AC Circuits
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I. Introduction

On this experiment we observed an alternating circuit under various impedance changes. An alternating circuit is a circuit driven by a varying voltage that changes with respect to time which can be observed using Kirchhoff's Loop Rule to break down the voltage across different components of the circuit as seen in the derivation of Equation 1.

$$\begin{aligned}\sum \Delta V &= 0 = V(t) - V_R - V_L - V_C \\ V(t) &= IR + L\frac{dI}{dt} + \frac{q}{C} \\ &= IR + L\frac{dI}{dt} + \frac{1}{C}\int I\ dt\end{aligned} \tag{1}$$

Since V(t) in our experiment is sinusoidal we may write the equations for V(t) and I(t) as:

$$V(t) = V_{\mathsf{max}} \sin \omega t + \phi_V \tag{2}$$

$$I(t) = I_{\mathsf{max}} \sin \omega t + \phi_I, \tag{3}$$

With our understanding of these equations the voltage equations for the individual parts of the circuit can be derived as shown with Equations 4, 5, and 6 by using the Equation 1, 2, and 3.

$$V_R = I(t) * R = I_{max} R sin(\omega t) \tag{4}$$

$$V_L = \omega L I_{max} cos(\omega t) = I_{max} X_L sin\left(\omega t + \frac{\pi}{2}\right) \tag{5}$$

$$V_C = \frac{q}{C} = \frac{1}{C}\int I(t)dt = I_{max} X_c sin\left(\omega t - \frac{\pi}{2}\right) \tag{6}$$

In the second part of the experiment, we observed phase differences between circuit elements. The phase difference between circuit components can be expected to remain constant but the difference φ between circuit elements and the driving V is dependent on angular frequency ω. V_R in our experiment is in phase with the current I and therefore can be used to estimate the phase difference φ.

II. Method

For the first part of the experiment, we set up an RLC circuit with function generator and digital oscilloscope. The digital oscilloscope was used to measure V_R waves initiated by the voltage created by the function generator. The inductance of our inductor L was 150mH, capacitor value C was 0.5μF, and the resistance value R was varied between 10 Ω, 50 Ω, and 500 Ω. A diagram of our set up is pictured in Figure 1 where V(t) represents the function generator.

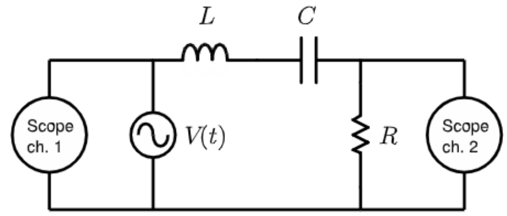

Figure 1: RLC circuit with function generator and oscilloscope

At the beginning of our experimental set up we set the function generator to 20V and adjusted the settings on the digital oscilloscope to display the appropriate voltage graphs where V_R could be observed. We calculated the theoretical resonant frequency ω_0 for our 3 frequency values by using Equation 7.

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{7}$$

The experimental value of resonant frequency ω_0 was found by tuning the function generator until the peak-to-peak max voltage was displayed on the function generator. We then recorded 10 more values above the resonant frequency and 10 values below resonant frequency. We converted the frequency recorded on the oscilloscope to angular frequency using Equation 8.

$$\omega = 2\pi f \tag{8}$$

Due to the peaking nature of V_R as we adjusted the frequency, we expected the data of V_R vs ω to first increase then decrease gradually after peaking.

We then replaced the 150 mH inductor with a large copper ring inductor and adjusted the frequency until we found ω_0. Using Equation 7 we measured the inductance L of the copper ring and compared it to the actual inductance L.

For the second part of the experiment, we set up the experiment as we did for the first part, shown in Figure 1, and reset the oscilloscope to show V_R and we found the resonant frequency as we did before with the resistor set to 30 Ω. We took similar measurements as we did before although fewer of them and also measured the total time for one cycle of the driving signal, T_D, and the time difference between the max of the driving signal and max of the signal across the resistor. We then calculated the phase difference for each data point using Equation 9 where X_L is the reactance from the inductor and X_C is the reactance from the capacitor.

$$tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R} \tag{9}$$

According to how Equation 9 is written, a positive phase difference would indicate that the driving voltage V_D peaks before the V_R whereas a negative phase difference indicates V_R peaking before V_D.

We then repeated this measurement process once each time for a very small frequency and once for a very large frequency in order to show how either the inductor or the capacitor dominated the circuit. Equation 4 represents that at a very large frequency the phase shift, φ, will increase asymptotically to π/2 and asymptotically to - π/2 at a very low frequency. This can be represented visually with the graph arctan(x).

$$tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} \tag{10}$$

III. Results & Analysis

The values of the capacitor and inductor used in the first part of the experiment were 150mH and 0.5 μF while the resistance value varied between 10 Ω, 50 Ω, and 500 Ω. Our uncertainty for the recorded frequency was 4Hz and 25.132 radians/second for angular frequency. The uncertainty for our voltage data points was 0.4V. We then normalized the voltage data points and plotted the points on a chart of V_R vs ω where a steep increase in voltage can be observed coinciding proportionally with increasing angular frequency then flattening out and peaking until the voltage begins gradually decreasing with angular frequency.

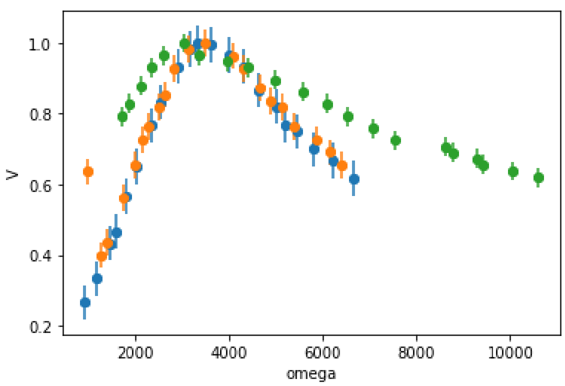

Figure 4: Normalized data of V_R vs ω

The peak-to-peak voltages of each of our resonant frequencies were 4.8V, 22V, and 116V. The resonant frequency values for the resistance value of 10 Ω is closest to the theoretical value of 3651.48 radians/second and the resonant frequency value becomes less close as the resistance value is increased to 500Ω. Although there are certain discrepancies the resonant frequency stays about the same as resistance R is increased. Potential sources for the discrepancies between the measured values and theoretical value are the large uncertainties for voltage and frequency due to the variability of the data on the oscilloscope at the peak voltage.

The width of the peaks show that the smaller resistance causes higher circuit sensitivity to changes in driving frequency. The narrowest full width at half maximum (FWHM) is the group of 10 Ω resistance points while the widest FWHM is the group of 500 Ω resistance points.

After replacing the smaller inductor with the large copper ring of capacitance 20 μF, and total resistance R of 80 Ω ,stemming from the resistor and function generator, we collected data for 5 data points in order to estimate the resonant frequency to be 10995.574 radians/second, and we found the estimated inductance to be a value of 16.542mH which was not super close to the actual value of 20mH.

A possible source of error could be in the measurement of the frequency since the uncertainty was large due to the range of values given by the oscilloscope at that point and nearby points. Another cause of error could be in the given capacitance value due to unknown factors in the circuit system.

In the second part of the experiment, we measured the phase shift between V_R and V_D for different driving frequencies. The circuit had 50 Ω of resistance from the function generator and 30 Ω from the resistor for a total of 80 Ω.

The measured ω_0 in this circuit was 3788.761 radians/second as shown in Table 1 below. All of the trials where the frequency is greater than the resonant frequency have positive phase shift whereas the negative phase shifts are when the resonant frequency is greater than the measured frequency at that point. In Table the extremes of frequency values, and their effect on the phase shift are shown. This shows how the inductor dominates the circuit at high frequencies whereas the capacitor will dominate the circuit at low frequencies. Also, the extremes shown in Table 2 seem to be approaching π/2 and – π/2 asymptotically representing the graph of arctan(x) as described by Equation 10.

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f (Hz)	ω (radians/sec)	φ (radians)
603	3788.76074	0
550	3455.751919	-0.29
508	3191.858136	-0.25
640	4021.238597	0.2
683	4291.415565	0.29

Table 1: Frequency values and phase and respective phase shift for AC circuit in second part

	High (10kHz)	Low (50Hz)
ω	62831.853	314.159
φ	1.421	-1.453
Xl	9424.778	47.124
Xc	0.3183	63.662

Table 2: Data from frequency extremes also showing whether X_L dominates X_C or vice versa

In Table 2 X_L is much greater than X_C when the frequency is very high and X_C is much greater than X_L when the frequency is very low therefore determining whether the inductor or capacitor dominates the circuit at either frequency extrema.

IV. CONCLUSION

In this experiment we observed an RLC circuit at three separate resistance levels where we observed that the resonant frequency stayed remotely near an expected value, and would skew based on measuring accuracy due to large uncertainties. The circuit at 10 Ω was closest to the expected value likely due to the level of sensitivity in the circuit system. Our measured inductance value of the large copper ring was close but not within error of the expected inductance value.

In the second part of the experiment, we observed phase shifts around a circuit's resonant frequency. The phase shift measurements were positive when the measured frequency was greater than the resonant frequency and negative when the resonant frequency exceeded the measured frequency at that point which is expected by the law of RLC circuits. We also took measurements for frequencies on the extreme and observed how they asymptotically approached $\pi/2$ and - $\pi/2$ which falls in line with our graphical analysis.

REFERENCES

[1] Department of Physics, "Experiments in Physics," Columbia University. New York, pp. 1 5 -22.