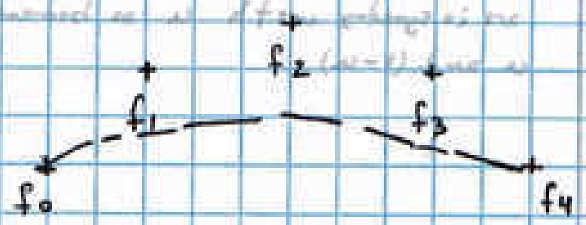
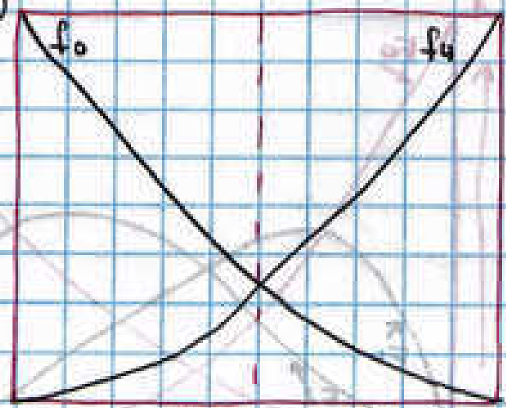
$n = 4$ $\quad f_0, f_1, \dots f_4$

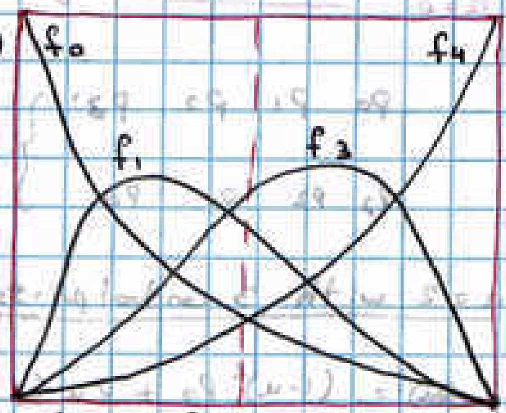


①



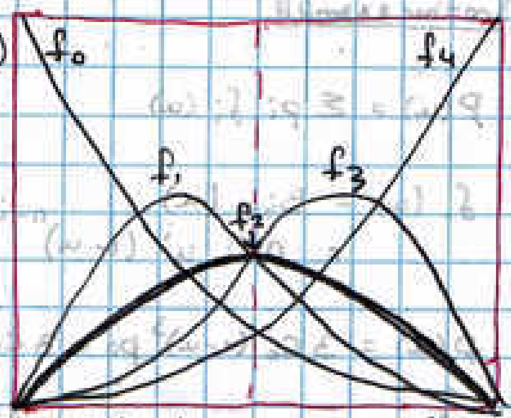
(f_0 and f_4 are symetric)

②



(f_1 and f_3 are also symetric)

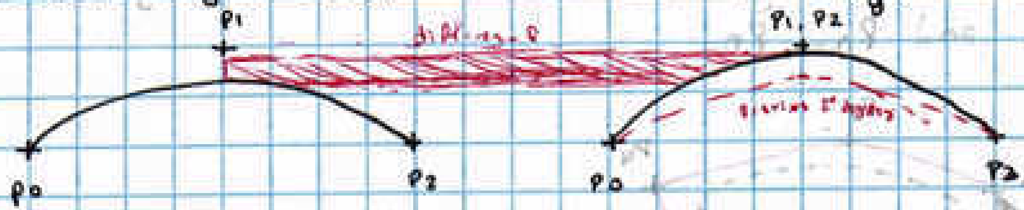
③



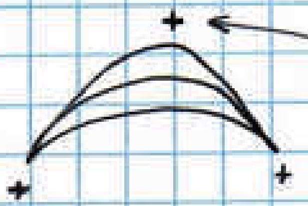
- All functions are symmetric to respect the center
- None of those functions would be zero except at their end points

- The curve chang base on the number of points (degree) in the same place.
  example

→ second degree curve → Third degree curve



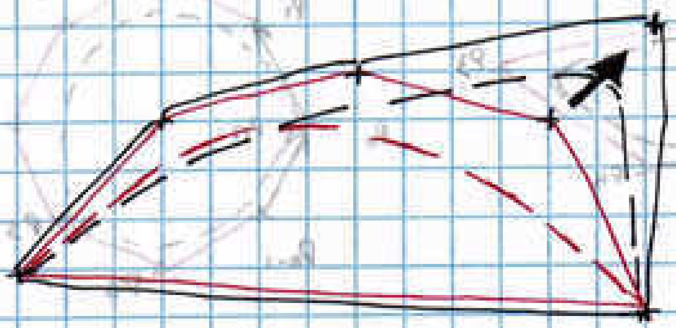
- The more points in the same place the sharper the curve will be.



So instead of moving this point, we can add points (degree) to the curve to sharpe the curve.

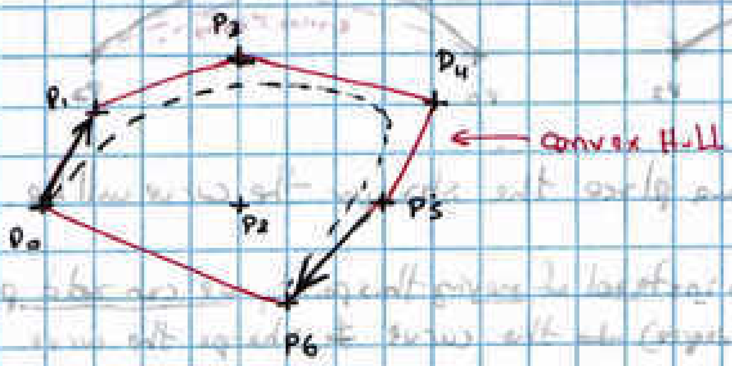
- If we have for example a polygon made by the points our curve would be inside this polygon ~~and~~

- If we would modify the polygon, our curve would be modified to.

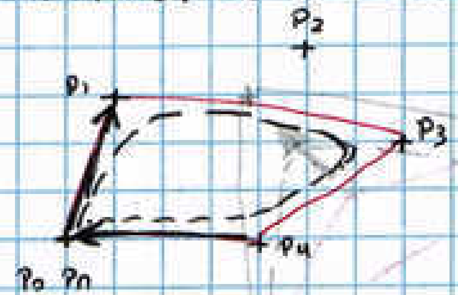


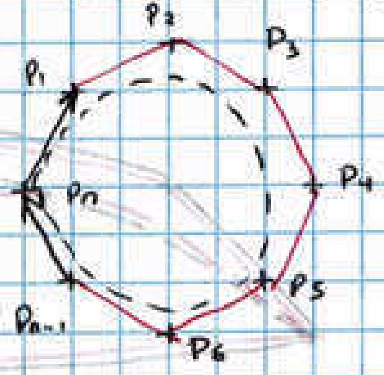
Properties

- End points of the curve are p_0 and p_n
- Slope of the tangent is the same as the line segments p_0, p_1 and $p_{n-1} p_n$

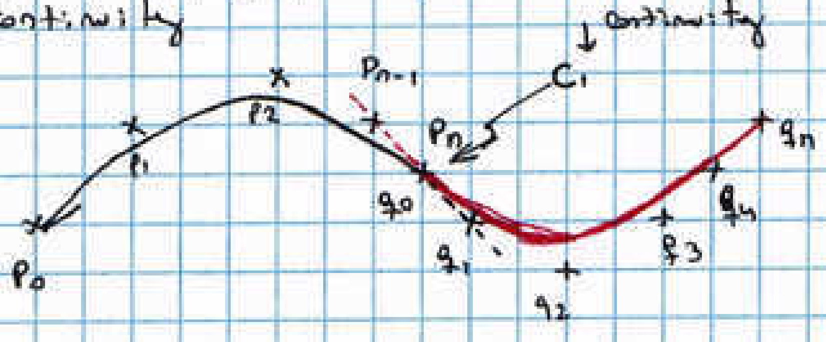


- Curves are contained in the convex hull
- They never oscilate wildly
- No slopes are implicit
- If p_0 and p_n are the same we have a curve that is be close





- If p_{n-1}, p_n, q_0, and q_1 are in line, then there are a continuity

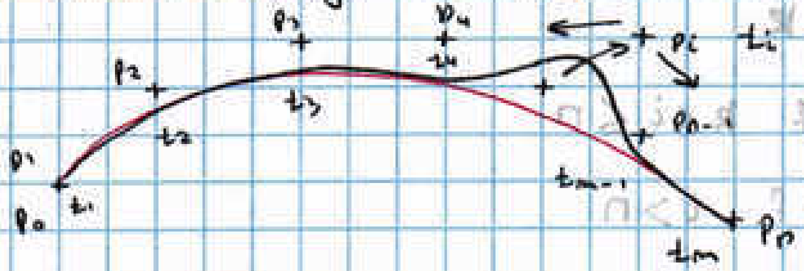


- Continuity C_k is defined by this points in line between curves the end points of previous curve p_{n-1} and p_n with the start points of next curve q_0 and q_1.

B-Spline Curve

- ~~B-spline~~ other curves have a property called "global control" which stablish that if ~~one point~~ control point is moved, the whole curve is affected.

- what we wish is "local control" in which the change of a point do not change the rests of the curve



- the question is In which part the point p_i affect the curve

$$p(u) = \sum p_i \underbrace{N_{i,k}(u)}_{\text{not depend of } n \text{ but depend of } k} \quad \Big| \quad \sum p_i \underbrace{B_{i,n}(u)}_{\text{depend of } n \text{ and } i}$$

$k-1 \leftarrow$ degree of the curve

- this mean the degree of ~~N_i~~ do not depend of n

- $N_{i,1}(u) = 1$ if $t_i \le u < t_{i+1}$

- $N_{i,1}(u) = 0$ otherwise

$u = \boxed{0}$ ~~now there we not~~

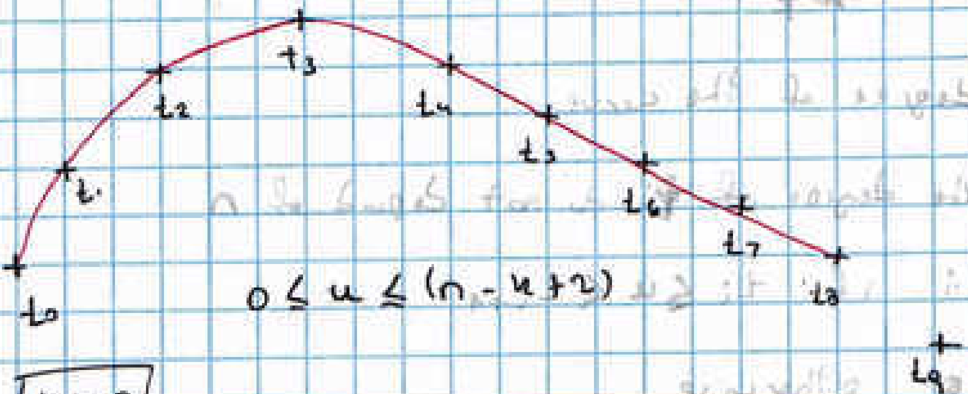
- Now u is not between 0 to 1 but between $0 \to (n-k+2)$

t_m

$u = \cancel{\boxed{1.0}} \leftarrow$ not

$u = (n-k+2) \leftarrow$ yes

- $t_i = 0$ if $i < k$
- $t_i = i - k + 1$ if $k \le i \le n$
- $t_i = n - k + 2$ if $i > n$
- $0 \le u \le (n-k+2)$
- ~~Letting n = 10 and~~
- Lets say we have 10 points and $k = 3$ then the last value would be u between $0 \to 9$



$0 \le u \le (n - k + 2)$

$\boxed{u = 0}$

$\boxed{u} = n - k + 2$
$= 10 - 3 + 2$
$= 7 + 2$
$= \boxed{9}$

- $n = 5$ $\quad$ $\overset{d}{\downarrow}$ $k = 1$

$t_0 = 0$ $\quad (i < k)$

$t_1 = 1$ $\quad (k \leq i \leq n)$

$t_2 = 2$ $\quad (k \leq i \leq n)$

$t_3 = 3$ $\quad (k \leq i \leq n)$

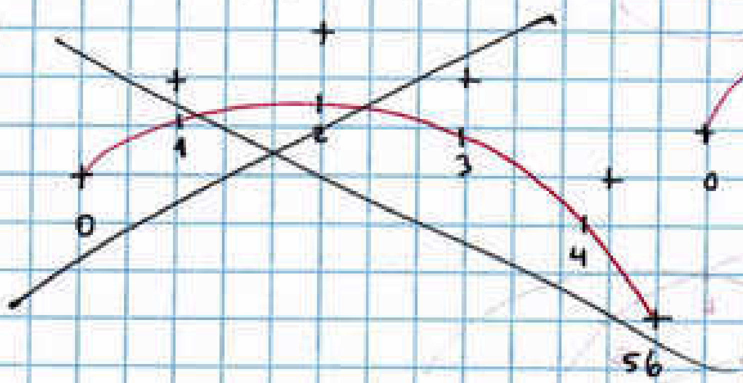
$t_4 = 4$ $\quad (k \leq i \leq n)$

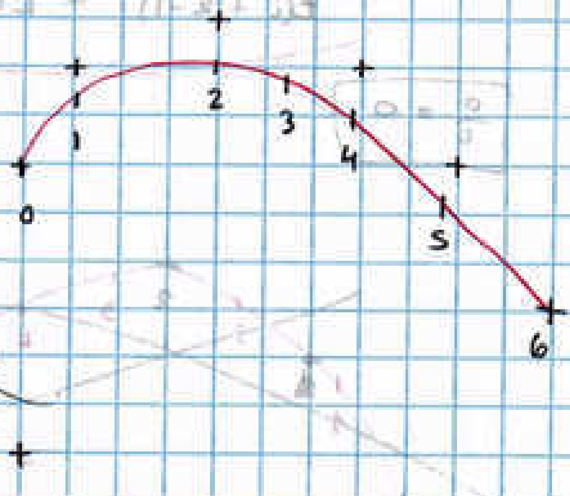
$t_5 = 5$ $\quad (k \leq i \leq n)$

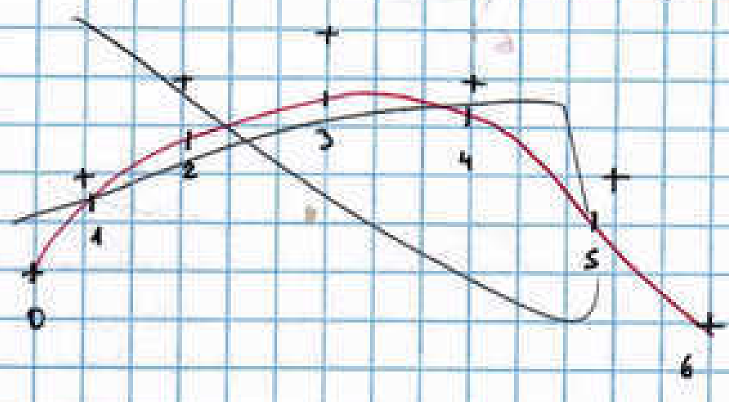
$t_6 = 6$ $\quad (i > n)$

- $t_i = 0$ if $i < k$
- $t_i = i - k + 1$ if $k \leq i \leq n$
- $t_i = i - k + 2$ if $i > n$

by $n = 5$ we know that we have six control points:

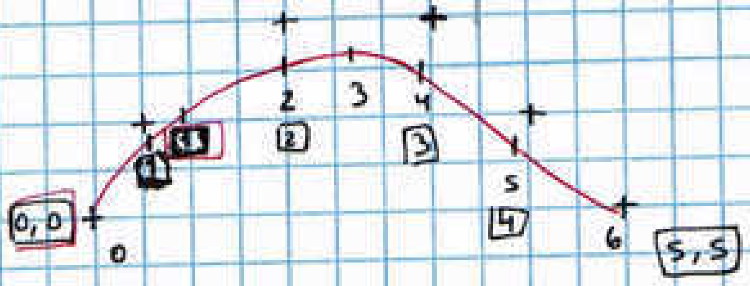






Let say $n=5$ and $k=2$ the [illegible]

$t_i's = 0, 0, 1, 2, 3, 4, 5, 5$

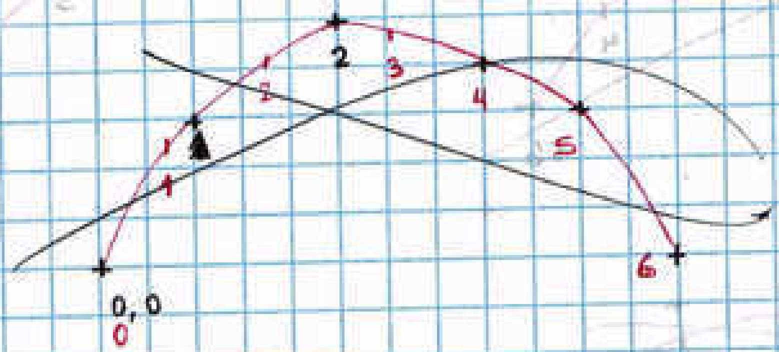


- to define $N_{i,k}$

Same [illegible] $\frac{0}{0}=0$

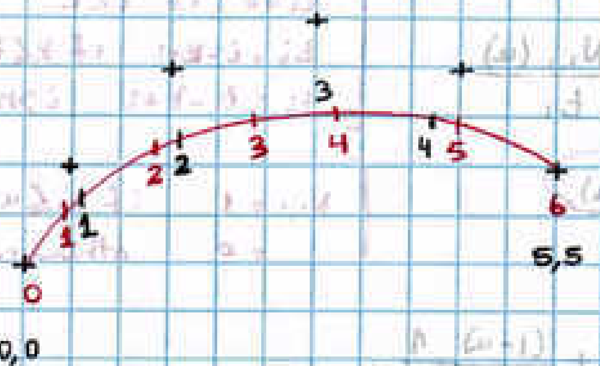
$$N_{i,k} = \frac{(u-t_i)\, N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k}-u)\, N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}}$$

$$\boxed{\frac{0}{0} = 0}$$



between: t_{i+k} , p_i , t_{i-k}

$n=5 \quad k=1$



$N_{i,1}(u) = 1$ if $t_i \leq u < t_{i+1}$

$= 0$ otherwise

$t_i = 0$ if $i < k$

$t_i = i-k+1$ if $k \leq i \leq n$

$t_i = n-k+2$ if $i > n$

$\boxed{n=5 \quad k=1}$

$t_0 = 0$

$t_1 = 1$

$t_2 = 2$

$N_{0,1} = 1 \quad 0 \leq u < 1$

$\quad\quad 0$ otherwise

$N_{1,1} = 1 \quad 1 \leq u < 2$

$\quad\quad 0$ otherwise

$N_{2,1} = 1 \quad 2 \leq u < 3$

$\quad\quad 0$ otherwise

$N_{3,1} = 1 \quad 3 \leq u < 4$

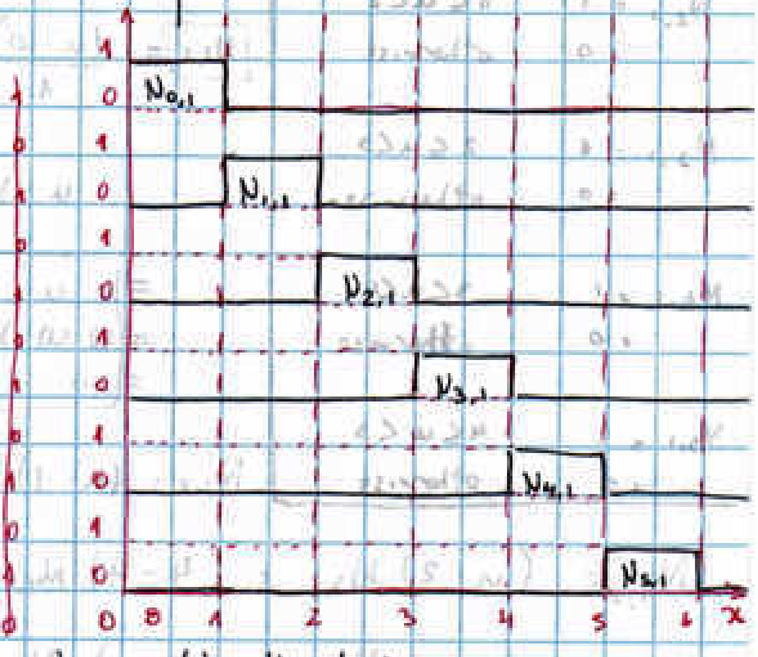
$\quad\quad 0$ otherwise

$N_{4,1} = 1 \quad 4 \leq u < 5$

$\quad\quad 0$ otherwise

$N_{5,1} = 1 \quad 5 \leq u < 6$

$\quad\quad 0$ otherwise



No need to define any more functions after this last one so there is no need to modify after n+1

$0 \leq u \leq 1 \quad p(u) = p_0$ (~~between 0→1~~)

$\boxed{\begin{array}{l} p(u) = \sum p_i N_{i,k}(u) \\ k-1 \rightarrow \text{degree of the curve} \end{array}}$

180

depends of ... and

$$N_{i,k} = \frac{(u - t_i)\cdot N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)\, N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}}$$

• $n = 5$, $k = 2$ $\qquad t_i$'s $= 0, 0, 1, 2, 3, 4, 5, \cancel{0, 0}$ 5

$t_i = 0$ if $i < k$
$t_i = i - k + 1$ if $k \le i \le n$
$t_i = n - k + 2$ $\quad i > n$

$N_{i,1} = 1$ if $t_i \le u < t_{i+1}$
$\quad = 0$ otherwise

$$\boxed{N_{0,2}} = \frac{(u - t_0)\, N_{0,1}(u)}{t_1 - t_0} + \frac{(t_2 - u)\, N_{1,1}(u)}{t_2 - t_1}$$

$$= \frac{(u - 0)\cdot N_{0,1}(u)}{0} + \frac{(1 - u)\cdot N_{1,1}(u)}{1}$$

$$= \cancel{\frac{(u-0)\cdot 0}{0}} + \frac{(1-u)\, 1}{1}$$

$$= \boxed{(1 - u) \quad 0 \le u < 1}$$

$$= \boxed{0 \quad \text{otherwise}}$$

$N_{0,1} = 1$ if $u = 0$
$\quad = 0$ otherwise

$N_{1,1} = 1$ if $0 \le u < 1$
$\quad 0$ otherwise

$N_{2,1} = 1$ $\quad 1 \le u < 2$
$\quad = 0$ otherwise

$N_{3,1} = 1$ $\quad 2 \le u < 3$
$\quad = 0$ otherwise

$N_{4,1} = 1$ $\quad 3 \le u < 4$
$\quad = 0$ otherwise

$N_{5,1} = 1$ $\quad 4 \le u < 5$
$\quad = 0$ otherwise

$$\boxed{N_{1,2}} = \frac{(u - 0)}{1} N_{1,1} + \frac{(2 - u)}{1} N_{2,1}$$

$$= u\, N_{1,1} + (2 - u)\, N_{2,1}$$

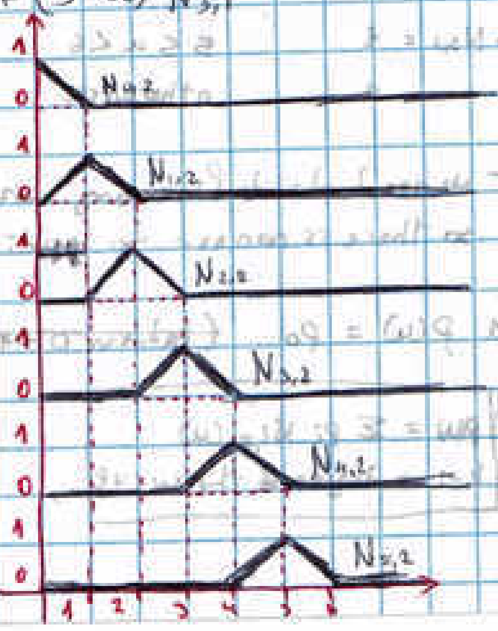
$= u\, N_{1,1}$ $\quad 0 \le u < 1$
$= (2 - u)\, N_{2,1}$ $\quad 1 \le u < 1$
$= 0$ $\quad$ otherwise

$N_{2,2} = (u - 1)\, N_{2,1} + (3 - u)\, N_{3,1}$

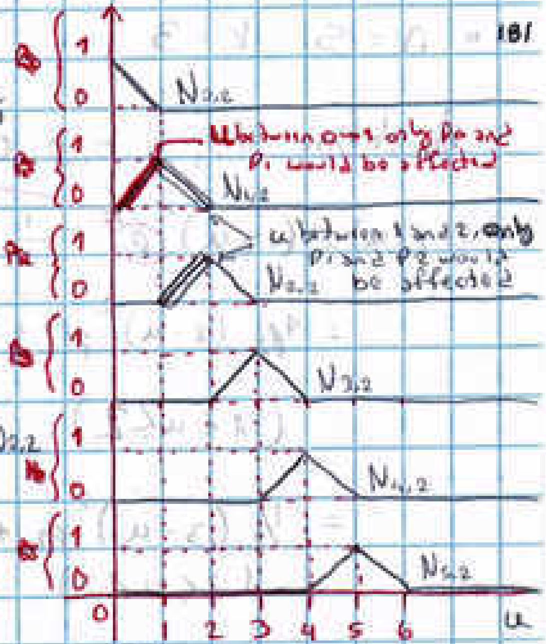
$N_{3,2} = (u - 2)\, N_{3,1} + (4 - u)\, N_{4,1}$

$N_{4,2} = (u - 3)\, N_{4,1} + (5 - u)\, N_{5,1}$

$N_{5,2} = (u - 4)\, N_{5,1}$



In this graphic, we can notice that at any point only two points are affected at a time. Example:



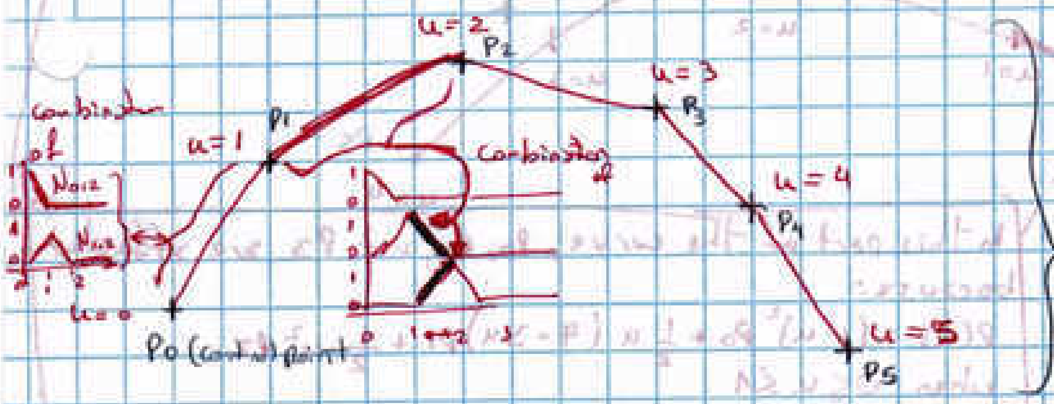
- Between (u) 0 → 1 (x-axis), in $N_{1,2}$, P_0 and P_1 would have effect

- Between u 1 → 2 (x-axis), in $N_{1,2}$ and $N_{2,2}$, P_1 and P_2 would have effect

$\boxed{\text{• At any point of the curve, two points \& } P_0 \to P_n \text{ are affected.}}$

- Between 2 → u → 3 (x axis), P_2 and P_3 would have effect

- These means that if we have a set of points the curve in this case would look like a set of straight lines



$n = 5$
$K = 2$

↳ linear continuity w/ linear functions

- next $n = 5$, $K = 3$

Continued ⇒

$n = 5 \quad K = 3$

1) define t_i's $= 0, 0, 0, 1, 2, 3, 4, 4, 4$ (nodes values)

affected when $0 \leq u \leq 1$

2) $p(u) = (1-u)^2 p_0 + \frac{1}{2} u\,(4-3u) p_1 + \frac{1}{2} u^2 p_2 \qquad (0 \leq u \leq 1)$

$= \frac{1}{2}(2-u)^2 p_1 + \frac{1}{2}(-2u^2+6u-3) p_2 + \frac{1}{2}(u-1)^3 p_3$

$(1 \leq u \leq 2)$

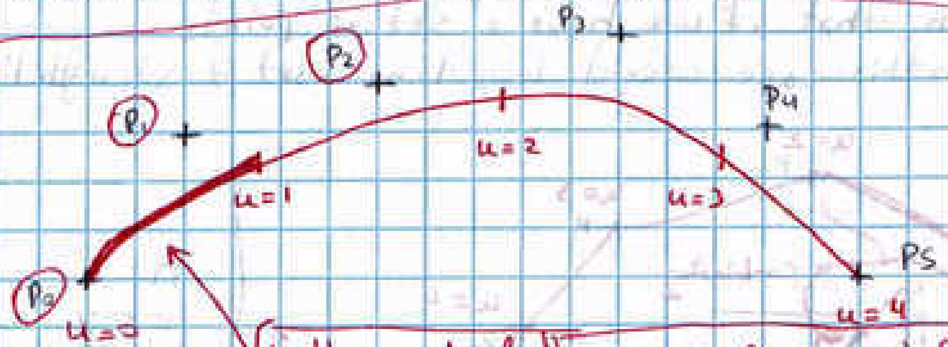
$= \frac{1}{2}(3-u)^2 p_2 + \frac{1}{2}(-2u^2+10u-11) p_3 + \frac{1}{2}(u-2)^2 p_4$

$(2 \leq u \leq 3)$

$= \frac{1}{2}(4-u)^2 p_3 + \frac{1}{2}(-2u^2+20u-32) p_4 + \frac{1}{2}(u-3)^2 p_5$

$(3 \leq u \leq 4)$

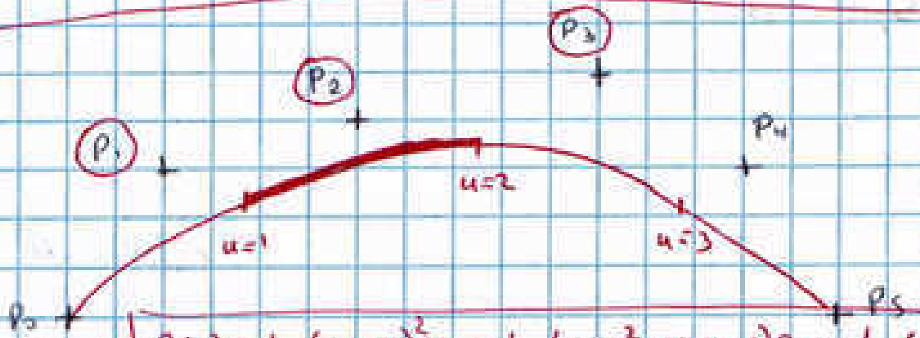
3) This would mean that if we have a curve as such:



In this part of the curve, p_0, p_1, and p_3 are affected because:

$p(u) = (1-u)^2 p_0 + \frac{1}{2} u (4-3u) p_1 + \frac{1}{2} u^2 p_2$

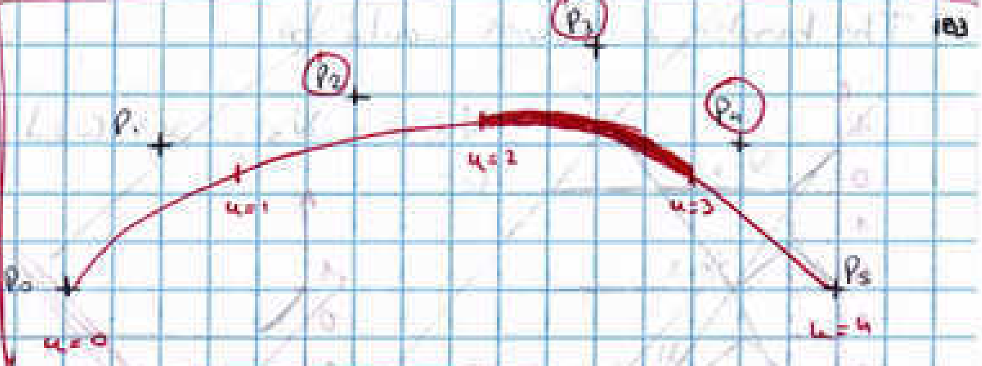
when $0 \leq u \leq 1$



$p(u) = \frac{1}{2}(2-u)^2 p_1 + \frac{1}{2}(-2u^2+6u-3) p_2 + \frac{1}{2}(u-1)^3 p_3$

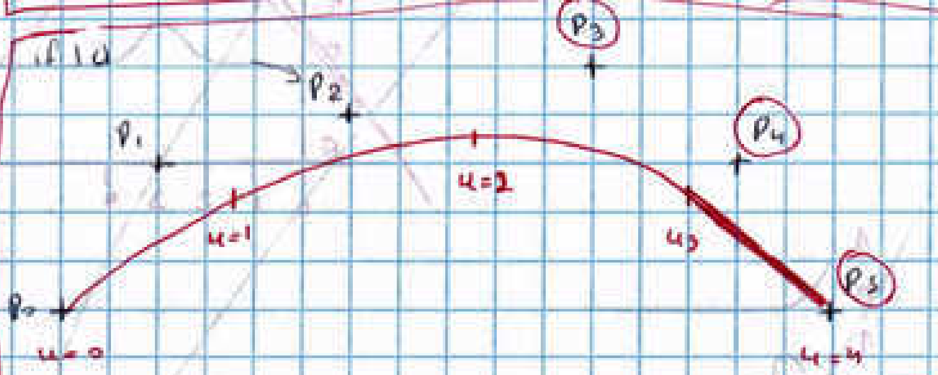
when $1 \leq u \leq 2$

3) Continued



$$P(u) = \frac{1}{2}(3-u)^2 p_2 + \frac{1}{2}(-2u^2+10u-11)p_3 + \frac{1}{2}(u-2)^2 p_4$$

when $2 \leq u \leq 3$

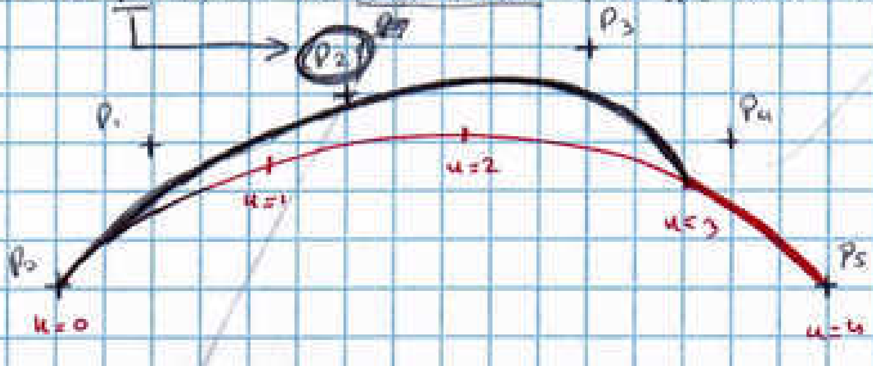


• If for example, I would change P2 of position, P3, P4, P5 would not change, therefore the last segment of the curve would not change either.

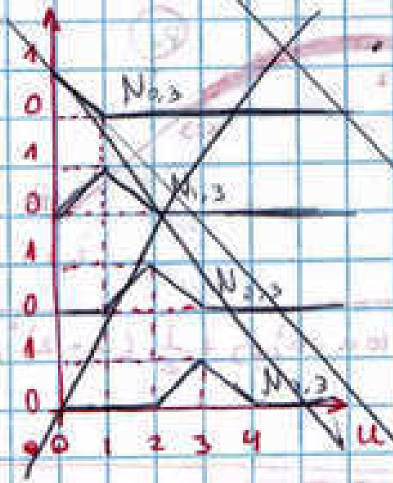
$$P(u) = \frac{1}{2}(4-u)^2 p_3 + \frac{1}{2}(-2u^2+20u-32)p_4 + \frac{1}{2}(u-3)^2 p_5$$

when $3 \leq u \leq 4$

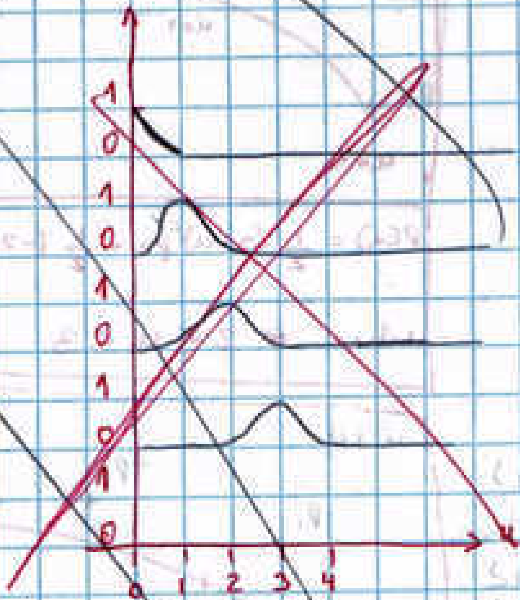
• If P2 would be modified then the curve would change:

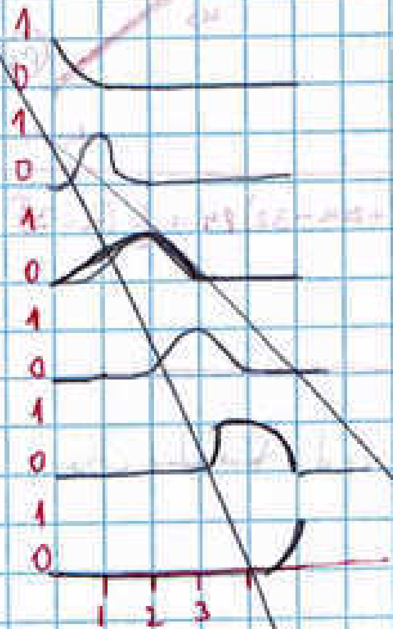


10. The blending functions would be:



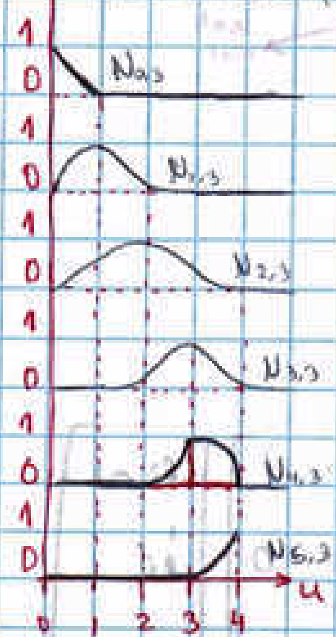
• $N_{0,3} \rightarrow N_{3,3}$ are quadratic functions so:





· The Blending functions are quadratic functions that would create the following graph:

$n = 5,\ k = 3$ (2^{nd} degree)



· If we increase the number of points (n) we can expect an increase of complexity

→ For n any given value and $k = 3$

~~$p(u) = \frac{1}{2}(i+1-u)^2 p_i + \frac{1}{2}\begin{bmatrix}(u-i+1)(i+1-u)\\(i+2-u)(u-i)\end{bmatrix}$~~

$-u^2 + u - u + 1 +$
$+2u - u^2 = -2u^2 + 2u + 1$

$$p(u) = \frac{1}{2}(i+1-u)^2 p_i + \frac{1}{2}\{[(u-i+1)(i+1-u)] + [(i+2-u)(u-i)]\} p_{i+1} + \frac{1}{2}(u-i)^2 p_{i+2}$$

$$(i \le u < i+1) \ \& \ (k \le i \le n)$$

· $u' = u - i$ $\quad \boxed{(u'^2 - 2u' + 1)}$ $\quad \boxed{-2u'?}$

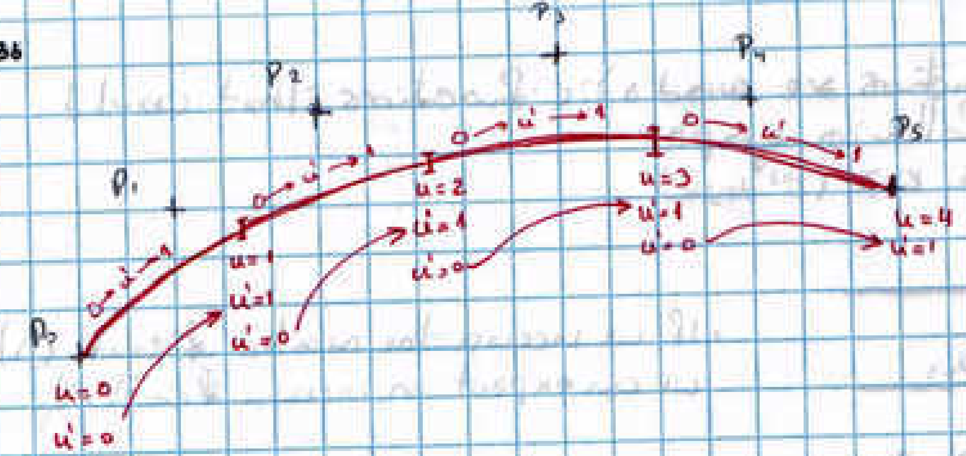
$$p(u') = \frac{1}{2}\left[(1-u')^2 p_i + (-2u'^2 + 2u' + 1) p_{i+1} + u'^2 p_{i+2}\right] \quad (0 \le u' < 1)$$

~~$p(u') = \frac{1}{2}[u^2 \quad u]$~~

$$p(u') = \underbrace{\frac{1}{2}\begin{bmatrix}u^2 & u & 1\end{bmatrix}}\begin{bmatrix}1 & -2 & 1\\-2 & +2 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}p_{i-1}\\p_i\\p_{i+1}\end{bmatrix}$$

geometry matrix

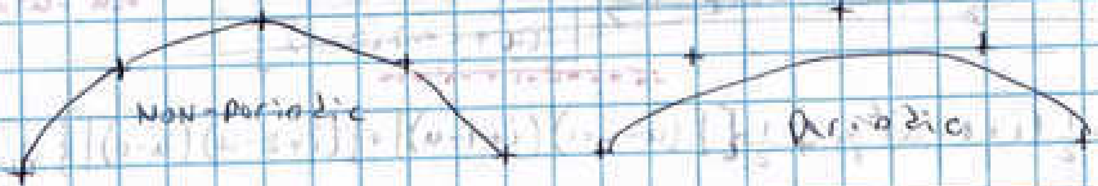
$= U_K\, M_K\, P_K$



if for example k = 4 then:

$$P(u) = u_k \; M_k \; P_k$$

$$= \frac{1}{6}\begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & 6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{bmatrix}$$



- Before we learned that for $k=3$

$$P(u) = \frac{1}{2}\begin{bmatrix} u^2 & u & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \end{bmatrix}$$ where $i \in [1, n-1]$ and $k=3$

- This means a set of control points in which we have a curve created by these set of control points, three of these points could define a segment of the curve

- This formula is for a periodic B-splines

- for $k=4$ $i \in [1, n-2]$

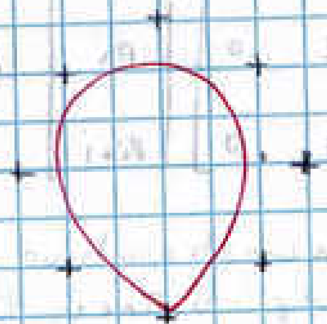
$$P(u) = \frac{1}{6}\begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & 6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix}$$ for $i \in [1, n-2]$ & $k=4$

→ Therefore as a general rule for any k:

$$\boxed{i \in [1, n-k+2]}$$

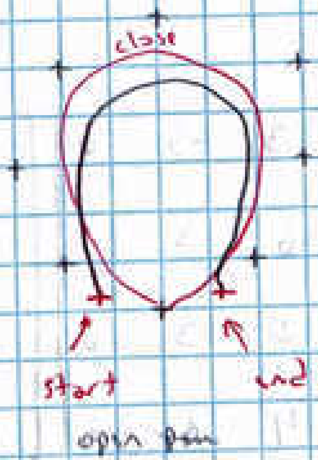
Let assume we have a circle defined by curves:

close Periodic



If we define another curve starts non-Periodic

(to differentiate close curves to open curves)



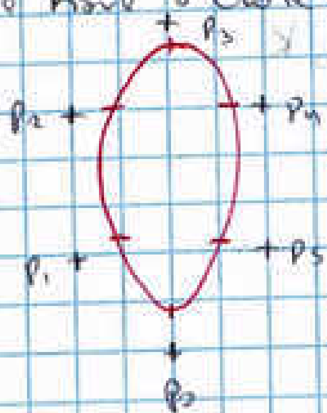
So we need to take three points in a cycle format

$p_{i-1} \mod (n+1)$

$p_i \mod (n+1)$

$p_{i+1} \mod (n+1)$

. So let say we have a close curve, with $n = 5$



$n = 5$

six control segments

$p_0 - p_3$

$p_1 - p_4$

$p_2 - p_5$

$p_3 \to p_6 \mod 6 = p_0$

$p_4 \to p_7 \mod 6$

$p_5 \to p_8 \mod 6$

So $u = 3$ $i \in [1, n+1]$

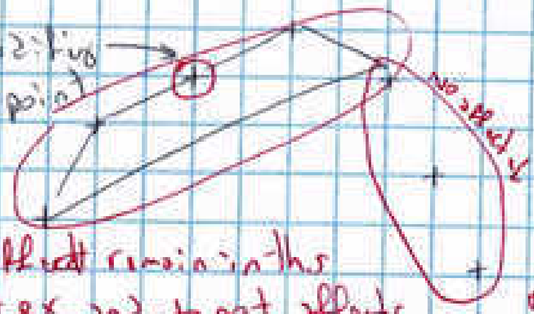
- NON Periodic

Properties

1. local control (true for all close, open, non periodic & periodic curves)
2. Contained within convex hull
3. Do not oscillate wildly

if modifying this point

then



the affect remain in this context and do not affects this other