

Sensitivity indexes of the Ishigami function

Ishigami function

In[9]:= $f[x1_ , x2_ , x3_] = \text{Sin}[x1] + a * \text{Sin}[x2]^2 + b * (x3)^4 * \text{Sin}[x1]$

Out[9]= $\text{Sin}[x1] + b x3^4 \text{Sin}[x1] + a \text{Sin}[x2]^2$

$X_1, X_2, X_3: U[-\pi, \pi]$

$P(X_i):$

In[10]:= $p = 1 / (2 * \pi);$

Computation of $f_0, f_1, f_2, f_3, f_{12}, f_{13}, f_{23}, f_{123}$:

In[11]:= $f_0 = p^3 * \text{Integrate}[f[x1, x2, x3], \{x1, -\pi, \pi\}, \{x2, -\pi, \pi\}, \{x3, -\pi, \pi\}]$

$f_1 = \text{Simplify}[p^2 * \text{Integrate}[f[x1, x2, x3], \{x2, -\pi, \pi\}, \{x3, -\pi, \pi\}] - f_0]$

$f_2 = p^2 * \text{Integrate}[f[x1, x2, x3], \{x1, -\pi, \pi\}, \{x3, -\pi, \pi\}] - f_0$

$f_3 = p^2 * \text{Integrate}[f[x1, x2, x3], \{x1, -\pi, \pi\}, \{x2, -\pi, \pi\}] - f_0$

$f_{12} = \text{Simplify}[p * \text{Integrate}[f[x1, x2, x3], \{x3, -\pi, \pi\}] - f_0 - f_1 - f_2]$

$f_{13} = \text{Simplify}[p * \text{Integrate}[f[x1, x2, x3], \{x2, -\pi, \pi\}] - f_0 - f_1 - f_3]$

$f_{23} = \text{Simplify}[p * \text{Integrate}[f[x1, x2, x3], \{x1, -\pi, \pi\}] - f_0 - f_2 - f_3]$

Out[11]= $\frac{a}{2}$

Out[12]= $\frac{1}{5} (5 + b \pi^4) \text{Sin}[x1]$

Out[13]= $-\frac{a}{2} + a \text{Sin}[x2]^2$

Out[14]= 0

Out[15]= 0

Out[16]= $-\frac{1}{5} b (\pi^4 - 5 x3^4) \text{Sin}[x1]$

Out[17]= 0

Computation of $v_t, v_1, v_2, v_3...$

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In[33]:= vt = Simplify[
  p^3 * Integrate[(f[x1, x2, x3] - f0)^2, {x1, -Pi, Pi}, {x2, -Pi, Pi}, {x3, -Pi, Pi}]
  v1 = Expand[Integrate[(f1)^2 * p, {x1, -Pi, Pi}]]
  v2 = Integrate[(f2)^2 * p, {x2, -Pi, Pi}]
  v3 = Integrate[(f3)^2 * p, {x3, -Pi, Pi}]
  v12 = Integrate[(f12)^2 * p, {x3, -Pi, Pi}]
  v13 = Integrate[f13^2 * p^2, {x1, -Pi, Pi}, {x3, -Pi, Pi}]
  v23 = Integrate[f23^2 * p^2, {x2, -Pi, Pi}, {x3, -Pi, Pi}]
  vt - v1 - v2 - v3 - v12 - v23;

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$$\text{Out[33]} = \frac{1}{2} + \frac{a^2}{8} + \frac{b \pi^4}{5} + \frac{b^2 \pi^8}{18}$$

$$\text{Out[34]} = \frac{1}{2} + \frac{b \pi^4}{5} + \frac{b^2 \pi^8}{50}$$

$$\text{Out[35]} = \frac{a^2}{8}$$

$$\text{Out[36]} = 0$$

$$\text{Out[37]} = 0$$

$$\text{Out[38]} = \frac{8 b^2 \pi^8}{225}$$

$$\text{Out[39]} = 0$$

Computation of S1, S2...

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In[26]:= s1 = v1 / vt
s2 = v2 / vt
s3 = v3 / vt
s12 = v12 / vt
s13 = v13 / vt
s23 = v23 / vt
s123 = Simplify[ (vt - v1 - v2 - v3 - v12 - v13) / vt]
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$$\text{Out[26]} = \frac{\frac{1}{2} + \frac{b \pi^4}{5} + \frac{b^2 \pi^8}{50}}{\frac{1}{2} + \frac{a^2}{8} + \frac{b \pi^4}{5} + \frac{b^2 \pi^8}{18}}$$

$$\text{Out[27]} = \frac{a^2}{8 \left(\frac{1}{2} + \frac{a^2}{8} + \frac{b \pi^4}{5} + \frac{b^2 \pi^8}{18} \right)}$$

$$\text{Out[28]} = 0$$

$$\text{Out[29]} = 0$$

$$\text{Out[30]} = \frac{8 b^2 \pi^8}{225 \left(\frac{1}{2} + \frac{a^2}{8} + \frac{b \pi^4}{5} + \frac{b^2 \pi^8}{18} \right)}$$

$$\text{Out[31]} = 0$$

$$\text{Out[32]} = 0$$