MODEL ORDER REDUCTION

RIDUZIONE DEL MODELLO

BASI RIDOTTE / REDUCED BASIS

$-\Delta u = f \qquad \Omega_h \to V_h \ni u_h \Leftarrow$ FEM / FD

SISTEMA LINEARE

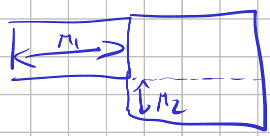
PARAMETRICO $\mu \in \mathcal{P} \subset \mathbb{R}^p$

μ → BOUNDARY CONDITIONS
→ PARAMETRI FISICI
→ GEOMETRIA

PARAMETRO FISICO

$-\nabla \cdot (K \cdot \nabla u) = f$

$K(x)$



- OBIETTIVO 1: RISOLVERE FEM/FD PER MOLTI PARAMETRI
- OBIETTIVO 2: TROVARE MODELLO SURROGATO DI FEM/FD CHE SIA PIÙ VELOCE DA RISOLVERE.

A) ⇒ MENO GRADI DI LIBERTÀ

B) ⇒ BUONA APPROSS. DEGLI FEM.

B) ✓ DOPO

⇒ A) UNO SPAZIO $V_N = \langle \{\psi_i\}_{i=1}^N \rangle \subset V_h$ — V_N è UNA BUONA APPROX. DI V_h PER IL NOSTRO PROB. PARAMETRICO

FEM $\quad a(\varphi_i, \sum \varphi_i u_i) = F(\varphi_i) \quad \forall i = 1, \dots, N_h$

ROM $\quad a(\psi_i, \sum_j \psi_j \hat{u}_j) = F(\psi_i) \quad \forall i = 1, \dots, N$

$N << N_h$

$\approx 10 \qquad \approx 10^6$

$V_h \ni \psi_i(x) = \sum_{j=1}^{N_h} \psi_{i,j} \cdot \varphi_j(x)$

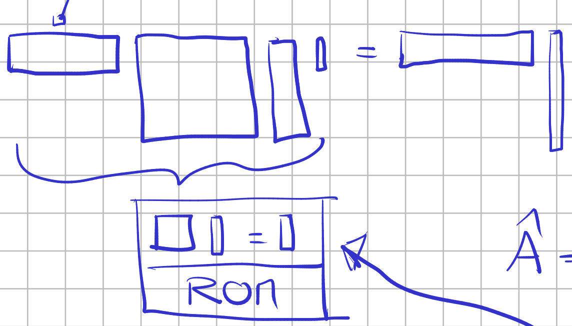
$\underline{\psi}_i = \begin{pmatrix} \psi_{i,1} \\ \vdots \\ \psi_{i,N_h} \end{pmatrix} \in \mathbb{R}^{N_h}$

$$\sum_j \cancel{\sum_k}\cancel{\sum_\ell} a\Big(\psi_{i,j}\cdot \underset{V_h}{\varphi_j}, \sum_k \sum_\ell \psi_{k,\ell}\cdot \underset{V_h}{\varphi_\ell(x)}\cdot \hat u_k\Big) = F\Big(\sum_j \psi_{i,j}\cdot \underset{V_h}{\varphi_j}\Big)$$

$$\forall\, i=1,\dots,N \qquad \sum_{j=1}^{N_h} \psi_{i,j}\, \overbrace{a(\varphi_j,\varphi_\ell)} \sum_{\ell=1}^{N_h}\sum_{k=1}^{N} \overbrace{\psi_{k,\ell}}\cdot \hat u_k = \sum_j \psi_{i,j} F(\varphi_j)$$

$$\underline{\underline{R}} \in \mathbb{R}^{N_h\times N} \qquad R = \overbrace{[\psi_1, \psi_2, \dots \psi_N]}^{N}\ \updownarrow N_h \qquad R = [\ \ ]$$

$$R^T \underset{\text{FEM}}{A}\cdot R\cdot \hat u = R^T\cdot F$$



FEM

RON

$$\hat A = R^T A R \qquad \hat F = R^T\cdot F$$

$$\hat A\, \hat u = \hat F$$

- $A(\mu)$ $\quad F(\mu)$ $\quad \rightsquigarrow \quad R^T A(\mu) R$
- SE C'È UNA DIPENDENZA AFFINE DI A e F DAL PARAMETRO μ

$$A(\mu) = \sum_{k=1}^{m_A} \underset{\mathbb{R}}{\Theta_k^a(\mu)}\cdot \underset{\text{INDIP. DA } \mu}{A_k} \qquad F(\mu) = \sum_{k=1}^{m_F} \underset{\mathbb{R}}{\Theta_k^F(\mu)}\cdot \underset{\text{INDIPEND. DA } \mu}{F_k}$$

$$R^T A(\mu)\cdot R =$$

$$= \sum_{k=1}^{m_A} R^T \Theta_k^a(\mu)\, A_k\cdot R = \sum_{k=1}^{m_A} \Theta_k^a(\mu)\, \underbrace{R^T A_k R}_{\hat A_k} = \sum_{k=1}^{m_A} \Theta_k^a(\mu)\, \underbrace{\hat A_k}_{\in \mathbb{R}^{N\times N}}.$$

$$R^T F(\mu) = \sum_{k=1}^{m_F} \Theta_k^F(\mu)\cdot \underbrace{R^T F_k}_{\hat F_k} = \sum_{k=1}^{m_F} \Theta_k^F(\mu)\cdot \underset{\mathbb{R}^N}{\hat F_k}$$

- COME SCEGLIERE V_N e LA SUA BASE $\{\psi_i\}_{i=1}^N \Rightarrow R$ MATRICE DI PROIEZ. ?

↳ 1. GENERARE DEGLI SNAPSHOTS $\{u(\mu_1), \dots, u(\mu_{m_{TRAIN}})\}$

↳ POD PROPER ORTHOGONAL DECOMPOSITION

SVD SINGULAR VALUE DECOMPOSITION

$$S = [u(\mu_1), \dots, u(\mu_{m_{TRAIN}})] \in \mathbb{R}^{N_h\times m_{TRAIN}}$$

$\mathrm{SVD}(S) = U \Sigma V = S$ $\qquad U \in \mathbb{R}^{N_h \times N_h}$

U, V ORTOGONALI $\qquad U^T U = I \qquad V^T V = I$

$\Sigma \in \mathbb{R}^{N_h \times m_{TRAIN}}$

$V \in \mathbb{R}^{m_{TR} \times m_{TRAIN}}$

Σ È DIAGONALE $\qquad \Sigma > 0$

SPAZIO RIDOTTO

$S \approx \hat{U} \hat{\Sigma} \hat{V}$ $\qquad \hat{\Sigma} \in \mathbb{R}^{N \times N}$

$R \leftarrow \hat{U} \in \mathbb{R}^{N_h \times N}$

$\hat{V} \in \mathbb{R}^{N \times m_{TRAIN}}$

$\hat{U}$ HA BUONA PARTE DELLE INFORMAZIONI DI S

$S - \hat{U}\hat{\Sigma}\hat{V} = $ PICCOLO

NON BORDO

$(\Theta_0^a(\mu) A_0 + \Theta_1^a \overset{\uparrow}{A_1}) u - \Theta_0^f \cdot F_0$

DIRI OMO $\Rightarrow \qquad A_{0,[i,:]} = [\,0 - 0\ 1\ 0 - 0\,] \qquad \forall i$ BORDO DIRI

$F_{0,i} = [0] \leftarrow$ OMOGENEO

$\underbrace{\Theta_0^a(\mu)}_{\neq 0} \cdot 1 \cdot u_i = \underset{\downarrow}{0} \Rightarrow u_i = 0$ $\qquad$ BENE COSÌ

DISOMOGENEO $\Rightarrow$ PIÙ COMPLICATO

$(R^T A_k R)_{ij} = (R^T (A_k R))$ $\qquad R_i^T A_k R_j \quad \forall i,j = 1,\dots,N$

$R_i^T \in \mathbb{R}^{N_h}$, $\quad A_k \in \mathbb{R}^{N_h \times N_h}$, $\quad R_j \in \mathbb{R}^{N_h}$

$= \sum_{l,z} (R^T)_{il} (A_k)_{lz} (R)_{zj}$

$(R_{:,i})^T A \cdot R_{:,j}$

$\underline{a_k(\psi_i, \psi_j)}$

$R_i^T \cdot (A_k R_j) = \hat{A}_{ij}$

$\mathbb{R}^{N_h}$

- LINEARE PROBLEMA
- DECOMPOSIZIONE AFFINE NEI PARAMETRI
- VALORI SINGOLARI DECADONO ESPONENZIALMENTE

- $\underline{\underline{\partial_t}} u - \underline{\underline{\Delta}} u = 0$
- $\partial_t u + a \partial_x u = 0$