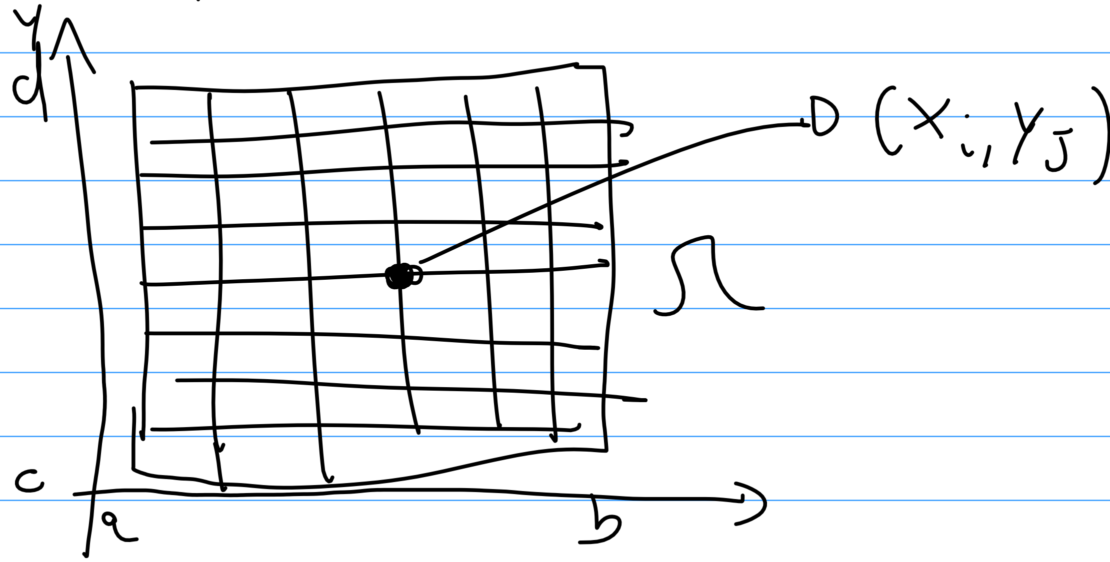
# FINITE DIFFERENCE IN 2D

$\Omega = [a,b] \times [c,d]$




$a = x_0 < x_1 \dots < x_{N-1} = b$ $\qquad x_{i+1} - x_i = \Delta x \quad \forall i$

$c = y_0 < \dots < y_{M-1} = d$ $\qquad y_{j+1} - y_j = \Delta y \quad \forall j$

$u_{ij} \approx u(x_i, y_j)$

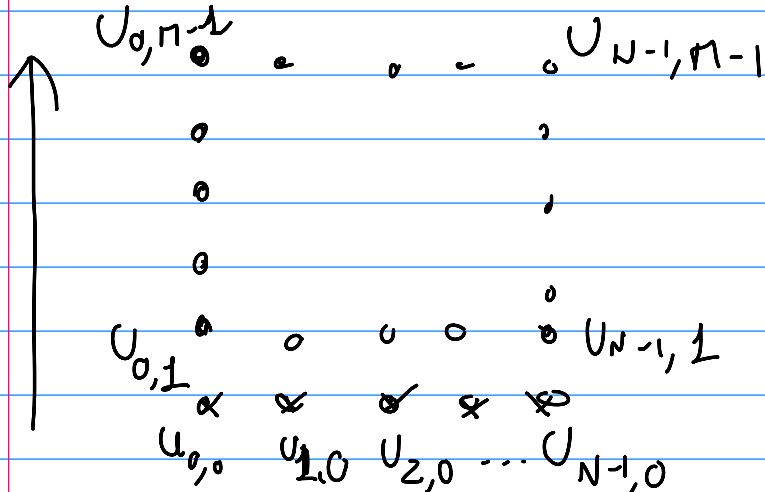
$$\Delta u = \partial_{xx} u + \partial_{yy} u \approx \left( (\delta^x_{\Delta x})^2 u_{i,j} + (\delta^y_{\Delta y})^2 u \right)_{ij}$$

$$= \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2}$$

$$\rightarrow \begin{array}{|ccc|} \hline & 1 & \\ 1 & -4 & 1 \\ & 1 & \\ \hline \end{array} \qquad \left( u_{i+1,j} + u_{i,j+1} - 4u_{ij} + u_{i-1,j} + u_{i,j-1} \right) / \Delta x^2$$

$\Delta x = \Delta y$

• SYS LIN $\underline{\underline{A}}\,\underline{u} = \underline{f}$




$N \cdot M$ pti e INCOGNITE

$$\begin{array}{c|c}
\alpha & \\
0 & U_{0,0} \\
1 & U_{0,1} \\
2 & U_{0,2} \\
\vdots & \vdots \\
M-1 & U_{0,M-1} \\
M & U_{1,0} \\
M+1 & U_{1,1} \\
 & U_{1,2} \\
\vdots & \vdots \\
M+M-1 & U_{1,M-1} \\
2M & U_{2,\dots} \\
 & \vdots \\
NM-1 & U_{N-1,M-1}
\end{array} = \underline{U}$$

$U_\alpha = u_{ij}$

$$\boxed{\alpha = iM + j}$$

$$A_{\alpha\alpha} = \frac{2}{\Delta x^2} + \frac{2}{\Delta y^2}$$

$\boxed{\alpha = iM + j}$

$\alpha + 1$

$\parallel$

$A_{\alpha\beta} = -\frac{1}{\Delta y^2}$ PER $\beta = iM + (j+1)$

$A_{\alpha\beta} = -\frac{1}{\Delta y^2}$ PER $\beta = \alpha - 1 = iM + (j-1)$

$$-\left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2}\right] \quad \forall \underbrace{ij}_{\alpha}$$

$$\sum_\beta \underline{\underline{A}}_{\alpha,\beta} \cdot \underline{u}_\beta = f_\alpha$$

$$\overbrace{iM + j}^{\alpha} + M = \alpha + M$$

$\parallel$

$A_{\alpha,\delta} = -\frac{1}{\Delta x^2}$ PER $\delta = (i+1)M + j$

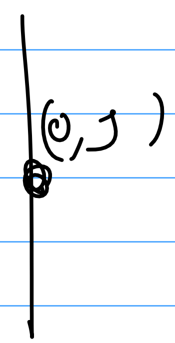
$A_{\alpha,\varepsilon} = -\frac{1}{\Delta x^2}$ PER $\varepsilon = (i-1)M + j = \alpha - M$

$$\sum_{i,j} \underbrace{A_{\underbrace{lk}, ij}} \cdot \underbrace{u_{i,j}} = \underbrace{f_{l,k}} \quad \forall l = 1,\dots,N \quad \forall k = 1,\dots,M$$

$$i,j \longmapsto \alpha = iM + j$$

$$\alpha \longmapsto i = \alpha // M$$
$$j = \alpha \% M$$

NEUMANN p.es. BORDO SX




$$\partial_x u_{0,j} = g_{0,j}$$

$$\Updownarrow$$

$$\frac{u_{1,j} - u_{0,j}}{\Delta x} = g_{0,j} \quad \forall j$$