Projectile Motion and Conservation of Energy (Sept 27th, 2022)

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I. Introduction

This experiment was meant to examine The Law of Conservation of Energy which is a significant law in physis that states that energy within a system remains constant unless some outside work is done on the system. This law hinges on the fact that energy can not be created or destroyed, but only moved from one medium to another. This law applies to any isolated system no matter the size or complexity including an isolated system on the scale of an entire galaxy or a system on a molecular scale.

$$K_1 + U_1 = K_2 + U_2 \quad (1)$$

For this experiment we focused on a mechanical system with a ball going through a tubed ramp and projectile launching off the end of the ramp onto the ground a horizontal distance x away from where it left the ramp. We used a metal and plastic ball to send through the ramp in order to observe the differences in landing positions compared to calculated landing positions due to differences in mass and material.

$$\Delta U = mgh \quad (2)$$

Equation 2 shows how potential energy is calculated which was useful in our experiment since we knew that the ball started from the top of the ramp with only potential energy.

$$K = K_{trans} + K_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{7}{10}mv^2 \quad (3)$$

$$\Delta K = W_f - \Delta U = mg\Delta h' - mg\Delta h = mg(h' - \Delta h) \quad (4)$$

Equations 3 and 4 can be derived from our knowledge of the experiment set up, and by using Equations 1 and 2. From here we can find the energy lost to friction along the ramp by setting our original heights in each part of the lab h'_1 and h'_2 so that each ball only goes up to the very edge of the ramp.

$$x(t) = x_0 + v_{0,x}t \quad (5)$$

$$y(t) = y_0 + y_{0,y}t - \frac{1}{2}gt^2 \quad (6)$$

Equations 5 and 6 describe the motion of an airborne object which can be used to derive the equations for finding the predicted position of each ball.

Once we had our predictions, we set up for our trials by changing the degree that the ramp was angled at. We then began performing our trials, repeating the same frictional energy calculations for the plastic ball as we did for the metal ball.

II. Method

For this experiment, we had a ramp tube for a small ball to travel through, and observed its landing position after having gone through the track. We used the Law of Conservation of Energy, which states that the total energy of a system is constant unless some outside work is performed on the system, to determine where the ball might land after having left the track.

We began by using the Law of Conservation of Energy to account for energy lost to friction along the track. We know that the energy of the metal ball at the top of the ramp is entirely potential energy, and that if the metal ball stops at the very end of the opposite end of the track that the energy at that point is also entirely potential energy. Therefore, we adjusted the angle of the ramp so that the metal ball stops at the very end of the launching part of the track, but doesn't actually launch. From here we are able to understand how the energy lost to friction relates to the kinetic and potential energy.

$$\Delta E = \Delta K + \Delta U - W_f = 0 \quad (7)$$

Since we knew that the ball started from rest, hence the initial kinetic energy was zero, we could manually calculate the change in potential energy and solve for the energy lost to friction using Equation 7. We then readjusted the ramp to a degree where the ball would leave the ramp and where Δh was at least twice $\Delta h'$. We were able to calculate the landing position and initial velocity of the ball after leaving the ramp after adjusting for the new heights of *h1* and *h2* using Equations 8 and 9 derived from Equations 3, 4, 5, and 6.

$$v_0 = \sqrt{\frac{10}{7}g([\Delta h] - [\Delta h'])} \quad (8)$$

$$\Delta x_{est} = v_{0,x}t = v_0\frac{D}{L}\frac{v_{0,y} + \sqrt{v_{0,y}^2 + 2gh_2}}{g} \quad (9)$$

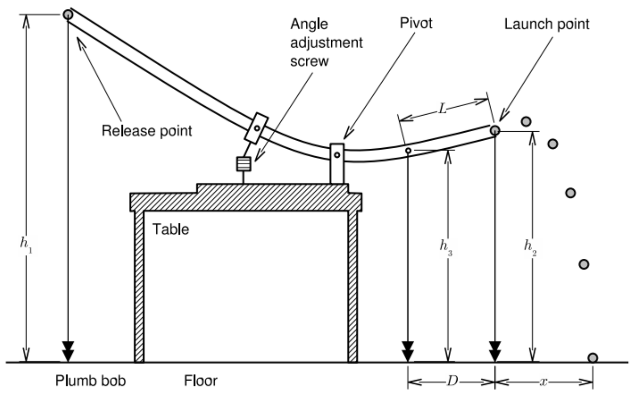

Figure 1: Ball Launching off Ramp

We then taped a piece of printer paper with cardinal axis's drawn on it where the origin was the measured landing position. A carbon paper was taped on top of it to track exactly what positions it landed at by imprinting the ball onto the printer paper underneath. The metal ball was sent along the ramp at the new heights for 20 times, and the visual data was recorded on the printer paper.

Once the trials for the metal ball were complete, we switched to a plastic ball where we then had to remeasure the energy lost to friction using the same procedure as the first ball. Then we readjusted for the heights again taking new measurements for *h1* and *h2* then measuring *h3, D, and L*. We then repeated the procedures of predicting the landing position and placing papers down to perform 20 trials of launching the plastic ball from the ramp.

III. Results & Analysis

We start our analyzations of the results by looking at the specific measurements describing the set up of all trials for the metal and plastic ball. *Table 1* shows the heights for *h1, h2*, and *h3* along with the dimensional measurements of the upwards pointing part of the ramp *D* and *L*. The $\Delta h'$ for the metal ball 0.020 ± 0.001 (m). Table 2 shows the same measurements but for the plastic ball with the $\Delta h'$ for the plastic ball being 0.060 ± 0.001 (m).

Metal Ball		
Variables	Trial 1	Trial 2
h_1(m)	1.280 ± 0.001	1.257 ± 0.001
h_2(m)	1.160 ± 0.001	1.149 ± 0.001
h_3(m)	1.066 ± 0.001	1.072 ± 0.001
L(m)	0.298 ± 0.001	0.298 ± 0.001
D(m)	0.268 ± 0.001	0.299 ± 0.001

Table 1: Initial measurements for Metal Ball

Plastic Ball		
Variables	Trial 1	Trial 2
h_1(m)	1.283 ± 0.001	1.273 ± 0.001
h_2(m)	1.132 ± 0.001	1.143 ± 0.001
h_3(m)	1.056 ± 0.001	1.064 ± 0.001
L(m)	0.298 ± 0.001	0.298 ± 0.001
D(m)	0.323 ± 0.001	0.334 ± 0.001

Table 2: Initial measurements for Plastic Ball

Table 2 shows are calculated landing positions for each ball in each trial calculated using Equations 8 and 9. As shown in the table the estimations for each second trial are less than the first trial since *h_1* was lowered after the first trial meaning the ball started with less potential energy. Also, the estimations for the plastic ball are generally greater than the estimations for the metal ball.

Trials	Metal Ball	Plastic Ball
Trial 1	0.559 ± 0.007	0.617 ± 0.007
Trial 2	0.564 ± 0.006	0.558 ± 0.005

Table 3: Estimated Displacement Δx_{est} (m)

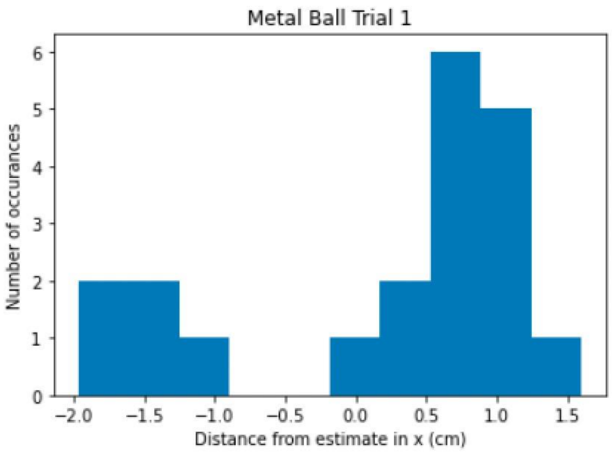

Figure 2: Spread of Metal ball in x direction for trial 1

Figure 2 shows the spread of data for the metal ball in the first trial. A histogram was used to represent the data to better examine how the spread of the data relates to the estimated landing position.

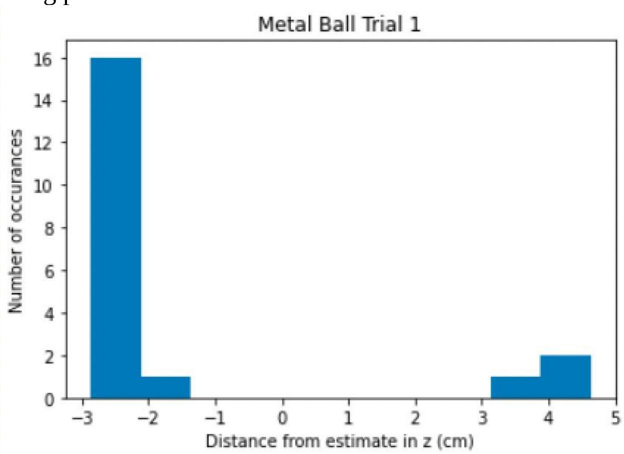

Figure 3: Spread of Metal ball in z direction for trial 1

The spread for the z position of the metal ball appears to have a more concentrated landing area than the x position. This is likely due to the fact that the ball experienced no major forces exerted on it in the z direction when traveling along the ramp or through the air.

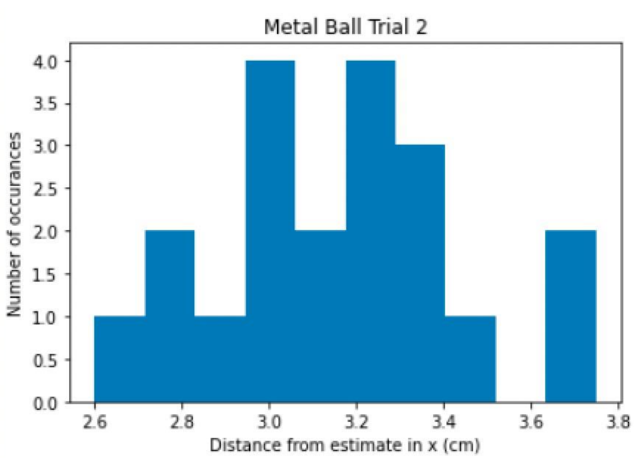

Figure 4: Spread of Metal ball in x direction for trial 2

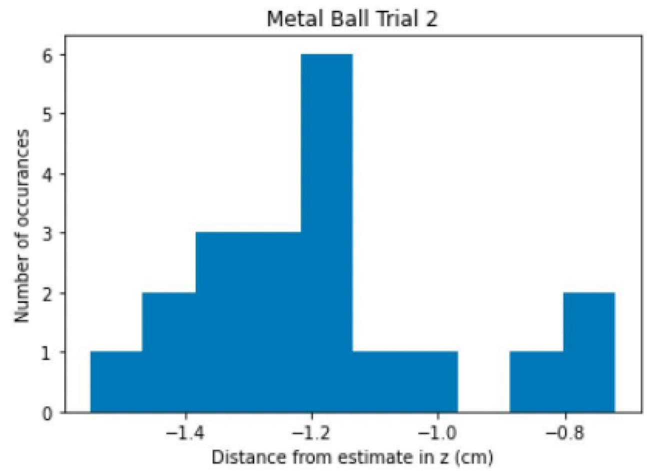

Figure 5: Spread of Metal ball in z direction for trial 2

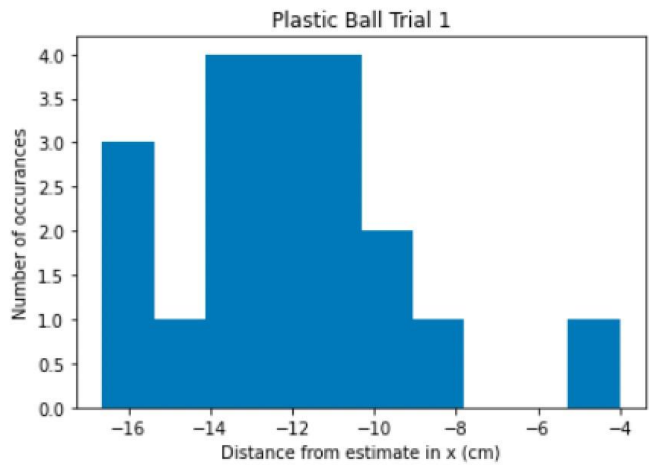

Figure 6: Spread of Plastic ball in x direction for trial 1

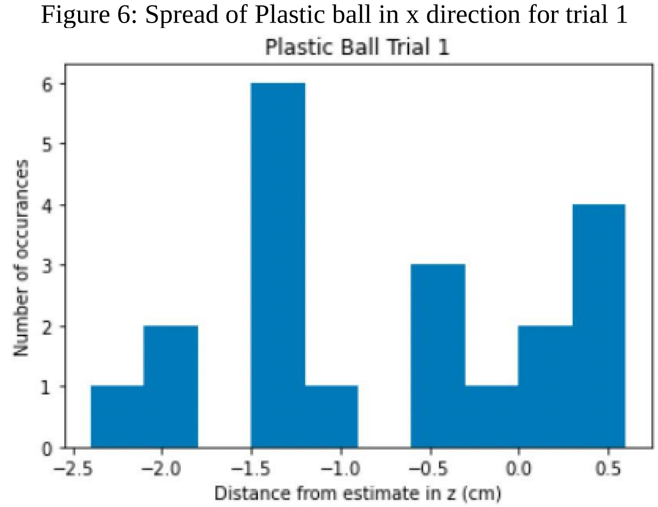

Figure 7: Spread of Plastic ball in z direction for trial 1

The histograms for trial 1 of the plastic ball's x and z direction from the predicted position clearly shows a much greater spread of data at larger distances from the calculated landing position. This can be due to the plastic ball, having been lighter and less dense, being under greater influence from external factors such as gusts of air or irregularities along the ramp. Notice in Figure 6 there is appears to be a cluster of data around the -14cm to -10cm mark which indicates that some unknown factor or factors, likely something influencing the air the ball was traveling through, were causing the plastic ball to lose horizontal speed as it was airborne. This could be due to the fact that our lab station was set up right next to an active radiator which was likely influencing the nearby air in ways that would not have happened in a more isolated setting. This would also help explain why the metal ball landed closer to the calculated landing position due to it being of a different material and mass. This experiment could be made better by making the environment of the experiment more isolated from external factors that create air resistance on the balls.

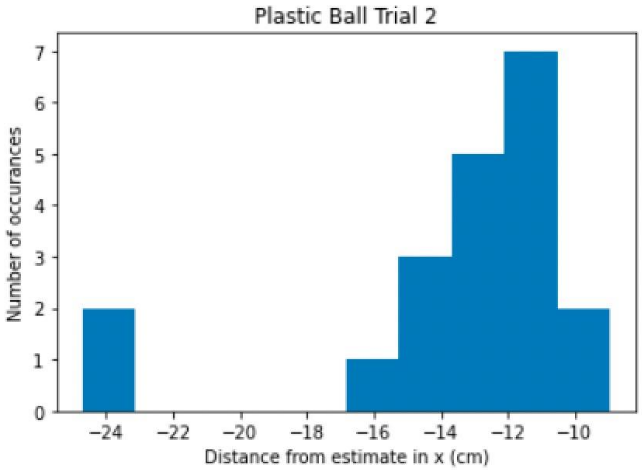

Figure 8: Spread of Plastic ball in x direction for trial 2

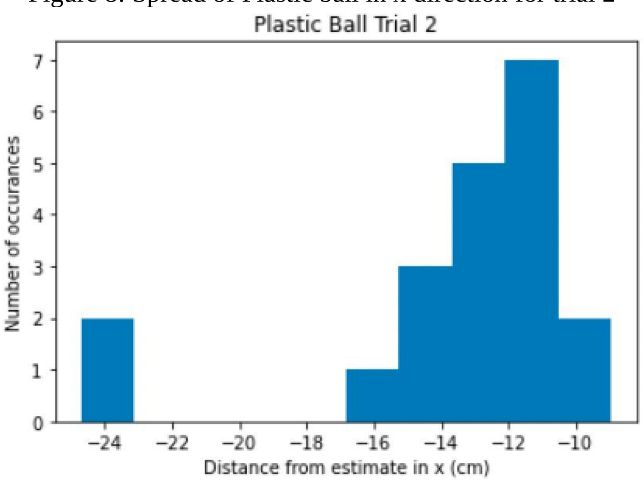

Figure 9: Spread of Plastic ball in z direction for trial 2

Every histogram appears to have a random distribution of landing position values around a particular value, even if the value does not match with the coordinates of the calculated landing position, indicating Gaussian distributions across the data.

Certain things that could have influenced the results of our experiment are the fact that the moments of inertia of the two balls might have been different due to the plastic balls hollowness which would affect our calculations since the rotational energy would have taken more energy away from the balls center of mass, kinetic energy. Also, we did not know of any imperfections on each ball which might have influenced the rotational energy, and then linear kinetic energy, on the ball. Another thing is that the energy lost to friction may not have been accurate due to the fact that the ball may not have been perfectly rolling at certain points along the ramp. This means that some kinetic frictional force may have been present in the trials which increases with speed, and the ball would have been moving faster through the ramp in the trials than when we were measuring for friction.

$$W = f_f l \tag{10}$$

Using Equation 10 and our understanding of the dimensions of the ramp we are able to estimate the value of the frictional force. Our value L which is 0.298 (m) represents about a sixth of the ramp's length. Knowing this we can estimate the total length of the ramp to be 1.788 (m). Using Equation 3 to solve for *f* along with the total length of the ramp the predicted frictional force is 0.006 N. The force of friction would affect the plastic ball much more than the metal ball given its reduced mass and hence reduced influence from the force of gravity.

Our Equations were derived with the assumption of perfect conditions which means no work done on the system from outside forces, uniformity of the sphere shapes of the balls, and uniformity inside the tube of the ramp. Environmental conditions such as the nearby radiator, gusts of air, particles in the air, or shifts in the position of the table could have all influenced how that metal and plastic balls landed during each iteration.

IV. Conclusion

This experiment demonstrated how the motion of an object in projectile motion is governed by The Law of Conservation of Energy. Our estimations for where the ball would land were not always the most accurate, but this can be explained by the influence of external forces and unaccounted for nonuniformities. Future iterations of this experiment should attempt to isolate for these influencing factors even more by creating more isolated environments for the ball to move in that remove air resistance. Also using more uniform and clean instruments would also ensure greater accuracy in the experiment. When estimating the positioning of a projectile outside of this experiment environmental factors like air resistance must be accounted for in order to obtain accuracy of where the object will go.

References

[1] Department of Physics, "Experiments in Physics," Columbia University. New York, pp. 1 5 -22.