- PINN

$$NN(x) : \mathbb{R}^{l_1} \to \mathbb{R}^{l_{out}}$$

$$NN(x) = \ldots A_3\sigma(A_2\sigma(A_1x + b_1) + b_2) + b_3 \ldots$$

σ ACTIVATION FUNCTION
FUNZIONE NONLINEARE SEMPLICE

$1+N$ = NUMERO DI LAYER

l_1 NODI DEL PRIMO LAYER (x IN INPUT) $\in \mathbb{R}^{l_1}$

l_2 NODI DEL SEC. LAYER

⋮

$l_{N+1} = l_{OUTPUT}$ # NODI LAYER OUTPUT

LAYER DA #2 AL #N-1
HIDDEN LAYER $K \subseteq \mathbb{R}^n$

- FEEDFORWARD NN ~~...~~ DA ~~...~~ $K \to \mathbb{R}$

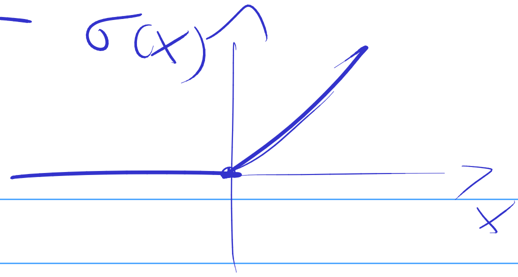
1 SOLO HIDDEN LAYER, DATA $f \in C(K;\mathbb{R})$

$$\forall \varepsilon > 0 \quad \exists\, l_2 \in \mathbb{N} :$$

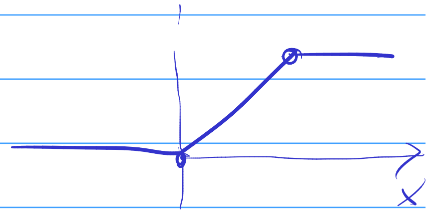
$$\max_{x \in K} |NN(x) - f(x)| < \varepsilon.$$

RELU RECTIFYING LINEAR UNIT

$$\sigma(x) = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$



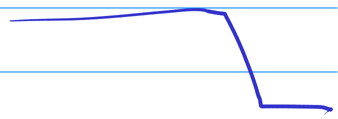
$$\sigma(x) - \sigma(1-x)$$



$x<0 \to 0 + 1-x$

$0 < x < 1$

$x \geq 1$



$$NN(x) = A_N \sigma(A_{N-1} \dots A_1 x + b_1) \dots + b_N$$

$$\Theta = (A_1, A_2, \dots, A_N, b_1, \dots, b_N) \in \mathbb{R}^z$$

CI POSSO OTTIMIZZARE PER APPROX LA MIA f.

$$A_1 \in \mathbb{R}^{l_1 \times l_2} \qquad A_2 \in \mathbb{R}^{l_2 \times l_3}$$

$$X_{train} = \{(x_i, y_i)\}_{i=1}^{N_{train}} \qquad y_i = f(x_i) + \text{errori}$$

$$\mathcal{L}(\Theta) = \frac{1}{N_{train}} \sum_{i=1}^{N_{train}} \underbrace{(NN(x_i) - y_i)^2}$$

LOSS FUNCTION

SCELTA COME MSE

$$\Theta^* = \arg\min_{\Theta} \mathcal{L}(\Theta)$$

- GRADIENT DESCENT (e VARIAZIONI SUL TEMA)

- STOCHASTIC GRADIENT DESCENT
- ADAM

$$\Theta_i^{m+1} = \Theta_i^m - \underbrace{\eta}_{\text{LEARNING RATE}} \nabla_{\Theta_i} \mathcal{L}(\Theta^m)$$

$$\Theta_i = \begin{cases} A_1 \\ A_2 \\ \vdots \\ b_1 \\ \vdots \\ b_N \end{cases}$$

1) FORWARD PASS $\mathcal{L}(\Theta)$

$h_0 := \underline{x}$

$$\begin{cases} f_k := A_k h_{k-1} + b_k \\ h_k := \underline{\underline{\sigma}}(f_k) \end{cases} \quad \forall k = 1, \dots, N$$

$NN(x) = h_N$

$(h_n - c)^2$

$$\mathcal{L}(\Theta) = \frac{1}{N_{TRAIN}} \sum_{i=1}^{N_{TRAIN}} (\overbrace{NN(x_i)}^{h_n} - y_i)^2$$

$\nabla_\Theta \mathcal{L}(\Theta)$

$$g_{h_N} := \nabla_{h_N} \mathcal{L}(\Theta) = 2(h_N - y_i) \frac{1}{N_{TRAIN}}$$

$$g_{f_N} := \nabla_{f_N} \mathcal{L}(\Theta) = \underbrace{\nabla_{h_N} \mathcal{L}(\Theta)}_{g_{h_N}} \cdot \nabla_{f_N} h_N$$

$$g_{h_N} \cdot \sigma'(f_N)$$

$$g_{b_N} := \nabla_{b_N} \mathcal{L}(\Theta) = \nabla_{f_N} \mathcal{L}(\Theta) \cdot \underbrace{\nabla_{b_N} f_N}_{=1}$$

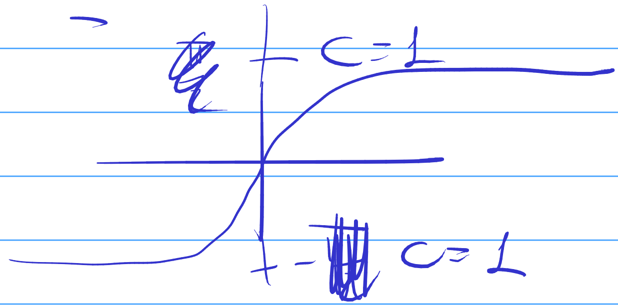
$$= g_{f_N}$$

$$g_{A_N} = \nabla_{A_N} \mathcal{L}(\theta) = \nabla_{f_N} \mathcal{L}(\theta) \cdot \underbrace{\nabla_{A_N} f_N}_{h_{N-1}}$$

$$= g_{f_N} \cdot h_{N-1}$$

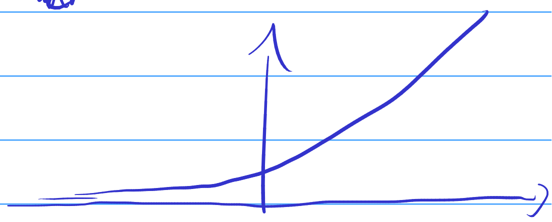
ACTIVATION FUNCTIONS

- $\mathrm{ReLU}(x) = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$ $\quad \mathrm{ReLU}'(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$



- $\tanh(x)$

- $\mathrm{softplus}(x) = \log(1 + e^x)$



- $\mathrm{sigmoid}(x) = \dfrac{1}{1+e^{-x}}$

$$P(u) = 0 \quad \text{PDE}$$

$$+ \quad u = g_D \quad \text{on } \partial\Omega$$

PINN 2017 $\quad$ NN $u^\theta : \Omega \to \mathbb{R}^s$ $\quad u^{ex} \sim u^\theta$

$$\mathcal{L}(\theta) = \frac{1}{N_Q} \sum_{q=1}^{N_Q} P(u^\theta)(x_q)|^2 + \frac{1}{N_B} \| u^\theta(x_b) - g_D(x_b^{\partial\Omega}) \|^2$$

es. $P(u) = -\Delta u - f$