Sharp Transitions of Collision Behavior in Multi-Robot Systems

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***Abstract*—This paper focuses on multi-robot environments and determines if there are any sharp transitions that occur in such a system given certain path planning and movement properties. There were no sharp transitions that occurred in the systems that were studied. As far as can be said, for these specific systems with these properties, there are not any sharp transitions.**

***Index Terms*—Sharp Transitions, Multi-Robot Systems, Multi-Agent**

I. Introduction

SHARP TRANSITIONS are key behavioral components of many systems and allow for predictions of the behavior of the system. Furthermore, it shows us where physical improvements should be made. For example, traffic flow has been shown, by Biham and Middleton [2], to have dynamic behavior, in which there are phases; there is a low-density dynamical phase and a high-density jammed phase in which there is slow car movement. If it reaches the high-density jammed phase, then we should improve the roads at that point to allow for smoother travel if and only if we can afford to overcome the point of transition.

Another example of an environment with a sharp transition is that of a Geometric Random Graph, where the connectivity is something that responds dramatically to a small change in the number of points placed in the graph, according to Han and Makowski [3]. Using these examples, we would like to see if Multi-Robot Systems have such transitions as well.

Since Multi-Robot environments are becoming much more popular, with self-driving cars and even the multi-robot systems that are used at amazon fulfillment centers, it is necessary to understand the limitations and properties of the systems in order to properly optimize its objective functions.

The objective function of such systems can be, in this case, minimizing the amount of time it takes for each robot to get its destination. Based on the objective function of the problem we can start to study what properties of the system affect the outcome most drastically. One example of such an objective function is to minimize the total path length of all robots as stated by Nagar et. al [5].

In order to simplify the problem, we are going to be looking at the case in which there is a robot that is represented as a disc in two dimensional space.

So far, Multi-Robot Path planning problems have been proven to be NP-Complete as by Yu [1]. This indicates that the problems of path planning for such systems quickly becomes intractable for a large amount of robots.

In the cases to be studied, we are simply looking at the level of interaction within these certain multi-robot systems. We will be looking at two different factors; one factor is the density of the system and the other is the locality of the system. The locality is defined for a continuous case as the maximum distance from the starting to ending point. For the discrete case, it is the maximum edge distance from the starting to ending points for each disc.

II. Method

We are going to be looking at three cases: Continuous, Discrete, and Discrete with collision avoidance behavior. The Continuous case will have the discs placed in the unit square. The Discrete case will take the discs from the Continuous domain and place them in a graph over the unit square. For the Discrete case with avoidance behavior, the discs will only choose a closer node to travel to if it is not occupied.

The reason why we avoid having path planning in the continuous domain is because of the combinatorial explosion of the search space with every new robot added, as stated by Ryan [4]. As such we only have discrete path planning, because the problem is more manageable.

For all cases, similar if not the same properties were given to the system. I.e. there was the same radius and density or number of discs. The changes are subtle; The continuous case has the same theoretical radius for the graph, but the locality is a different metric since it is no longer the edge distance. For all cases, collisions are going to be used to observe the level of interaction that occurs in the system.

A. Discrete Case

For the discrete case, a hexagonal grid was created with 1000 nodes. The path planning is based on placing discs onto the grid and assigning them a goal node. If a locality is specified, the edge distance between the starting node and ending node of each node is at most a certain distance away. Then a breadth first search is done from all goal nodes and the nodes distances from the goals are marked for all discs. This is

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what gives the discs their direction in the graph.

There are also a few rules for the types of movement that can occur. Firstly, the discs cannot use the same edge in the same time step. They take their steps in order to greedily get their goal node. The priority in the movement of the discs in a time step is random. Furthermore, if there are multiple nodes that a disc can travel to and benefit from, it will randomly choose an unoccupied node. If a disc happens to be surrounded by discs and a disc also moves into the spot of that disc, it is considered to be a collision and the node stays in the same place as it was until the next time step. The number of pairwise collisions is then recorded

B. *Continuous Case*

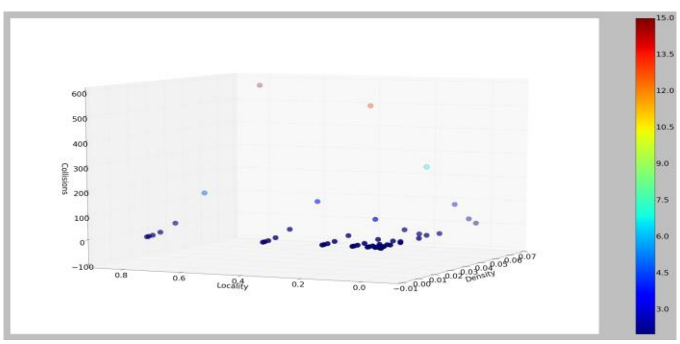
The continuous case was constructed as so, a certain number of discs are placed into the unit square that all travel at the same speed and radius size. They have a randomly assigned and non-overlapping starting point and ending point in a unit square. The radius used was .0128 units. This was decided as the radius for the continuous case, based on the theoretical maximum size of the radius of a disc in the discrete case determined as so.

$$Number\ of\ Columns\ in\ grid = n = 40$$
$$Length\ of\ Edge = E = \frac{2}{(n-1) * \sqrt{3}}$$
$$Max\ Radius = R = \frac{1}{(n-1) * 2} = \frac{1}{78} = .0128$$

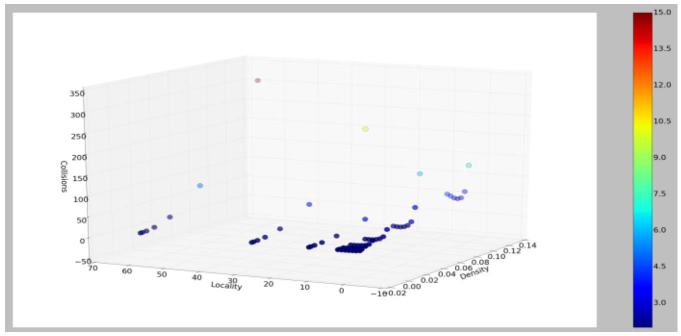
They are not allowed to be initially touching each other. Each disc is placed with its ending location defined that also cannot cross over with another discs. Then, the number of pairwise collisions are counted. Collisions can also occur between a moving disc and a disc that has reached its goal point.

III. Results

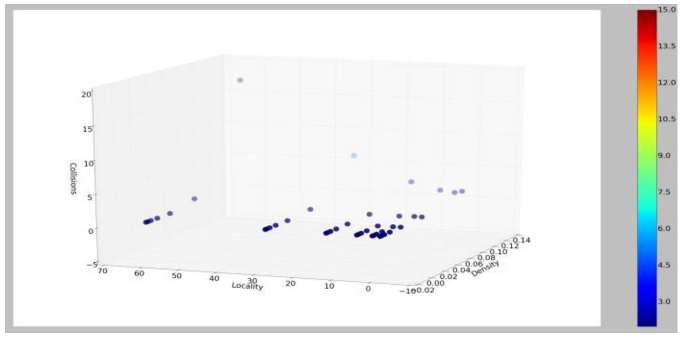
A. *Continuous – No Avoidance Behavior*



B. *Discrete – No Avoidance Behavior*



C. *Discrete – With Avoidance Behavior*



IV. Discussion/Conclusion

Based on the simulations that were ran, there was no evidence of any sharp transition behavior. Furthermore, there seemed to be no difference in the collision behavior based on the density of discs in the system. There were no phase changes in these systems.

One interesting behavior is that when the locality is extremely low in the discrete case, there is unexpected behavior with the number of interaction between the discs. This could be because with a low locality if two discs are placed next to each other, they have a high chance of their paths overlapping, which leads to a greater chance of collision.

While there are no sharp transitions in the system, it does not follow that there are no sharp transitions in multi-robot systems. The work done so far has not revealed any unique phase changes. There was however some increased interaction with lower locality.

With the simulating environments created, a lot of functionality can be added. So, with this in mind, it would be useful to expand on the code to determine what additional attributes can be added so that a sharp transition can be created. Furthermore, we can work to better mimic the real world systems.

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REFERENCES

[1] Jingjin Yu (2015, December). Intractability of Optimal Multi-Robot Path Planning on Planar Graphs. Published in Robotics and Automation Letters [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=7342901&tag=1

[2] Ofer Biham and A. Alan Middleton. (1992, November). Self-organization and dynamical transition in traffic-flow models. Presented at American Physical Society. [Online]. Available: http://journals.aps.org/pra/abstract/10.1103/PhysRevA.46.R6124

[3] Guang Han and Armand M. Makowski. (2007, January) A Very Strong Zero-One Law for Connectivity in One-Dimensional Geometric Random Graphs. Presented in IEE Communication letters [Online]. Available: http://www.isr.umd.edu/~armand/documents/2006-2010/A%20very%20strong%20zero-one%20law%20for%20connectivity%20in%20one-dimensional%20geometric%20random%20graphs-2007.pdf

[4] Malcolm R. K. Ryan (2008, March). Exploiting Subgraph Structure in Multi-Robot Path Planning. Published in Journal of Artificial Intelligence Research [Online]. Available: http://www.jair.org/media/2408/live-2408-3752-jair.pdf

[5] Preetha Bhuttacharjee, Pratyusha Rakshit, Indrani Goswami, Amit Konar, Atulya K. Nagar (2011, October) Multi-Robot Path-Planning Using Artificial Bee Colony Optimization Algorithm. Presented at Salamanca Conference [online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1041624