR values of 10 portfolios designated by Portfolio Optimization method based on the average variance, it is elaborated that whether investors and researchers will be in the search of lowering their uncertainty about the future by using both methods in parallel with each other.

2. Literature Search

There are substantial studies in the literature about both subjects which consists of the foundation of this study: Portfolio Optimization based on Markowitz average variance and the concept of VaR.

Markowitz (1952) exhibited that it is not possible to reduce the risk just by following portfolio diversification, and that the direction and degree of the relationship among the securities included in a portfolio have also significant importance in reduction of the overall risk by means of his “Average - Variance Model”. Markowitz guided his many successors. In his study, Sharpe (1964) referred the Markowitz’s work and carried his theory one step away by introducing the “Financial Markets Pricing Model (CAPM)” which means that he accomplished modeling a chaotic environment. Together with his model, Sharpe separated systematical and non-systematical risk occurred in financial markets from each other; and attributed the market behaviors to an econometric model. Roll and Ross (1984) used Sharpe’s study as ground and went one step further away in security pricing model. Roll and Ross (1984) showed that markets can also be affected by several macro-economic variables; and risk concept can be configured based on these macro-economic variables. They built an econometric multi-variable model that explains financial markets by means of “Arbitrage Pricing Model”. There have been numerous studies to reduce risk and uncertainty at financial markets. In general, these studies have taken studies of Markowitz (1952), Sharpe (1964) and Roll and Ross (1984) as a foundation.

VaR was built as a method to determine minimum amount of capital that should be hold by banks against market risk. However, there are other studies to use it as a risk measure on financial markets. In their studies, Duffie and Pan (1997) were focused on risk variables of a portfolio composed of stocks and bonds. In this study, with the 99% probability measure, VaR values were calculated by daily and bi-weekly scenarios; and parallel results were achieved by the volatility of the portfolio. Artzner et al. (1998) focused on both systematic and non-systematic risks concepts in their studies. In the study in which there were many risk measures were employed, it was also shown that VaR values can be used as sort of measure for risk concept. Linsmeier and Pearson (2000) revealed that the VaR value is an alternative risk measure through their studies in which they provide alternative methods to calculate VaR. Basak and Sharpiro (2001) developed their own model by using VaR value as a risk measure of optimum portfolios. In their study, they proved that the developed model made up the deficiencies of VaR approach and even provided more effective results empirically. Mandaci (2003) emphasized risk measuring techniques of banks when they faced financial crisis in his study. It was stated that in the VaR approach, employing more than one calculation method in the risk measuring leads more objective results. Akkaya et al. (2008) used VaR values and stress tests in the evaluation of market risk. As a result of their study, VaR and stress test used in calculation of market risk were presented as methods that can be used jointly. Tas and Iltuzer (2008) calculated VaR value by using Monte-Carlo Simulation method on a portfolio consisting stocks listed in ISE-30 index. Consequently, they exhibit that VaR value can be used as a mean to measure portfolio risk.

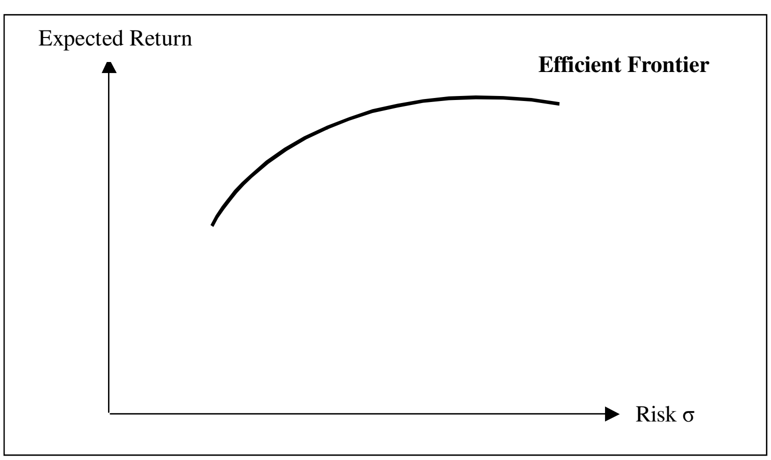
On the other hand, there are also studies in the literature presenting dissident point of view about VaR. These studies have usually focused on the issue that VaR values mislead investors especially during financial crisis periods. Therefore, they aim to provide alternative approaches to VaR. Bozkus (2005) defines the expected loss method as a consistent risk measuring mean and an appropriate alternative for the conditions in which VaR approach present deviation. In his study where he used daily the US. dollar, Euro and ISE-100 index data, he made comparison by emphasizing strengths and weaknesses of the method. In their studies, Gianopoulos and Tunaru (2005) include issues caused by failure in fulfilling majority of the assumptions of VaR. They stress their model to dismiss these issues. Zmeskal (2004) shows the derivation and application possibilities of select hedging strategies. The author uses five basic hedging strategies; delta hedging, minimum variance, minimum value at risk, maximum expected utility value, and minimum shortfall. All the strategies are derived for two asset portfolios consisting of risk assets and hedged assets. The author suggests that several applications are suitable for small open economies that lack liquid capital market with limited secondary derivative market.

3. Methodology

a. Portfolio Optimization based on Average Variance

If we mention portfolio in terms of securities, it is referred as a pool portfolio which consists at least two securities to lower risk and to increase highest possible return based on the desired risk (Ercan, 2010: 189). Investors aim to allocate their funds among current securities to attain the highest possible return at certain level of risk; or to expose lowest possible level of risk at the same level of return (Atan, 2005). Portfolio Optimization is acquired by creating most suitable security mixture under the framework of expected return, risk and investment constraints. Markowitz state in his theory that created optimum portfolios are placed on an efficient frontier due to expected return and risk perception. All portfolios on the efficient frontier curve are considered as the portfolios with highest possible return at a certain level of risk (Figure-1).

Figure 1: Efficient Frontier of Portfolio Optimization based on Average Variance



Portfolio Optimization model seeks to find a portfolio with minimum variance (minimum risk) at a targeted expected return level.

Model:

$$\min \sum_{i=1}^{n} \sum_{i=1}^{n} x_i \quad x_j \quad \sigma_{ij} \tag{1}$$

Constraints:

$$\sum_{i=1}^{n} x_j \quad \mu_i \quad \geq \quad R \tag{2}$$

$$\sum_{i=1}^{n} x_i \quad = \quad 1 \tag{3}$$

$$0 \quad \leq \quad x_i \leq \quad 1, \qquad i = 1, 2, \ldots, \quad n \tag{4}$$

Where,

n: Number of security,

μ_i: Expected return on ith security (i=1,2,…,n)

σ_{ij}: Covariance values between ith and jth securities (i=1,2,…,n), (j=1,2,…,n), (for i = j, variance value of the ith security).

R: Target expected return rate

x_i: Weight of the ith security in the portfolio (i=1,2,…,n)

As a result of optimization, for the portfolios that are expected to yield highest return at a determined risk level, weights of securities, their expected returns and expected risk levels are calculated. According to their risk perceptions, investors (risk taker or risk aversive) can make investments into those designated portfolios.

b. Value at Risk (VaR)

Risk can be defined as a measure of uncertainty caused by the future of an investment (Gitman and Zutter, 2010: 310). Risk is grouped into two groups as systematic and non-systematic. While systematic risks are considered as risk that are affecting all securities in an economy and that can not be dismissed by investment diversification; non-systematic risks are considered as risks that is caused by the security's itself and that can be adjusted by diversification of investment. On the other hand, financial risk is measure of possible changes in portfolios in a period now an in the future. In other words, financial risk is that individual and corporate entities' change in their asset and liability position against price variations in the market (Usta, 2005: 234).

VaR is a statistical concept referring the highest loss that may arise as a result of a variation in interest rate of security, in exchange rate and in stock prices relevant with a portfolio or an asset by taking a confidence interval and a period into account (Mandacı 2003). This is most common methodology employed while calculating market risk (Tas and Iltuzer, 2008). It is possible to present VaR in terms of model shown in the basic Equation 5 as follows:

$$VaR = M * a * \sigma * \sqrt{T} \tag{5}$$

Where,

M : Market value of the Portfolio

α : Confidence level (*α=0.01, 0.05, 0.10*)

σ : Portfolio standard deviation

T : Duration

We can summarize what VaR means with simple expression as "Tomorrow, we can be sure by X% probability that we will not lose more than N Turkish Lira". There have been several methods developed to calculate VaR. Each method owns their distinct advantage and disadvantages within. VaR calculation methods are below:

- Delta-Normal (Variance-Covariance) method
- Historical data method
- Monte Carlo Simulation Method

Within the nature of the Delta-Normal method, which is also called Parametric method, there is certain distribution assumption. Financial corporations usually apply this method due to this character; and they assume in their calculations that returns present normal distribution. Based on the normal distribution assumption, it is possible to calculate the VaR value of a portfolio return as a linear function of standard deviations of asset returns (Bozkus, 2005). This method is presented as most advantageous model in terms of calculation convenience and time. However, because of the fact that most of the financial series has comatic aberration rather than normal distribution, this may cause to calculate VaR value less than its actual level (Bolgun and Akcay, 2005). In the historical data method, an empirical distribution is created by using historical data; and according to this, VaR is estimated. This is a non-parametric method. Since this method is based on historical data, it takes variations in the sampling into account, but it ignores some risks because it does not take different possible variations that may be experienced in the future into account (Zenti, Pallotta, 2001). Monte Carlo method is another non-parametric VaR method. On the scenarios side which is created for calculations, they are created randomly from a certain distribution in Monte Carlo method. This method is known as the one that bears the most comprehensive and the highest risk. Furthermore, it requires longest time and it is hardest method (Tas and Iltuzer, 2008).

4. Analysis

The purpose of this study is to compare VaR and Markowitz efficient frontier approaches in terms of portfolio risks. To that end, the annual data acquired from daily closing prices of 91 stocks listed in ISE 100 index in 2011 were used. Stocks used in the study were listed in Table-1.

Table 1: Stocks listed in ISE 100 in 2011

ADNAC	BAGFS	ENKAI	IHLAS	MGROS	TEKST
AEFES	BANVT	EREGL	IPEKE	MNDRS	TEKTU
AFYON	BIMAS	FENER	ISCTR	MUTLU	THYAO
AKBNK	BJKAS	FROTO	ISFIN	NETAS	TIRE
AKENR	BOYNR	GARAN	ISYHO	NTHOL	TKFEN
AKFEN	BRISA	GLYHO	ITTFH	NTTUR	TOASO
AKGRT	BRSAN	GOLDS	IZMDC	OTKAR	TRCAS
AKSA	DEVA	GOLTS	KARSN	PETKM	TRKCM
AKSEN	DOAS	GOODY	KARTN	PRKME	TSKB
ALARK	DOCO	GSDHO	KCHOL	SAHOL	TSPOR
ANSGR	DOHOL	GSRAY	KONYA	SASA	TTKOM
ARCLK	DYHOL	GUBRF	KOZAA	SISE	TTRAK
ASELS	ECILC	HALKB	KOZAL	SKBNK	TUPRS
ASYAB	ECZYT	HURGZ	KRDMD	TAVHL	ULKER
AYGAZ	EGGUB	IHEVA	METRO	TCELL	VAKBN
					YKBNK

The basic purpose in the analysis stage is to create 10 optimum portfolios with the current stocks; and to calculate the expected risk rates and VaRs that belong to the portfolios. Along with this purpose, the analysis will be composed of several stages below:

By means of portfolio optimization based on average variance, determine portfolios whose VaRs are going to be calculated,

- Calculate expected risk levels of the created optimal portfolios,
- Obtain data about risk factors and last year's annual daily stock prices,
- Create variance/covariance matrix,

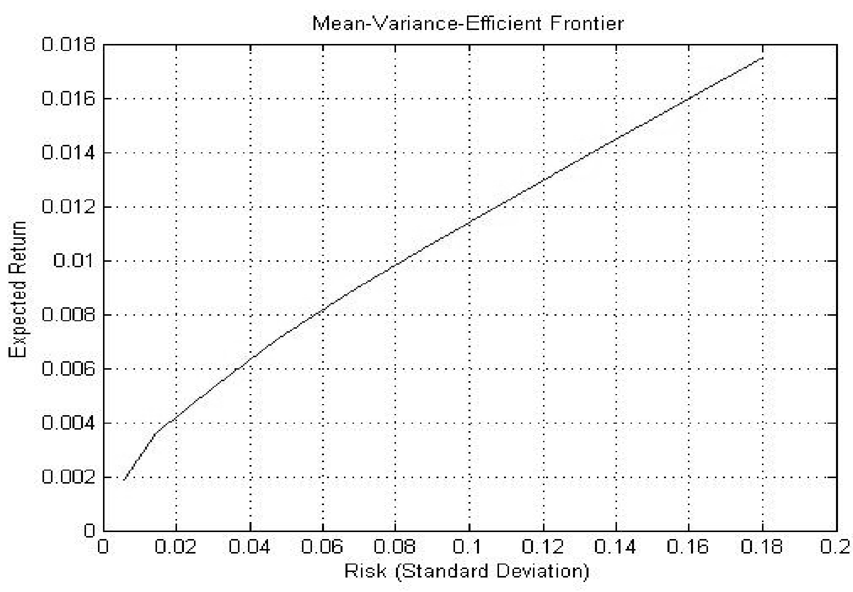
- Generate random numbers as much as determined number of simulation,
- Calculate Cholesky Decomposition matrix from the covariance matrix,
- Multiply generated random number matrix and Cholesky matrix,
- Calculate portfolio returns according to security weights,
- At the selected confidence level, calculate Monte-Carlo VaRs.

First of all, Portfolio Optimization calculations based on Markowitz's average variance and 91 stocks listed in ISE Ulusal-100 Index in 2011 were completed. At the end, there are 10 different portfolios with separate risk levels at the efficient frontier curve. Expected return and risk levels of the created portfolios were exhibited on Table-2; and the efficient frontier chart was presented on Figure-2. Returns and risk levels of portfolios increase from the first one to the tenth. The important thing here is that portfolio risks are presented in terms of rate. For instance, risk level of Port-8 can be presented as 13.41%.

Table 2: Portfolios created by portfolio optimization based on average variance.

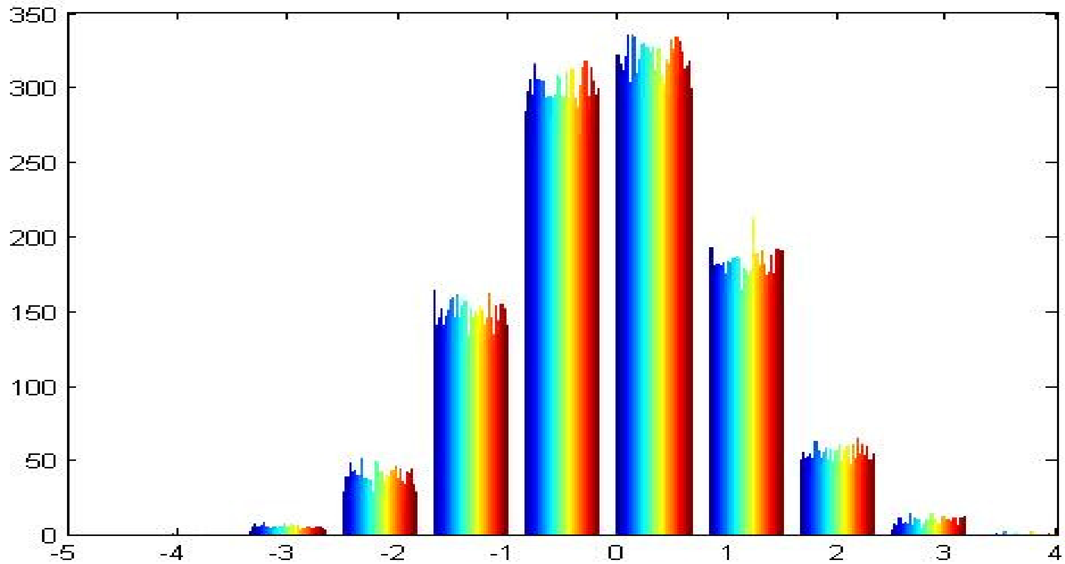
	Risk(σ)	**Return (E)**
Portfolio-1	0.0055	0.0019
Portfolio -2	0.0145	0.0036
Portfolio -3	0.0306	0.0053
Portfolio -4	0.0476	0.0071
Portfolio -5	0.0674	0.0088
Portfolio -6	0.0890	0.0106
Portfolio -7	0.1113	0.0123
Portfolio -8	0.1341	0.0140
Portfolio -9	0.1570	0.0158
Portfolio -10	0.1801	0.0175

Figure 2: Efficient Frontier plotted as a result of portfolio optimization



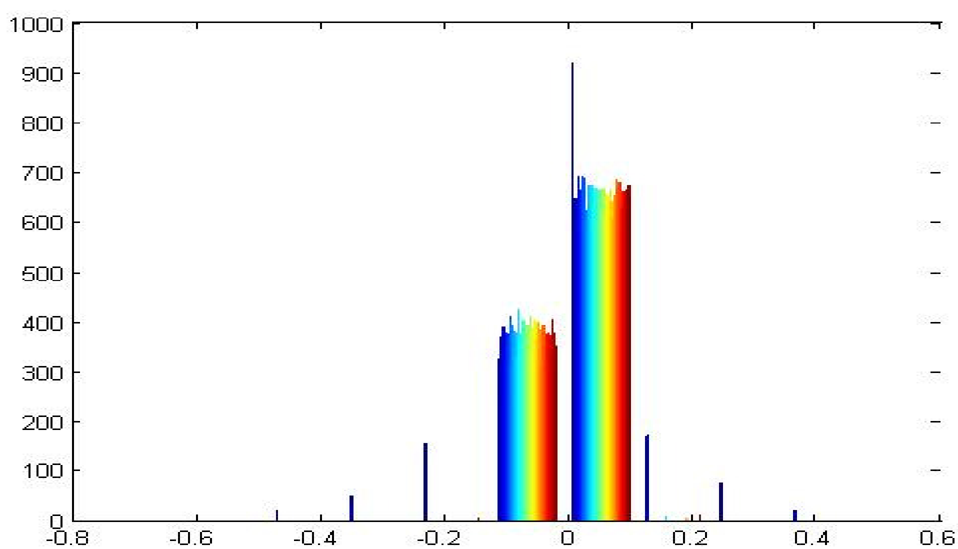
During the stage of calculation of VaR values, Monte Carlo Simulation method is used. For the simulation, confidence level is determined as 95% ($\alpha = 0.05$). In the following stage, the annual data about the returns of the stocks from daily closing prices included in the portfolios were collected for 2011; and a 252 X 91 "return matrix" was acquired. By using acquired return matrix, a 91 X 91 variance-covariance matrix was calculated, which belongs to the variables. In the stage of Monte Carlo simulation application, an integer number set that presents normal distribution was created. Number of simulation for each portfolio was set as 1000 run. Then, 10 separate 1000 X 91 random number matrixes were generated. The distribution of generated random numbers was presented on Figure-3.

Figure 3: Monte Carlo simulation random number matrix scatter chart



In the next stage, Cholesky decomposition matrix is required to be created. Cholesky decomposition matrix simply means that dismissing the non-symmetrical part of the variance-covariance matrix. The purpose of this operation is to make sure that variance-covariance matrix which will be multiplied by the random number matrix will not cause any misleading result because of recurring numbers in itself. As result of the multiplication of the created Cholesky decomposition matrix and random number matrix, 10 different 100 X 91 return probability matrixes which simulated by means of the real data from 91 stocks. The distribution graph of the simulated return matrix was exhibited in Figure-4.

Figure 4: Simulated return probabilities matrix scatter chart



At the last stage of the analysis, to make comparison between optimum portfolios and simulated portfolios, return of these 10 optimum portfolios are calculated by multiplying weights of securities and return probability matrix. Through the calculated returns and other data, VaR values were calculated by means of Equation 5 and Monte Carlo simulation method. The calculated VaR values were presented on Table-3.

Table 3: Portfolios' Value at Risk calculated based on Monte Carlo simulation method

Portfolio VaR (TL)	
VaR1	-0.0087
VaR2	-0.0241
VaR3	-0.0515
VaR4	-0.0789
VaR5	-0.1110
VaR6	-0.1466
VaR7	-0.1827
VaR8	-0.2173
VaR9	-0.2499
VaR10	-0.2885

The data presented on Table-3 show that how much loss those portfolios would incur in Turkish Lira on the next day by 95% probability. For instance, as a result of the analysis, it can be said that "the investor who invested in 7^{th} portfolio would not lose more than 18.27 tomorrow by 95% probability". The important point related with VaR values is that they represent the monetary values of the values. They should not be confused with risk rates.

5. Results

In the study, through the annual daily closing values of 91 stocks listed in ISE-100 index in 2011, expected returns and risk levels of 10 optimal portfolios were calculated by means of Portfolio Optimization method based on average variance. Determined 10 portfolios' VaR were calculated by means of the Monte-Carlo simulation method. The purpose of the study is to test whether the risk level of investor caused by uncertain future can be reduced or not by employing the VaR method and Portfolio Optimization method based on the average variance jointly. The results of the analysis were summarized on Table-4.

Table 4: Summary of analysis result

	VaR(TL)	Risk(σ)
Portfolio-1	-0.0087	0.0055
Portfolio -2	-0.0241	0.0145
Portfolio -3	-0.0515	0.0306
Portfolio -4	-0.0789	0.0476
Portfolio -5	-0.1110	0.0674
Portfolio -6	-0.1466	0.0890
Portfolio -7	-0.1827	0.1113
Portfolio -8	-0.2173	0.1341
Portfolio -9	-0.2499	0.1570
Portfolio -10	-0.2885	0.1801

Obtained results show that both Portfolio Optimization method based on average variance and VaR method present parallel results. As the risk rates of 10 different portfolios calculated by the Portfolio Optimization method based on average variance increases, VaR values calculated by relevant method increase as well. This means that the investor may face higher losses at high risk levels. In other words, about the portfolios located on the efficient frontier and whose risk level increases, we can be 95% sure that their losses in Turkish Lira also will increase. Consequently, Portfolio Optimization based on average variance and VaR methods provide solution for investor against their decision-making problem caused by uncertain future when they are used jointly.

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