Linear Regression Part 2

DATA 606 - Statistics & Probability for Data Analytics

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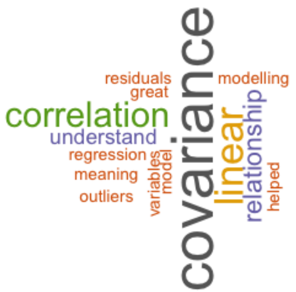
April 14, 2021

One Minute Paper Results

What was the most important thing you learned during this class?



What important question remains unanswered for you?

Homework Presentations

- 8.22 Tage Singh
- 8.25 Henry Owens

NYS Report Card

NYS publishes data for each school in the state. We will look at the grade 8 math scores for 2012 and 2013. 2013 was the first year the tests were aligned with the Common Core Standards. There was a lot of press about how the passing rates for most schools dropped. Two questions we wish to answer:

1. Did the passing rates drop in a predictable manner?
2. Were the drops different for charter and public schools?

```
load('../course_data/NYSReportCard-Grade7Math.Rda')
names(reportCard)
```

```
##  [1] "BEDSCODE"      "School"        "NumTested2012" "Mean2012"      "Pass2012"      "Charter"       "GradeSubject
##  [8] "County"        "BOCES"         "NumTested2013" "Mean2013"      "Pass2013"
```

`reportCard` Data Frame

Show 3 entries Search:

BEDSCODE	School	NumTested2012	Mean2012	Pass2012	Charter	GradeSubject	County	BOCES	NumTested2013	Mean2013
010100010020	NORTH ALBANY ACADEMY	47	649	13	false	Grade 7 Math	Albany	BOCES ALBANY-SCHOH-SCHENECTADY-SARAT	45	268
010100010030	WILLIAM S HACKETT MIDDLE SCHOOL	212	652	30	false	Grade 7 Math	Albany	BOCES ALBANY-SCHOH-SCHENECTADY-SARAT	250	279
010100010045	STEPHEN AND HARRIET MYERS MIDDLE SCHOOL	262	670	50	false	Grade 7 Math	Albany	BOCES ALBANY-SCHOH-SCHENECTADY-SARAT	256	284

Showing 1 to 3 of 1,362 entries

Previous 1 2 3 4 5 … 454 Next

Descriptive Statistics

```
summary(reportCard$Pass2012)
```

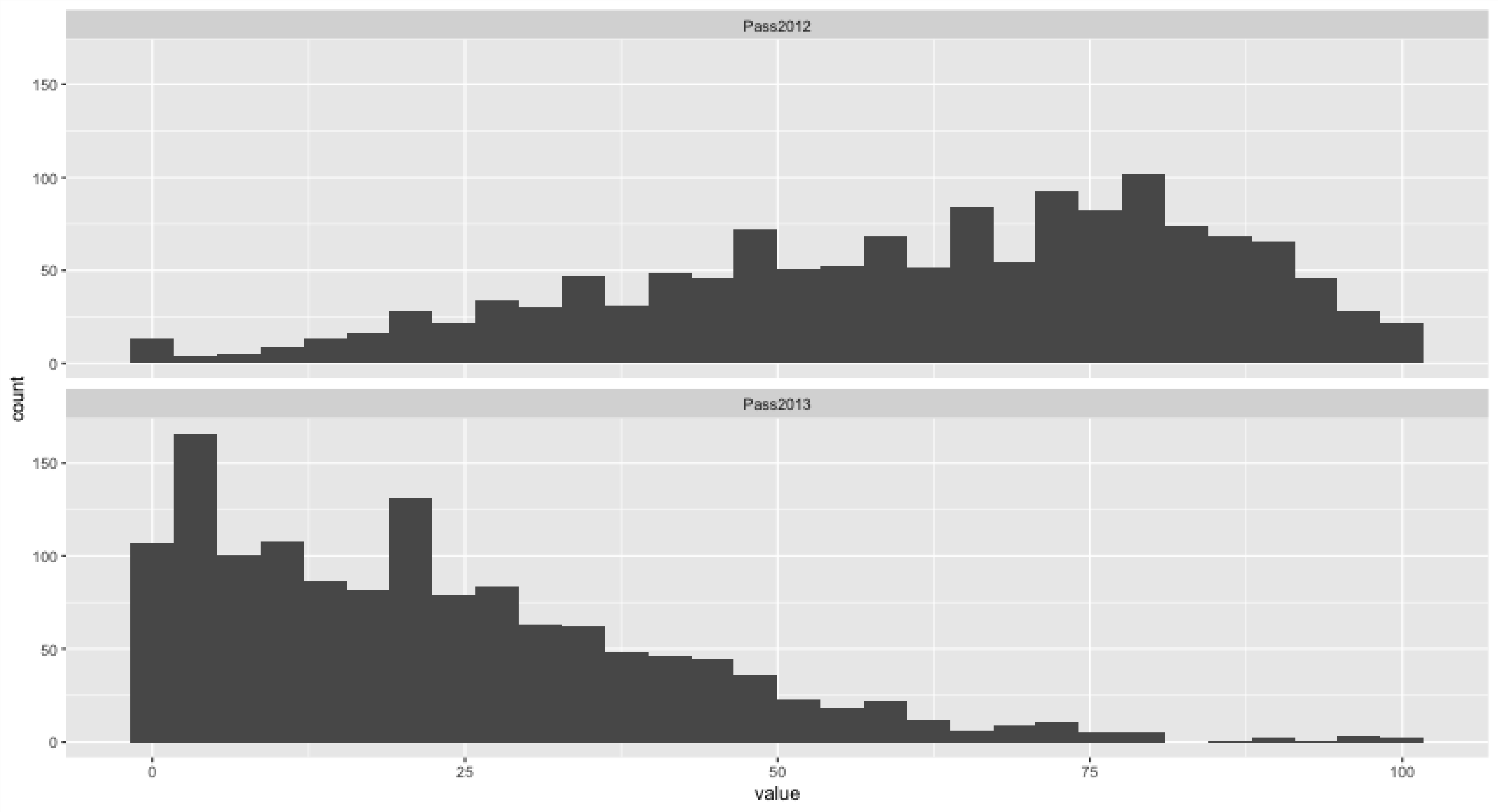
```
##     Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     0.00   46.00   65.00   61.73   80.00  100.00
```

```
summary(reportCard$Pass2013)
```

```
##     Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     0.00    7.00   20.00   22.83   33.00   99.00
```

Histograms

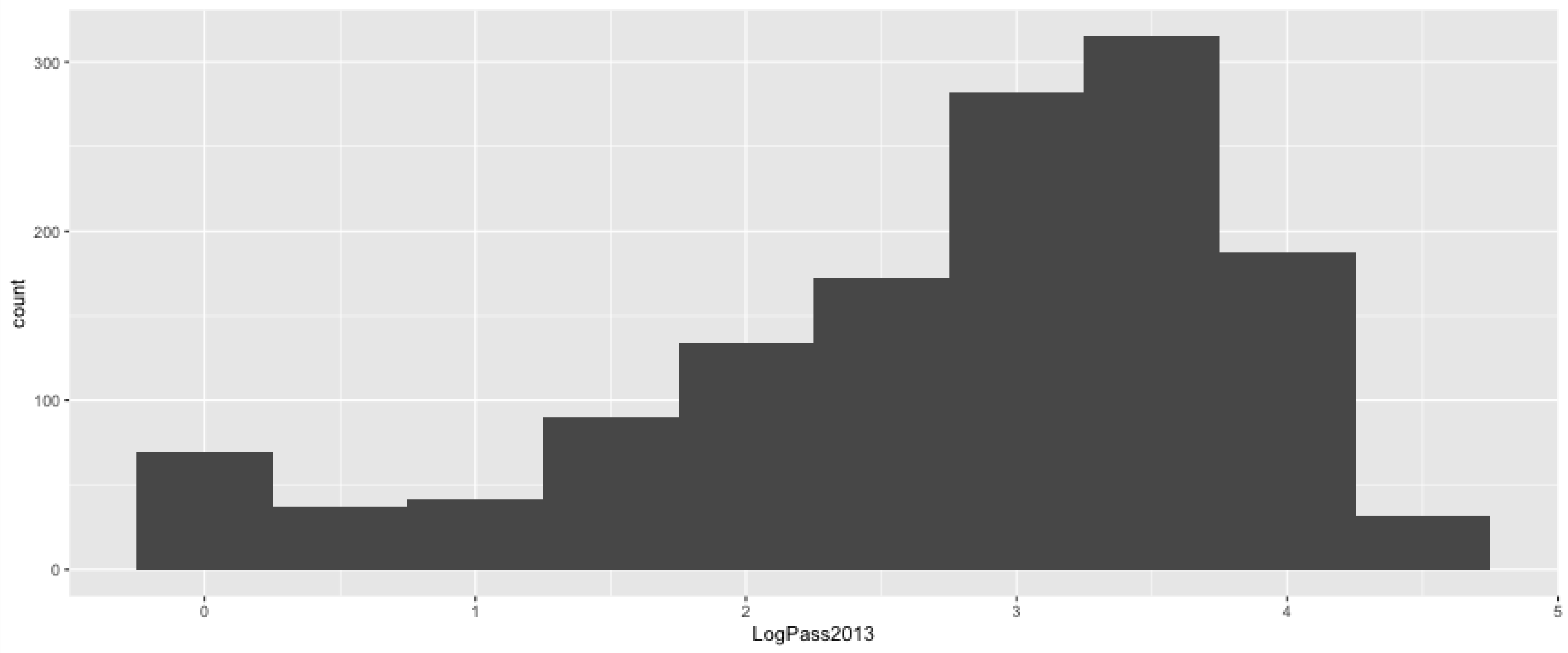
```r
melted <- melt(reportCard[,c('Pass2012', 'Pass2013')])
ggplot(melted, aes(x=value)) + geom_histogram() + facet_wrap(~ variable, ncol=1)
```



Log Transformation

Since the distribution of the 2013 passing rates is skewed, we can log transfor that variable to get a more reasonably normal distribution.

```
reportCard$LogPass2013 <- log(reportCard$Pass2013 + 1)
ggplot(reportCard, aes(x=LogPass2013)) + geom_histogram(binwidth=0.5)
```



Scatter Plot

```
ggplot(reportCard, aes(x=Pass2012, y=Pass2013, color=Charter)) +
    geom_point(alpha=0.5) + coord_equal() + ylim(c(0,100)) + xlim(c(0,100))
```

Scatter Plot (log transform)

```r
ggplot(reportCard, aes(x=Pass2012, y=LogPass2013, color=Charter)) +
    geom_point(alpha=0.5) + xlim(c(0,100)) + ylim(c(0, log(101)))
```

Correlation

```
cor.test(reportCard$Pass2012, reportCard$Pass2013)
```

```
## 
##  Pearson's product-moment correlation
## 
## data:  reportCard$Pass2012 and reportCard$Pass2013
## t = 47.166, df = 1360, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.7667526 0.8071276
## sample estimates:
##       cor 
## 0.7877848
```

Correlation (log transform)

```
cor.test(reportCard$Pass2012, reportCard$LogPass2013)
```

```
##
##     Pearson's product-moment correlation
##
## data:  reportCard$Pass2012 and reportCard$LogPass2013
## t = 56.499, df = 1360, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.8207912 0.8525925
## sample estimates:
##       cor
## 0.8373991
```

Linear Regression

```
lm.out <- lm(Pass2013 ~ Pass2012, data=reportCard)
summary(lm.out)
```

```
##
## Call:
## lm(formula = Pass2013 ~ Pass2012, data = reportCard)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -35.484  -6.878  -0.478   5.965  51.675
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -16.68965    0.89378  -18.67   <2e-16 ***
## Pass2012      0.64014    0.01357   47.17   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.49 on 1360 degrees of freedom
## Multiple R-squared:  0.6206,    Adjusted R-squared:  0.6203
## F-statistic:  2225 on 1 and 1360 DF,  p-value: < 2.2e-16
```

Linear Regression (log transform)

```
lm.log.out <- lm(LogPass2013 ~ Pass2012, data=reportCard)
summary(lm.log.out)
```

```
## 
## Call:
## lm(formula = LogPass2013 ~ Pass2012, data = reportCard)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -3.3880 -0.2531  0.0776  0.3461  2.7368 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 0.307692   0.046030   6.685 3.37e-11 ***
## Pass2012    0.039491   0.000699  56.499  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.5915 on 1360 degrees of freedom
## Multiple R-squared:  0.7012,	Adjusted R-squared:  0.701 
## F-statistic:  3192 on 1 and 1360 DF,  p-value: < 2.2e-16
```

Did the passing rates drop in a predictable manner?

Yes! Whether we log tranform the data or not, the correlations are statistically significant with regression models with R^2 creater than 62%.

To answer the second question, whether the drops were different for public and charter schools, we'll look at the residuals.

```
reportCard$residuals <- resid(lm.out)
reportCard$residualsLog <- resid(lm.log.out)
```

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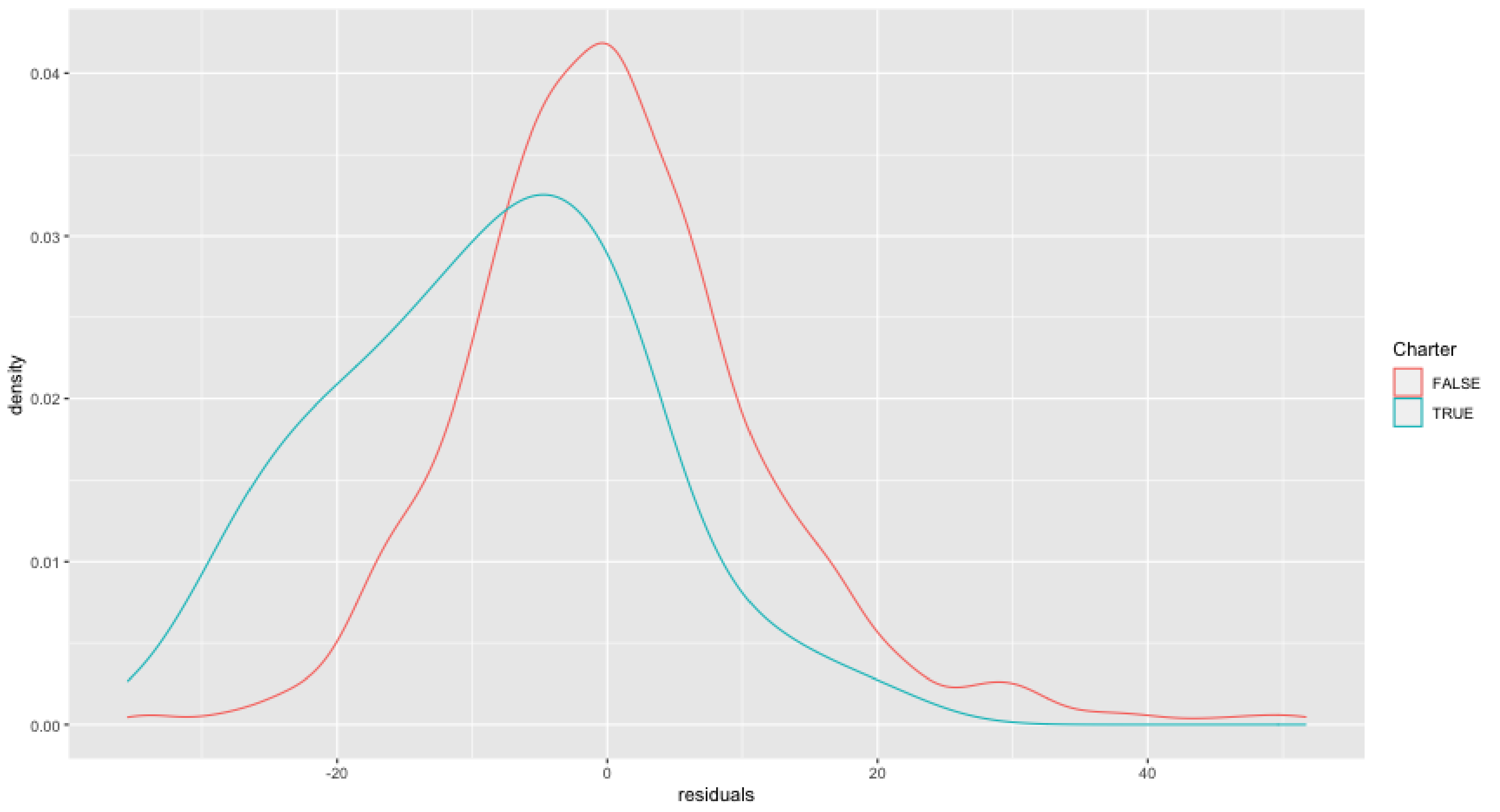
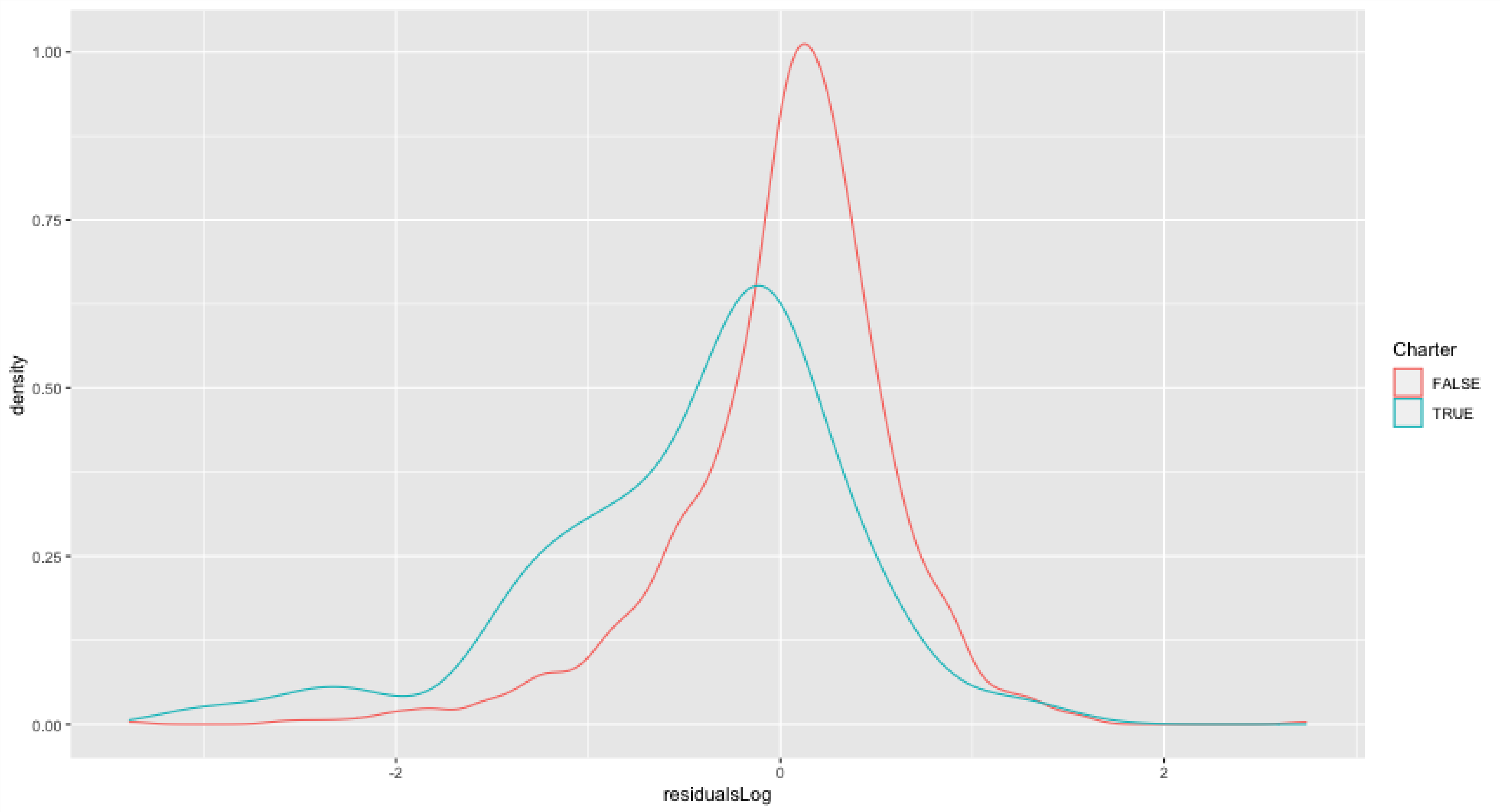
Distribution of Residuals

```
ggplot(reportCard, aes(x=residuals, color=Charter)) + geom_density()
```



Distribution of Residuals

```
ggplot(reportCard, aes(x=residualsLog, color=Charter)) + geom_density()
```



Null Hypothesis Testing

H_0: There is no difference in the residuals between charter and public schools.

H_A: There is a difference in the residuals between charter and public schools.

```
t.test(residuals ~ Charter, data=reportCard)
```

```
## 
##     Welch Two Sample t-test
## 
## data:  residuals by Charter
## t = 6.5751, df = 77.633, p-value = 5.091e-09
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##   6.411064 11.980002
## sample estimates:
## mean in group FALSE  mean in group TRUE 
##            0.479356           -8.716177
```

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Null Hypothesis Testing (log transform)

```
t.test(residualsLog ~ Charter, data=reportCard)
```

```
##
##      Welch Two Sample t-test
##
## data:  residualsLog by Charter
## t = 4.7957, df = 74.136, p-value = 8.161e-06
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.2642811 0.6399761
## sample estimates:
## mean in group FALSE  mean in group TRUE
##          0.02356911         -0.42855946
```

Polynomial Models (e.g. Quadratic)

It is possible to fit quatric models fairly easily in R, say of the following form:

$$y = b_1 x^2 + b_2 x + b_0$$

```
quad.out <- lm(Pass2013 ~ I(Pass2012^2) + Pass2012, data=reportCard)
summary(quad.out)$r.squared
```

```
## [1] 0.7065206
```

```
summary(lm.out)$r.squared
```

```
## [1] 0.6206049
```

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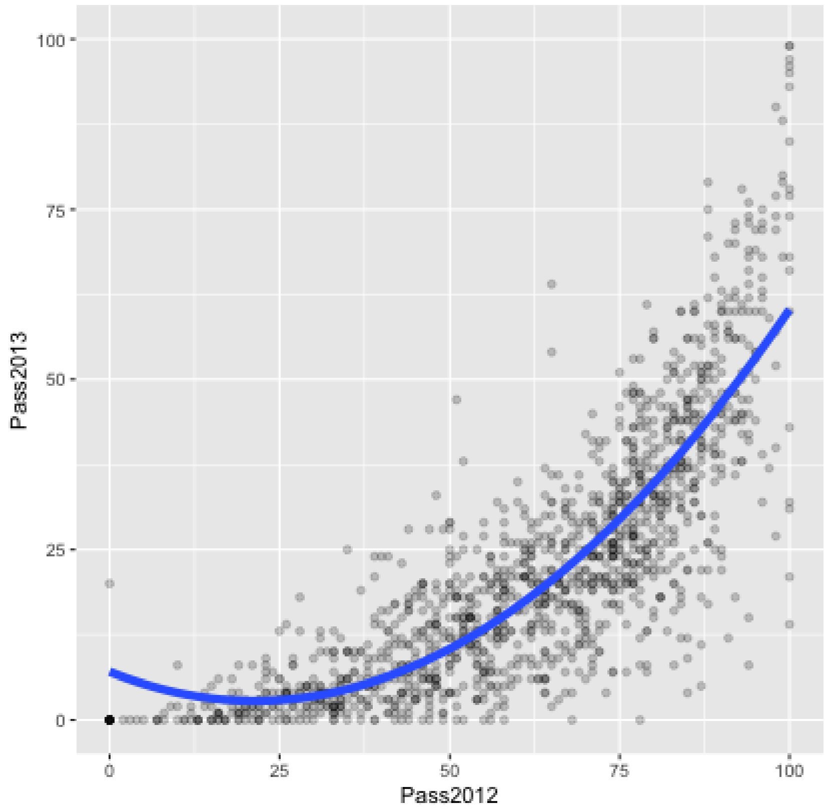
Quadratic Model

```r
summary(quad.out)
```

```
##
## Call:
## lm(formula = Pass2013 ~ I(Pass2012^2) + Pass2012, data = reportCard)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -46.258  -4.906  -0.507   5.430  43.509
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)    7.0466153  1.4263773   4.940 8.77e-07 ***
## I(Pass2012^2)  0.0092937  0.0004659  19.946  < 2e-16 ***
## Pass2012      -0.3972481  0.0533631  -7.444 1.72e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.11 on 1359 degrees of freedom
## Multiple R-squared:  0.7065,    Adjusted R-squared:  0.7061
## F-statistic:  1636 on 2 and 1359 DF,  p-value: < 2.2e-16
```

Scatter Plot

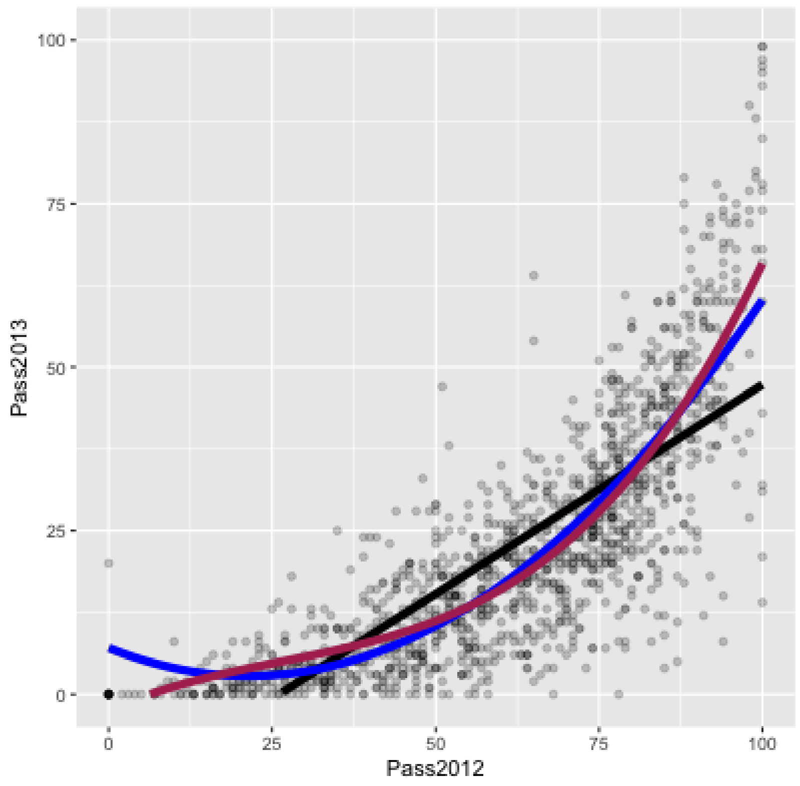
```
ggplot(reportCard, aes(x=Pass2012, y=Pass2013)) + geom_point(alpha=0.2) +
    geom_smooth(method='lm', formula=y ~ I(x^2) + x, size=2, se=FALSE) +
    coord_equal() + ylim(c(0,100)) + xlim(c(0,100))
```



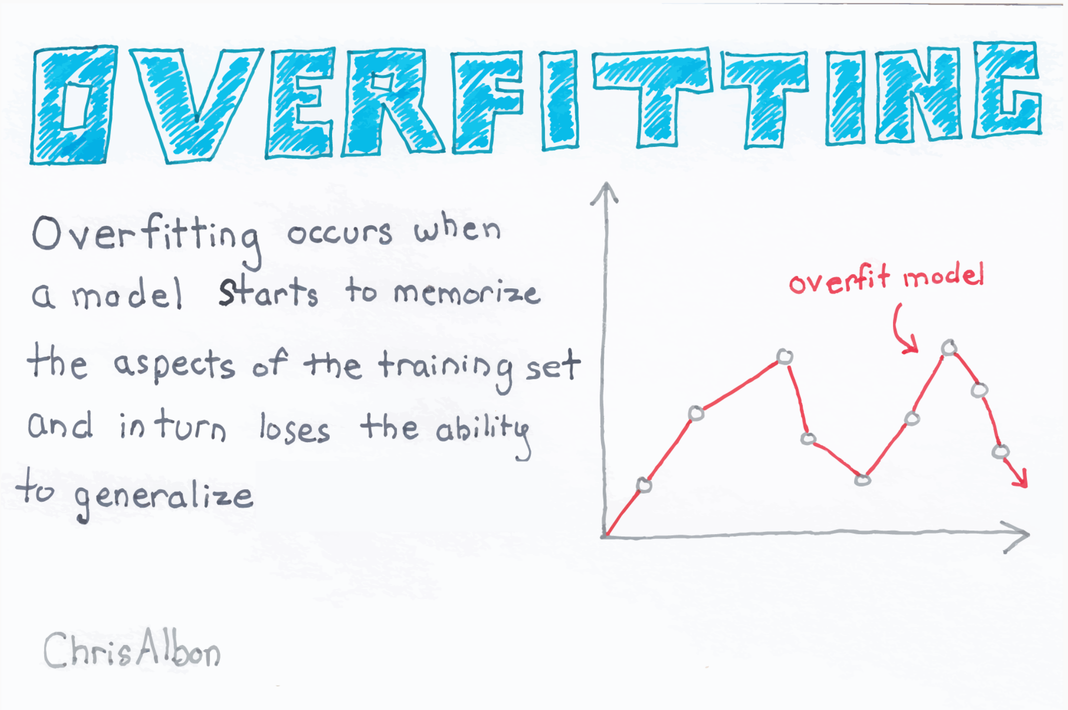
Let's go crazy, cubic!

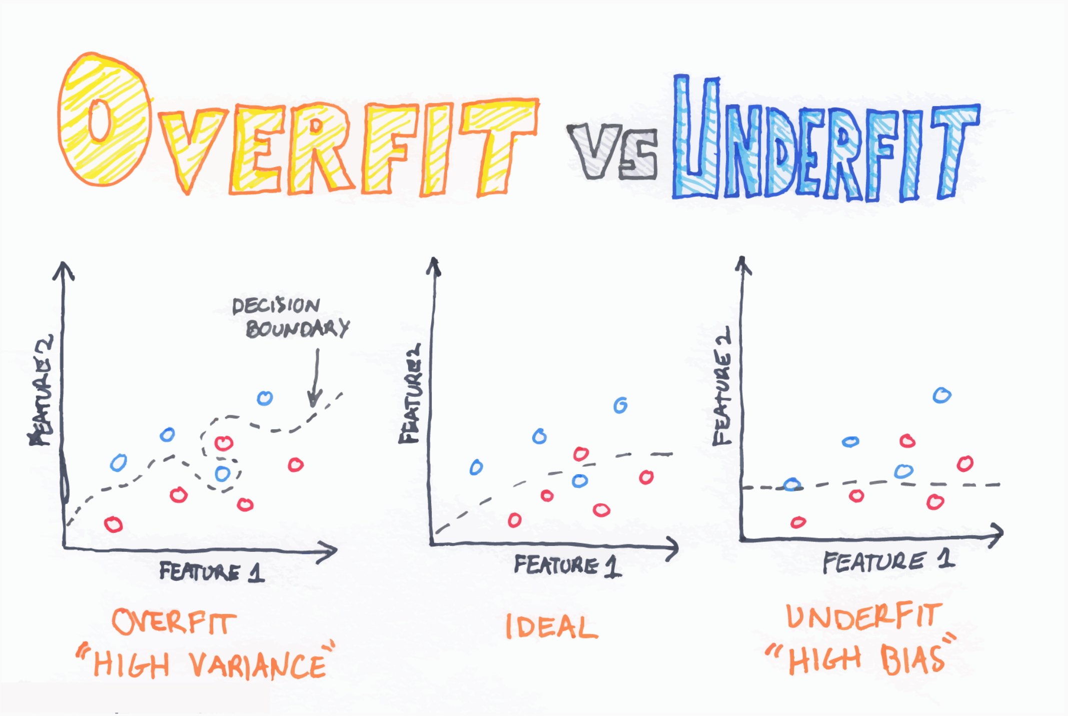
```
cube.out <- lm(Pass2013 ~ I(Pass2012^3) + I(Pass2012^2) + Pass2012, data=reportCard)
summary(cube.out)$r.squared
```

```
## [1] 0.7168206
```



Be careful of overfitting...

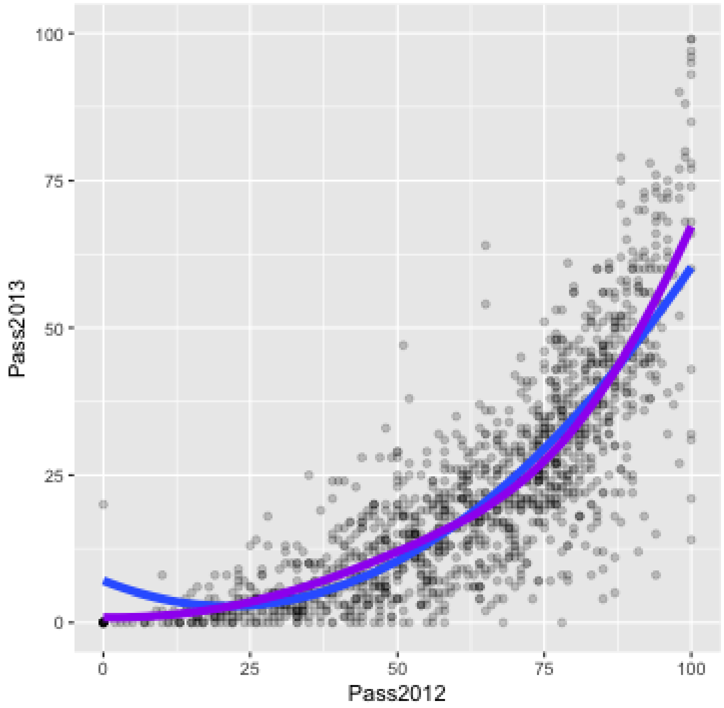




Source: Chris Albon @chrisalbon MachineLearningFlashCards.com

Loess Regression

```
ggplot(reportCard, aes(x=Pass2012, y=Pass2013)) + geom_point(alpha=0.2) +
    geom_smooth(method='lm', formula=y~poly(x,2,raw=TRUE), size=2, se=FALSE) +
    geom_smooth(method='loess', formula = y ~ x, size=2, se=FALSE, color = 'purple') +
    coord_equal() + ylim(c(0,100)) + xlim(c(0,100))
```



See this site for more info: http://varianceexplained.org/files/loess.html

Shiny App

```
shiny::runGitHub('NYSchools','jbryer',subdir='NYSReportCard')
```

See also the Github repository for more information: https://github.com/jbryer/NYSchools

One Minute Paper

Complete the one minute paper:

https://forms.gle/gY9SeBCPggHEtZYw6

1. What was the most important thing you learned during this class?
2. What important question remains unanswered for you?