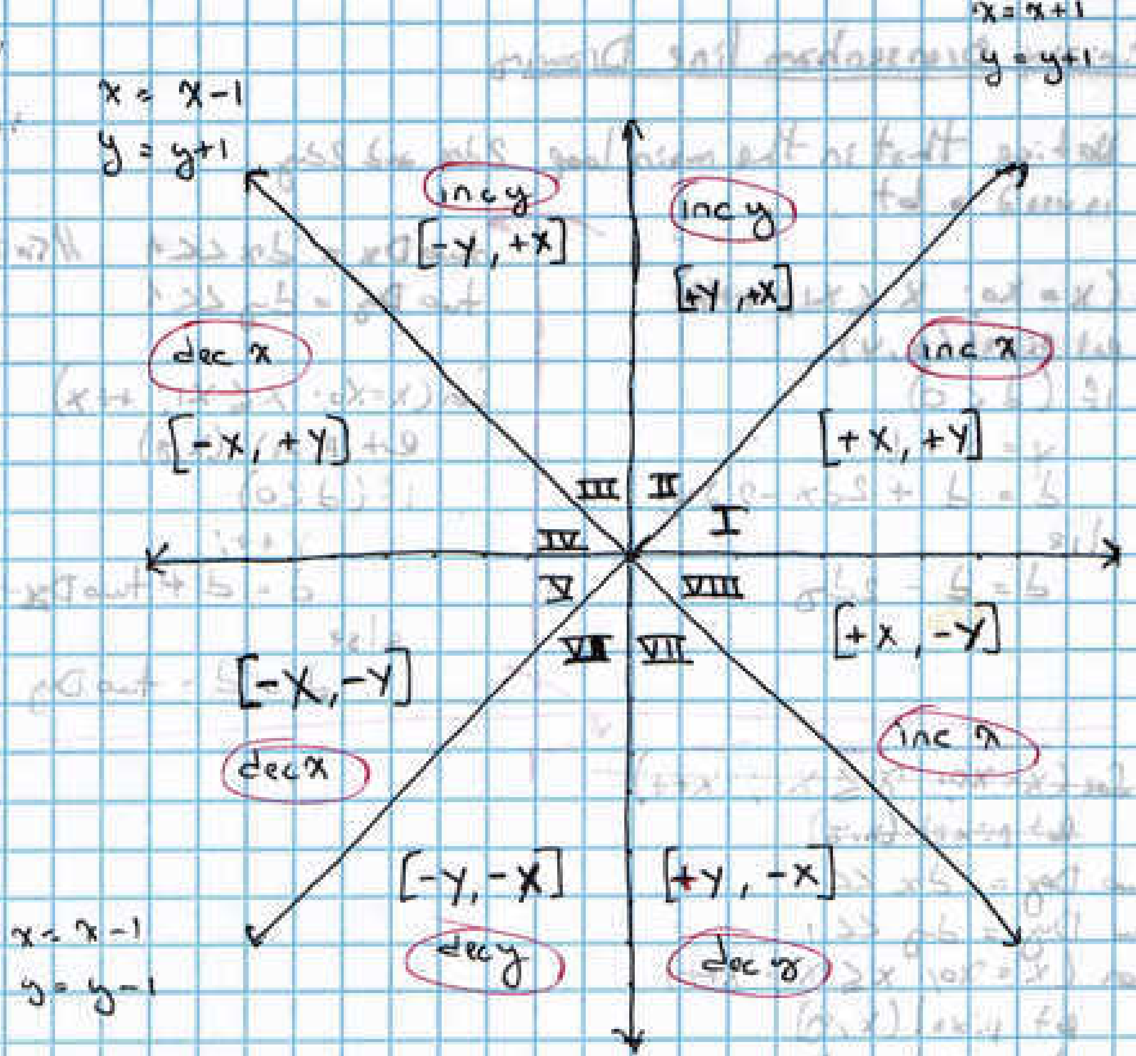

octant	dx dy Ratio	slope	swapping	Correct swapping
I	dy > dx	+	Yes. No swapping	
II	dy < dx	+	No swapping	
III	dy < dx	−		
IV	dy > dx	−		
V	dy > dx	+	No swapping	
VI	dy < dx	+		
VII	dy < dx	−		
VIII	dy > dx	−		

is_negative_slope && d > 0 || !is_negative_slope && d < 0

0	0		0	0
0	0		0	1
1	0		1	0
1	1		1	1

neg =	0	0	$d \leq 0$
	0	1	$d > 0$
	1	0	$d \leq 0$
neg	1	1	$d > 0$

Neg	d > 0 ($d \leq 0$)	d < 0 ($d \geq 0$)
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

neg & d>0
!neg & d<0

A && B ||
$\bar{A}$ && C

$AB + \bar{A}C$

neg	d > 0	d < 0
0	0	1
1	1	0

1

isnegativeslope (d & 0x8000 000)

(is_negative && d>0) ||

!is_negative && d<0)

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$d == 0$

neg = true

$d > 0$ = true

neg = ~~true~~ false

$(d<0)$.true

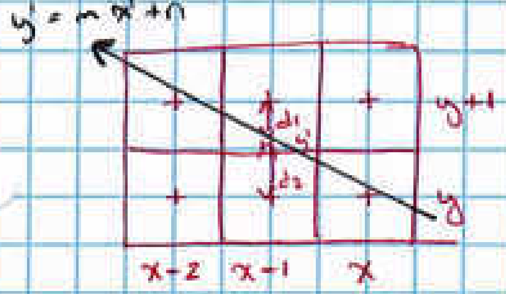
neg	$d>0$	$d<0$	$d=0$
1	1	0	0
0	0	1	0

$d<0$

$\boxed{-}$

$d>0 \;||\; d=0$

$\boxed{+}$



$$d_1 = y + 1 - y'$$

$$d_2 = y' - y$$

$$d_1 - d_2 = y + 1 - y' - y' + y$$

$$= 2y + 1 - 2y'$$

$$f(x,y) = d_1 - d_2$$

$$\begin{cases} f(x,y) = 2y + 1 - 2y' = 2y + 1 - 2(mx' + n) \\ y' = mx' + n \end{cases}$$

$$= 2y + 1 - 2mx' + 2n$$

$$= -2$$

$$\begin{cases} f(x,y) = 2y + 1 - 2mx' + 2n = 2y + 1 - 2(x-1)m + 2n \\ x' = x - 1 \end{cases}$$

$$= 2y + 1 - 2xm - 2m + 2n$$

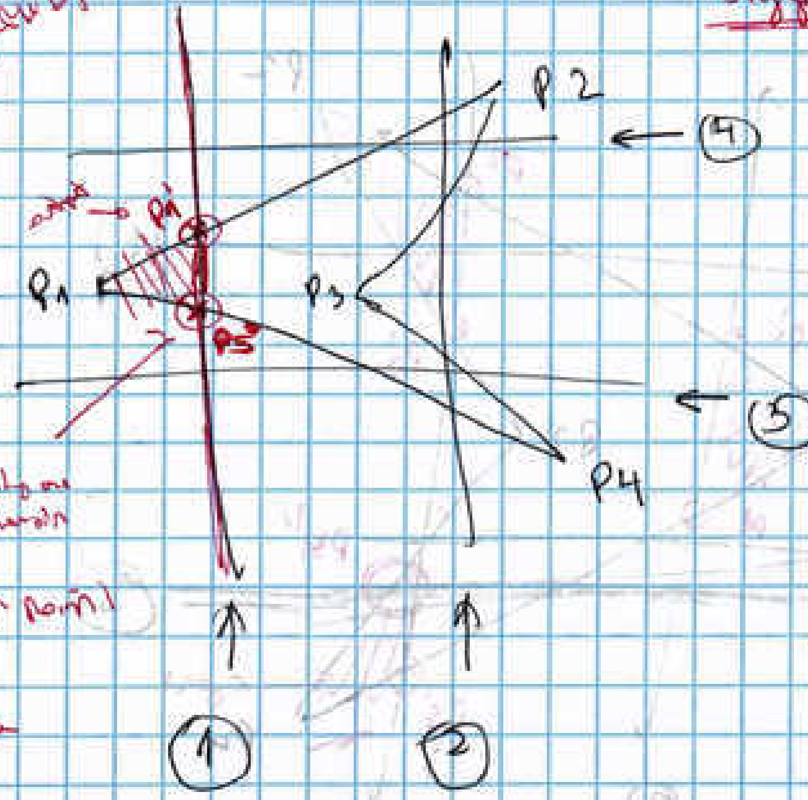
$$\begin{cases} f(x,y) = 2y + 1 - 2xm - 2m + 2n = 2y + 1 - 2\frac{dy}{dx}x - 2\frac{dy}{dx} + 2n \\ m = \frac{dy}{dx} \end{cases}$$

$$d_1 - d_2 = 2y + 1 - 2\frac{dy}{dx}x - 2\frac{dy}{dx} + 2n$$

$$\left[dx(d_1 - d_2)\right] = 2y + 1 - 2dyx - 2dy + 2n$$

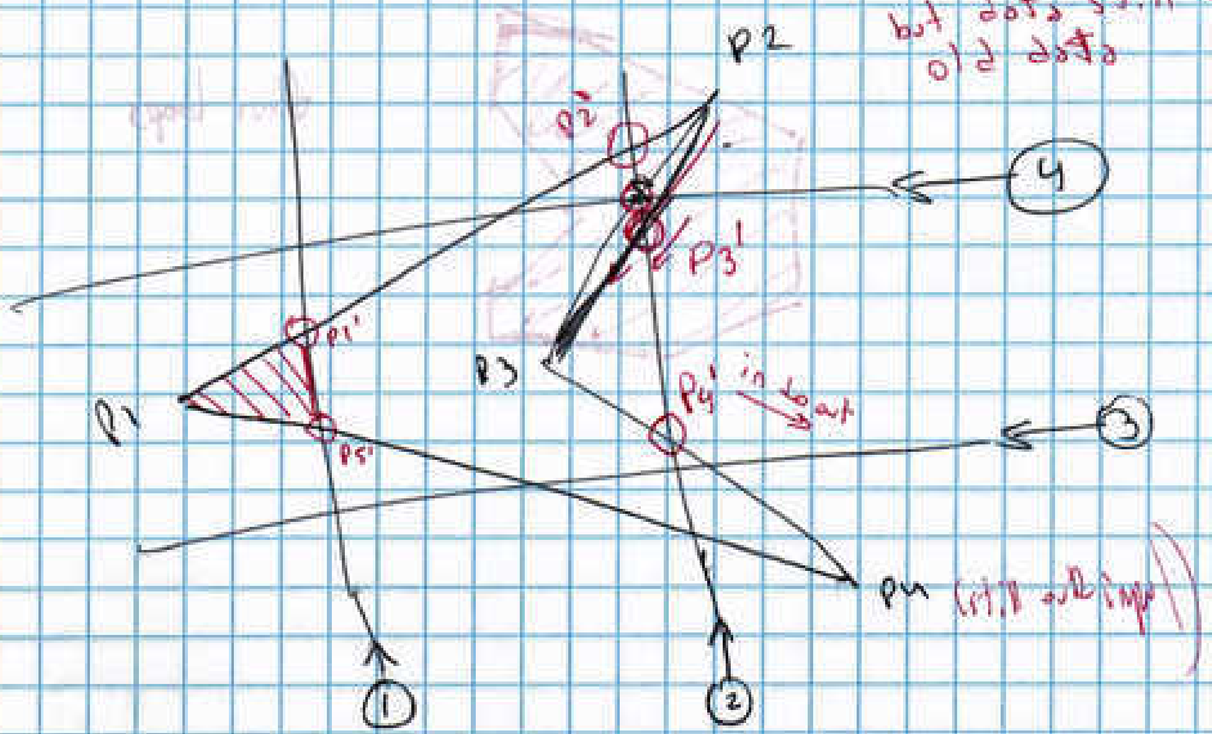
Polygon Clipping

↳ code on board



intersection point
↳ slope
↳ region

update P2' to P2'
but data still the
old data



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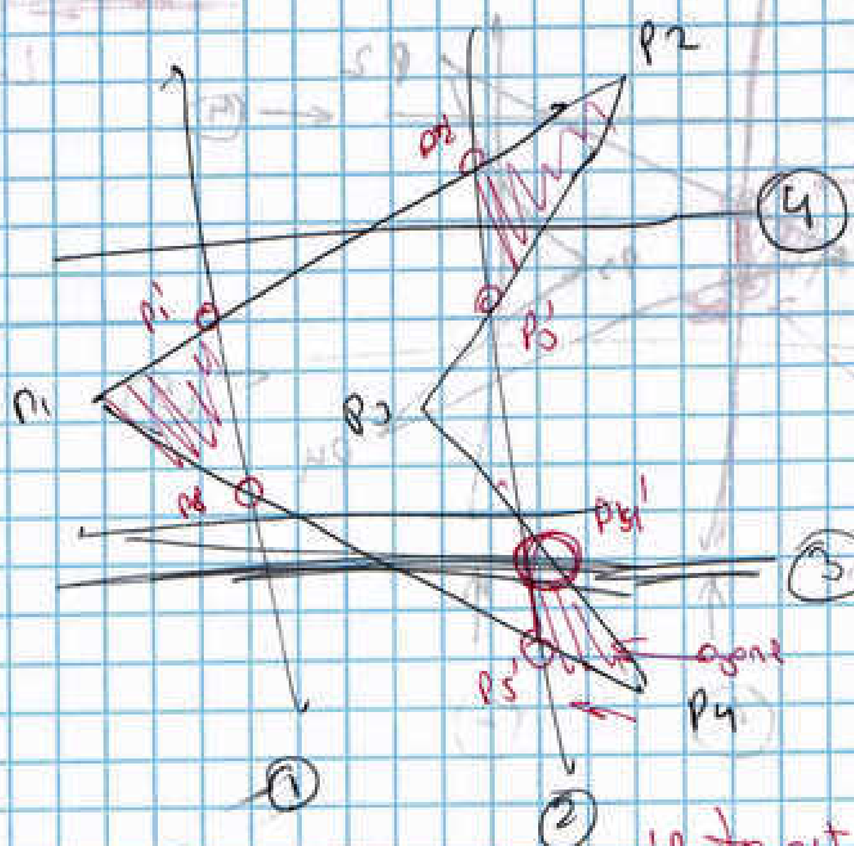
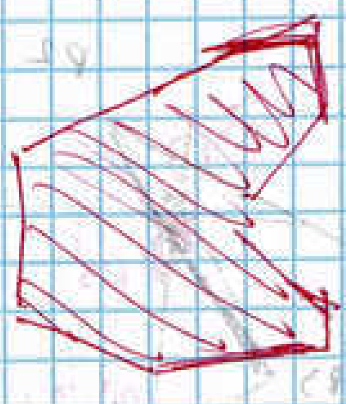


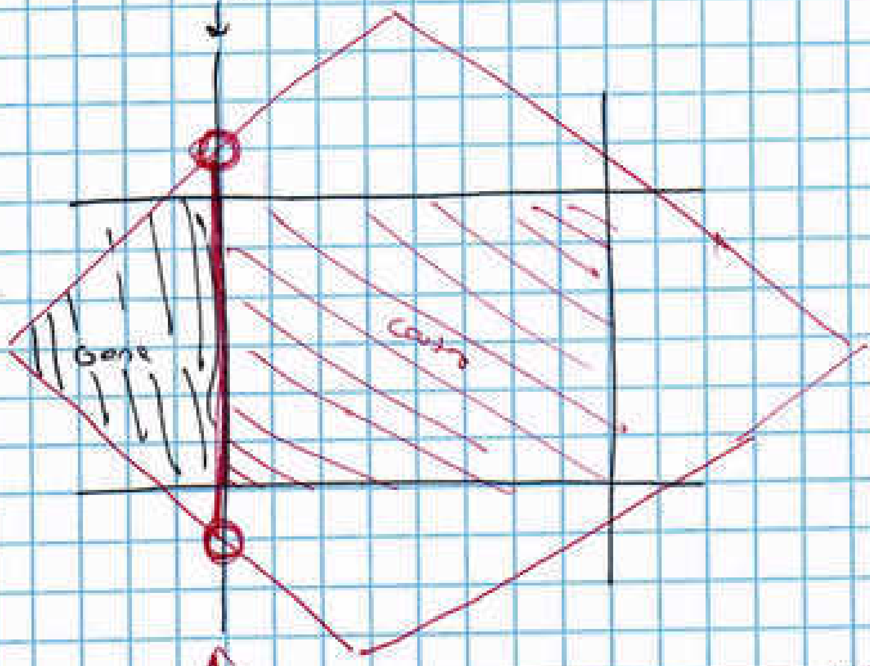
in to out



four loops

Sutherland-Hodgman

Exercise



clipping contain the outside boundary of the clipping

this case only 4 parts

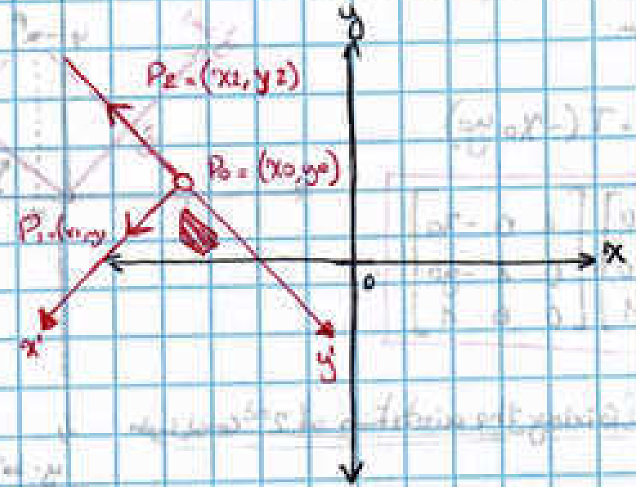
One line against one plain instead of one plain against one line

2D Transformation, Clipping, and filling

- 2D Translation
- clipping
- filling
- odd-rule parity
- Polygon clipping
- Convex polygon
- Sutherland - Hodgman Algorithm
- Weiler - Atherton Algorithm
- General Scan-line polygon - fill Algorithm
- odd - even segments
- ~~Weiler~~ Region filling

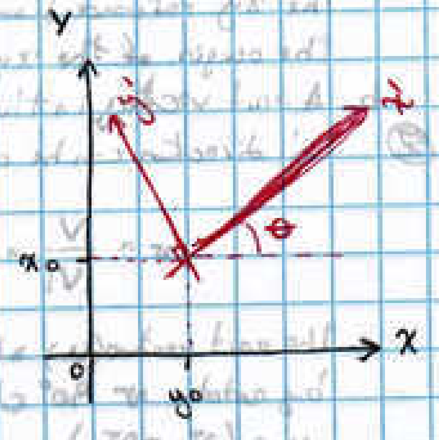
Problem

1) Given two perpendicular vectors P_0P_1 and P_0P_2:
   a. Derive a transformation matrix which can transform object destination from xy coordinate system to x'y' coordinate system



Theory: Transformation Between coordinate systems

Here we show two Cartesian Systems:
1) the coordinate origin at (0,0) and (x_0, y_0) w/ orientation angle θ between x and x' axes.
2) to transform object description from xy to x'y'
   a) set up a transformation that superimposes x'y' axes onto xy axes. two steps
      i) translate so that origin (x_0, y_0) of x'y' is moved to the origin of xy
      ii) Rotate the x' axis onto the x axis



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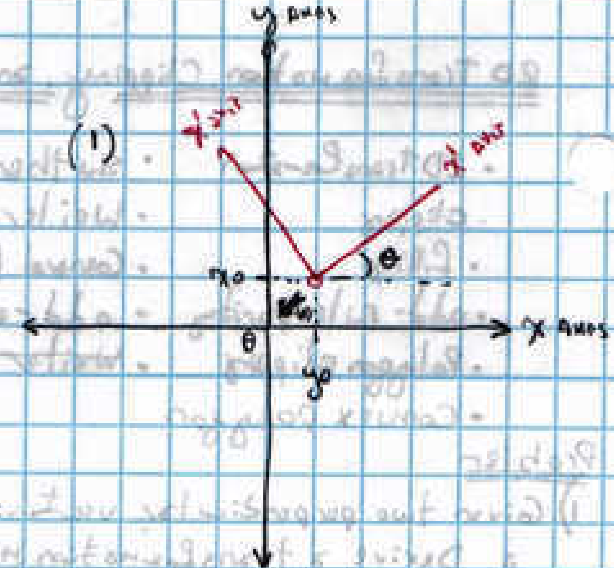
④ Transaction of coordinate origin

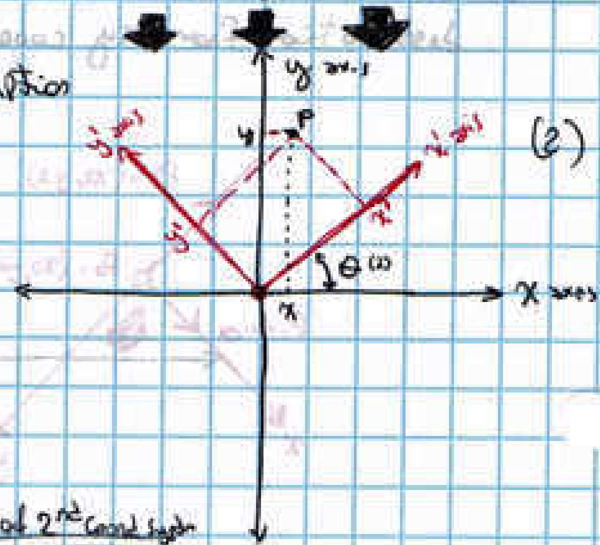
$$T(-x_0,-y_0) = \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$



② Get both systems axis into coincidence perform clockwise rotation

$$R(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(1&2) This would give us a complete composite matrix for transformation object description from $xy \rightarrow x'y'$ system:



$$M_{xy,x'y'} = R(-\theta)\cdot T(-x_0\, y_0)$$

$$\boxed{M_{xy,x'y'}} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

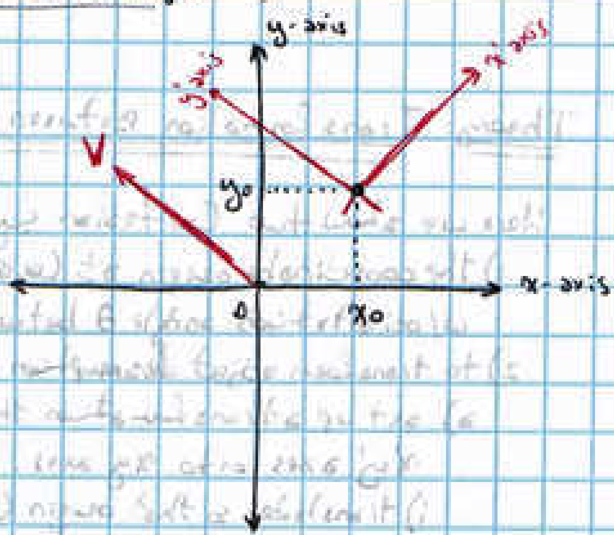
Alternative Method of Giving the orientation of 2nd Coord system

① specify a vector V that indicates the direction for the positive y' axis

a. vector V is specified as a point in the xy reference frame relative to the origin of the xy system

b. A unit vector (v) in the ~~xy~~ y' direction can be obtained as

② $$v = \frac{V}{|V|} = \cancel{(v_x, v_y)} = (v_x, v_y)$$



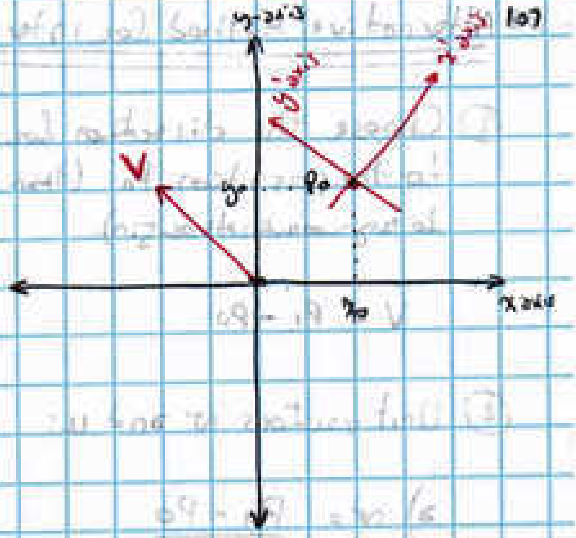
c. The unit vector (u) along x' axis by rotating v 90° clockwise

$$u = (v_y, -v_x) = (u_x, u_y)$$

Alternative method (continuation)

$$\mathbf{v} = \frac{V}{|V|} = (v_x, v_y)$$

$$u = (v_y, -v_x) \quad \text{(90° clockwise)}$$
$$= (u_x, u_y)$$



③ elements of ~~a set~~ comp rotation matrix could be expressed as elements of a set of orthogonal unit vector.

The matrix to rotate the x'y' system into coincidence w/ the xy system: ~~can be~~

$$R = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example

1. we choose orientation for y' axis:
   $V = (-1, 0)$
2. the x' axis is in the positive y direction and the rotation transformation matrix is:

$$R = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{v} = \frac{V}{|V|} = \frac{(-1,0)}{|(-1,0)|} = (-1,0)$$

$$u = (v_y, -v_x)$$
$$= (0, -(-1))$$
$$= (0, 1)$$
$$= (u_x, u_y)$$

~~Note that the elements~~

108 Alternative Method for interactive Application

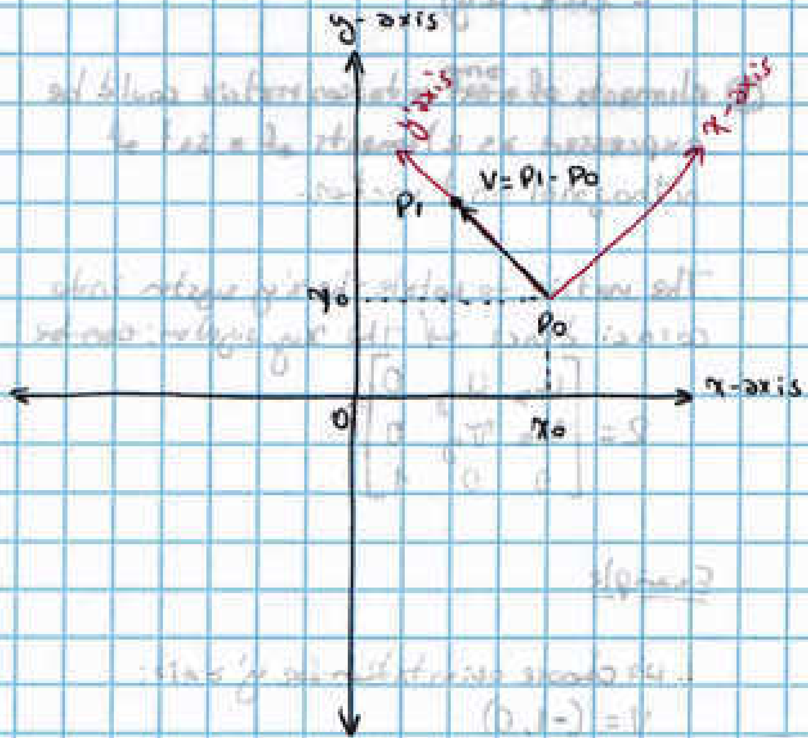
① Choose the direction for vector V relative to the position P_0 (then it is to specify it relative to xy-coordinate origin).

$$V = P_1 - P_0$$

② Unit vectors v and u:

a) $$v = \frac{P_1 - P_0}{|P_1 - P_0|}$$

b) $$u = (v_y, -v_x) = (u_x, u_y)$$

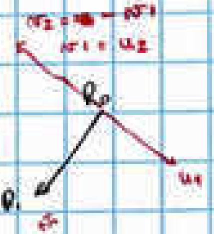


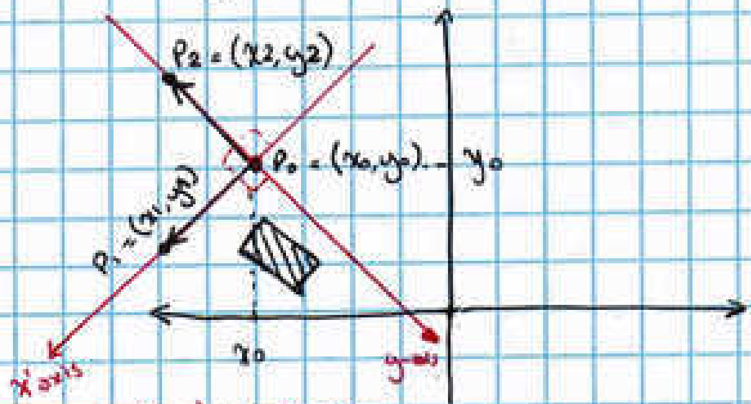
Problem 1

Part A

Problem 1

Given two perpendicular vectors P_0P_1 and P_0P_2.
Derive a transformation matrix which can transform object description from xy coordinate system to $x'y'$ coordinate system





$V_1 \perp V_2$

①

$V_1 = P_1 - P_0$ (indicate the direction of positive x-axis)

$V_2 = P_2 - P_0$ (indicate the direction of negative y-axis)

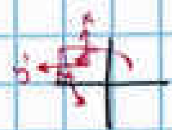
$v_1 = \dfrac{P_1 - P_0}{|P_1 - P_0|} = (v_{1x}, v_{1y})$ | $v_2 = \dfrac{P_2 - P_0}{|P_2 - P_0|} = (v_{2x}, v_{2y})$

$u_1 = (v_{1y}, -v_{1x}) = (u_{1x}, u_{1y})$ | $u_2 = (v_{2y}, -v_{2x}) = (u_{2x}, u_{2y})$

~~$v_1 \measuredangle 90° \; v_2$~~

~~Note: $u_1 \measuredangle 90° \; v_1$, $u_2 \measuredangle 90° \; v_2$~~

~~if $v \measuredangle 90° \; u$ in this case $v_1 \measuredangle 90° \; u_1$, $u_1 \measuredangle -180 \; u_2$~~



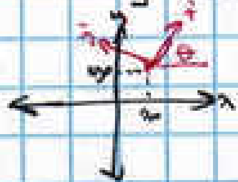
②

$$M_{xy,x'y'} = R(-\theta)\cdot T(-x_0, -y_0)$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

③

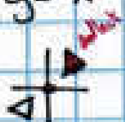
$$R = \begin{bmatrix} +u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



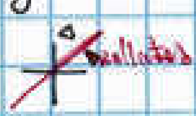
(only for vector points to positive y axis)

④ Reflexion

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad y = x$$



$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad y = -x$$



$-v_2$ $(y'\text{-axis}) = (v_{2x}, v_{2y})$

$+v_1$ $(x'\text{-axis}) = (v_{1x}, v_{1y})$

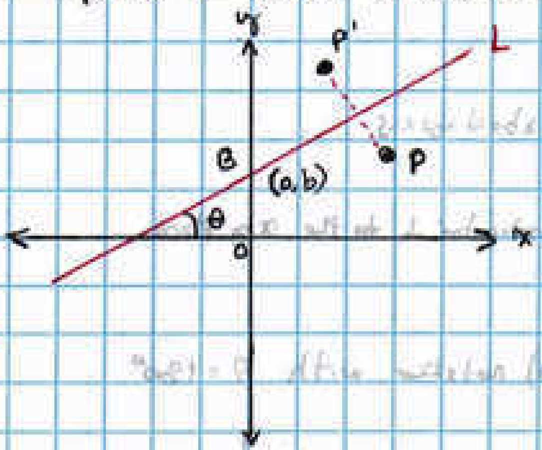
$$R = \begin{bmatrix} v_{1x} & v_{2y} & 0 \\ -v_{2x} & -v_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

???

Problem 2

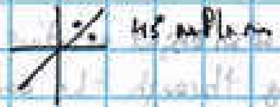
Let line L have a y-intercept (0,b) and an angle θ
(with respect to the x-axis)

1) Describe the transformation M which reflect an arbitrary point P about a Line L (eg. reflect P to P')



Plan: ①

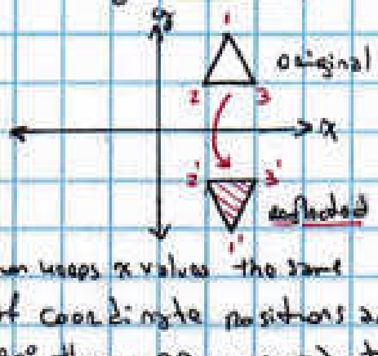
②



Theory: Reflexion

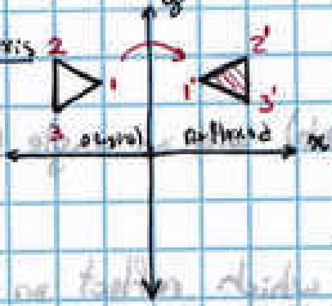
1) Reflexion is a transformation that produces a mirror image of an object.
2) For 2D Reflexion is generated relative to the axis of reflexion by rotating the object 180° about the reflexion axis.
3) We can choose a reflexion in the xy plane or perpendicular ⊥ to the xy Plane.
4) Reflexion in the xy plane:
   a) the rotation path about this axis is a plane perpendicular to the xy plane
5) Reflexion perpendicular to xy plane:
   a) the rotation path is in the xy Plane
6) Reflexion about line $y = 0$ (x-axis):

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



a) transformation keeps x values the same
b) y values of coordinate positions are "flipped"
c) equivalent 180° through 3D space about x axis

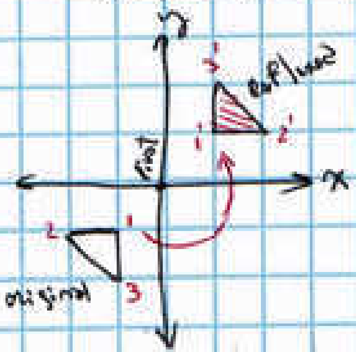
112 7) <u>Reflexion y-axis</u>



$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

8) a) keep y coordinates
b) flip x coordinates
c) reflected about line $x=0$
d) equivalent to $180°$ throug 3D space about y axis

8) Reflexion of an object relative to an axis perpendicular ⊥ to the xy plane and passing through the coordinate origin



a) ~~R(θ) = 180°~~ $R(\theta)$ rotation with $\theta = 180°$

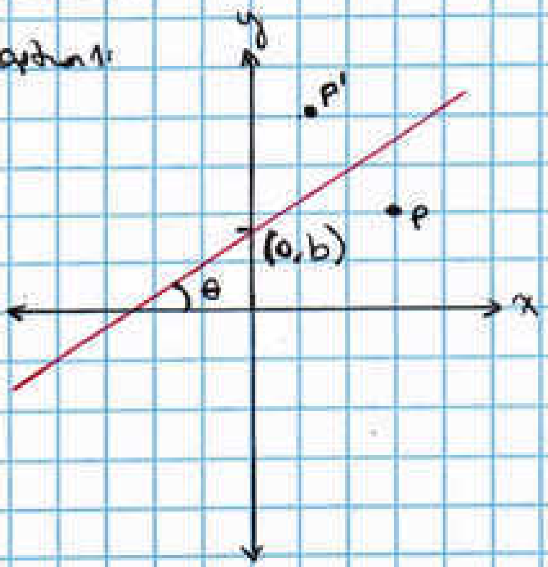
$$\cancel{R(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$R(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

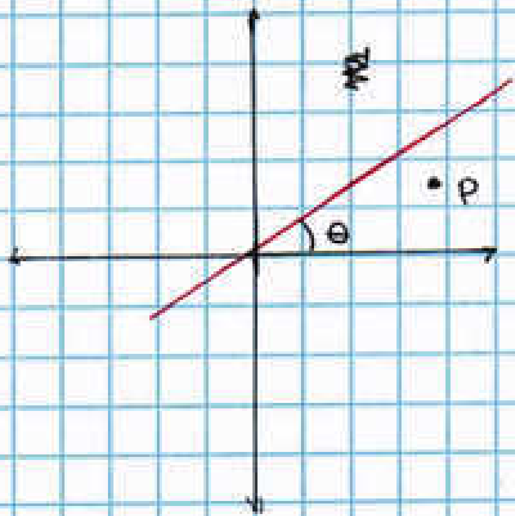
9)

Problem 2

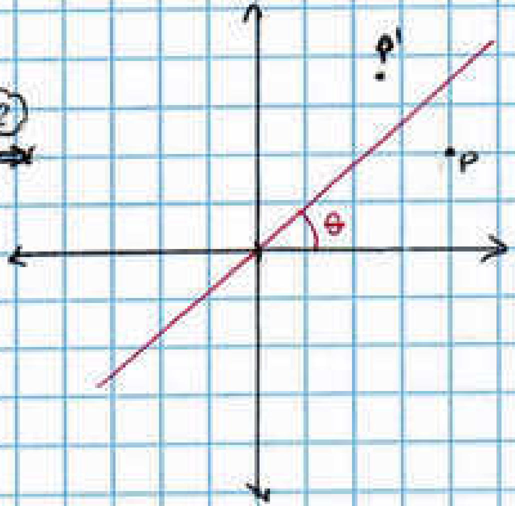
option 1:



1 ⇒

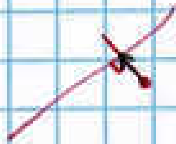


2 ⇒

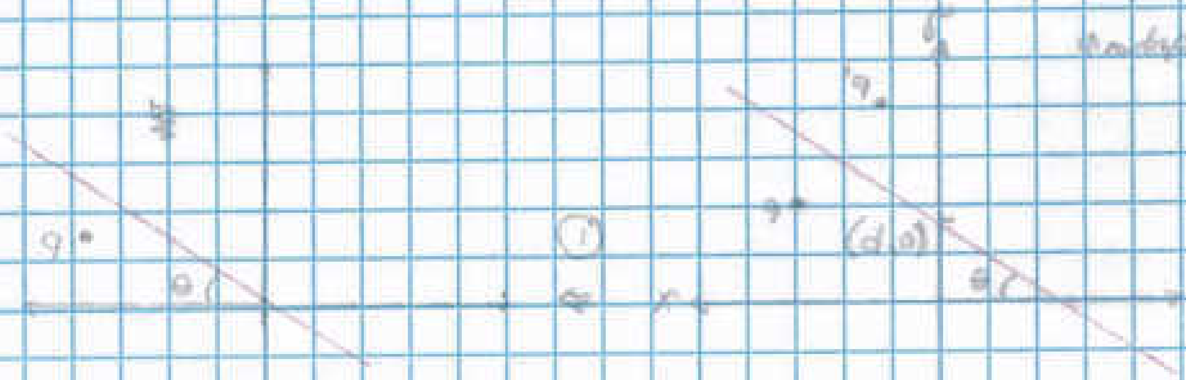


option 2

$$y = mx + b$$



$$y - b = mx$$


q
θ
q
(d,0)
θ