Arturo
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HW# 6
C 341
proofs

E1 1. Claim: a full m-ary with n nodes has $(n-1)/m$ internal nodes and $((m-1)n+1)/m$ leaves.

pf. Base Case: 1 nodes, $n(t)=1$ leaves $=1$, internal nodes $=0$

$(n-1)/m = 0$ $\qquad$ $(m-1)1+1 = 1$

$m = 0$ $\qquad$ $m = 1$

Induction case for Internal nodes

Using nod$(d, (t_1, t_2, t_3), m)$ where $t_1, t_2, \ldots, t_m$ are subtrees with $N(t_1)=n_1, \ldots, n(t_m) = n_m$. So, $n = 1 + n_1 + n_2 + \ldots + n_m$

Assume claim is true for T_1 to t_m

By induction, $I(t) = I(t_1) + I(t_2), \ldots, I(t_m) + 1$

$\frac{n_1-1}{m} + \frac{n_2-1}{m} + \ldots + \frac{n_m-1}{m}$

So, internal nodes $= \frac{nm-1}{m}$

Induction Case for leaves

Since we've proven $I(t)$, and we know $L(t) = N(t) - I(t)$

$L(t) = n - \frac{n-1}{m}$

$\frac{nm}{m} - \frac{n-1}{m} = \frac{nm-n-1}{m}$

2. Claim: a full m-ary tree with i internal nodes has mi+1 nodes and (m-1)i+1 leaves

Base Case: see #1

Induction Step:

$I(t) = N(t) - L(t)$

$I(t) = \quad n \quad = \frac{nm - n + 1}{m}$

Induction step for leaves:

we know that $L(t) = N(t) - I(t)$

we also know that $l = (m-1)i + 1$

$l/i = \frac{(mi+1)(m-1)+1}{m}$

$m^2 i + m - mi - 1 + 1$

$= m + 1 - i$

$= m + 1 - i$

$= mi + 1 - i$

$= i(m-1) + 1 = (m-1)i + 1$

3. Claim: a full m-ary tree with l leaves has $(ml-1)/(m-1)$ nodes and $(m-1)i+1$ leaves and $(l-1)/(m-1)$ internal nodes.

Pf. We know that $L(t) = N(t) - I(t)$

we know that $I = (m-1)i+1$

we know that $n = (m(m-1)i+1)/(m-1)$

we can solve n by saying that

$$n = \frac{(m(m-1)i+1)-1}{(m-1)}$$

for nodes: we know that $N(t) = L(t) + I(t)$

so,

$$n = l + \frac{n-1}{m}$$

$$n = \frac{lm}{m} + \frac{n-1}{m}$$

$$n = \frac{lm + n - 1}{m}$$

$$= \frac{ln + (n-1)}{m}$$

$$nm = lm + n - 1$$

$$nm - n = lm - 1$$

$$n(m-1) = lm - 1 \quad = n = \frac{(lm-1)}{m-1}$$

For internal nodes: Since we have $n - l = i$

$$i = (lm-1)/(m-1) - l(m-1)/m-1$$

$$i = \frac{(lm-1) - l(m-1)}{m-1}$$

$$i = \frac{lm - 1 - lm + l}{m-1}$$

$$i = \frac{lm - 1 - lm + l}{m-1}$$

$$i = \frac{1+l}{m-1}$$

$$i = \frac{l+1}{m-1}$$

4. $I(t) = N(t) - L(t)$

$i = (9 \times 100 - 1) / (9-1) = 133$

$l = 100$

$(L-1)/(m-1) = {}^{99}/_{3} = 33$ sending

$100 + 33 = 133$ people who saw the letter