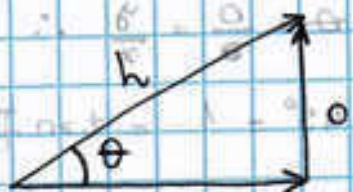


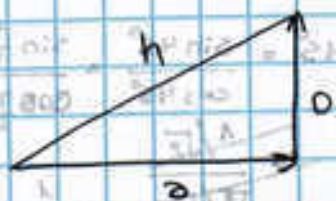
soh cah toa



$$\sin \theta = \frac{y}{h} = \frac{y}{r}$$

$$\cos \theta = \frac{x}{h} = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



$$h^2 = x^2 + y^2$$

$$r^2 = x^2 + y^2$$

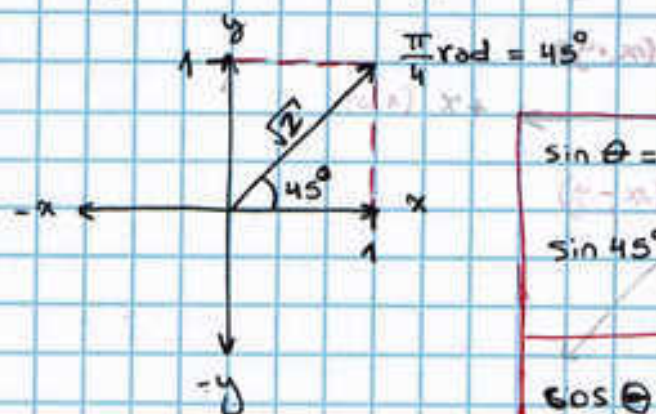
$$x = r \cos \theta = x = h \cos \theta$$

$$y = r \sin \theta = y = h \sin \theta$$



$$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2 + y^2}$$

$$\text{if } x=1 \text{ \& } y=1 \therefore r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$



$$\sin \theta = \frac{o}{h} = \frac{x}{r} \therefore$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$$

$$\cos \theta = \frac{a}{h} = \frac{y}{r} \therefore$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4}$$

$$\tan \theta = \frac{o}{a} = \frac{y}{x} \therefore$$

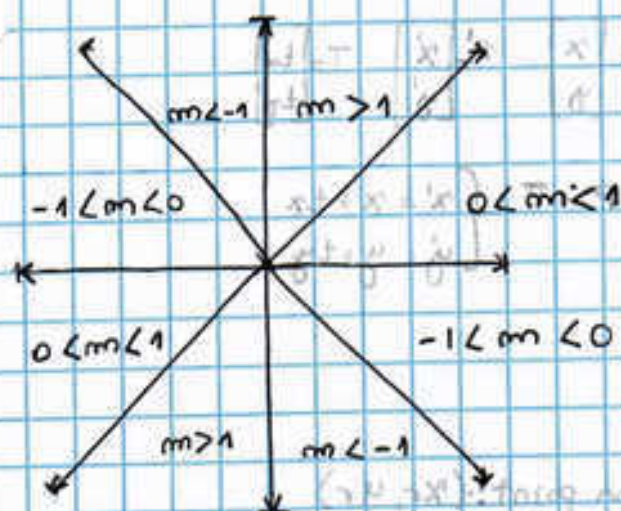
$$\tan 45^\circ = 1 = \tan \frac{\pi}{4}$$

$$\tan \theta = \frac{o}{a} = \frac{x \cdot \sin \theta}{x \cdot \cos \theta} = \frac{\sin \theta}{\cos \theta}$$

$$\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} = \frac{1/\sqrt{2}}{1/\sqrt{2}}$$

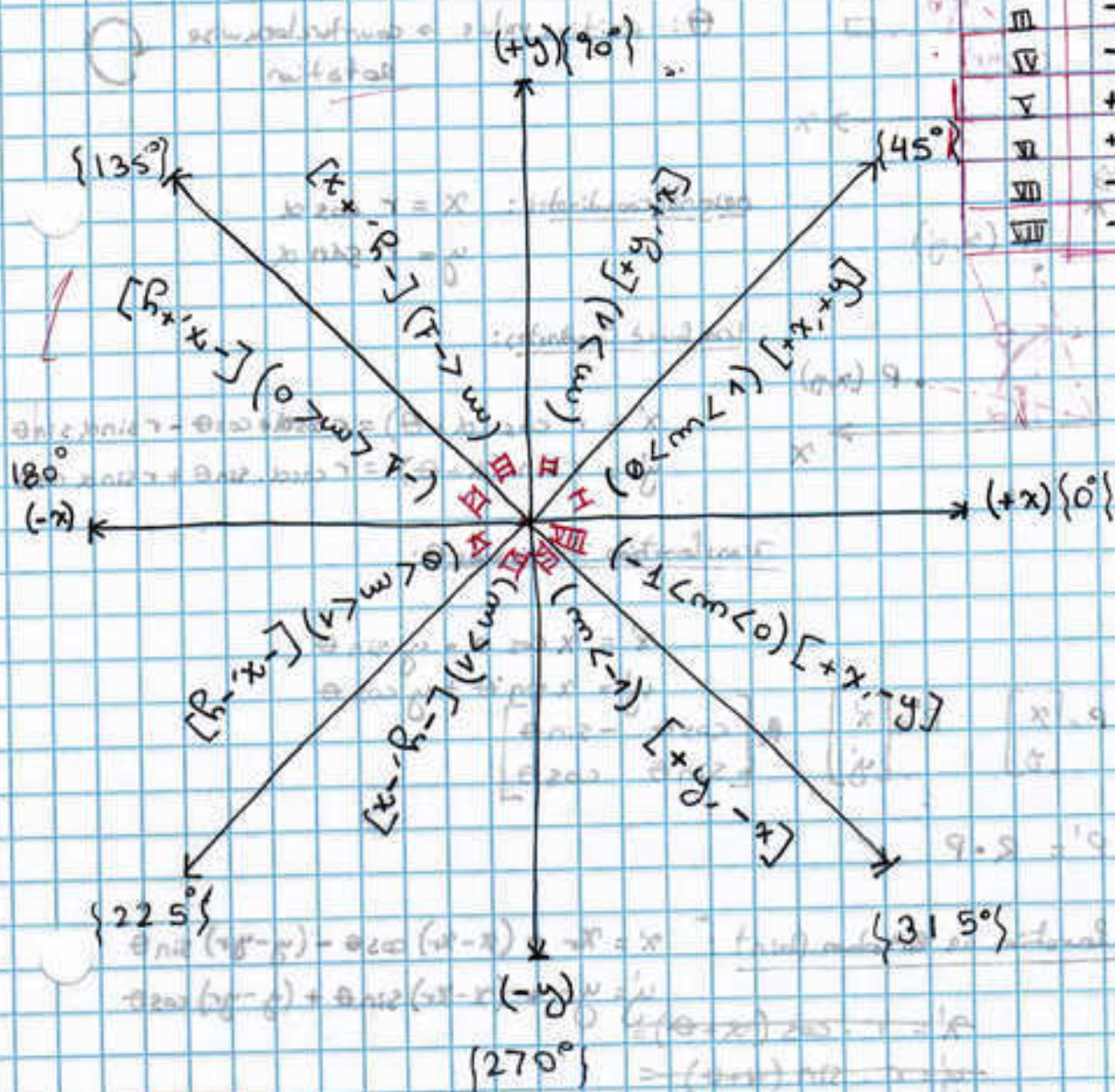
$$= \frac{1/\sqrt{2}}{1/\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{1} = 1$$

(slope) $m = \frac{dy}{dx}$

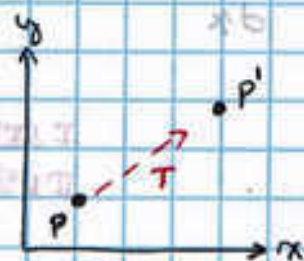


I & III $\Delta y \Delta x$ (x, y) (x, y)
II & IV $\Delta y \Delta x$ (x, y) (x, y)

Quadrant	Slope	dy/dx
I	+	$dy > dx$
II	+	$dy < dx$
III	-	$dy < dx$
IV	-	$dy > dx$
V	+	$dy > dx$
VI	+	$dy < dx$
VII	-	$dy < dx$
VIII	-	$dy > dx$



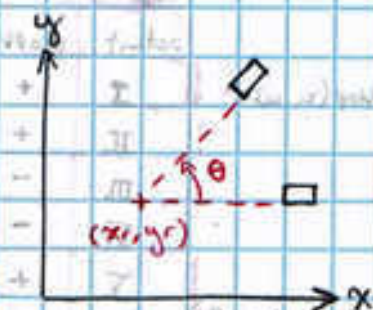
Two Dimensional Translation



$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad T = \begin{bmatrix} tx \\ ty \end{bmatrix}$$

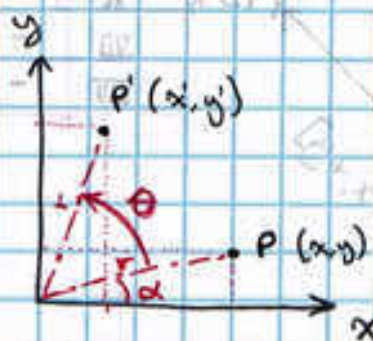
$$P' = P + T \quad \begin{cases} x' = x + tx \\ y' = y + ty \end{cases}$$

Two Dimensional Rotation



Rotation point: (x_r, y_r)
(Pivot Point)

θ : positive value \rightarrow counterclockwise rotation



original coordinates: $x = r \cos \alpha$
 $y = r \sin \alpha$

transformed coordinates:

$$\begin{aligned} x' &= r \cos(\alpha + \theta) = r \cos \alpha \cos \theta - r \sin \alpha \sin \theta \\ y' &= r \sin(\alpha + \theta) = r \cos \alpha \sin \theta + r \sin \alpha \cos \theta \end{aligned}$$

translation through angle θ :

$$\begin{aligned} x' &= x \cos \theta - y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}$$

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$P' = R \cdot P$$

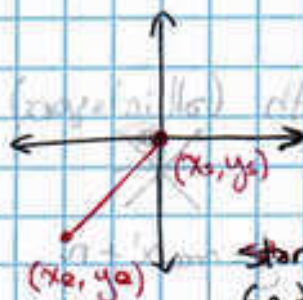
Translation on rotation point: $x' = x_r + (x - x_r) \cos \theta - (y - y_r) \sin \theta$

$$y' = y_r + (x - x_r) \sin \theta + (y - y_r) \cos \theta$$

$$x' = r \cdot \cos(\alpha + \theta)$$

$$y' = r \cdot \sin(\alpha + \theta)$$

①



Start point: (x_s, y_s)
End point: (x_e, y_e)

Slope



$$\tan \alpha = \frac{y}{x}$$

② Bresenham's algorithm Octant I



Grid

128 x 128 grid, and all

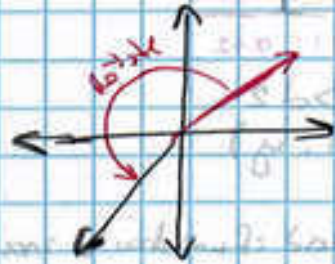
③ Obtain angle β of line β



④ Calculate β' so $0^\circ < \beta' < 45^\circ$

⑤ Use Bresenham's Algorithm to store all points coordinate

⑥ Use 2D rotation to rotate points to β



32. Bresenham's Line Draw Algorithm

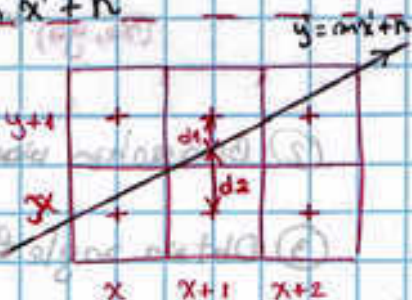
Positive slope

- Draw line without using floating point math (all integers)
- Basic algorithm draws a line at 2nd Octant



1. We wish to draw a line which equation is $y = mx + n$

2. Draw a pixel on position (x, y) & verify which closest point pixel to (x, y) belong to the line. $(x', y') \in \text{line}$

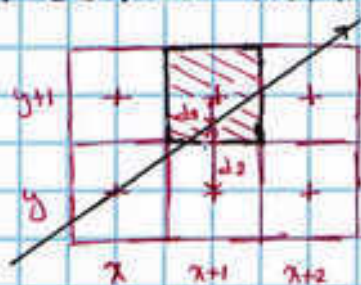


3. Move along x-axis, use algorithm to decide which next point on y-axis to draw.

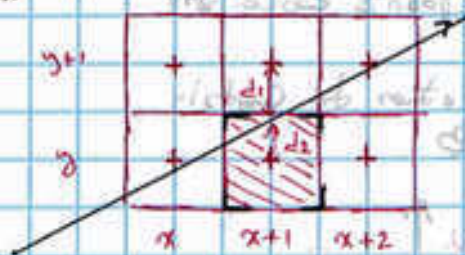
• Do $(x+1, y)$ pixel or $(x+1, y+1)$ pixel belong to line $y = mx + n$?

- if point is not in the center of the pixel, we must find the second best point who's distance from the line is the smallest ($d_1 > d_2$ or $d_2 < d_1$)

4. if $d_1 < d_2$ then next point to (x, y) should be $(x+1, y+1)$
if $d_1 > d_2$ then next point to (x, y) should be $(x+1, y)$



$d_1 < d_2$
 $(x+1, y+1)$



$d_1 > d_2$
 $(x+1, y)$

→ For every increment of x coordinate we must find if we should increment y coordinate

Bresenham's Line Drawing Algorithm (continuation) [Positive slope]

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$$5 \quad d1 = (y+1) - y'$$

$$d2 = y' - y$$



$$\begin{aligned} d1 - d2 &= (y+1) - y' - (y' - y) \\ &= y+1 - y' - y' + y \\ &= 2y - 2y' + 1 = 2y + 1 - 2y' \end{aligned}$$

$$6. \quad \begin{cases} d1 - d2 = 2y + 1 - 2y' \\ y' = mx' + n \end{cases} \Rightarrow \begin{cases} d1 - d2 = 2y + 1 - 2y' \\ y' = mx' + n \end{cases} = \begin{cases} d1 - d2 = 2y + 1 - 2(mx' + n) \end{cases}$$

$$7. \text{ For every iteration } x' = x+1 \text{ and slope } m = \frac{dy}{dx} = \frac{(y+1) - y}{x+1 - x}$$

$$\begin{cases} d1 - d2 = 2y + 1 - 2(mx' + n) \\ x' = x+1 \end{cases}$$

$$\begin{aligned} d1 - d2 &= 2y + 1 - 2(mx' + n) \\ &= 2y + 1 - 2[m(x+1) + n] \\ &= 2y + 1 - 2[mx + m + n] \\ &= 2y + 1 - 2mx - 2m - 2n \\ &= 2y + 1 - 2m(x+1) - 2n \end{aligned}$$

$$\begin{cases} d1 - d2 = 2y + 1 - 2m(x+1) - 2n \\ m = \frac{dy}{dx} \end{cases}$$

$$d1 - d2 = 2y + 1 - 2\left(\frac{dy}{dx}\right)(x+1) - 2n$$

$\frac{dy}{dx}$ - we wish to eliminate division

$$dx(d1 - d2) = dx(2y + 1 - 2\frac{dy}{dx}(x+1) - 2n)$$

$$\begin{aligned} dx(d1 - d2) &= 2y dx + dx - 2dy(x+1) - 2n dx \\ &= 2y dx + dx - 2x dy - 2dy - 2n dx \end{aligned}$$

8. Let $f(x, y) = dx(d_1 - d_2) =$
 $[2y dx + dx - 2x dy - 2dy - 2n dx]$
 $f(x, y) = 2y dx + dx - 2x dy - 2dy - 2n dx$
 a. $f(x, y)$ determine how far a point is from the line
 without using division

b. $f(x, y)$ is the function that determine the distance
 from the pixel we want to draw (x, y) to the line $y = mx + n$

9. Let (x_i, y_i) be the last point we draw
 Let (x_{i+1}, y_{i+1}) be the next point we will draw
 For each iteration $f(x_{i+1}, y_{i+1}) - f(x_i, y_i)$

$$f(x_i, y_i) = 2y_i dx + dx - 2x_i dy - 2dy - 2n dx$$

$$f(x_{i+1}, y_{i+1}) = 2y_{i+1} dx + dx - 2x_{i+1} dy - 2dy - 2n dx$$

$$f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = [2y_{i+1} dx + dx - 2x_{i+1} dy - 2dy - 2n dx] - [2y_i dx + dx - 2x_i dy - 2dy - 2n dx]$$

$$= 2y_{i+1} dx - 2x_{i+1} dy - 2dy - 2n dx - 2y_i dx + dx + 2x_i dy + 2dy + 2n dx$$

$$f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2y_{i+1} dx + dx - 2x_{i+1} dy - 2dy - 2n dx - [2y_i dx + dx - 2x_i dy - 2dy - 2n dx]$$

$$= 2y_{i+1} dx + dx - 2x_{i+1} dy - 2dy - 2n dx - 2y_i dx - dx + 2x_i dy + 2dy + 2n dx$$

$$= 2y_{i+1} dx - 2x_{i+1} dy - 2y_i dx + 2x_i dy$$

$$= 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i)$$

Bresenham line drawing Algorithm (continued) [Positive slope] 85

$$10. \cancel{f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i)}$$

$$\cancel{f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i)}$$

• For each increment of the x coordinate each iteration is $x_{i+1} = x_i + 1$

$$\begin{cases} f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i) \\ x_{i+1} = x_i + 1 \end{cases}$$

$$\begin{aligned} \boxed{f(x_{i+1}, y_{i+1}) - f(x_i, y_i)} &= 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i) \\ &= 2dx(y_{i+1} - y_i) - 2dy(\cancel{x_i + 1} - \cancel{x_i}) \\ &= \boxed{2dx(y_{i+1} - y_i) - 2dy} \end{aligned}$$

11. $\cancel{f(x_{i+1}, y)}$

$$\cancel{f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_{i+1} - y_i) - 2dy}$$

- if $d_1 < d_2$ then next point of (x,y) should be (x_{i+1}, y_{i+1})
therefore $y_{i+1} = y_i + 1$

$$\begin{cases} f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_{i+1} - y_i) - 2dy \end{cases}$$

$$\begin{aligned} \boxed{f(x_{i+1}, y_{i+1}) - f(x_i, y_i)} &= 2dx(y_{i+1} - y_i) - 2dy \\ &= 2dx(\cancel{y_i + 1} - \cancel{y_i}) - 2dy \\ &= \boxed{2dx - 2dy} \end{aligned}$$

- if $d_1 > d_2$ then next point of (x,y) should be (x_{i+1}, y)
therefore $y_{i+1} = y_i$

$$\begin{cases} f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_{i+1} - y_i) - 2dy \end{cases}$$

$$\begin{aligned} \boxed{f(x_{i+1}, y_{i+1}) - f(x_i, y_i)} &= 2dx(y_{i+1} - y_i) - 2dy \\ &= 2dx(\cancel{y_i} - \cancel{y_i}) - 2dy \\ &= \boxed{-2dy} \end{aligned}$$

12. Find out $f(x, y)$ for the initial start point:

eg Let assume the initial start point is $(0, 0)$

(Even if $f(0, 0)$ is not on the line we only care for the slope and direction)

(A) $f(x, y) = 2y dx + dx - 2x dy - 2dy - 2n dy$

(B) $y = mx + n$

(C) $m = \frac{dy}{dx}$

we need to find this = n

1. $\begin{cases} y = mx + n \\ m = \frac{dy}{dx} \end{cases} \Rightarrow \boxed{y = \frac{dy}{dx} x + n}$

2. $y = \frac{dy}{dx} x + n$

~~$f(x, y) = 2y dx + dx - 2x dy - 2dy - 2n dy$~~

1. $\begin{cases} y = mx + n \\ m = \frac{dy}{dx} \end{cases} \Rightarrow \boxed{y = \frac{dy}{dx} x + n} \Rightarrow \boxed{n = y - \left(\frac{dy}{dx}\right)x}$

2. $n = y - \left(\frac{dy}{dx}\right)x$

~~$f(x, y) = 2y dx + dx - 2x dy - 2dy - 2n dy$~~

$\boxed{f(x, y) = 2y dx + dx - 2x dy - 2dy - 2n dy}$

$= 2y dx + dx - 2x dy - 2dy - 2 \left[y - \left(\frac{dy}{dx}\right)x \right] dy dx$

$= 2y dx + dx - 2x dy - 2dy - 2y dy + 2 dx \left(\frac{dy}{dx}\right) x$

$= \cancel{2y dx} + dx - \cancel{2x dy} - 2dy - \cancel{2y dy} + \cancel{2x dy}$

$= \boxed{dx - 2dy} \Rightarrow \boxed{f(x, y) = f(0, 0) = dx - 2dy}$

Bresenham line drawing algorithm (continued) [Pos. v slope]

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void draw_bresenham_line(x_0, y_0, x_1, y_1)

$$dx = x_1 - x_0$$

$$dy = y_1 - y_0$$

$$d = dx - 2dy \quad // f(x,y) = f(0,0)$$

$$y = y_0$$

for ($x = x_0$; $x \leq x_1$; $++x$)

draw pixel (x, y)

if ($d < 0$)

$$y = y + 1$$

$$d = d + 2dx - 2dy$$

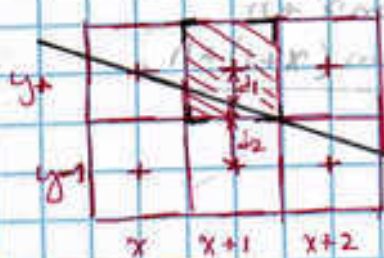
else

$$d = d - 2dy$$

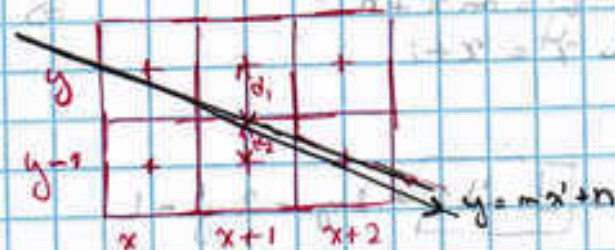
(Note: This Algorithm is missing octants I & III or octant II only) *

Bresenham line Drawing Algorithm for negative slope

1. We increment x value and decide if y value should be decremented or not



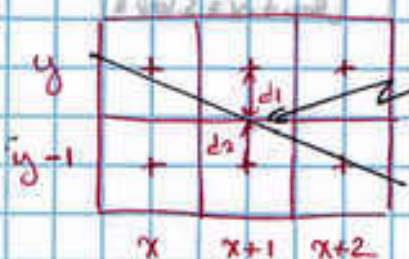
$$d_1 < d_2$$
$$(x+1, y)$$



$$d_1 > d_2$$
$$(x+1, y-1)$$

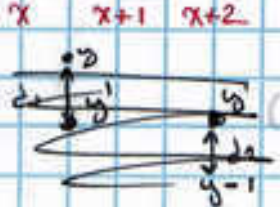
- if $d_1 < d_2$ then the next point to (x, y) should be $(x+1, y)$
- if $d_1 > d_2$ then the next point to (x, y) should be $(x+1, y-1)$

88 2.

 (x, y')

$$d_1 = y - y'$$

$$d_2 = y' - (y - 1)$$



$$3. \quad d_1 = y - y'$$

$$d_2 = y' - (y - 1)$$

$$\boxed{d_1 - d_2} = y - y' - [y' - (y - 1)]$$

$$= y - y' - y' + (y - 1)$$

$$= y - y' - y' + y - 1$$

$$= \boxed{2y - 2y' - 1}$$

4. For the next increment of x , $x' = x + 1$

$$\begin{cases} d_1 - d_2 = 2y - 2y' - 1 \\ y' = mx' + n \\ x' = x + 1 \end{cases}$$

$$\Rightarrow \boxed{y' = mx' + n}$$

$$= \boxed{m(x + 1) + n}$$

$$\Rightarrow \boxed{d_1 - d_2} = 2y - 2y' - 1$$

$$= 2y - 2[m(x + 1) + n] - 1$$

$$= 2y - 2m(x + 1) - 2n - 1$$

$$\boxed{2y - 2mx + 2m - 2n - 1}$$



Bresenham line Drawing Algorithm for negative slope (continued)

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4. (continued)

$$\begin{cases} d_1 - d_2 = 2y - 2mx + 2m \Rightarrow 2n - 1 \\ m = \frac{dy}{dx} \end{cases}$$

$$d_1 - d_2 = 2y - 2mx + 2m \Rightarrow 2n - 1$$

$$= 2y - 2\left(\frac{dy}{dx}\right)x + 2\left(\frac{dy}{dx}\right) \Rightarrow 2n - 1$$

we can't want divisions

so multiply both side by dx

$$dx[d_1 - d_2] = dx\left[2y - 2\left(\frac{dy}{dx}\right)x + 2\left(\frac{dy}{dx}\right) \Rightarrow 2n - 1\right]$$

$$= 2y dx - 2x dy + 2dy \Rightarrow 2n dx - dx$$

$$\begin{cases} f(x, y) = dx(d_1 - d_2) \\ dx(d_1 - d_2) = 2y dx - 2x dy + 2dy \Rightarrow 2n dx - dx \end{cases}$$

$$f(x, y) = 2y dx - 2x dy + 2dy \Rightarrow 2n dx - dx$$

90 5. Let $f(x_i, y_i)$ be the last point we draw and let $f(x_{i+1}, y_{i+1})$ be the next point to draw so the increment for the start point to the next point is $f(x_{i+1}, y_{i+1}) - f(x_i, y_i)$

$$f(x_i, y_i) = 2y_i dx - 2x_i dy + 2dy - 2n dx - dx$$

$$f(x_{i+1}, y_{i+1}) = 2y_{i+1} dx - 2x_{i+1} dy + 2dy - 2n dx - dx$$

$$\begin{aligned} f(x_{i+1}, y_{i+1}) - f(x_i, y_i) &= 2y_{i+1} dx - 2x_{i+1} dy + 2dy - 2n dx - dx - [2y_i dx - 2x_i dy + 2dy - 2n dx - dx] \\ &= 2y_{i+1} dx - 2x_{i+1} dy + 2dy - 2n dx - dx - 2y_i dx + 2x_i dy - 2dy + 2n dx + dx \\ &= 2y_{i+1} dx - 2x_{i+1} dy - 2y_i dx + 2x_i dy \\ &= 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i) \end{aligned}$$

6. For every iteration in x coordinate such that $x_{i+1} = x_i + 1$

$$\begin{aligned} \begin{cases} x_{i+1} = x_i + 1 \\ f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i) \\ = 2dx(y_{i+1} - y_i) - 2dy(x_{i+1} - x_i) \\ = 2dx(y_{i+1} - y_i) - 2dy \end{cases} \end{aligned}$$

7. If $d_1 < d_2$ then the next point of (x, y) should be (x_{i+1}, y_i)
so $y_{i+1} = y_i$

$$f(x_{i+1}, y_{i+1}) - f(x_i, y_i) = 2dx(y_i - y_i) - 2dy = -2dy$$

If $d_1 > d_2$ then the next point of (x, y) should be (x_{i+1}, y_{i-1})
so $y_{i+1} = y_i - 1$

$$\begin{aligned} f(x_{i+1}, y_{i+1}) - f(x_i, y_i) &= 2dx(y_i - 1 - y_i) - 2dy \\ &= -2dx - 2dy \end{aligned}$$

Brensenham line drawing Algorithm for negative slope (continued)

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8. Find out initial start point $f(x,y)$

Let's assume $f(x,y)$ start at point $(0,0)$

$$(A) f(x,y) = 2y dx - dx - 2x dy - dy - 2n dx - dx$$

$$(B) y = mx + n$$

$$(C) m = \frac{dy}{dx}$$

$$\begin{cases} y = mx + n \\ m = \frac{dy}{dx} \end{cases} \Rightarrow y = \left(\frac{dy}{dx}\right)x + n \Rightarrow \boxed{n = y - \left(\frac{dy}{dx}\right)x}$$

$$\begin{aligned} \boxed{f(x,y)} &= 2y dx - dx - 2x dy - dy - 2n dx - dx \\ &= 2y dx - dx - 2x dy - dy - 2\left[y - \left(\frac{dy}{dx}\right)x\right] dx - dx \\ &= 2y dx - dx - 2x dy - dy - 2y dx + 2\left(\frac{dy}{dx}\right)x dx - dx \\ &= \cancel{2y dx} - dx - 2x dy - dy - \cancel{2y dx} + 2x dy - dx \\ &= \boxed{-2dy - dx} \quad \boxed{-2dx - dy} \end{aligned}$$

Therefore, for $f(x,y) = -2dy - dx$
 $f(0,0) = -2dy - dx$

draw-bransham-line-negative (x_0, y_0, x_1, y_1)

$$dx = x_1 - x_0$$

$$dy = y_1 - y_0$$

$$d = -dx - 2dy$$

$$y = y_0$$

for ($x = x_0$; $x \leq x_1$; $++x$)

put pixel (x, y)

if ($d > 0$)

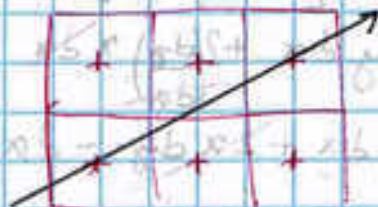
$$y = y - 1$$

$$d = d - 2dx - 2dy$$

else

$$d = d - 2dy$$

bransham line drawing for positive and negative slope



positive slope



negative slope

$$m = \frac{dy}{dx} = \frac{y_1 - y_0}{x_1 - x_0}$$

if $dy > 0$ and $dx > 0$ or $dy < 0$ and $dx < 0$
slope m is positive

if $dy < 0$ and $dx > 0$ or $dy > 0$ and $dx < 0$
then slope is negative

draw-bransham-line (x_0, y_0, x_1, y_1)

int dy

if $dy = y_1 - y_0$

$dx = x_1 - x_0$

if ($dy > 0 \&\& dx > 0$) || ($dy < 0 \&\& dx < 0$)

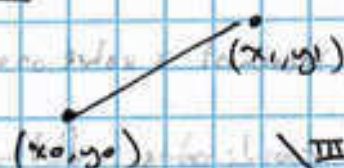
draw-bransham-line (x_0, y_0, x_1, y_1)

else

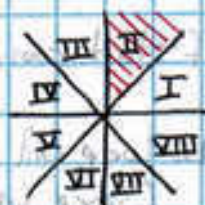
draw-bransham-line-negative (x_0, y_0, x_1, y_1)

Bresenham line drawing w/ octants

1. start point (x_0, y_0)
end point (x_1, y_1)

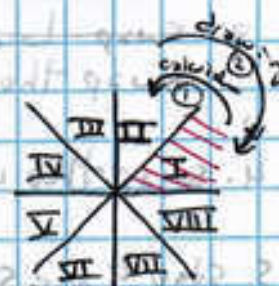


2. Bresenham line algorithm for positive and negative was for octant II



• Bresenham line drawing in First Octants

1. line needs to be transformed to the second octant for calculation and transformed back for drawing



- 2a. line is in the first octant * $dy > dx$

- 2b. Algorithm increment on each iteration and increment the value of y only if needed $(x, y) \Rightarrow (x+1, y+1)$
 $(x, y) \Rightarrow (x+1, y)$

- 2c. Algorithm cannot run when $dy > dx$ because loop will end before it get to last y value, because # of iteration in x -axis is shorter than

3. to overcome this we need to swap x and y transform the line to 2nd octant so $dy < dx$ but

4. However, when drawing x and y should be swapped back to represent real line.

swapped = 0;

if ($\text{abs}(y_1 - y_0) > \text{abs}(x_1 - x_0)$)

swap (x_0, y_0);

swap (x_1, y_1);

swapped = 1;

} calculation

if (!swapped) put Pixel (x, y)

else

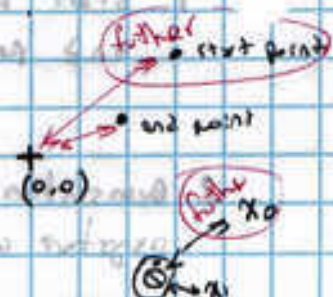
put Pixel (y, x)

} drawing

• Bresenham line drawing when start coordinates is further than end coordinate

1. The loop increment x value on each iteration
2. If start coordinates is further than the end coordinate

2a. loop will never start
(since we are looping from start point to end point)



3. ~~swap loop condition~~

swap the x values & keep incrementing the x value

4. swap the y values to match as well

5. step 4 and 5 obtain the same line w/
the starting point closer than the end point

6. All lines can be treated as lines in the
1st octant or 2nd Octant after many swaps
 x coordinate of the starting point is
closer to 0 (zero) than the x coordinate
of the end point end point

• Bresenham line drawing for Vertical and Horizontal lines

if ($x_0 = x_1$)

if ($y_0 > y_1$)

swap (y_0, y_1)

for ($y = y_0; y \leq y_1; y++$)

put pixel (x_0, y); return;

if ($y_0 = y_1$)

if ($x_0 > x_1$)

swap (x_0, x_1)

for ($x = x_0; x \leq x_1; x++$)

put pixel (x, y_0)

return;

• Optimizing Bresenham line Drawing

1. Notice that in the main loop $2dx$ and $2dy$ is used a lot

```
for (x = x0; x <= x1; x++)
```

```
  putPixel(x, y)
```

```
  if (d < 0)
```

```
    y = y + 1;
```

```
    d = d + 2dx - 2dy;
```

```
  else
```

```
    d = d - 2dy;
```

```
twoDx = 2dx << 1 // shift left
```

```
twoDy = 2dy << 1
```

```
for (x = x0; x <= x1; x++)
```

```
  putPixel(x, y)
```

```
  if (d < 0)
```

```
    y++;
```

```
    d = d + twoDx - twoDy;
```

```
  else
```

```
    d = d - twoDy;
```

```
for (x = x0; x <= x1; x++)
```

```
  putPixel(x, y)
```

```
  twoDy = 2dy << 1
```

```
  twoDx = 2dx << 1
```

```
  for (x = x0; x <= x1; x++)
```

```
    putPixel(x, y)
```

```
    d = twoDy
```

```
    if (d < 0)
```

```
      y++;
```

```
      d += twoDx;
```

2. We can improve the tests of negative slope

A. since we are dealing w/ signed integers we can use the most significant bit (MSB) to know if the value is positive or negative

B. if MSB is negative if MSB is set (1) then negative else positive

C. XORing dx with dy would set the MSB if dx and dy is negative

D. masking it with $0xB0000000$ will zero all bits except the MSB

AB	Z
00	0
01	1
10	1
11	0

is Negative Slope = $(dy > 0 \& \& dx > 0) \vee (dy < 0 \& \& dx < 0)$

is Negative Slope = $(dx \wedge dy) \& 0xB0000000$