The University Of Sheffield.

Data Provided:
NONE

DEPARTMENT OF COMPUTER SCIENCE **Spring Semester 2019-2020**

NETWORK PERFORMANCE ANALYSIS **2 hours**

Answer BOTH questions

All questions carry equal weight. Figures in square brackets indicate the percentage of available marks allocated to each part of a question.

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1. a) Consider an $M/M/m$ queue in which there is one queue and m servers. The steady state probabilities for this system are

$$P_k = \begin{cases} P_0\left(\frac{(m\rho)^k}{k!}\right) & k < m \\ P_0\left(\frac{m^m\rho^k}{m!}\right) & k \geq m \end{cases}, \qquad \rho = \frac{\lambda}{m\mu}.$$

(i) What do λ and μ represent? Also, state the restriction on the value of ρ. [15%]

(ii) Derive an expression for P_0 in terms of m and ρ. Simplify the expression as much as possible. [20%]

b) Consider a switch in a computer network that has one input port and three output ports, and is modelled as an $M/M/3$ queue.

(i) Write down the expressions for the probabilities P_k for $k < 3$ and $k \geq 3$, and the restriction on the value of ρ. [10%]

(ii) Show that [15%]

$$P_0 = \left[1 + 3\rho + \frac{9}{2}\rho^2 + \frac{9\rho^3}{2(1-\rho)}\right]^{-1}.$$

(iii) Show that the average number of packets in the system is

$$P_0 \sum_{k=0}^{2} \frac{k(3\rho)^k}{k!} + \frac{9P_0}{2} \sum_{k=3}^{\infty} k\rho^k$$

and that this expression simplifies to

$$P_0\left[\frac{9\rho}{2(1-\rho)^2} - \frac{3\rho}{2}\right].$$

[40%]

2. a) The Poisson process is the arrival process that is most frequently used to model the behaviour of queues.

   (i) Derive expressions for the mean and variance of the Poisson distribution at a specific time in terms of the rate λ. [25%]

   (ii) What is the probability that there are no arrivals in the time interval T? [5%]

   (iii) What is the probability that there is at least one arrival in the time interval T? [5%]

b) Consider an M/M/1 queue for which the arrival and service rates at state k are

$$\begin{aligned} \lambda_k &= \lambda\alpha^k, \qquad && k \geq 0, \quad 0 \leq \alpha < 1 \\ \mu_k &= \mu, && k \geq 1 \end{aligned}$$

   (i) Calculate the probability P_k that there are k people in the system. Express your answer in terms of P_0. State the restriction, if any, on the value of α. [20%]

   (ii) Deduce an expression for P_0 and calculate the probability that there are two or more people in the system. [15%]

   (iii) Calculate the average arrival rate and explain why the restriction on the value of α in 2b.(i) is necessary. [10%]

   (iv) Show that if $\frac{\lambda}{\mu} < 1$, then

$$P_0 > 1 - \frac{\lambda}{\mu}$$

[10%]

   (v) Is the condition $\frac{\lambda}{\mu} < 1$ necessary for a steady state solution to exist? Can this solution exist for $\frac{\lambda}{\mu} \geq 1$? Explain your answer. [10%]

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