Sensitivity indexes of the test function with fixed exponent

Test function

```
ln[37] = f1[W_, X_, Z_] := a * log[W] * log[X] + b * log[Z] * log[X^(2)]
```

```
W,X,Z: U[0.5, 1.5]
```

P:

Computation of f for f1:

```
In[42]= f0 = Simplify[p^3*Integrate[f1[W, X, Z], {W, 1, h}, {X, 1, h}, {Z, 1, h}]]
Out[42]= 1.17841 a + 2.35683 b

In[43]= fw = Simplify[p^2*Integrate[f1[W, X, Z], {X, 1, h}, {Z, 1, h}] - f0]
Out[43]= -1.17841 a - 4.26326 × 10<sup>-14</sup> b + 1.08555 a Log[W]

In[44]= fx = Simplify[p^2*Integrate[f1[W, X, Z], {W, 1, h}, {Z, 1, h}] - f0]
Out[44]= -1.17841 a - 2.35683 b + 1.08555 a Log[X] + 1.08555 b Log[X²]
In[45]= fz = Simplify[p^2*Integrate[f1[W, X, Z], {W, 1, h}, {X, 1, h}] - f0]
Out[45]= 0. - 2.35683 b + 2.1711 b Log[Z]
In[46]= fwx = Simplify[p*Integrate[f1[W, X, Z], {Z, 1, h}] - f0 - fw - fx]
Out[46]= 1.17841 a + 4.26326 × 10<sup>-14</sup> b - 1.08555 a Log[X] + a Log[W] (-1.08555 + 1. Log[X]) - 1.95399 × 10<sup>-14</sup> b Log[X²]
In[47]= fwz = Simplify[p*Integrate[f1[W, X, Z], {X, 1, h}] - f0 - fw - fz]
Out[47]= 0. + 4.26326 × 10<sup>-14</sup> b + 2.22045 × 10<sup>-16</sup> a Log[W] - 3.90799 × 10<sup>-14</sup> b Log[Z]
```

```
lo[48]:= fxz = Simplify[p*Integrate[f1[W, X, Z], {W, 1, h}] - f0 - fx - fz]
Out[48] = 0. + 2.35683 \text{ b} + 2.22045 \times 10^{-16} \text{ a} \text{ Log}[X] - 2.1711 \text{ b} \text{ Log}[Z] + \text{b} \text{ Log}[X^2] (-1.08555 + 1. \text{ Log}[Z])
```

Computation of vt, vw, vx, vz...

```
log_{[49]} = vt = Simplify[p^3 * Integrate[(f1[W, X, Z] - f0)^2, \{W, 1, h\}, \{X, 1, h\}, \{Z, 1, h\}]]
Out[49]= 1.12192 a^2 + (1.91405 - 8.19593 \times 10^{-16} \text{ i}) a b + (4.48767 - 7.57452 \times 10^{-16} \text{ i}) b^2
 ln[50] = vw = Simplify[p * Integrate[(fw)^2, {W, 1, h}]]
Out[50]= 0.478514 \ a^2 + 6.77369 \times 10^{-28} \ a \ b + 1.81754 \times 10^{-27} \ b^2
 ln[51]= vx = Simplify[p * Integrate[(fx)^2, {X, 1, h}]]
Out[51]= 0.478514 a^2 + 1.91405 a b + 1.91405 b^2
 ln[52] = vz = Simplify[p * Integrate[(fz)^2, {Z, 1, h}]]
Out[52]= 1.91405 b^2
 ln[53]:= vwx = Simplify[p^2*Integrate[(fwx)^2, \{W, 1, h\}, \{X, 1, h\}]]
Out[53]= (0.164889 + 5.6492 \times 10^{-17} \text{ i}) \text{ a}^2 +
                   \left(\textbf{5.04871} \times \textbf{10}^{-29} + \textbf{7.62306} \times \textbf{10}^{-43} \ \dot{\mathbb{1}} \right) \ \textbf{a} \ \textbf{b} + \ \left(\textbf{6.20201} \times \textbf{10}^{-28} + \textbf{2.20081} \times \textbf{10}^{-43} \ \dot{\mathbb{1}} \right) \ \textbf{b}^2
 ln[54]:= vwz = Simplify[p^2 * Integrate[(fwz)^2, {W, 1, h}, {Z, 1, h}]]
\text{Out} \text{[54]= } \textbf{7.81209} \times \textbf{10}^{-32} \, \textbf{a}^2 \, + \, \left( \textbf{1.01002} \times \textbf{10}^{-31} \, + \, \textbf{8.75719} \times \textbf{10}^{-45} \, \, \dot{\textbf{1}} \, \right) \, \textbf{a} \, \textbf{b} \, + \, \left( \textbf{6.20201} \times \textbf{10}^{-28} \, + \, \textbf{2.20081} \times \textbf{10}^{-43} \, \, \dot{\textbf{1}} \, \right) \, \textbf{b}^2 \, , \\ \text{Out} \text{[54]= } \textbf{7.81209} \times \textbf{10}^{-32} \, \textbf{a}^2 \, + \, \left( \textbf{1.01002} \times \textbf{10}^{-31} \, + \, \textbf{8.75719} \times \textbf{10}^{-45} \, \, \dot{\textbf{1}} \, \right) \, \textbf{a} \, \textbf{b} \, + \, \left( \textbf{6.20201} \times \textbf{10}^{-28} \, + \, \textbf{2.20081} \times \textbf{10}^{-43} \, \, \dot{\textbf{1}} \, \right) \, \textbf{b}^2 \, , \\ \text{Out} \text{[54]= } \textbf{7.81209} \times \textbf{10}^{-32} \, \textbf{a}^2 \, + \, \left( \textbf{1.01002} \times \textbf{10}^{-31} \, + \, \textbf{8.75719} \times \textbf{10}^{-45} \, \, \dot{\textbf{1}} \, \right) \, \textbf{a} \, \textbf{b} \, + \, \left( \textbf{6.20201} \times \textbf{10}^{-28} \, + \, \textbf{2.20081} \times \textbf{10}^{-43} \, \, \dot{\textbf{1}} \, \right) \, \textbf{b}^2 \, .
 log_{55} = vxz = Simplify[p^2 * Integrate[(fxz)^2, {X, 1, h}, {Z, 1, h}]]
Out[55]= 7.81209 \times 10^{-32} \text{ a}^2 + \left(3.98358 \times 10^{-31} - 1.67645 \times 10^{-31} \pm\right) \text{ a b} + \left(0.659557 + 2.25968 \times 10^{-16} \pm\right) \text{ b}^2
```

Computation of Sw, Sx, Sz...

```
ln[56]:= a = -8;
        b = 1.7;
In[58]:= SW = VW / Vt
Out[58]= 0.521355 - 7.95017 \times 10^{-17} \text{ i}
In[59]:= sx = vx / vt
Out[59]= 0.172373 - 2.62853 \times 10^{-17} \text{ i}
In[60]:= Sz = vz / vt
Out[60]= 0.0941698 - 1.436 \times 10^{-17} i
In[61]:= SWX = VWX / Vt
Out[61]= 0.179652 + 3.41545 \times 10^{-17} i
```