

## The Effect Angle of Attack has on Lift and Drag Forces

To what extent does the angle of attack of an airfoil influence its lift and drag forces?

IB Physics

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## Introduction

Although just bicycle shop owners, the Wright Brothers were the first to design and build an aircraft that could be controlled in the air. Their method of controlling an aircraft was the first of its kind, and they became pioneers of aviation because of it. Their journey spans from failed gliders to controlled, sustained planes, and their innovative minds allowed us a platform to progress from. To this day, machines like submarines and spacecraft use the controls designed by Orville and Wilbur<sup>1</sup>. They were the first to include yaw, roll and pitch in an aircraft, and their research established the foundations of modern aerospace engineering.

My personal experience in flight has motivated me to investigate the extent to which the angle of attack influences the forces of flight. Currently, I'm learning how to fly with the goal of getting my private pilot's license in the future. In this experience I've learned that controlling an aircraft requires a lot of experience and focus, so being aware of the point to which a plane can pitch is necessary. Although explored in the past, I wish to know the critical angle of attack of the airplane I practice in, a Piper Warrior II PA-28-161, through this research question: to what extent does the angle of attack of an airfoil influence its lift and drag forces?

This research question is worthy of investigation because of its relevance in the field of aviation, specifically to aspiring flyers. This question also serves to demonstrate the functionality of simulations of airfoils in lift production and drag forces. In essence, the research question addresses a significant concern in aviation training and highlights the effectiveness of simulations in theoretical knowledge and application.

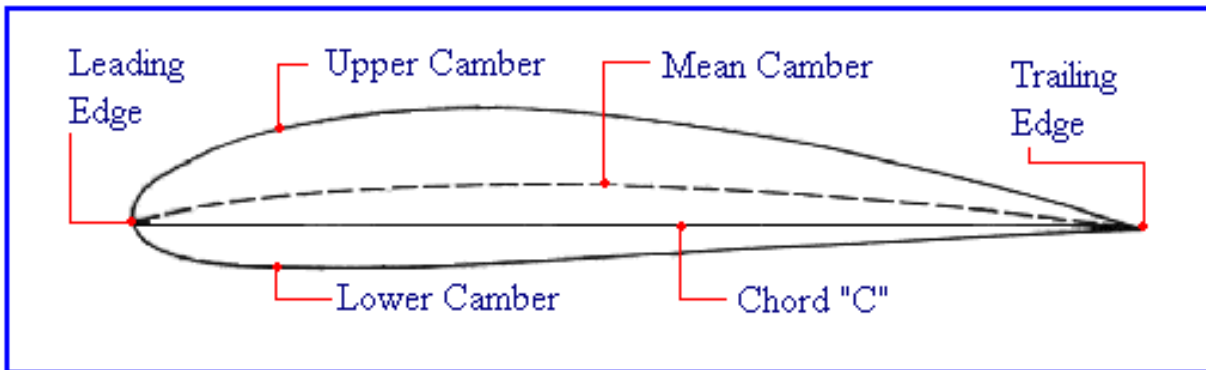
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<sup>1</sup> Wright Brothers Aeroplane Company, and Aviation History Wing. "The Wright Story." *Wright-Brothers.org*, [https://www.wright-brothers.org/History\\_Wing/Wright\\_Story/Wright\\_Story\\_Intro/Wright\\_Story\\_Intro.htm](https://www.wright-brothers.org/History_Wing/Wright_Story/Wright_Story_Intro/Wright_Story_Intro.htm). Accessed 7 December 2023.

## Background Information

### The Airfoil

“An airfoil (or aerofoil in British English) is any structure designed to manipulate the flow of a fluid to produce a reaction, which in an aircraft’s case, is aerodynamic lift”<sup>2</sup>.



*Figure 3-1 Cross section of an airfoil*

**Figure 1**, diagram of the various aspects of an airfoil<sup>3</sup>.

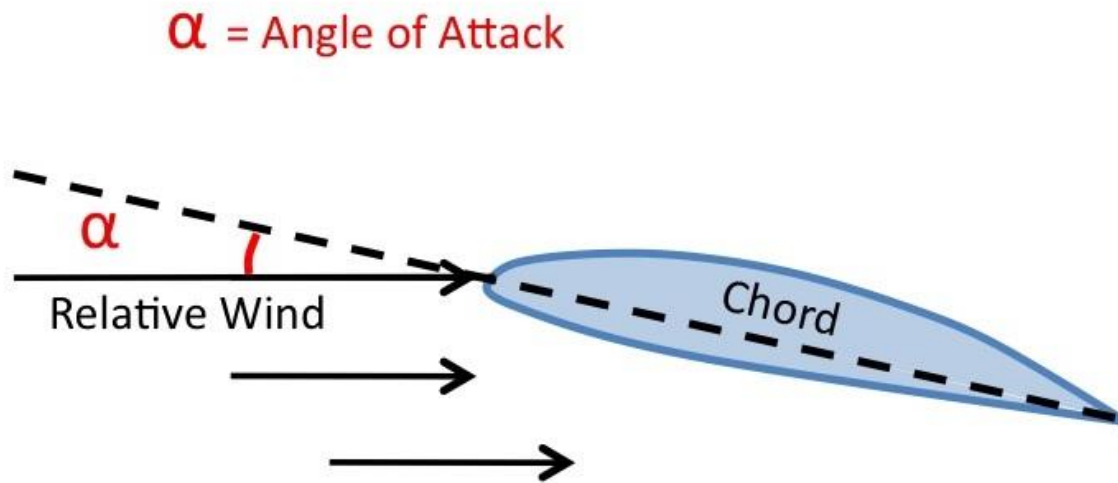
The *leading edge* of an airfoil is the portion that meets incoming air first, and the *chord* is the imaginary straight that is drawn from the *leading edge* to the *trailing edge*<sup>3</sup>. This is important to the subject because the angle of attack is measured using the degree at which the *chord* meets relative wind<sup>4</sup>. There are many geometric factors that make up a three-dimensional airfoil. Later in the exploration, airfoil measurements like the chord, wing aspect ratio and others will become important control variables and inputs to the tool of investigation.

<sup>2</sup> “Airfoil Design 101: What Is an Airfoil?” *National Aviation Academy*, 14 March 2022, <https://www.naa.edu/airfoil-design/>. Accessed 28 October 2023.

<sup>3</sup> Civil Air Patrol. “Airfoils.” *Aeronautics - Principles of Flight (AIRFOILS) - Level 2*, 23 February 1999, <https://web.eng.fiu.edu/allstar/flight31.htm>. Accessed 26 November 2023.

<sup>4</sup> “Angle of Attack (AOA).” *SKYbrary*, <https://skybrary.aero/articles/angle-attack-aoa>. Accessed 28 October 2023.

## Angle of Attack



**Figure 2**, the difference between where an airfoil is pointing and the incoming wind<sup>4</sup>.

An airfoil is associated with many metrics such as the angle of attack. An airfoil's angle of attack describes the angle at which the leading edge meets relative wind. The angle is formed between the chord line of the airfoil and direction of the relative wind<sup>4</sup>. The angle of attack can be influenced directly by the pilot pitching the plane up by pulling the yoke towards themselves. Pulling the yoke will raise the elevator, a part of the plane that controls pitch, and the plane will pitch up.

## The Four Forces of Flight

The four forces of flight govern the movement and dynamics of an aircraft in flight. They include lift, thrust, weight (gravity) and drag. Now, lift is the upward force that overcomes the weight of the aircraft to stay suspended in the air. Weight opposes lift as it is the downward pull caused by gravity. Next, thrust is the forward force produced by engines, similar to a car and its acceleration. The resistance of thrust is drag which opposes the aircraft's motion in the air. Drag

is caused by friction and air pressure that the airfoil confronts<sup>5</sup>. All these forces can change within flight. Weight decreases as fuel burns up, lift and drag can change depending on the angle of attack, and thrust can change based on engine's power input.

The focus of this investigation is lift and drag. "The wings provide lift by creating a situation where the pressure above the wing is lower than the pressure below the wing. Since the pressure below the wing is higher than the pressure above the wing, there is a net force upwards"<sup>6</sup>. A factor that can influence this situation is the angle of attack, the independent variable of this paper. Drag is also influenced by the change in an airfoil's angle of attack, "An increase in angle of attack results in an increase in both lift and induced drag, up to a point"<sup>7</sup>. The investigation will cover the relationship between the angle of attack and lift and drag.

### Bernoulli's Principle

In aviation, Bernoulli's principle attempts to explain the theory of flight and how an airfoil can generate lift. Because of the teardrop structure of an airfoil, the wind travelling above the airfoil moves faster than the air travelling underneath. This creates a reduced pressure above the airfoil and a high pressure below. In turn, the pressure difference results in the production of lift which explains the theory of flight<sup>8</sup>. This principle is well known and popular in modern science as its theory explains the production of lift. For the investigation, it is a big part of the reasoning behind the hypothesis.

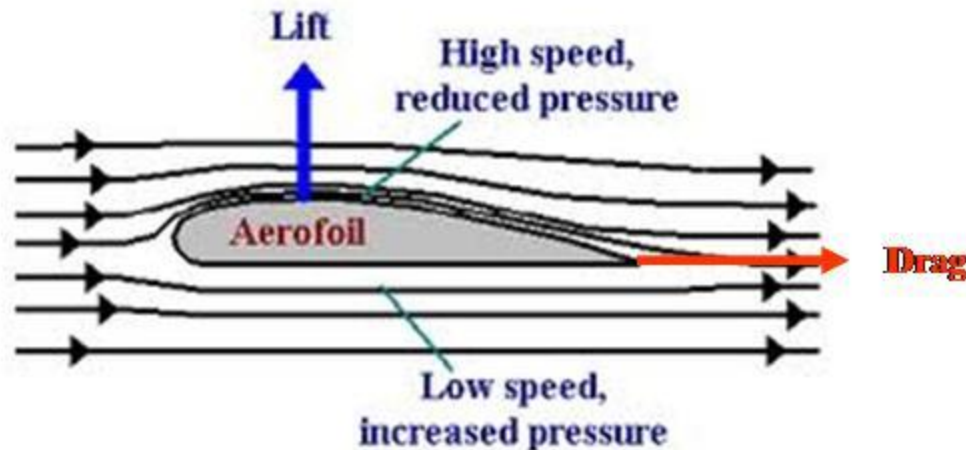
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<sup>5</sup> "Four Forces on an Airplane | Glenn Research Center | NASA." *NASA Glenn Research Center*, 21 July 2022, <https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/four-forces-on-an-airplane/>. Accessed 28 October 2023.

<sup>6</sup> Nakamura, Mealani. "2.972 How An Airfoil Works." *MIT*, <https://web.mit.edu/2.972/www/reports/airfoil/airfoil.html>. Accessed 28 October 2023.

<sup>7</sup> Civil Air Patrol.

<sup>8</sup> "Bernoulli's Principle." *SKYbrary*, <https://skybrary.aero/articles/bernoullis-principle>. Accessed 26 November 2023.



**Figure 3**, a depiction of Bernoulli's principle and production of lift<sup>9</sup>.

### Flow Attachment

In *Figure 3*, there is an illustration of the contour lines and how they follow the curvature of the airfoil. This tendency is known as *Flow Attachment* (or the *Coanda Effect*), and in the context of flight, it is when the airflow sticks to the wing surface and follows the airfoil to the sharp trailing edge. The sharp trailing edge allows the separate airflow traveling above and below the airfoil to converge smoothly. In this effect, the air continues to flow diagonally downward to create an opposite reaction of lift on the airfoil. Newton's Third Law states that for every action of motion there is an equal and opposite reaction, and in the case of *Flow Attachment*, the action is the wind's downward push, and the reaction is the lift that results from it.

### Boundary Layer

The *boundary layer* is the flow that develops very closely over the surface of an airfoil as its viscosity effects are confined to the surface of the airfoil<sup>10</sup>. These viscosity effects are crucial in

<sup>9</sup> "Bernoulli's Principle." *SKYbrary*, <https://skybrary.aero/articles/bernoullis-principle>. Accessed 26 November 2023.

the production of lift<sup>11</sup>. Since air has an extremely low viscosity, it is more likely to conform to the shape of the airfoil and develop the boundary layer. Air's low viscosity is the reason for the tendency of *flow attachment* which supports the production of lift.

The boundary layer also has an influence on drag forces. “One effect produced by the boundary layer development over a body is to create *shear stresses* on its surface, the cumulative

manifestation of shear being *skin friction*

*drag*”<sup>12</sup>. Skin friction

drag is the resistance

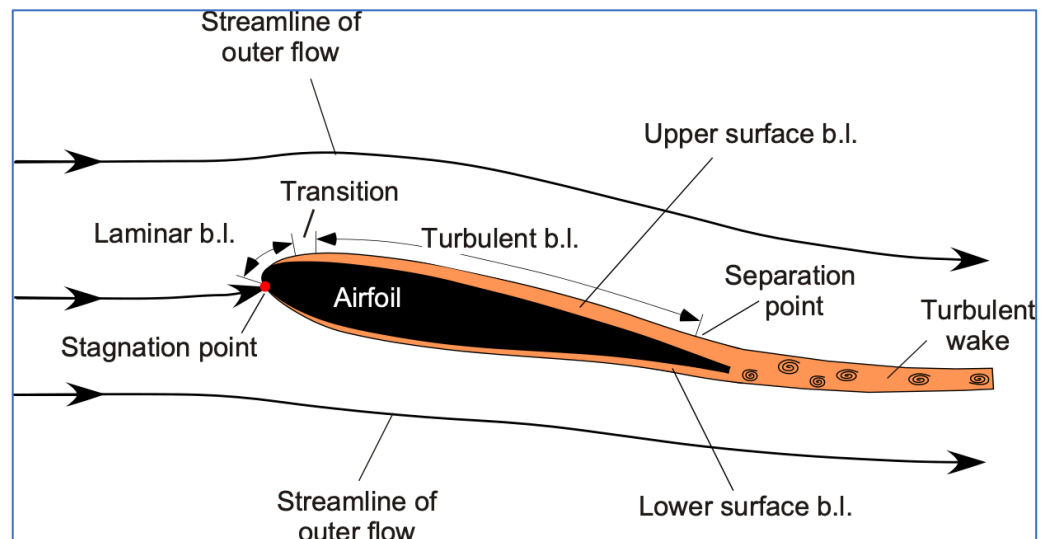
the airfoil opposes

moving in a fluid

(airflow); it accounts

for up to half of the

total drag of an airfoil.



**Figure 4,** depiction of the boundary layer produced by an airfoil<sup>12</sup>.

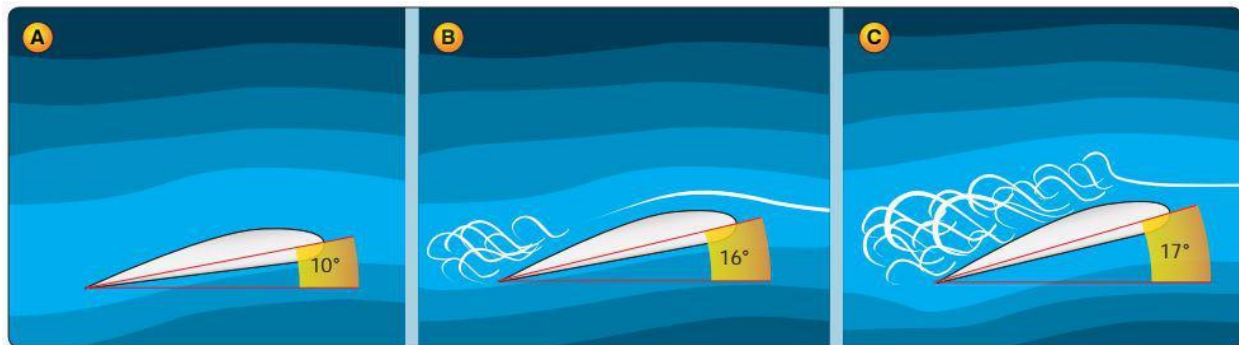
<sup>10</sup> Lewis, Mark J. “Boundary Layers – Introduction to Aerospace Flight Vehicles.” *Eagle Pubs*, <https://eaglepubs.erau.edu/introductiontoaerospaceflightvehicles/chapter/introduction-to-boundary-layers/>. Accessed 7 December 2023.

<sup>11</sup>Nakamura, Mealani. “2.972 How An Airfoil Works.” *MIT*, <https://web.mit.edu/2.972/www/reports/airfoil/airfoil.html>. Accessed 28 October 2023.

<sup>12</sup> Lewis, Mark J.

## Critical Angle of Attack

The critical angle of attack is when the airflow separates from the airfoil and becomes turbulent, causing the loss of lift. In this event, flow attachment is no longer in effect and the plane reaches its maximum lift coefficient before stalling<sup>13</sup>.



**Figure 5**, a representation of flow separation as the angle of attack reaches its critical point.

*Flow separation* or *boundary layer separation* is a phenomenon that occurs when the boundary layer detaches from the airfoil under specific conditions, characterized by a loss of lift and an increase in drag<sup>14</sup>. In aviation, preventing flow separation and stalls is essential because of the risk placed on the pilot and passengers when flying. When an aircraft reaches a critical angle of attack, boundary layer separation and stalls are more likely to occur.

<sup>13</sup> "What Is Angle of Attack? — Three Critical Angles." *Aviation Performance Solutions*, 13 October 2009, <https://apstraining.com/resource/three-critical-angles/>. Accessed 6 December 2023.

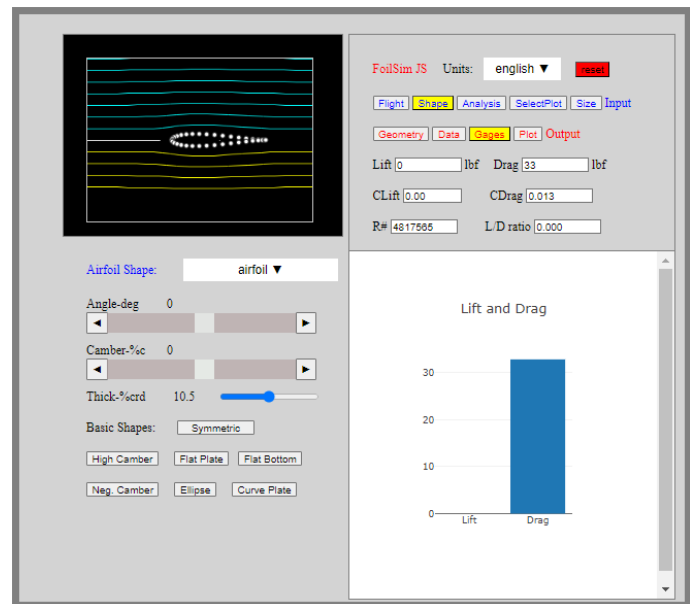
<sup>14</sup> Lewis, Mark J. "Boundary Layers – Introduction to Aerospace Flight Vehicles." *Eagle Pubs*, <https://eaglepubs.erau.edu/introductiontoaerospaceflightvehicles/chapter/introduction-to-boundary-layers/>. Accessed 7 December 2023.

## Information about FoilSim

“FoilSim” is an interactive software program, created by the NASA Glenn Center, that investigates the different properties of airfoils such as the chamber line, chord line, and angle of attack in effect to the forces acted upon the airfoil like lift and drag<sup>15</sup>. The simulator allows users to edit different proportions and

measurements of the airfoil like the angle of attack as mentioned before. Then, it will provide an output of lift drag based on such changes. This simulator was chosen for this experiment for its practicality and ease of use. “With this simulator you can investigate how an aircraft

wing produces lift and drag by changing the values of different factors that affect lift and the factors that affect drag”<sup>16</sup>.



**Figure 6,** design and structure of the airfoil simulator.

## Hypothesis

As the angle of attack increases, the airfoil offers a greater surface area to oncoming air. The lift equation is a representation of this idea because there is a positive relationship between  $A$  and  $L$ :

<sup>15</sup> “Aeronautical Engineer for a Day.” *Heriot-Watt University*, <https://www.hw.ac.uk/campaigns/dubai/files/old/Aeronautical.pdf>. Accessed 26 November 2023.

<sup>16</sup> NASA Glenn Center. “Student Airfoil Interactive.” *NASA Glenn Research Center*, 24 June 2019, <https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/foilsimstudent/>. Accessed 21 December 2023.

$$L = \frac{1}{2} \rho v^2 A C_l$$

**Equation 1**, the lift equation

“The lift equation states that lift **L** is equal to the lift coefficient **Cl** times the density **rho** ( $\rho$ ) times half of the velocity **V** squared times the wing area **A**”<sup>17</sup>. The critical angle of attack is when the airfoil generates the maximum lift and drag coefficient.

Similarly, wing area has the same positive relationship with drag. The drag equation is identical to **equation 1**, the only differences being that drag replaces lift, and the drag coefficient replaces the lift coefficient:

$$D = \frac{1}{2} \rho v^2 A C_d$$

**Equation 2**, the drag equation

“The drag equation states that drag **D** is equal to the drag coefficient **Cd** times the density **rho** ( $\rho$ ) times half of the velocity **V** squared times the reference area **A**”<sup>18</sup>.

Both the lift and drag equations plus past theories propose that as an airfoil’s angle of attack increases, there will be an increase in both lift and drag up to a certain point. If the angle of attack increases, then there will be a simultaneous increase in both lift and drag forces up to approximately 18 degrees. Beyond the critical angle of attack, the lift will increase, and the drag force will increase significantly.

<sup>17</sup> Hall, Nancy. “Lift Equation | Glenn Research Center | NASA.” *NASA Glenn Research Center*, 20 November 2023, <https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/lift-equation-2/>. Accessed 7 December 2023.

<sup>18</sup> Hall, Nancy. “Drag Equation | Glenn Research Center | NASA.” *NASA Glenn Research Center*, 22 November 2023, <https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/drag-equation-2/>. Accessed 7 December 2023.

## Methodology

This investigation focuses on how lift and drag forces generated by an airfoil are impacted at different angles of attack (AoA). The use of the simulator is to produce data for analyzation at different angles of attack.

### Variables

#### Independent Variable

The angle of attack of the airfoil is the independent variable for this investigation. The boundaries of the angle of attack are from  $-20^\circ$  to  $20^\circ$  and increase in increments of  $0.5^\circ$ . The boundary is from  $-20^\circ$  to  $20^\circ$  because it is the extent to which the simulator can function. The boundaries go into extreme conditions to analyze the lift and drag forces past the stall point.

#### Dependent Variables

1. Lift is symbolized with  $L$  and measured in newtons (N).  $L$  will be provided as an output of the simulator, dependent on the change in angle of attack. Lift is proportional to drag, and the two will demonstrate similar behavior up to a certain point.
2. Drag is symbolized with  $D$  and measured in newtons (N).  $D$  is also provided as an output in the simulator.

#### Control Variables

First of the control variables is climate: the atmosphere in the simulation is constant, set to “Standard Earth Atmosphere – Average Day”

Another constant is the specifications of the airfoil, set to very closely mimic that of a Piper Warrior II PA-28-161 with a span of 10.7 m, surface area of  $16.05 \text{ m}^2$ , chord of 1.5 m, a camber of 0% chord and thickness set to 10.5% chord.

Some of the control variables of the flight needed to be in place for the simulator such as an altitude of 12,500 ft, speed of 160 km/h, temperature of -56°C, density of 0.2864 kg/cu m and pressure of 17.796 kPa.

A few of the control variables listed could have an influence on the lift and drag production of the airfoil, so keeping them constant is an important part of preventing the data from being skewed. Weight and fuel levels were not factors provided in the simulator; however, the simulator is set up to match the specifications of the Piper's airfoil, not the entire plane. Unfortunately, due to rounding, the airfoil in the simulator does not exactly match the specifications of the Piper Warrior II, but they are still very similar:

The actual specifications of the Piper<sup>19</sup> had a wingspan of 10.67 meters, wing chord at root of 1.60 m, wing chord at tip 1.07 meters, wing aspect ratio of 7.24, and a wing gross area of 15.8 meters squared. Since there were two wing chord measurements given, they're supposed to be averaged to get of wing chord of 1.335 meters. Compared to input given to the simulator, the two airfoils are different but still very similar given most of their disparities are by decimal places.

## Procedures

1. The simulator was set to the wing specifications of the Piper Warrior II PA-28-161.
2. Set the **output** to **data**, so the dependent variables are provided numerically and not in a plot.
3. Change the altitude from 0 ft to 12,500 ft.
4. Starting at the angle -20°, set the angle of attack.
5. Record the data from the simulator.

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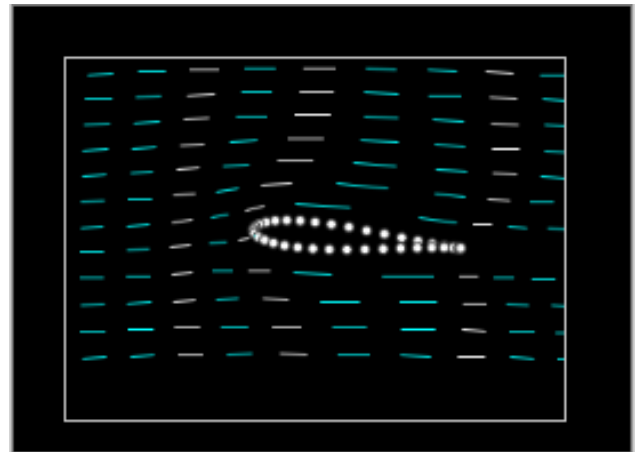
<sup>19</sup> "Piper PA-28-161 Warrior II four-seat low-wing monoplane." *Skytamer Images*, [https://www.skytamer.com/Piper\\_PA-28-161.html](https://www.skytamer.com/Piper_PA-28-161.html). Accessed 24 December 2023.

6. Repeat steps 3 and 4 in increments of  $5^\circ$  beginning from  $-20^\circ$  to  $20^\circ$ .
7. After completing, calculate the uncertainty by decimal places for each output of the simulator.

## Data Collection, Processing, and Analysis

### Qualitative Observations

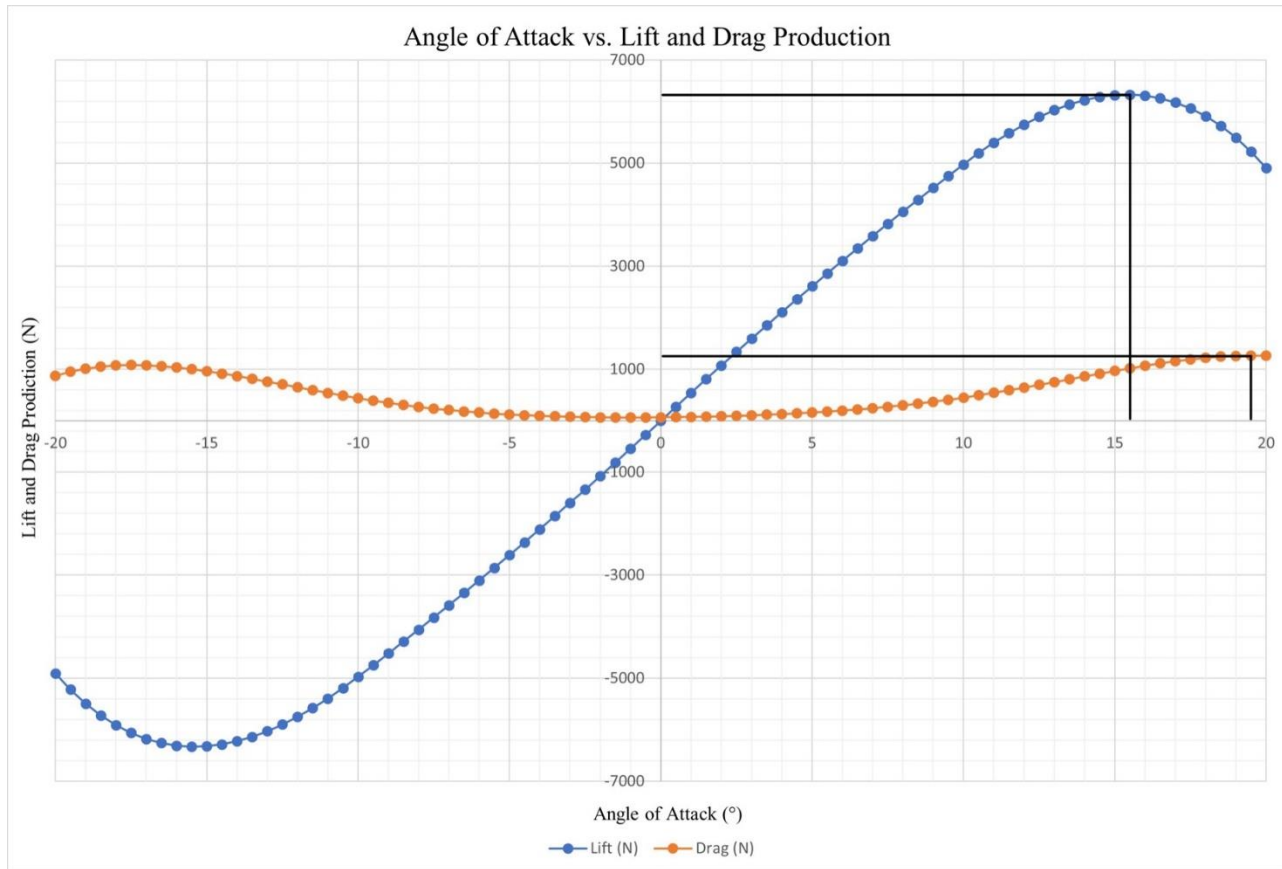
In the simulator, the only qualitative observation available was the **view window**. The view window offered a depiction of the airfoil with oncoming streamlines of airflow. As the angle of attack changed, the contour of the streamlines followed—a representation of flow attachment. Although it is not a detailed illustration of the airflow, the window provides a likely point when *boundary layer separation* occurs ( $\sim 16^\circ$ ).



**Figure 7**, view window of FoilSim depicting an airfoil simulated with oncoming airflow.

### Quantitative Results

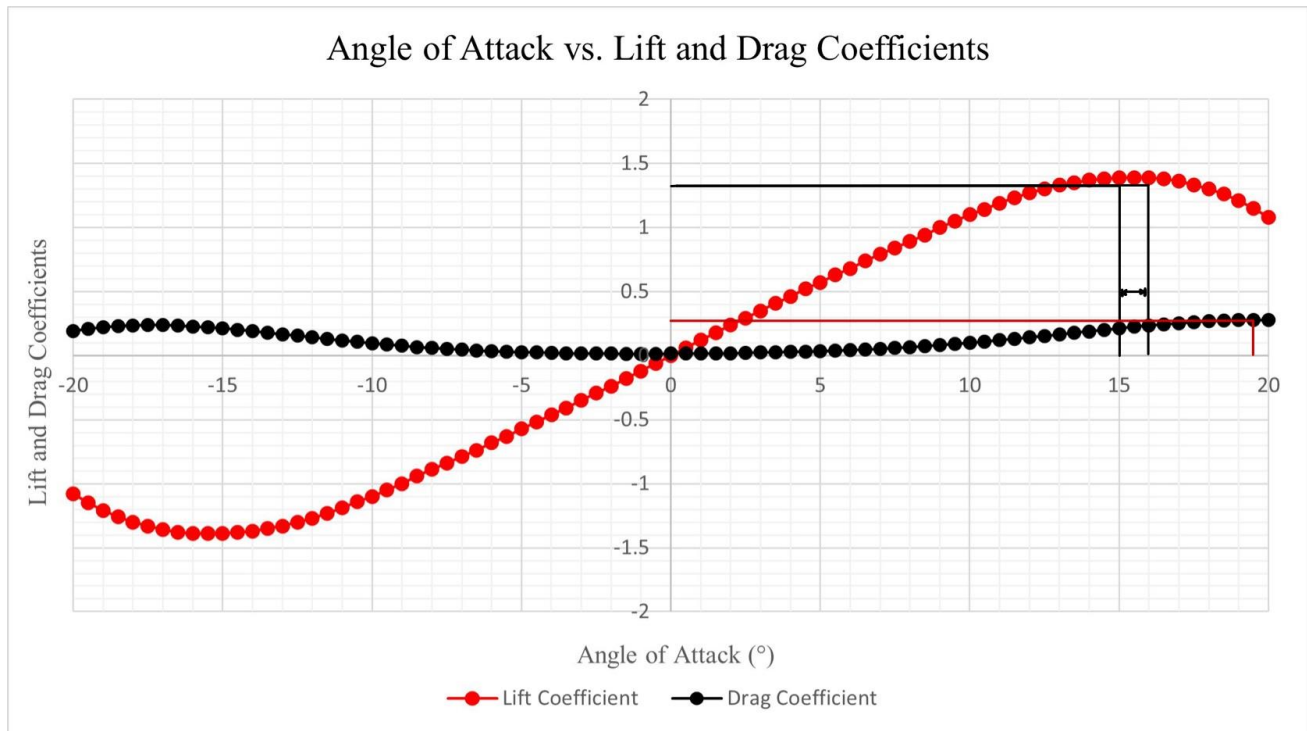
For the analyzation of this experiment, I chose to use Google sheets to develop a line graph based on the data outputted by the simulator. The program used the data to graph a smooth line function with estimates of other points of data based on the trend. In the **Appendix**, the raw data of the lift, drag and their coefficients are present. Both  $L$  and  $D$  have an uncertainty of plus or minus 0.5 N.



**Figure 8**, function of the lift and drag forces based on the angle of attack.

Visually, it can be inferred that drag is a reflection of itself across the y-axis. Also, lift is a reflection of itself across the x-axis and y-axis. Because of the proportionality of  $A$  and  $L$  in **equation 1**, and  $A$  and  $D$  in **equation 2**. Another interesting detail is that drag never reaches zero which is because no matter how aerodynamic an airfoil is, there will always be skin friction drag. Although not visible, each point of data has an uncertainty or error bar of  $\pm 0.5$  N. The application used made it difficult to see as the difference of scale was so big.

Now, at the point where the lift is at its maximum, is where the critical angle of attack lies. The highest point in the data has an angle of attack of  $15.5^\circ$  and a lift of 6328 newtons. After this point, there is a loss of lift and increase of drag which is a representation of a stall in the Piper Warrior II.



**Figure 9**, function of the lift and drag coefficients based on the angle of attack.

The lift and drag coefficient graphs look identical to **Figure 8**; they are simply different scales as the boundaries for the coefficients are limited from -2 to 2. Because the simulator rounds its data, the highest lift coefficient is 1.39 from a 15° to 16°. This range is where boundary layer separation will occur, and the plane is placed in immediate risk of stalling<sup>20</sup>.

For drag, the force is at a maximum of 1265 N at a 19.5° angle of attack. The increase in drag comes from the boundary layer separation; and similar to lift, the drag begins to decrease after a point. In addition, the drag coefficient is at its highest point of 0.279 at 19.5°.

<sup>20</sup> Federal Aviation Administration. "Angle of Attack Awareness." *Angle of Attack Awareness.pdf*, <https://www.faa.gov/sites/faa.gov/files/2022-01/Angle%20of%20Attack%20Awareness.pdf>. Accessed 8 December 2023.

## Conclusion

In answering the research question, to what extent does the angle of attack of an airfoil influence its lift and drag forces, the investigation used a simulator FoilSim with the airfoil of the Piper Warrior II to explore the effects the angle of attack has on the lift and drag forces of an airfoil. As explained through research and seen on the graph as well as the **appendix**, the hypothesis was supported because as the angle of attack increased, the lift and drag forces of the airfoil increased up to a specific point of  $15.5^\circ$ . Bernoulli's principle, Newton's third law, flow attachment and other phenomena of flight support the hypothesis because when the angle of attack increases, the surface area of the airfoil also increases which influences wind velocity travelling over the wing and influences the pressure difference over and under the wing. Seen in **Figure 8 and 9**, there is an increase of lift to a maximum called the critical angle of attack, then a loss of lift and production of drag soon after. In the two figures, the critical angle of attack is represented as the x-value of the maximum on the curve. This loss of lift and increase in drag is the stalling of the aircraft which can cause death to all involved if not carefully and swiftly resolved.

To limit the margin of error in this investigation, I decided to use a simulator instead of performing the experiment with an apparatus. The simulator offers prevention of external influences such as varied wind velocity or changing atmospheres—factors that are difficult to control if not performed in a wind tunnel. Even though, the simulation was helpful in generating data, the use of computational fluid dynamics would improve the accuracy as well as give a more in depth understanding of the data. “Computational fluid dynamics (CFD) is a science that, with the help of digital computers, produces quantitative predictions of fluid-flow phenomena based

on the conservation laws (conservation of mass, momentum, and energy) governing fluid motion”<sup>21</sup>. The modelling of the Piper in CFDs will prove beneficial through its use of physics theories, conservation laws and more. It could be used to calculate pressure differences, wind velocity over and under the airfoil and other specific details, and it could do so at a faster rate with machine learning. However, it proved difficult to use since it is a tool that requires programming skills and a high-level education in fluid mechanics. Another limitation of the simulator was the rounding. The values had a limit to the number of decimal values that could be included in the input. In addition, the outputs of the simulation were rounded and only went up to three decimal places. This could have potentially made the data inconsistent as the results are not as accurate as they could have been.

There are a few potential ways I would have extended this project. The first option is to include multiple variables in the influence of lift and drag forces. There were plenty controlled variables that could have been independent in this research. The extension of this project to include multiple variables of flight would have been more relevant to contemporary applications. Additionally, the exploration of other dependent variables such as turbulence has the same effect in the extension of this investigation. Lastly, studying the effects the angle of attack has on different airfoil designs rather than a single design would give insight to various aircrafts and the purpose of their designs.

As we look at the real-world application of this investigation, understanding the relationship between the angle of attack and forces of flight is essential for the safety in aviation. As mentioned, going past the critical angle of attack in an aircraft causes it to stall, and although

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<sup>21</sup> “Chapter 10 - Computational Fluid Dynamics.” *ScienceDirect*, 16 June 2023, <https://www.sciencedirect.com/science/article/abs/pii/B9780123821003100101>. Accessed 8 December 2023.

pilots can recover, it is still a dangerous situation if not trained properly. For new flyers like me that do not understand the effects of pitch and other controls, it is crucial that they learn the influence the angle of attack or pitch has on the aircraft.

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## Appendix A

Angle of Attack (°)	Lift (N)	Lift Coefficient	Drag (N)	Drag Coefficient
-20	-4905	-1.08	874	0.193
-19.5	-5222	-1.15	953	0.21
-19	-5493	-1.21	1012	0.223
-18.5	-5722	-1.26	1051	0.231
-18	-5911	-1.3	1073	0.236
-17.5	-6062	-1.33	1081	0.238
-17	-6177	-1.36	1076	0.237
-16.5	-6258	-1.38	1060	0.233
-16	-6308	-1.39	1035	0.228
-15.5	-6328	-1.39	1001	0.22
-15	-6319	-1.39	961	0.212
-14.5	-6284	-1.38	916	0.202
-14	-6222	-1.37	867	0.191
-13.5	-6137	-1.35	814	0.179
-13	-6029	-1.33	760	0.167
-12.5	-5899	-1.3	705	0.155
-12	-5749	-1.27	649	0.143
-11.5	-5581	-1.23	594	0.131
-11	-5394	-1.19	540	0.119
-10.5	-5192	-1.14	487	0.107
-10	-4974	-1.1	437	0.096
-9.5	-4748	-1.05	390	0.086
-9	-4520	-1	346	0.076
-8.5	-4289	-0.94	306	0.067
-8	-4057	-0.89	270	0.059
-7.5	-3821	-0.84	237	0.052
-7	-3584	-0.79	208	0.046
-6.5	-3344	-0.74	182	0.04
-6	-3102	-0.68	159	0.035
-5.5	-2857	-0.63	140	0.031

-5	-2610	-0.57	123	0.027
-4.5	-2360	-0.52	108	0.024
-4	-2108	-0.46	96	0.021
-3.5	-1853	-0.41	86	0.019
-3	-1596	-0.35	78	0.017
-2.5	-1337	-0.29	72	0.016
-2	-1074	-0.24	68	0.015
-1.5	-810	-0.18	65	0.014
-1	-542	-0.12	64	0.014
-0.5	-273	-0.06	65	0.014
0	0	0	67	0.015
0.5	273	0.06	70	0.015
1	542	0.12	75	0.017
1.5	810	0.18	81	0.018
2	1074	0.24	88	0.019
2.5	1337	0.29	97	0.021
3	1596	0.35	107	0.024
3.5	1853	0.41	118	0.026
4	2108	0.46	131	0.029
4.5	2360	0.52	145	0.032
5	2610	0.57	161	0.035
5.5	2857	0.63	178	0.039
6	3102	0.68	198	0.044
6.5	3344	0.74	220	0.048
7	3584	0.79	244	0.054
7.5	3821	0.84	271	0.06
8	4057	0.89	300	0.066
8.5	4289	0.94	333	0.073
9	4520	1	369	0.081
9.5	4748	1.05	408	0.09
10	4974	1.1	450	0.099
10.5	5192	1.14	496	0.109
11	5394	1.19	544	0.12

11.5	5581	1.23	593	0.131
12	5749	1.27	645	0.142
12.5	5899	1.3	698	0.154
13	6029	1.33	752	0.165
13.5	6137	1.35	806	0.177
14	6222	1.37	860	0.189
14.5	6284	1.38	914	0.201
15	6319	1.39	967	0.213
15.5	6328	1.39	1018	0.224
16	6308	1.39	1067	0.235
16.5	6258	1.38	1113	0.245
17	6177	1.36	1154	0.254
17.5	6062	1.33	1190	0.262
18	5911	1.3	1221	0.269
18.5	5722	1.26	1244	0.274
19	5493	1.21	1259	0.277
19.5	5222	1.15	1265	0.279
20	4905	1.08	1260	0.277

## Appendix B

This data table includes the uncertainties of the calculations done by the simulator:

Angle of Attack (°)	Lift (N)	Lift Coefficient	Drag (N)	Drag Coefficient
-20	-4905 ± 0.5	-1.08 ± 0.005	874 ± 0.5	0.193 ± 0.0005
-19.5	-5222 ± 0.5	-1.15 ± 0.005	953 ± 0.5	0.21 ± 0.005
-19	-5493 ± 0.5	-1.21 ± 0.005	1012 ± 0.5	0.223 ± 0.0005
-18.5	-5722 ± 0.5	-1.26 ± 0.005	1051 ± 0.5	0.231 ± 0.0005
-18	-5911 ± 0.5	-1.3 ± 0.05	1073 ± 0.5	0.236 ± 0.0005
-17.5	-6062 ± 0.5	-1.33 ± 0.005	1081 ± 0.5	0.238 ± 0.0005
-17	-6177 ± 0.5	-1.36 ± 0.005	1076 ± 0.5	0.237 ± 0.0005
-16.5	-6258 ± 0.5	-1.38 ± 0.005	1060 ± 0.5	0.233 ± 0.0005

-16	-6308 ± 0.5	-1.39 ± 0.005	1035 ± 0.5	0.228 ± 0.0005
-15.5	-6328 ± 0.5	-1.39 ± 0.005	1001 ± 0.5	0.22 ± 0.005
-15	-6319 ± 0.5	-1.39 ± 0.005	961 ± 0.5	0.212 ± 0.0005
-14.5	-6284 ± 0.5	-1.38 ± 0.005	916 ± 0.5	0.202 ± 0.0005
-14	-6222 ± 0.5	-1.37 ± 0.005	867 ± 0.5	0.191 ± 0.0005
-13.5	-6137 ± 0.5	-1.35 ± 0.005	814 ± 0.5	0.179 ± 0.0005
-13	-6029 ± 0.5	-1.33 ± 0.005	760 ± 0.5	0.167 ± 0.0005
-12.5	-5899 ± 0.5	-1.3 ± 0.05	705 ± 0.5	0.155 ± 0.0005
-12	-5749 ± 0.5	-1.27 ± 0.005	649 ± 0.5	0.143 ± 0.0005
-11.5	-5581 ± 0.5	-1.23 ± 0.005	594 ± 0.5	0.131 ± 0.0005
-11	-5394 ± 0.5	-1.19 ± 0.005	540 ± 0.5	0.119 ± 0.0005
-10.5	-5192 ± 0.5	-1.14 ± 0.005	487 ± 0.5	0.107 ± 0.0005
-10	-4974 ± 0.5	-1.1 ± 0.05	437 ± 0.5	0.096 ± 0.0005
-9.5	-4748 ± 0.5	-1.05 ± 0.005	390 ± 0.5	0.086 ± 0.0005
-9	-4520 ± 0.5	-1 ± 0.5	346 ± 0.5	0.076 ± 0.0005
-8.5	-4289 ± 0.5	-0.94 ± 0.005	306 ± 0.5	0.067 ± 0.0005
-8	-4057 ± 0.5	-0.89 ± 0.005	270 ± 0.5	0.059 ± 0.0005
-7.5	-3821 ± 0.5	-0.84 ± 0.005	237 ± 0.5	0.052 ± 0.0005
-7	-3584 ± 0.5	-0.79 ± 0.005	208 ± 0.5	0.046 ± 0.0005
-6.5	-3344 ± 0.5	-0.74 ± 0.005	182 ± 0.5	0.04 ± 0.005
-6	-3102 ± 0.5	-0.68 ± 0.005	159 ± 0.5	0.035 ± 0.0005
-5.5	-2857 ± 0.5	-0.63 ± 0.005	140 ± 0.5	0.031 ± 0.0005
-5	-2610 ± 0.5	-0.57 ± 0.005	123 ± 0.5	0.027 ± 0.0005
-4.5	-2360 ± 0.5	-0.52 ± 0.005	108 ± 0.5	0.024 ± 0.0005
-4	-2108 ± 0.5	-0.46 ± 0.005	96 ± 0.5	0.021 ± 0.0005
-3.5	-1853 ± 0.5	-0.41 ± 0.005	86 ± 0.5	0.019 ± 0.0005
-3	-1596 ± 0.5	-0.35 ± 0.005	78 ± 0.5	0.017 ± 0.0005
-2.5	-1337 ± 0.5	-0.29 ± 0.005	72 ± 0.5	0.016 ± 0.0005
-2	-1074 ± 0.5	-0.24 ± 0.005	-1074 ± 0.5	0.015 ± 0.0005
-1.5	-810 ± 0.5	-0.18 ± 0.005	-810 ± 0.5	0.014 ± 0.0005
-1	-542 ± 0.5	-0.12 ± 0.005	64 ± 0.5	0.014 ± 0.0005
-0.5	-273 ± 0.5	-0.06 ± 0.005	65 ± 0.5	0.014 ± 0.0005
0	0	0	67 ± 0.5	0.015 ± 0.0005

0.5	273 ± 0.5	0.06 ± 0.005	70 ± 0.5	0.015 ± 0.0005
1	542 ± 0.5	0.12 ± 0.005	75 ± 0.5	0.017 ± 0.0005
1.5	810 ± 0.5	0.18 ± 0.005	81 ± 0.5	0.018 ± 0.0005
2	1074 ± 0.5	0.24 ± 0.005	88 ± 0.5	0.019 ± 0.0005
2.5	1337 ± 0.5	0.29 ± 0.005	97 ± 0.5	0.021 ± 0.0005
3	1596 ± 0.5	0.35 ± 0.005	107 ± 0.5	0.024 ± 0.0005
3.5	1853 ± 0.5	0.41 ± 0.005	118 ± 0.5	0.026 ± 0.0005
4	2108 ± 0.5	0.46 ± 0.005	131 ± 0.5	0.029 ± 0.0005
4.5	2360 ± 0.5	0.52 ± 0.005	145 ± 0.5	0.032 ± 0.0005
5	2610 ± 0.5	0.57 ± 0.005	161 ± 0.5	0.035 ± 0.0005
5.5	2857 ± 0.5	0.63 ± 0.005	178 ± 0.5	0.039 ± 0.0005
6	3102 ± 0.5	0.68 ± 0.005	198 ± 0.5	0.044 ± 0.0005
6.5	3344 ± 0.5	0.74 ± 0.005	220 ± 0.5	0.048 ± 0.0005
7	3584 ± 0.5	0.79 ± 0.005	244 ± 0.5	0.054 ± 0.0005
7.5	3821 ± 0.5	0.84 ± 0.005	271 ± 0.5	0.06 ± 0.005
8	4057 ± 0.5	0.89 ± 0.005	300 ± 0.5	0.066 ± 0.0005
8.5	4289 ± 0.5	0.94 ± 0.005	333 ± 0.5	0.073 ± 0.0005
9	4520 ± 0.5	1 ± 0.5	369 ± 0.5	0.081 ± 0.0005
9.5	4748 ± 0.5	1.05 ± 0.005	408 ± 0.5	0.09 ± 0.005
10	4974 ± 0.5	1.1 ± 0.05	450 ± 0.5	0.099 ± 0.0005
10.5	5192 ± 0.5	1.14 ± 0.005	496 ± 0.5	0.109 ± 0.0005
11	5394 ± 0.5	1.19 ± 0.005	544 ± 0.5	0.12 ± 0.005
11.5	5581 ± 0.5	1.23 ± 0.005	593 ± 0.5	0.131 ± 0.0005
12	5749 ± 0.5	1.27 ± 0.005	645 ± 0.5	0.142 ± 0.0005
12.5	5899 ± 0.5	1.3 ± 0.05	698 ± 0.5	0.154 ± 0.0005
13	6029 ± 0.5	1.33 ± 0.005	752 ± 0.5	0.165 ± 0.0005
13.5	6137 ± 0.5	1.35 ± 0.005	806 ± 0.5	0.177 ± 0.0005
14	6222 ± 0.5	1.37 ± 0.005	860 ± 0.5	0.189 ± 0.0005
14.5	6284 ± 0.5	1.38 ± 0.005	914 ± 0.5	0.201 ± 0.0005
15	6319 ± 0.5	1.39 ± 0.005	967 ± 0.5	0.213 ± 0.0005
15.5	6328 ± 0.5	1.39 ± 0.005	1018 ± 0.5	0.224 ± 0.0005
16	6308 ± 0.5	1.39 ± 0.005	1067 ± 0.5	0.235 ± 0.0005
16.5	6258 ± 0.5	1.38 ± 0.005	1113 ± 0.5	0.245 ± 0.0005

17	$6177 \pm 0.5$	$1.36 \pm 0.005$	$1154 \pm 0.5$	$0.254 \pm 0.0005$
17.5	$6062 \pm 0.5$	$1.33 \pm 0.005$	$1190 \pm 0.5$	$0.262 \pm 0.0005$
18	$5911 \pm 0.5$	$1.3 \pm 0.05$	$1221 \pm 0.5$	$0.269 \pm 0.0005$
18.5	$5722 \pm 0.5$	$1.26 \pm 0.005$	$1244 \pm 0.5$	$0.274 \pm 0.0005$
19	$5493 \pm 0.5$	$1.21 \pm 0.005$	$1259 \pm 0.5$	$0.277 \pm 0.0005$
19.5	$5222 \pm 0.5$	$1.15 \pm 0.005$	$1265 \pm 0.5$	$0.279 \pm 0.0005$
20	$4905 \pm 0.5$	$1.08 \pm 0.005$	$1260 \pm 0.5$	$0.277 \pm 0.0005$