Type of textures

- There are two kind of classification:

    1. Explicit Texture Map: Consist of a regular bitmap.
    we require to provide the texture coordinates and a subsystem for the texturing processing

    2. Procedural Texture: output of the program which computes the texture map.
    The texture is descomposed in primitive equations and functions.
    The advantage over explicit texture mapping is that they are resolution independent, they are usually not as repetitive.
    However, they can take system resources

- Another classification is between static and dynamic texture maps.
while dynamic maps are computed in real time, static maps are created once. Using a sequence of bitmaps to create a fire effect that can be wrapped around an object is an example of dynamic and explicit texturing mapping.

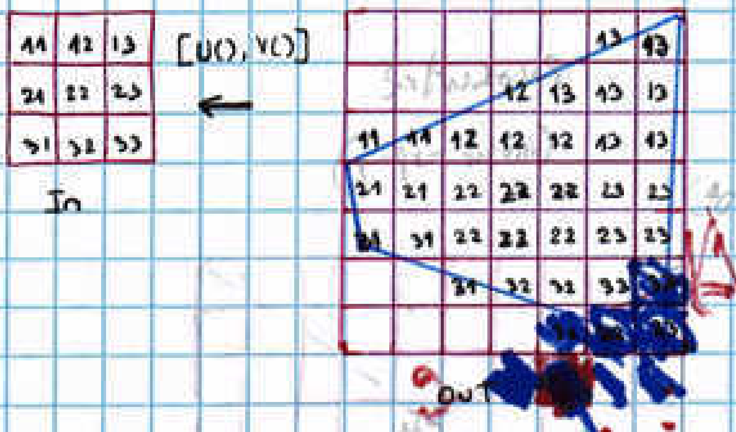
- Textures can be 1D, 2D, and 3D data set. Example: 3D textures are called volume textures

- Texturing mapping is the specification of how the texture map should streeth and wrap the object.

In the case of 2D textures, the texture mapping is a function that goes from (x, y, z) to (u, v) where (u, v) are mapping coordinates. This deffines the correspondence between vertices on the geometry and the pixels in the map. Example:

- $u = x + z$
  $v = y$

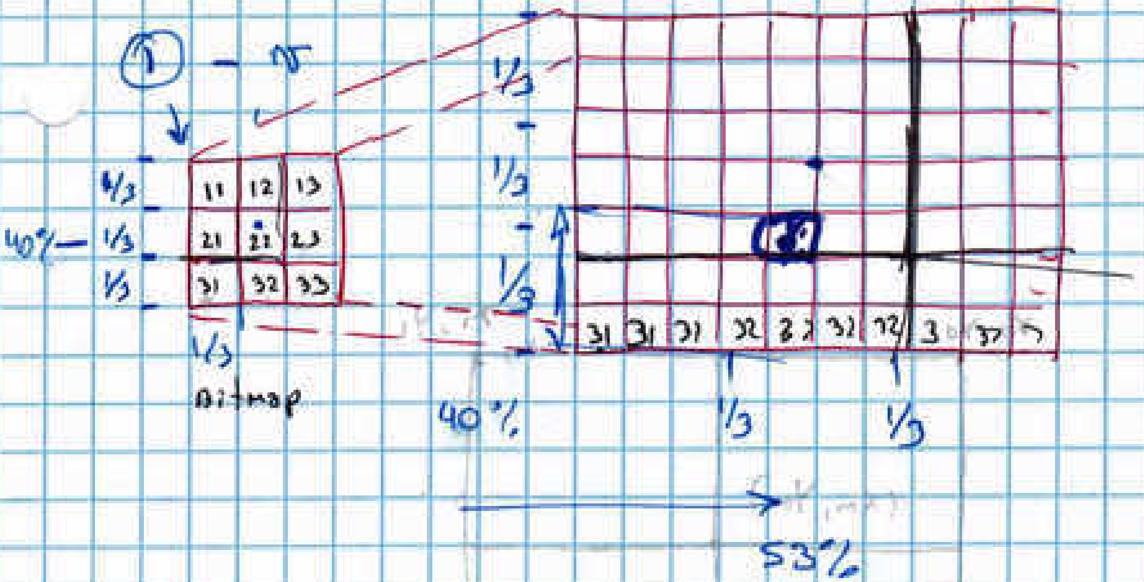
154 **XYZ Mapping**

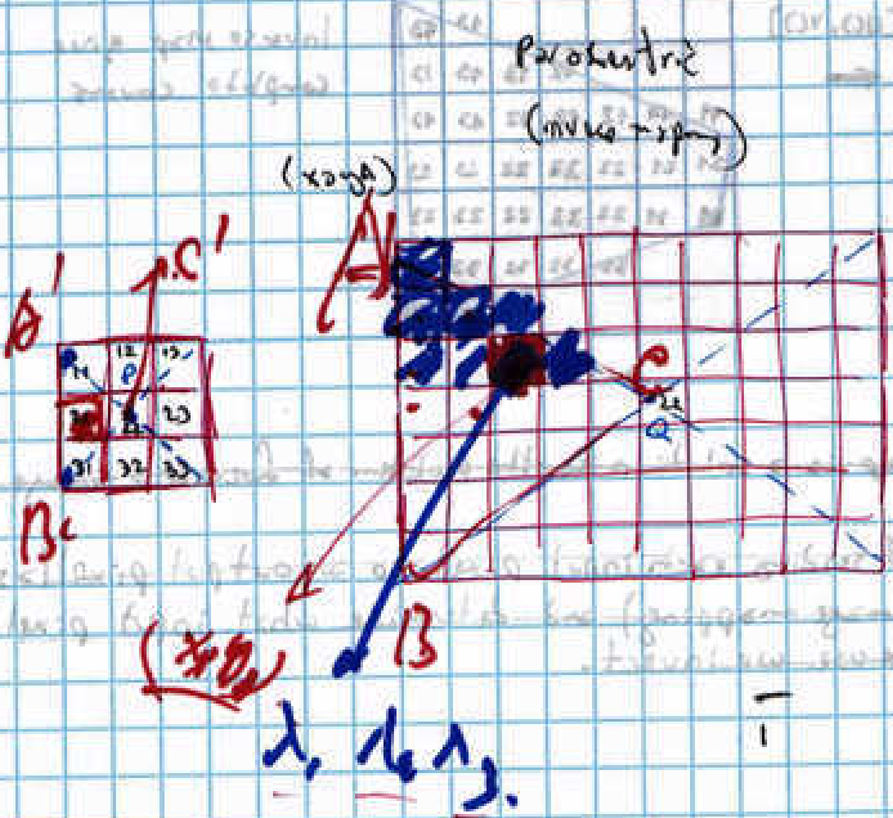


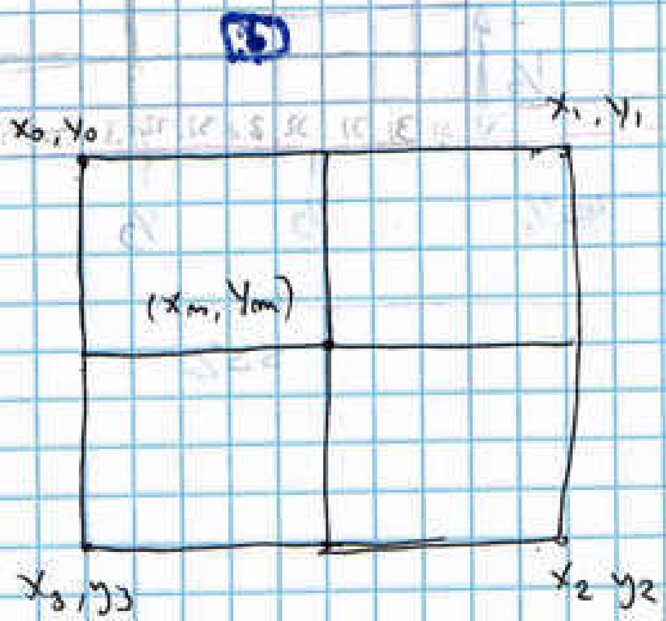
Inverse map give complete covering

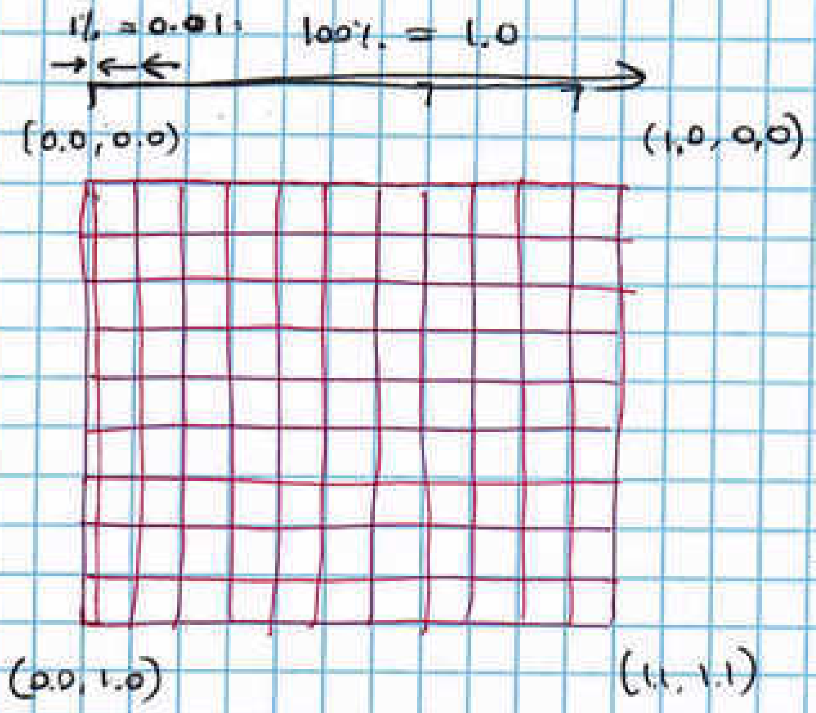
~~Inverse map is a solution to the problem of forward image mapping process~~

~~Instead of sending each input pixel to an output pixel (as in forward image mapping) and determine what input pixel(s) map should be use, we invert.~~




Parametric
(xa,ya)
A
B'
C'
11
12
13
21
22
23
31
32
33
C
B
λ1 λ2 λ3


x0, y0
x1, y1
(xm, ym)
x3, y3
x2 y2



→ 10px
↓ 10px

400 cars — 100%
$\frac{400}{100}$ 4 cars — 1%
48 — 12%

0.01 = [+]
0.1px

0.0 → 1.0
10p

512 | 100
12 5.

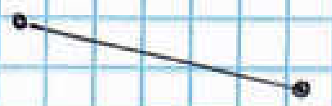
100 = 1
$\frac{10}{1000}$ = 0.1

512px — 1
5.12px — 0.1

(width . height)

① vertex 2f (x, y)

vertex 2f (x_2, y_2)



② DRAW

③ Mouse.x   Mouse.y

④ Vector

$\boxed{N/A}$

for(1){

$\boxed{X, Y}$

vrtx2f(x, y)

}

vrtx(mouse.x , ms y)

MoveMouse ≠ vertex 2f

x y
x y

M.x	M.y

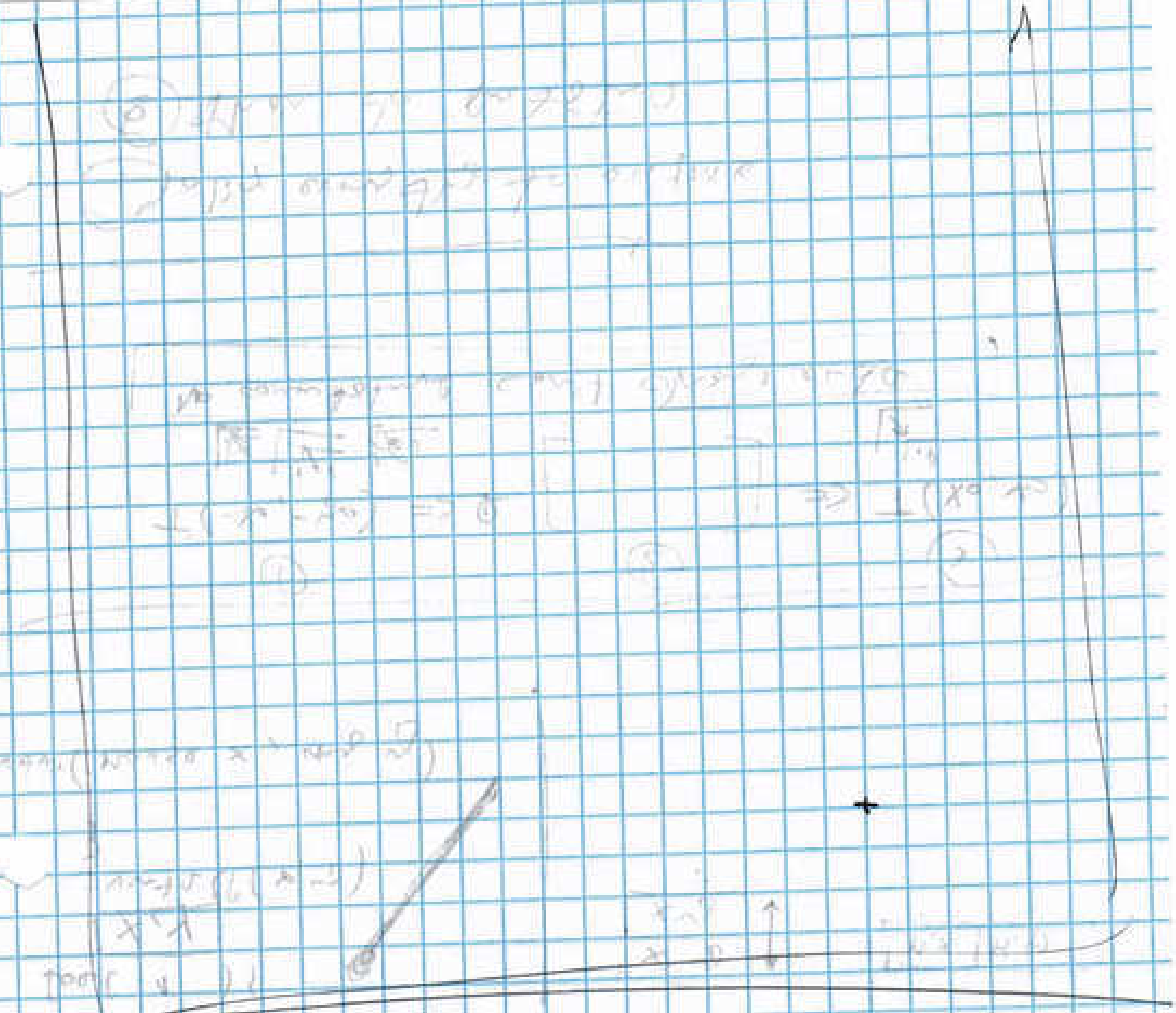
① ② ③

$T(-x_0, -y_0) \Rightarrow R \begin{bmatrix} & \end{bmatrix} \Rightarrow T(x_0, y_0)$

$\boxed{\text{No commutative can't change order}}$

① align everything to origin

② then do rotation



draw
for (loop
vertex (x, y)

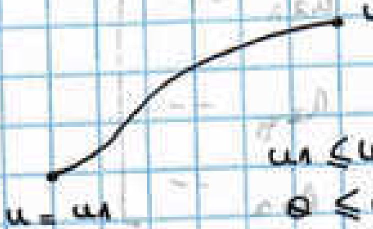
[x, y]

}

(vertex(mouse.x , mouseY))

Modeling of Curves

- If we have any ~~curve~~ general 2D curve, we can express this curve in ~~an~~ parameter form:



$u_1 \leq u \leq u_2$

$0 \leq u \leq 1$

$$\left.\begin{aligned} x &= X(u) \\ y &= Y(u) \\ z &= Z(u) \end{aligned}\right\} \text{(cubic expression)}$$

-OR-

$$f(x, y, z) = 0$$

○ some examples:

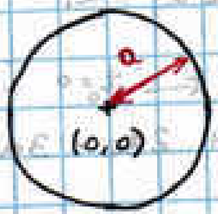
a.

$x = a + lu$

$y = b + mu$

$z = c + nu$

b.



$x = a\cos\theta$

$y = a\sin\theta$

$x^2 + y^2 = a^2$

- Parametric Cubic Curves (PC curves)

$$\vec{f}(u) = \vec{a}_0 + \vec{a}_1 u + \vec{a}_2 u^2 + \vec{a}_3 u^3$$

constants

where $0 \leq u \leq 1$ and $a_0 \ldots a_3$ are constants vectors

-OR-

$$x(u) = a_{0_x} + a_{1_x} u + a_{2_x} u^2 + a_{3_x} u^3$$

$$y(u) = a_{0_y} + a_{1_y} u + a_{2_y} u^2 + a_{3_y} u^3$$

$$z(u) = a_{0_z} + a_{1_z} u + a_{2_z} u^2 + a_{3_z} u^3$$

$$\vec{f} = [u^3\ u^2\ u\ 1]\,[\vec{a}_3\ \vec{a}_2\ \vec{a}_1\ \vec{a}_0]^T$$

$f = \underset{\text{matrix}}{U} \cdot \underset{\text{matrix}}{A}$

3 components each

$\underbrace{\begin{bmatrix} \vec{a_3} & a_2 & a_1 & a_0 \end{bmatrix}}_{4\ \text{vectors}} = 12$ components

$$\begin{bmatrix} a_{3x} & a_{2x} & \cdots \\ a_{3y} & a_{2y} & \cdots \\ \vdots & a_{1x} & \cdots \end{bmatrix}$$

- $\vec{f} = [u^3\ u^2\ u\ 1]\,[\vec{a_3}\ \vec{a_2}\ \vec{a_1}\ \vec{a_0}]^T$

   o. Start point: $u = 0 \Rightarrow f(0) = \vec{a_0}\ \cancel{+ \vec{a_1} + \vec{a_2} + \vec{a_3}}$

   $f(1) = \vec{a_0} + \vec{a_1} + \vec{a_2} + \vec{a_3}$

Tangent Vectors: $f(u) = [u^3\ u^2\ u\ 1]\,[a_3\ a_2\ a_1\ a_0]^T$

$f'(u) = [3u^2\ \ 2u\ \ 1\ \ 0]\,[a_3\ a_2\ a_1\ a_0]^T$

$= 3u^2 a_3 + 2u\, a_2 + a_1$

so $f'(0) = a_1 \leftarrow t_0 = 0$ (starting vector)

$f'(1) = a_1 + 2a_2 + 3a_3 \leftarrow t_1 = 1$ (ending vector)

$a_0 = f(0)$
$a_1 = t_0$
$a_2 = -3f(0) + 3f(1) - 2f'(0) - f'(1)$
$a_3 = 2f(0) - 2f(1) + f'(0) + f'(1)$

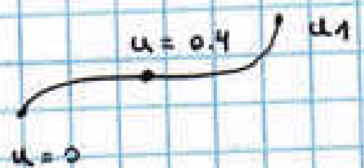
$f = UA \Longleftrightarrow$ Algebraic Form

$f(u) = a_3 u^3 + a_2 u^2 + a_1 u + a_0$
$= (2u^3 - 3u^2 + 1)\cdot f(0) + (-2u^3 + 3u^2)\, f(1) +$
$(u^3 - 2u^2 + u)\cdot f'(0) + (u^3 - u^2)\cdot f'(1)$

$$f(u) = [F_1(u) \quad F_2(u) \quad F_3(u) \quad F_4(u)] \begin{bmatrix} f_0 \\ f_1 \\ t_0 \\ t_1 \end{bmatrix}$$

Blending functions

- Along the length of the curve, at any specific point, we need to define the blending functions for this point to follow vectors f_0, f_1, t_0, and t_1 as move along the curve. so the blending function will change



$$f(u) = [F_1(u) \quad F_2(u) \quad F_3(u) \quad F_4(u)] \cdot B$$

$$B = [f_0 \quad f_1 \quad t_0 \quad t_1]^T$$

Geometric Matrix

$$f = FB$$

standard Geometric Matrix

$$F = \begin{bmatrix} 2u^3 - 3u^2 + 1 \\ -2u^3 + 3u^2 \\ u^3 - 2u^2 + u \\ u^3 - u^2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}}_{U} \overbrace{\underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{M}}^{\text{coefficients}}$$

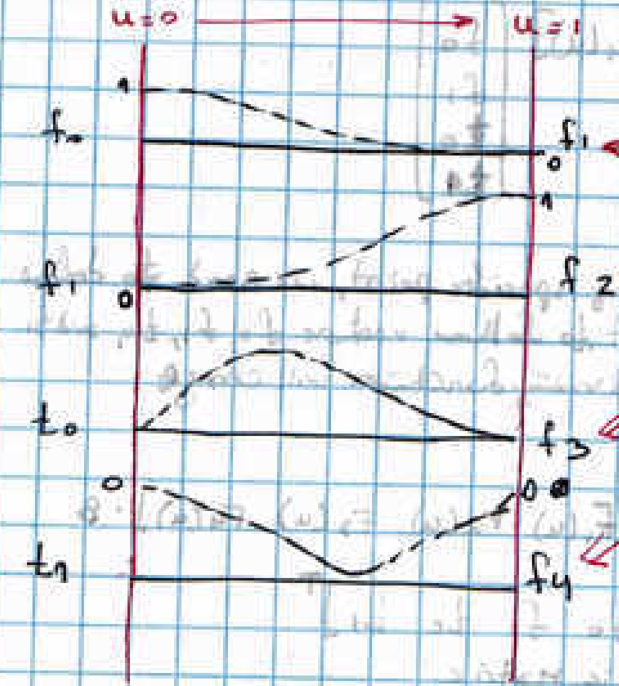
$$= UM$$

$$\left.\begin{aligned} f &= UA \\ &= FB \\ &= UMB \end{aligned}\right\} \quad A = MB$$

$$B = [f_0 \quad f_1 \quad t_0 \quad t_1]^T$$

$$= [f_0 \quad f_1 \quad k_1 \hat{t_0} \,,\ k_2 \hat{t_1}]^T$$

These functions change as we go from $u = 0$ to $u = 1$



$$\begin{bmatrix} 2u^3 - 3u^2 + 1 \\ -2u^3 + 3u^2 \\ u^3 - 2u^2 + u \\ u^3 - u^2 \end{bmatrix}^T$$

$$f(u) = F_1 f_0 + F_2 f_1 + F_3 t_0 + F_4 t_1$$

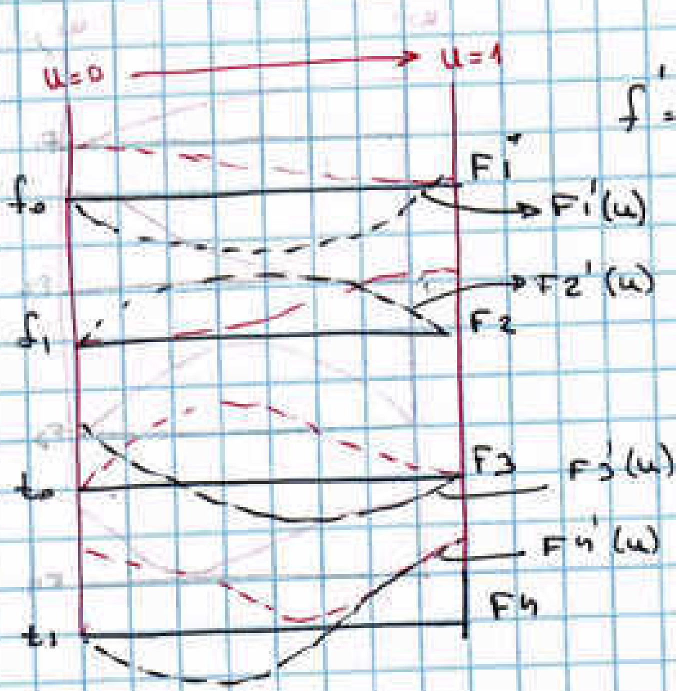
$$f'(u) = F_1' f_0 + F_2' f_1 + F_3' t_0 + F_4' t_1$$

$$= F' B$$

$$f = U \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{M} B = \cancel{UM}$$

$$=$$

$$F' = \begin{bmatrix} 6u^2 - 6u \\ -6u^2 + 6u \\ 3u^2 - 4u + 1 \\ 3u^2 - 2u \end{bmatrix}^T$$



$$f' = \begin{bmatrix} 6u^2 - 6u \\ -6u^2 + 6u \\ 3u^2 - 4u + 1 \\ 3u^2 - 2u \end{bmatrix}^T \quad F_1'(u)$$

$f'(u) = F'B = UM'B$ where M' are the coeficients of this matrix

$$M' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 6 & -6 & 3 & 3 \\ -6 & +6 & -4 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

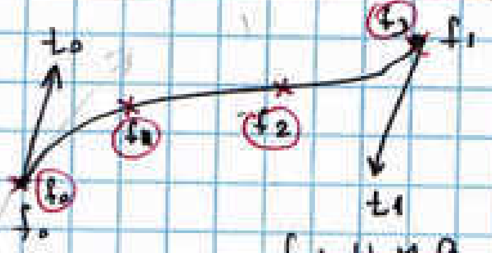
$f''(u) = F''B = UM''B$

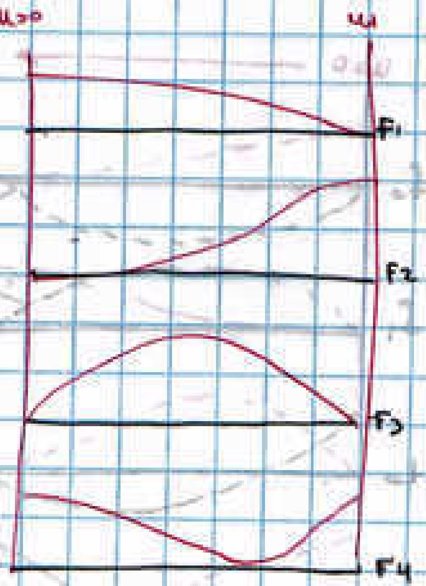
- $f = UA$

$= UMB$

$= FB$

$$\begin{bmatrix} f_0 \\ f_1 \\ t_0 \\ t_1 \end{bmatrix}$$





$f = UMB$

$u = u_0 \Rightarrow f_0 = U_0 MB = [u_0^3 \;\; u_0^2 \;\; u_0 \;\; 1]\, MB$

$u = u_1 \Rightarrow f_1 = U_1 MB = [u_1^3 \;\; u_1^2 \;\; u_1 \;\; 1]\, MB$

$u = u_2 \Rightarrow f_2 = U_2 MB = [u_2^3 \;\; u_2^2 \;\; u_2 \;\; 1]\, MB$

$u = u_3 \Rightarrow f_3 = U_3 MB = [u_3^3 \;\; u_3^2 \;\; u_3 \;\; 1]\, MB$

$$P = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} u_0^3 & u_0^2 & u_0 & 1 \\ u_1^3 & u_1^2 & u_1 & 1 \\ u_2^3 & u_2^2 & u_2 & 1 \\ u_3^3 & u_3^2 & u_3 & 1 \end{bmatrix} MB$$

I want to find out B

$B = M^{-1} U^{-1} P$

$= M^{-1} [U_0 \; U_1 \; U_2 \; U_3]^{T-1} \; P$

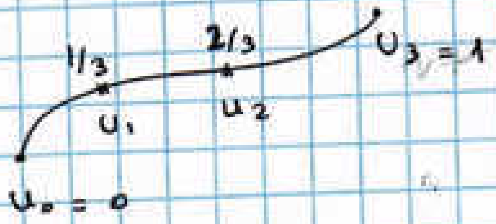
$= KP$

$[f_0 \; \cancel{f_1 \; f_2 \; f_3}]$

$$\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

f_1 given

← given



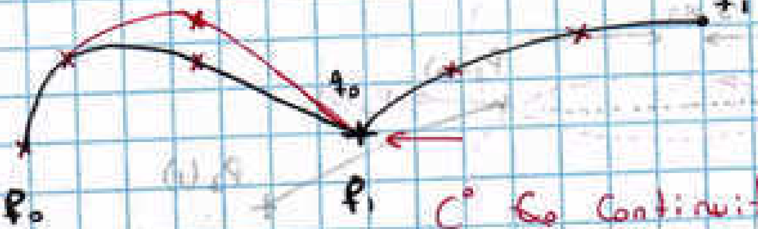
$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -11/2 & 9 & -9/2 & 1 \\ -1 & 9/2 & -9 & 9/2 \end{bmatrix}$$

$B = KP$

~~$K = M^{-1}C$~~

$$= M^{-1}\left[U_0 \ U_1 \ U_2 \ U_3\right]^{T-1}$$

Composite Curves

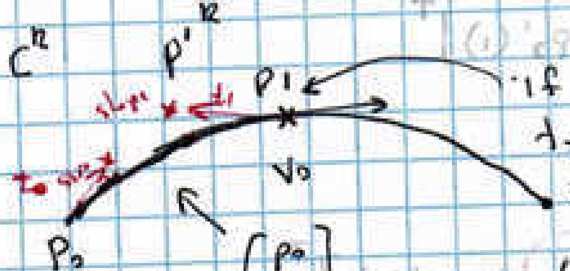


C^0 G_0 Continuity - Common End-Point $\quad p$

C^1 G_1 Continuity - Common tangent $\quad p''$

C^2 - Common Curvature $\quad p'''$

C^2



If they have the same continuity the tangent line would be the same at this point

$$B = \begin{bmatrix} p_0 \\ p_1 \\ t_0 \\ t_1 \end{bmatrix}$$

What kind of changes would affect the continuity

p' $\quad$ 2nd degree curve

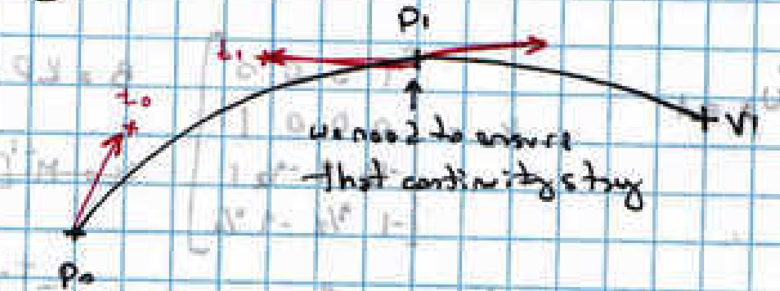
p'' $\quad$ 1st degree curve

p''' $\quad$ constant

p'''' $\quad$ 0

p'^{R}

C^k C^1



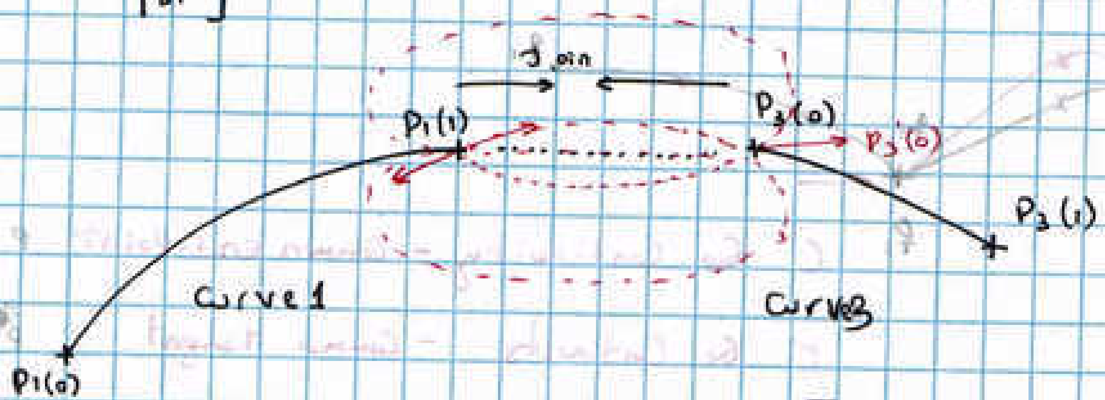
$$B = \begin{bmatrix} p_0 \\ p_1 \\ t_0 \\ t_1 \end{bmatrix}$$

continuity $\longrightarrow$ p' 2^{nd} degree

$\longrightarrow$ p'' 1^{st} degree

p''' constant

p^{iv} 0



$$B_1 = [\,P_1(0) \quad P_1(1) \quad P_1'(0) \quad P_1'(1)\,]^T$$

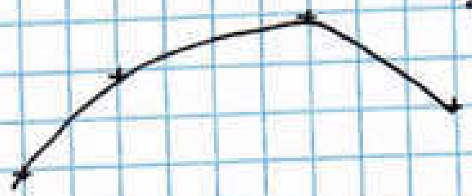
$\vdots$

$$B_3 = [\,P_3(0) \quad P_3(1) \quad P_3'(0) \quad P_3'(1)\,]^T$$

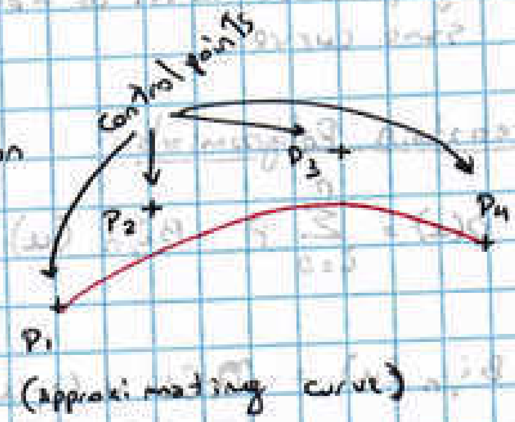
Defining B_2:

$$B_2 = [\,P_1(1) \quad P_3(0) \quad k_1 P_1'(1) \quad k_2 P_3'(0)\,]^T$$

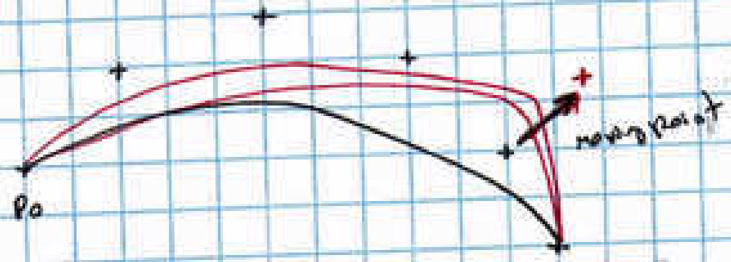
BEZIER Curves



approximation of these point =>

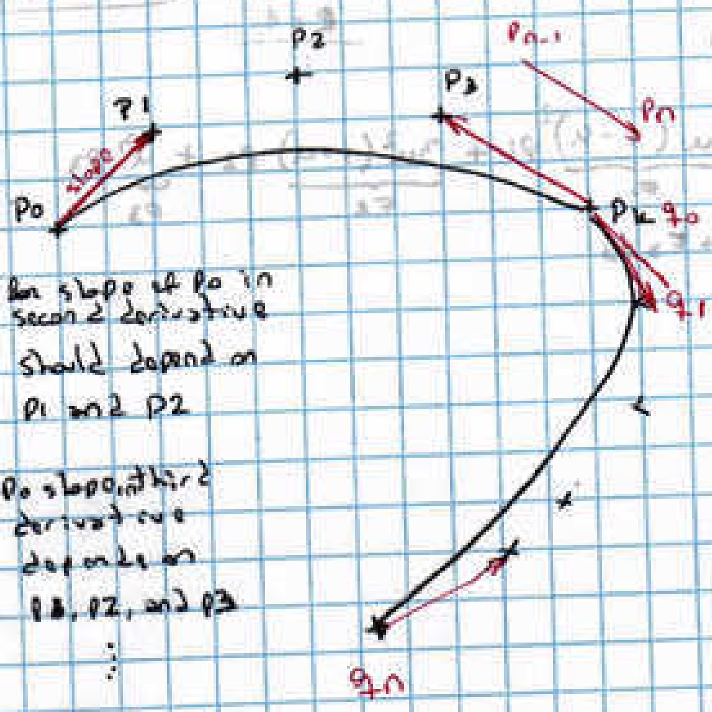


- Basic property of curves:
  - ◇ curve should start and end at the first and last point $P_0 \to P_n$



◇ slope at p_0 should be $\overrightarrow{p_0 p_1}$ and slope p_n is given by $\overrightarrow{P_n P_{n-1}}$

P_n is given by $\overrightarrow{P_n P_{n-1}}$ $\overleftarrow{P_{n-1} P_n}$



for slope of P_0 in second derivative should depend on P_1 and P_2

P_0 slope in third derivative depends on P_1, P_2, and P_3

⋮

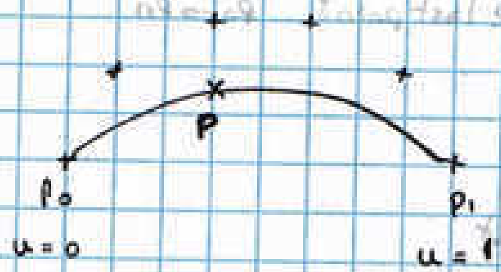
◇ R^{th} derivative at end points should depend on next R-points

17.9 ❖ Symmetric w.r.t depends on u and (1-u) which means that going from P_0 to P_n or from P_n to P_0 is would produce the same curve.

Brenstain Polynomials

$$p(u) = \sum_{i=0}^{n} P \cdot B_{i,n}(u)$$

$$B_{i,n}(u) = \underbrace{{}^{n}C_i \; u \,(1-u)^{n-i}}_{\text{Brezier expression}} \quad \text{where } \forall \; 0 \le u \le 1$$



3rd Degree Bezier Curves

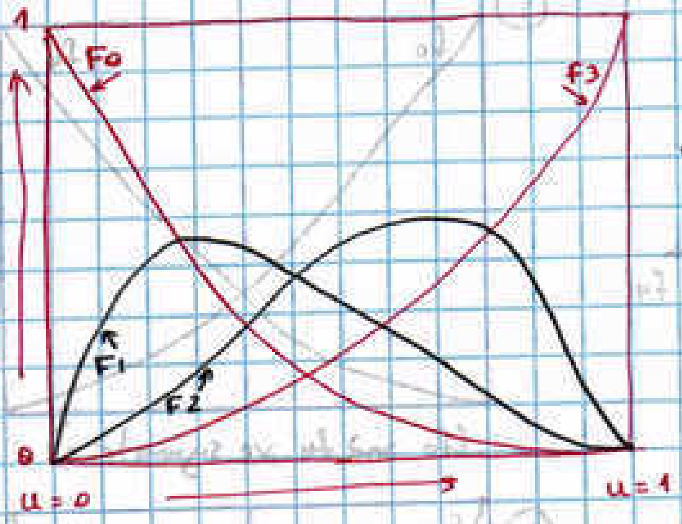
n = 3   Points: 0, 1, 2, 3

$$\frac{n=3}{\begin{matrix} n=3 \\ i=1 \\ \vdots \end{matrix}} \qquad \boxed{{}^{n}C_i \; B_{i,n} (1-u)^{n-i}}$$

$$p(u) = \sum P_i \; B_{i,n}(u)$$

$$= P_0 \underbrace{(1-u)^3}_{F_0} + \underbrace{3u(1-u)^2}_{F_1} p_1 + \underbrace{3u^2(1-u)}_{F_2} p_2 + \underbrace{u^3}_{F_3} p_3$$

$$= F_0 p_0 + F_1 p_1 + F_2 p_3 + F_3 p_3$$



All these Bezier functions $F_0 \ldots F_3$ are in symmetry with u in between u and $(1-u)$

$$\left.\begin{array}{llll} p_0 & p_1 & p_2 & p_3 \\ p_3 & p_2 & p_1 & p_0 \end{array}\right\} \text{same curves}$$

- **n = 2 with 3 control points**

$$P(u) = (1-u)^2 P_0 + 2u(1-u) P_1 + u^2 P_2$$

- **Another example**

$$P(u) = \Sigma p_i f_i(u) \qquad n = 3$$

$$f_i(u) = B_{i,n}(u) = {}_nC_i \, u^i (1-u)^{n-i}$$

$$P(u) = {}_3C_0 (1-u)^3 p_0 + {}_3C_1 \, u (1-u)^2 p_1 + {}_3C_2 \, u^2 (1-u) p_2 + {}_3C_3 \, u^3 p_3$$