Written in JHEP style

Everything S-Matrix

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Abstract: These are personal notes written in the style of "lecture notes" as a way to help organize in my own mind the various topics associated with the S-matrix. These notes are a WORK IN PROGRESS and are not meant to be any definitive source material on the various subjects found within. Please check any results for yourself, as well as the cited source material.

Contents

1 A Guide on the Literature

[**AC:**] Fill-in later, but briefly: trust chapters 1 and 4 of Eden's et al. textbook, and start with Mizera's notes since they are amazing [1, 2].

2 General Properties of Amplitudes

> The best assumptions are the ones you can't prove.
>
> *Xi Yin*
> PHY253b

The title of the section should really be "*Some* General Properties of Amplitudes," since the vastness of the literature on S-matrix results is staggering and ever changing. However, the core results from the 60's and 70's, being re-discovered and re-formulated will never change (only the efficiency of their derivation will). It is these core results that we will be looking at in this section by taking examples of physical setups that can be extended to a more general case.

2.1 Examples: $2 \to 2$ Scattering of Identical Massless Scalars

Consider the $2 \to 2$ scattering amplitude of identical massive scalar particles. The s- and t-channel can be written in terms of the scattering angle θ as

$$s = E^2 = 4(k^2 + m^2) \text{ and } t = -2k^2(1 - \cos(\theta)). \tag{2.1}$$

We know that the amplitude $A(s,t)$ can be analytically continued away from the physical region $s, t \in \mathbb{R}$ with $s \geq 4m^2 - t$ with $t \leq 0$ [1]. Consider fixing $t < 0$ and analytically continue s into the complex plane. This can be see in figure 1.

[1]We will still denote the ampliude $\mathcal{A}(s,t)$

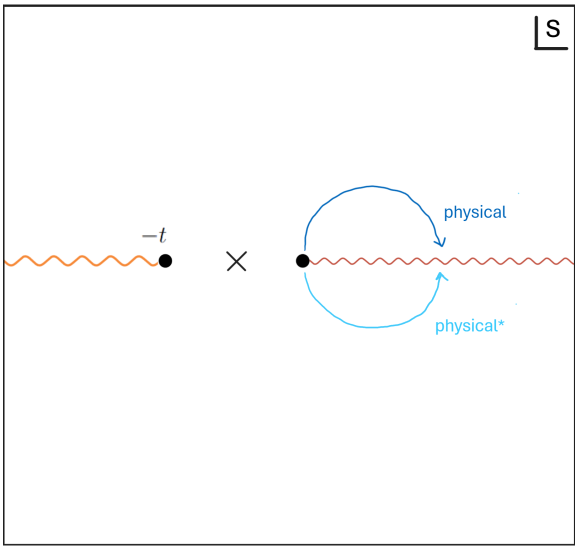

Figure 1. Complex graph of s with branch cuts on s-channel and u-channel with possible bound state poles denoted as x's.

The *branch cut* on the complex graph of s denotes all the physical states of the theory based on the bounds of s. The physical amplitude can be written as

$$\text{physical amplitude} = \lim_{\epsilon \to 0^+} A(s + i\epsilon, t). \tag{2.2}$$

The limit of (2.2) is taken such that we analytically continue from above towards the branch cut. If we instead went from 0^-, then we would have analytically continued from below, finding the complex conjugate of the physical amplitude. Analytically continuing the amplitude $A(s,t)$ is expected to obey the *real analyticity* condition

$$A(s^*, t^*) = (A(s,t))^*, \tag{2.3}$$

which can also be thought of as a reality condition [2]. This is the first assumption we will make about the amplitude. The graph in 1 of the s-channel is on the *first Riemann sheet* or the *physical sheet*. The cut on the LHS is a u-channel cut that starts at $-t$. The crosses denote possible bound state poles. The resonance poles are heinding on the *second sheet* which can be found by translating between momentum to energy [3].

The second assumption we make about the amplitude $A(s,t)$ is that it has a *crossing symmetry*. Crossing symmetry, in terms of the channels, relates the different scattering channels s, t, and u to each other via analytically continuing to different regions of s, t, and u. [**AC:**] Insert Mandelstam graph from [1]. Note that all of the assumptions being made about $A(s,t)$ hold non-perturbatively.

The third assumption we shall make is about unitarity. [**AC:**] Stopped here.

[2] In perturbation theory,this follows from the reality of coupling constants in the Lagrangian.

[3] A classical example from quantum mechanics is when graphing complex p and finding the second sheet by using $p^2/2m = E \implies p = \pm i\sqrt{-2mE}$.

2.2 Comment on Polynomial Boundedness

A Derivations of such

Please always give a title also for appendices.

Acknowledgments

Something something some cool people and such.

Note added. If we mess something up.

References

[1] R.J. Eden, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne, The analytic S-matrix, Cambridge Univ. Press, Cambridge (1966).

[2] S. Mizera, Physics of the analytic S-matrix, Phys. Rept. **1047** (2024) 1 [2306.05395].