e/m of the Electron
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I. INTRODUCTION

In 1897 J.J. Thomson made a foundational discovery in physics when he used cathode rays to discover the electron by observing how the rays behave as beams of particles with a standard charge to mass ratio *e/m.* For this experiment we will be also measuring the charge to mass ratio of electron particles as they move in a beam.

Charged particles will be sent through an electric potential V and bent along a circular path with radius r inside of a magnetic field with strength B. The charged particles will be moving along this circular path with velocity *v*. Since the charged particles will be moving in a circular path, we can use the equations of uniform circular motion along with our knowledge of energy conservation, and magnetic fields to help calculate the charge to mass ratio. The accuracy of our charge to mass ratio will likely be dependent on our ability to isolate the strength of the magnetic field which may hinder on factors outside of our experimental control.

II. METHOD

For this experiment we had an electric gun emitting charged particles into a glass vacuum tube. This was also then surrounded by conducting wire coils with current *I* producing a magnetic field, B, perpendicular to the motion of the emitted charged particles.

The charged particle experienced a deflection force by the present magnetic field that can be described by Equation 1.

$$\vec{F}_{mag} = e\vec{v} \times \vec{B} \tag{1}$$

Since the magnetic field and its deflection force are perpendicular to the charged particles' direction of velocity, we can treat the deflection force as the centripetal force to a particle in uniform circular motion. Hence, we can now also apply Equation 2 to our understanding of the motion of the particle.

$$evB = m\frac{v^2}{r} \tag{2}$$

We were not able to take direct measurements of *v* although we were able to redefine *v* in terms of a known V as the particle accelerated. Using the kinetic and potential energy equations described in Equation 3 and our derived Equation 2 we can derive an Equation for the ratio of *e/m* using known and measured values.

$$U_{kin} = \frac{1}{2}mv^2 \qquad and \qquad U_{pot} = eV \tag{3}$$

$$\frac{e}{m} = \frac{2V}{B^2r^2} \tag{4}$$

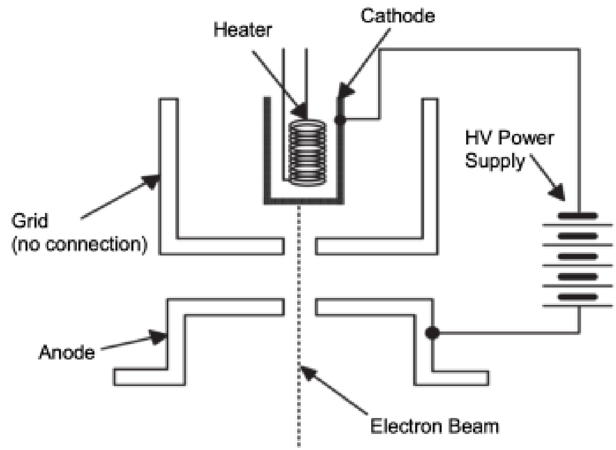

Figure 1: Electric gun in glass tube diagram

In order to determine the total magnetic field B acting on the emitted charged particles we had to consider both the magnetic field created by the coils, B_I, and the Earth's magnetic field, B_E. In order to factor out the Earth's magnetic field from the experiment we laid a compass in front of the apparatus and made the turned-off electric gun parallel with the north-south axis of the compass. Since we only needed to account for B_I, we were then able to conclude Equation 5 as representing the total magnetic field due to the positioning of the coils in the apparatus. Since we knew that N=132 turns and R=0.1475m we were able to calculate the constant C as $8.05*10^{-4}$ ($T\ A^{-1}$) which is used later to help find *e/m.*

$$B_I = \frac{\mu_0 R^2 N I}{(R^2 + (R/2)^2)^{3/2}} = \left(\frac{4\pi \times 10^{-7} N}{R(1+1/4)^{3/2}}\right) I = C \cdot I \tag{5}$$

After making the physical adjustments to our apparatus we were then able to turn on the apparatus and had to wait 15 minutes for it to heat up and produce a beam of charged particles. We then manipulated I and V directly using the knobs attached to the bottom of the apparatus which would also manipulate r and B. Understanding the relationships between the controlled and dependent variables we can see how Equation 6 is derived using Equations 4 and 5 to produce a I as a function of *r*.

$$I \simeq \left(\frac{1}{C}\sqrt{\frac{2V}{e/m}}\right)\frac{1}{r} \tag{6}$$

For given values of V (100, 200, 300, 400, and 500 V) we took 5 separate measurements of I for standard values of r (6, 7, 8, 9, and 10 cm).

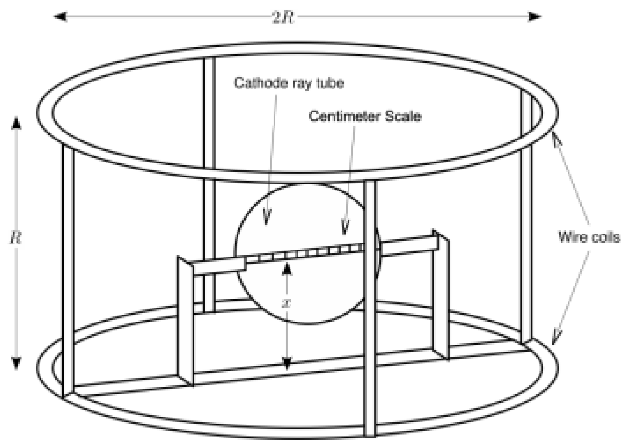

Figure 2: Cathode ray in glass tube with surrounding Helmholtz coils

III. Results & Analysis

Using the measurements obtained for each accelerating potential V, and the associated currents I and radii *r* for each separate V we were able to plot the data as a function I of 1/*r* with separate linear fits on the graph representing the different trials of V. The function from Equation 7 can be simplified down to a linear approximation with Equation 7 which represents the linear fits for each set of data on our graph in Figure 3.

$$I = A \cdot \frac{1}{r} + D \tag{7}$$

The values A and D in Equation 7 represent the slope and y-intercept for each of the linear fits shown in the graph.

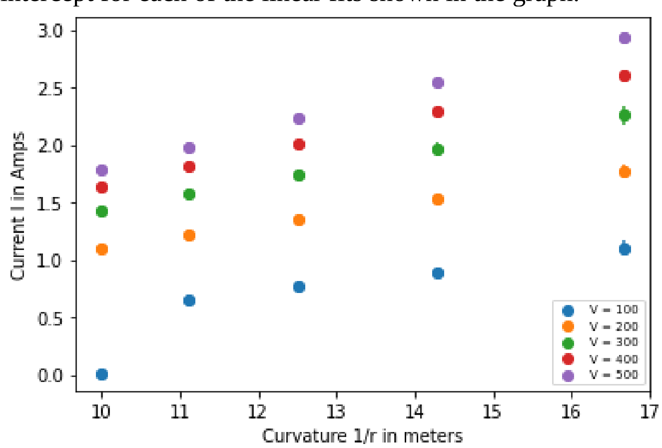

Figure 3: Scatter plot of I vs 1/r

Also, the data for when the potential was 100V is more sporadic likely due to the fact that under weaker currents, I, magnetic fields outside of the apparatus like Earth's magnetic field and the magnetic field from nearby computers would have more interference on the system.

In order to better analyze our data, we did a weighted linear regression to the data points on our scatter plot and for each set of potentials V shown in Figure 4.

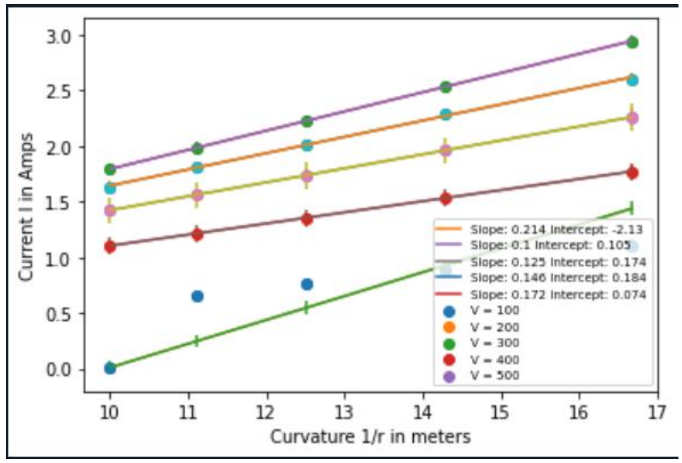

Figure 4: Linear Fits for each trial

For the trial with the potential at 100V the current became too week to properly collect data past a certain radius *r*, so the linear fit is less approximate for that set of data. This could likely be due to the lower kinetic energy that the charged particles were released at when emitted by the electric gun causing the loop to be fainter.

As the potential was raised the linear relationship between I and 1/*r* becomes much stronger since the magnetic field produced by the current strengthens. Also, as the potential difference increased throughout the experiment the necessary current needed to bend the ray in the magnetic field also grew because the speed at which the charged particles were fired from the electric gun grew hence also increasing the centripetal force.

Across our linear regressions the error tends to trend downward as r decreases and the slope of each line tends to trend upward with greater potential, with certain acceptations to each. One potential reasoning for why the error trends downward as the radius of the cathode ray increases is that the emitted charged particles have to travel farther for to reach to desired radius meaning that they will face greater interference along the way. This causes the loop to have less precise lines and for measurements to be more off.

We used the slopes and y-intercepts found in the weighted linear regressions to help calculate our charge to mass ratios shown in Table 1.

Average	Error
7.89E+10	4.25E+09
5.20E+10	1.52E+09
4.73E+10	1.42E+09
4.72E+10	1.32E+09
4.80E+10	1.36E+09

Table 1: Weighted averages and corresponding error

Our final value for the charge to mass ratio ended up being 4.98×10^{10} C kg^{-1} with an error of $7x10^{8}$ which was not within the statistical error of the real value of 1.758×10^{11} C kg^{-1}.

Our y-intercepts, or values of D and average D, do not agree with the expected values because the expected would be closer to zero in a true vacuum.

Certain things that could have thrown our data off could have been the Earth's magnetic field, moving computer and phones around the room, or non-uniformities in parts of the apparatus, like the wire coil, that we were unaware of.

IV. Conclusion

For this experiment we made an approximated estimate for the charge to mass ratio of an electron using the same type of experimental analysis used in J.J. Thomson's original experiment when he discovered the electron. We got to have an insight into the ideas and variety of physics concepts that went into one of the greatest discoveries in physics at the time. Even though our calculated charge to mass ratio, 4.76×10^{10} $C\ kg^{-1}$,was far off from the actual measurement, $1.758 \times 10^{11}\ C\ kg^{-1}$, we can use our knowledge gained from this experiment to inform future reiterations of the experiment on how better to isolate the magnetic field created within the experiment apparatus. Also, increasing the lower potential values to measure I and *r* in order to help prevent outside interference so that one can get a more accurate *e/m.*

References

[1] Department of Physics, "Experiments in Physics," Columbia University. New York, pp. 1 5 -22.