Capacitance and Oscilloscope
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I. Introduction

Capacitors are circuit components simply made of metal plates that store energy and electric charge, and are important pieces of physics and engineering. The potential difference of the capacitor is directly proportional to the stored electric charge inside the capacitor, and its charge can be gradually increased or deceased by connecting its metal plates to a circuit system where in the current flow is in the positive or negative direction of the charged plates.

We observed the charging and discharging of capacitors in this experiment using a Large and Small RC set up. For the first part, a Large RC set up was used to measure the current flow on an ammeter with respect to time as the capacitance of the capacitor was changed to different values. We were able to obtain time constants and time constant ratios from these measurements that when compared to estimated values were not always within the margin of error.

For the final part of the experiment, we had a small RC set up and digital oscilloscope that was used to display a graphical curve of voltage signals over time. We were able to obtain the time constant with different frequencies using the oscilloscope and compared it to the estimated time constant.

II. Method

A charge Q can be measured in a capacitor by examining the potential difference across the capacitor V and capacitance C of the capacitor as shown in Equation 1. Depending on the direction of the flow of current through the capacitor we may also observe the discharging or charging of the capacitor.

$$Q = CV \tag{1}$$

For this experiment we started out with a circuit that had a capacitor with three switches in parallel which allowed us to modify the capacitance from 10 μF to 20 μF to 30 μF. We then used the attached ammeter to measure the resulting current I as a function of time by keeping track of a constant interval of seconds that passed while recording each corresponding current value. A switch was used to set and reset the current flow in the circuit for each time that we measured it. The basic set up for the apparatus is shown in Figure 1.

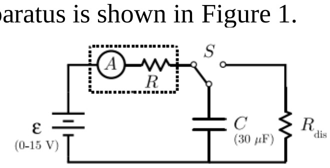

Figure 1: Large RC using a resistor R, EMF, an ammeter in series A, and switch S

When charging a capacitor, a battery with EMF is used in the first part of this experiment to fill the plates of the capacitor with charge, q, which can be represented with equations representing the charge and current with respect to time derived from Kirchhoff's Loop Rule where RC is the time constant as seen in Equations 2 and 3.

$$q(t) = Q\left(1 - e^{-t/RC}\right) \tag{2}$$

$$I(t) = \frac{\varepsilon}{R}\, e^{-t/RC} \tag{3}$$

When discharging the capacitor, the EMF form the battery is removed from the loop and the capacitor has an initial charge Q=Cε that declines gradually with time. This can be represented also using equations derived from Kirchhoff's Loop Rule that describe the charge and current over time.

$$q(t) = Q\, e^{-t/RC} = C\varepsilon\, e^{-t/RC} \tag{4}$$

$$I(t) = -\frac{\varepsilon}{R} e^{-t/RC} \tag{5}$$

A resistor R_{dis} was used to discharge the capacitance between measurements whereas the internal resistor of the ammeter R was used to determine the time constant τ=RC. After we measured charging of the capacitor, we then altered the apparatus slightly to measure discharging of the capacitor in the Large RC.

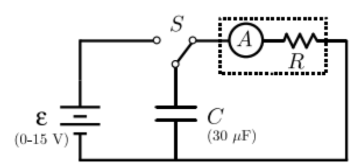

Figure 2: Large RC set up used to meausre discharging of capacitor

Figure 3 represents the set up for our observations of a small RC where a resistor of 10kOhm and capacitor of 0.082μF were used alongside a digital oscilloscope to observe signals that vary rapidly. We used the square wave setting with frequencies at 100Hz, 110Hz, 120Hz, and 200Hz. At each of these frequencies values we measured time at the peak and time at the 37% of the peak V to get our time constant Δt.

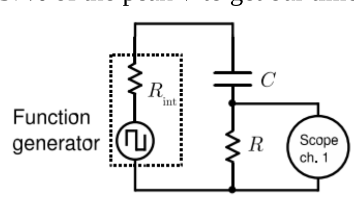

Figure 3: Small RC ciruit apparatus

III. Results & Analysis

To charge and discharge the capacitor 3 separate capacitance values (10 µF, 20 µF, and 30 µF) were used by turning the switches on the capacitor. Different current values were recorded for different amounts of time then we created graphs of ln(I) vs time(s) for each capacitance value on charging and discharging of the capacitor. The plotted data and linear fits are displayed in Figure 4 and 5.

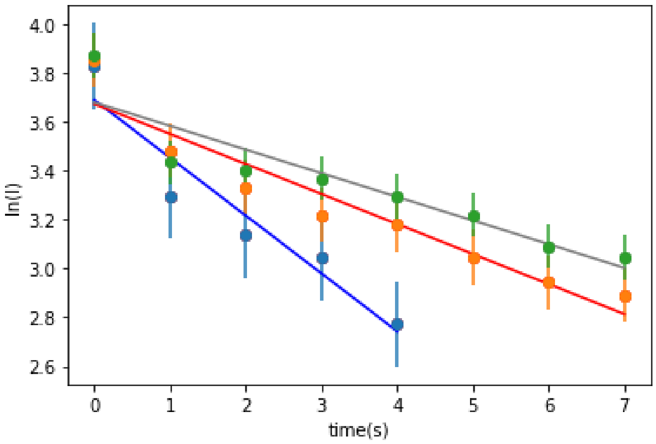

Figure 4: ln(I) as a function of time(s) for charging capacitor in Large RC

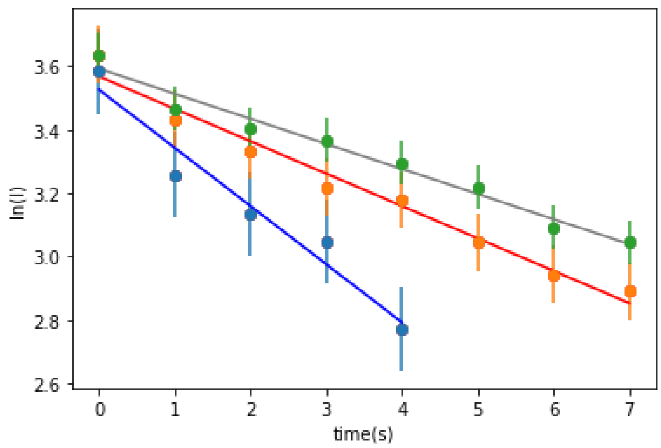

Figure 5: ln(I) as a function of time(s) for discharging capacitor in Large RC

We plotted the data on a logarithmic scale as a function of time in order to derive a time constant where in the exponential function in Equation 3 could be linearized as seen in Equation 6.

$$\begin{aligned} \ln I &= \ln\left(I_0\, e^{-t/RC}\right) \\ &= \ln I_0 - \left(\frac{1}{RC}\right) t \end{aligned} \tag{6}$$

From Equation 6 we can derive Equation 7 for the time constant where m is the slope of the linearized graph.

$$\tau = -\frac{1}{m} \tag{7}$$

Table 1 shows the slopes and uncertainties we got from our data for both charging and discharging of the capacitor, and for each tested capacitance value.

Experiment	**Capacitance (µF)**	**Slope (m)**	**Slope Uncertainty (σ_m)**
Charging	10 uF	-0.23634	0.04306
	20 uF	-0.12269	0.01527
	30 uF	-0.09667	0.01654
Discharging	10 uF	-0.18354	0.02319
	20 uF	-0.10217	0.00651
	30 uF	-0.07909	0.00533

Table 1: Slope and slope uncertainty values for each linear fit on both charging and discharging capacitor graphs

Table 2 has the time constants and their errors compiled for the tested capacitance values of both charging and discharging the capacitor.

Experiment	***C*** **(µF)**	**Time Constant τ(s)**	**Error of τ (s)**
Charging	10 uF	4.231	0.0428
	20 uF	8.151	0.0371
	30 uF	10.344	0.0532
Discharging	10 uF	5.448	0.0657
	20 uF	9.788	0.0739
	30 uF	12.644	0.0681

Table 2: Time constants and their uncertainties derived from measured data

Our measured time constant for the small RC agreed with the estimated time constant of 1153.2 µs within standard error. The time constants measured in both charging and discharging of the capacitor in the same Large RC set up did not tend to agree within error of one another which are likely due to observer errors made when reading the ammeter corresponding with the appropriate time in seconds. Another cause could be an unknown resistance in the circuit caused by moving of the set up or internal resistance in the wires.

In order to gain a better understanding of the data we took ratios of the different time constants against each other and compared them to expected values. The time constant ratios of the charging capacitor were $\tau_{30uF}/\ \tau_{20uF} = 1.269$, $\tau_{30uF}/\ \tau_{10uF} = 2.445$, $_{20uF}/\ \tau_{10uF} = 1.926$, and the time constant ratios of the discharging capacitor were $\tau_{30uF}/\ \tau_{20uF} = 1.292$, $\tau_{30uF}/\ \tau_{10uF} = 2.321$, $_{20uF}/\ \tau_{10uF} = 1.797$. These values are noticeably off from expected theoretical values of $\tau_{30uF}/\ \tau_{20uF} = 3/2$; $\tau_{30uF}/\ \tau_{10uF} = 3$; $\tau_{20uF}/\ \tau_{10uF} = 3$.

For the final part of the experiment, we used a digital oscilloscope to measure the time constant on a displayed graph for a small RC set up. We set our frequency at different values between 100Hz and 150Hz and measured the time difference between the peak of the curve to 37% of the peak. This measured time difference represented our time constant for the set up.

Frequency (Hz)	Delta Time (µs)
100	1100
110	1180
120	1160

130	1150
Average Time	1135
Standard Error	17.01714821

Table 3: Frequency and associated Time Differences of Small RC

Potential sources of error for this part of the experiment include possible mechanical errors stemming from poor tolerances in the circuit system affecting the impedance which would therefore influence the voltage.

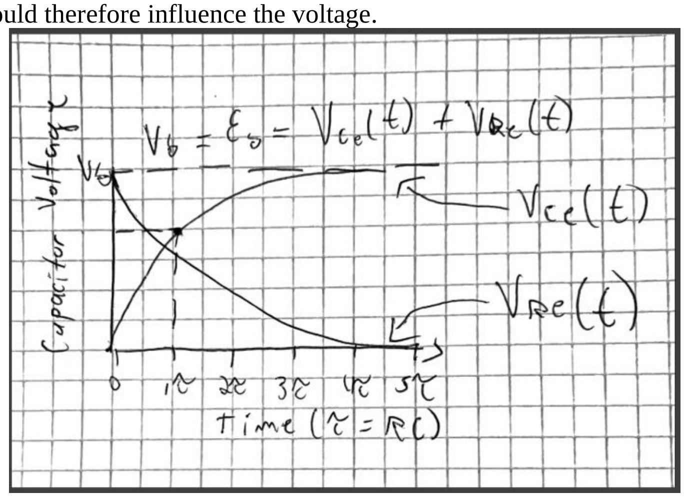

Figure 6: Drawing of time dependent behavior of voltage difference in capacitor

We sketched a graph of the voltage difference across the capacitor with respect to time noting that the capacitor's voltage was not instantaneously equal to the battery's voltage. The graph shows how the capacitors voltage, V_c, will asymptotically approach the battery's initial voltage V_b. In the first time constant the voltage the capacitor charges to 63% of initial V_b and 99% after 5 time constants.

Kirchhoff's Voltage Loop Law states that the total voltage in a loop should add up to 0, therefore, the sum voltage across the resistor and capacitor would be equal to the function generator voltage.

IV. Conclusion

For this experiment we observed the charging and discharging of a capacitor. In the first part of the experiment, we used a Large RC set up with an ammeter in order to observe current flow for different capacitance values in the capacitor. We did this for both charging and discharging of the capacitor to in order to find a time constant. The time constants calculated did not agree within error likely due to observer error when trying to keep track of the time and ammeter instantaneously.

In the second part of the experiment, we had a small RC set up with smaller resistance and capacitance values, and took our measurements for the time constant using a digital oscilloscope based on measured time differences of square waves under varying frequencies between 100Hz and 150Hz. The measured time constant from this part of the experiment did agree with the estimated time constant. In the future a much greater number of data points could be taken for each part to increase accuracy along with video taping the ammeter and looking back at the recording in order to gather more instantaneous data points.

References

[1] Department of Physics, "Experiments in Physics," Columbia University. New York, pp. 1 5 -22.