3 – Barlat-Lian plasticity for rolled metal sheets under plane stress states

Examination guidelines

The exam for the course 'Introduction to Theory of Materials' consists of three parts:

- A programming task is assigned to each group. The participants are expected to implement the models assigned to them, create meaningful and representative results, and verify their implementation and results.
- The examination comprises a presentation ($\approx$ 10 minutes).
- Oral examinations ($\approx$ 5 minutes per person) are conducted separately for each group. Participants will be required to discuss the contributions of the individual group members, as well as linking the presented work to the lecture materials.

To give the other students insight into every examination problem, the presentations are usually given in a seminar-like manner and attended by the other groups as well. However, this is not mandatory, and groups can opt out of this arrangement by notifying the examiners at least one week in advance. The complete exam will take place online via *zoom*. We kindly ask all participants to make sure they can use a webcam (e.g. by using their phone). Please be prepared to show your student ID so we can identify participants.

An exemplary workflow for the assignment is presented below:

1. Implementation of the assigned problem in *python* (other programming languages are possible, but require consultation)
2. Group discussion concerning meaningful numerical examples (in addition to the results required by the assignment, if applicable)
3. Creation of representative results (e.g. plots for exemplary problems) to showcase model behaviour
4. Preparation of a presentation, including slides
5. Review of the theoretical aspects of the project with attention to the lecture content

The following deadlines hold:

- The examination will take place on Sep. 7$^{\text{th}}$.
- The program source code has to be submitted on Sep. 4$^{\text{th}}$, 8:00 am *at the latest*, to tim.furlan@tu-dortmund.de.
- The presentation slides (.pdf, .ppt/x, .odp) have to be submitted at the day of the examination, 8:00 am *at the latest*, to tim.furlan@tu-dortmund.de. Since the presentation will take place online, participants have to use *screen sharing* for the presentation.

While we encourage you to ask questions that may arise during the assignment, please note that in the grading process the ability to work independently and on your own responsibility is taken into account. This particularly applies to the application of the supervisors' recommendations and feedback to your questions. Furthermore, the code handed in is expected to be well structured (make use of subroutines), well documented (comments), and easily comprehensible (meaningful naming of functions and variables).

A typical presentation includes the following sections:

- Title (course name, group members, date, ...)
- Motivation of the problem
- Relation between the assigned problem and the theory presented in the lecture, including specific model description
- Algorithmic implementation
- Representative results, including verification of the implementation (if applicable)
- Conclusion

Some guidelines for suitable presentations:

Do	Don't
• Only show core equations	• Extensive mathematical conversions
• Use flowcharts to represent program structure	• Line-by-line presentation of program code
• Axes labels etc. must be readable (font size)	• Use low resolution images
• Use proper citations for sources	• Forget to credit images
• Use appropriate tools to display mathematical formulae (e.g. LaTeX beamerclass, IguanaTeX)	• Use multiple text fonts and colours without a clear concept
• Use an appropriate layout for each slide	• Show nearly empty or overfull slides

For the preparation of the oral examination, sample questions are provided in a separate document. These are intended for your own review of the lecture material, therefore no exemplary answers will be provided either in written form or in person. While all members of your group will be present during the oral examination, the supervisors may choose to direct questions at individual persons. The intention of the oral examination is to determine the contribution of each group member to the project, as well as to ensure that no program parts were adopted/copied without proper understanding. Furthermore, you shall be able to put your project work into context w.r.t. the lecture.

Introduction and Theory

The standard von Mises flow criterion is not capable of modelling anisotropic material behaviour, which can be very important for some applications. This holds e.g. for materials that undergo the excessive formation of an anisotropic texture, such as rolled or extruded goods.

In the lectures [1], we already introduced Hill's criterion [2] as an anisotropic flow function. The latter extends the von Mises criterion (yield surface is a cylinder in principal stress state) by a directional dependency, resulting into an elliptical cylinder representing the yield surface.

Barlat and Lian introduced a flow function to model the anisotropic (orthotropic) behaviour of metal sheets under plane stress states which reads

$$\Phi(\boldsymbol{\sigma}, \sigma_{11}^{y}) = f(\boldsymbol{\sigma}) - 2\,[\sigma_{11}^{y}]^{M} \qquad \text{with} \tag{1}$$

$$f(\boldsymbol{\sigma}) = a\,|K_1 + K_2|^{M} + a\,|K_1 - K_2|^{M} + [2 - a]\,|2\,K_2|^{M} \qquad \text{and} \tag{2}$$

$$K_1 = \frac{\sigma_{11} + h\,\sigma_{22}}{2}, \qquad K_2 = \sqrt{\left[\frac{\sigma_{11} - h\,\sigma_{22}}{2}\right]^{2} + b^{2}\,\sigma_{12}^{2}}, \tag{3}$$

with material parameters a, b, h and M. The initial uniaxial flow stress of the first principal direction of orthotropy is denoted as σ_{11}^{y}.

The yield surface of this criterion in (six-dimensional) stress space is, depending on the choice of material parameters, always convex and represents again an isotropic criterion for $a = b = h = 1$. For the latter case, the choice of $M = 2$ represents the von Mises surface, whereas for $M = 1$ and $M = \infty$ the Tresca yield surface (hexagonal cylinder in principal stress space) is recovered.

For further details on the Barlat-Lian orthotropic criterion for plane stress cases, refer to [3].

For the elastic region, opposite to the anisotropic plastic behaviour, linear isotropic behaviour with the elasticity tensor

$$\mathbf{E}^{\mathrm{e}} = 2\,G\,\mathbf{I}_{\mathrm{dev}}^{\mathrm{sym}} + K\,\boldsymbol{I} \otimes \boldsymbol{I}. \tag{4}$$

shall be assumed.

Task

1. Derive the necessary algorithmic relations for the above Barlat-Lian yield function, assuming *linear isotropic hardening* as shown in [3], pp. 427 ff.

2. Implement the above material model in Matlab/Octave based on the constitutive driver used in the tutorials, making use of the following material parameters

   $$E = 69\,\text{GPa}, \quad \nu = 0.33, \quad H = 2000\,\text{MPa}, \quad M = 8, \quad a = 1.24, \quad b = 1.02, \quad h = 1.15.$$

   Analyse the material response to linear increasing as well as cyclic loads for

   a) equi-biaxial

   b) general biaxial

   strain loads. Furthermore, analyse the influence of the hardening modulus H by varying this parameter in sensible bounds and by applying it to a load case of your choice.

3. After successful implementation, investigate and interpret the results from the above model and plot the results in an illustrative manner, such as the total ε_{11}, ε_{22} or plastic strains $\varepsilon_{11}^{\mathrm{p}}$, $\varepsilon_{22}^{\mathrm{p}}$ over time t, strains ε_{11}, ε_{22} or stresses σ_{11}, σ_{22}.

4. Compare the convergence rates of a numerically determined tangent modulus and the exact algorithmic tangent modulus provided in [3].

Hints

1. Due to the structure of the yield function, the *explicit* calculation of the consistency parameter is no longer possible, i.e. $\Phi_{n+1} = 0$ cannot be solved explicitly with respect to $\Delta\lambda_{n+1}$. The latter is to be determined iteratively, using a Newton-Raphson iteration scheme known from the tutorials.

2. Note that, in order to apply biaxial load cases, the function `partition()` used in the stress driver routine has to be modified accordingly!

3. To test your implementation of the material routine, first use a numerically determined tangent operator based on a forward differential as shown, e.g., in the routine `moduli_num_kauderer()` in the tutorials before implementing the algorithmically consistent tangent operator. Note that, in order to implement a numerical tangent, it is convenient to implement two separate functions for the calculation of stresses and the tangent operator, respectively.

References

[1] A. Menzel. Theory of Materials. Lecture Notes, TU Dortmund, Institute of Mechanics.

[2] R. Hill. The Mathematical Theory of Plasticity. Oxford University Press, 1950.

[3] A. de Souza Neto, D. Perić and DRJ. Owen. Computational Methods for Plasticity: Theory and Applications. John Wiley & Sons, 2008