

2019-03-06 Stable Roommates

Wednesday, March 6, 2019 4:15 PM

- Very similar to stable marriage
 - Difference is that all entities are in the same set
 - Unlike stable marriage, there may not be a stable outcome with stable roommates algorithm
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- Set up very similar to stable marriage, except just one table

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

General Algorithm

- In sequence, each person proposes to top choice
 - If this person is higher on acceptor's list of choices, accept
 - Otherwise reject
- Continue until everyone has a matched proposal and acceptance, then go to next step (discussed later)

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

○ = proposal
○ = accept

Arnold proposes to Bob, Bob accepts

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

Bob proposes to Doug, Doug accepts

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Bob proposes to Doug, Doug accepts

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

Charlie proposes to Doug. Doug accepts and notifies Bob that he no longer wants to be roommates. Revisit Bob, who propose now to Erik (accepted).

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

Doug's turn. He proposes to Frank (accepted).

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

Erik proposes to Frank (rejected). Erik then proposes to Charlie, who accepts.

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

Frank proposes to Arnold, who accepts.

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

With everyone having both made and accepted a proposal, we now symmetrically cross out all accepted that are less than currently acceptance. Symmetric means "If A crosses out B, B must cross out A"

Arnold	Bob	Doug	Frank	Charlie	Erik
Bob	Doug	Erik	Frank	Arnold	Charlie
Charlie	Doug	Erik	Frank	Arnold	Bob
Doug	Frank	Charlie	Arnold	Erik	Bob
Erik	Frank	Charlie	Doug	Bob	Arnold
Frank	Arnold	Bob	Doug	Charlie	Erik

Reduced table...

Arnold	Bob		Frank		
Bob		Erik	Frank	Arnold	
Charlie	Doug	Erik			
Doug	Frank	Charlie			
Erik		Charlie		Bob	
Frank	Arnold	Bob	Doug		

Term: 2nd preference is the less preferable pairing.

Eliminate all 2nd preferences symmetrically starting with 2nd element in chain. This creates a circular chain.

Arnold -> Frank -> Doug -> Charlie -> Erik -> Bob -> Arnold *eliminate*

Arnold			Frank		
Bob		Erik	Frank		
Charlie	Doug				
Doug		Charlie			
Erik				Bob	
Frank	Arnold	Bob			

We're not done because at least one person has more than one remaining choice.

Repeat above process on people that have multiple choices.

Bob -> Frank -> Bob

Arnold			Frank		
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Bob		Erik			
Charlie	Doug				
Doug		Charlie			
Erik				Bob	
Frank	Arnold				

If during proposal phase a person runs out of choices, then no stable match will exist for that set.