Reduced Order Models on a Variational Multi-Scale Model of Navier–Stokes

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ABSTRACT

In this work, we study parametric incompressible flows given by Navier–Stokes equations. At the discrete level, we use continuous finite element (FE) method. The discrete model relies on a variational multi-scale (VMS) approach, which separates the large, sub-filter and small scales. The first two are resolved, while the last is not. The model includes sub-grid eddy viscosity to take care of the interaction between sub-filter and small scales. A local projection stabilization (LPS) term is introduced onto the sub-filter terms to provide stability. This term is based on interpolation and projection operators that penalizes the oscillations on the sub-filter scale[1, 2]. Moreover, in order to deal with no-slip boundary conditions without dramatically refining the mesh close to boundary layers, we resort to wall laws, which take into account the effect of the boundary already at a small distance from the boundary.
The computational costs of such simulations, though being faster than other models as the small scales are not resolved, are still large, in particular in the time dependent case. Moreover, dealing with parametric problems in a multi-query context makes the computational burden unbearable. We propose a Galerkin projection of the equations onto a POD-generated reduced basis space as reduced order model (ROM). To take care of the nonlinearities of the problem, different hyper-reduction techniques are studied in order to obtain a reduced model that is independent of the dimension of the FE space.
We provide simulations in two dimensions to validate the ROM and to prove the computational advantage of the approach.

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