~~ELEMENTI FINITI~~

NORME ED ERRORI

$$\rightarrow \|\underline{u}\|_2 = \sqrt{\sum_i u_i^2}$$

$$\|u\|_{L^2} = \sqrt{\int_0^1 u^2\,dx}$$

$$\|u\|_2 = \sqrt{\frac{1}{N}\sum_{i=1}^{N} u_i^2}$$

$$\downarrow N \to \infty$$

$$\|u\|_{L^2}$$

$$\frac{\|u - u^e\|_2 \cancel{\sqrt{\frac{1}{N}}}}{\|u^e\|_2 \cancel{\sqrt{\frac{1}{N}}}}$$

~~NORME~~ CONSISTENTI

$$\left[\frac{\|u - u^e\|_{L^2}}{\|u^e\|_{L^2}}\right]$$

$$u_h = \sum u_i \cdot \varphi_i(x)$$

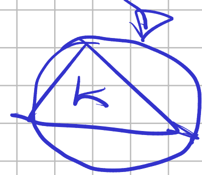
$$u_h'(x) = \sum u_i \varphi_i'(x)$$

$$\approx \sum \underbrace{u_i'}\ \underbrace{\varphi_i(x)}_{\text{LAGRANGIALE}}$$

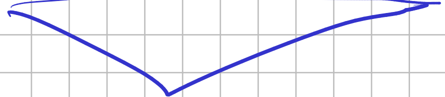
$$\underline{u_i'} = u_h'(x_i)$$

2D

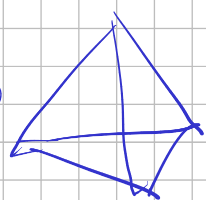
$$h = \max_{K \in T_h} \operatorname{diam}(K)$$



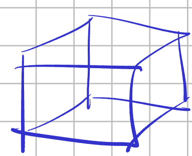
NON CI PIACE



$\rightarrow$

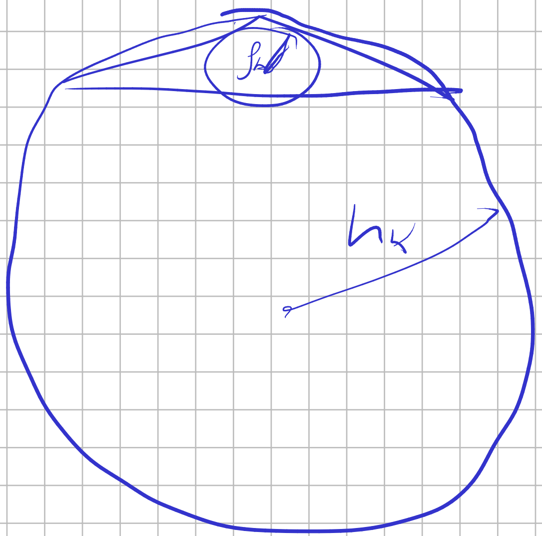


3D



BELLO

$\frac{h_K}{\rho_K}$ $\begin{matrix} \to \text{diam circonf. circoscritta} \\ \to \text{diam circ. inscritta} \end{matrix}$

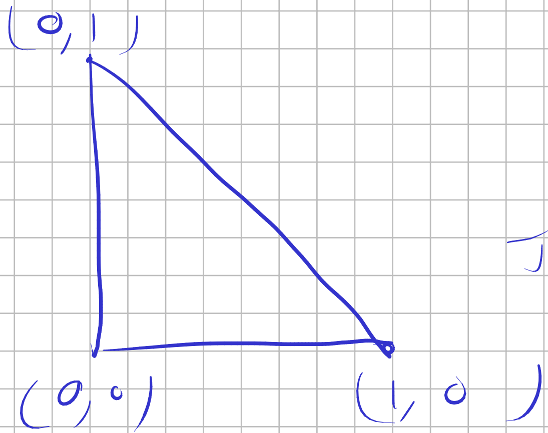


$$\frac{h_K}{\rho_K} \le \delta$$

ERRORE DIPENDE DA δ

ERRORE $\uparrow$ $\delta \uparrow$

TRIANGOLO DI RIFERIMENTO



$$\mathbb{P}^p = \left\{ f(x,y) : \sum_{\substack{i,j \\ i+j \le p}} a_{ij} x^i y^j : a_{ij} \in \mathbb{R} \right\}$$

TRIANGOLI

$$\boxed{\mathbb{P}^2} \ni 1, x, x^2, xy, y^2$$

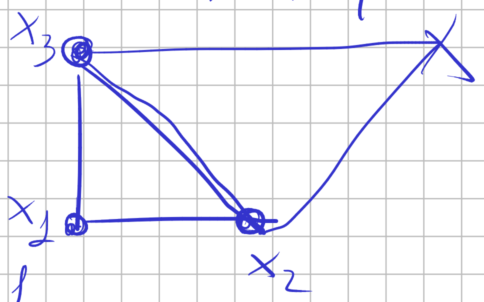
$$\not\ni x^2 y$$

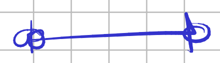
$$\mathbb{Q}^p = \left\{ f(x,y) : \sum_{\substack{i,j \\ i \le p \\ j \le p}} a_{ij} x^i y^j : a_{ij} \in \mathbb{R} \right\}$$

$$\boxed{\mathbb{Q}^2} \ni 1, x^2, y^2, xy, x^2y, x^2y^2, xy^2$$

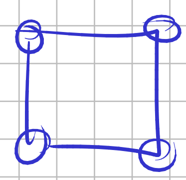
QUADRILATERI

$\mathbb{P}^1 \ni \{1, x, y\}$ $\leftarrow$ 3 BASI LAGRANGIANE





$\mathbb{Q}^1 \ni 1, x, y, xy$ $\longrightarrow$

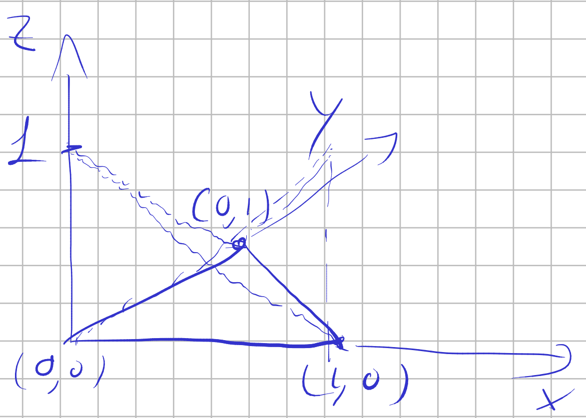


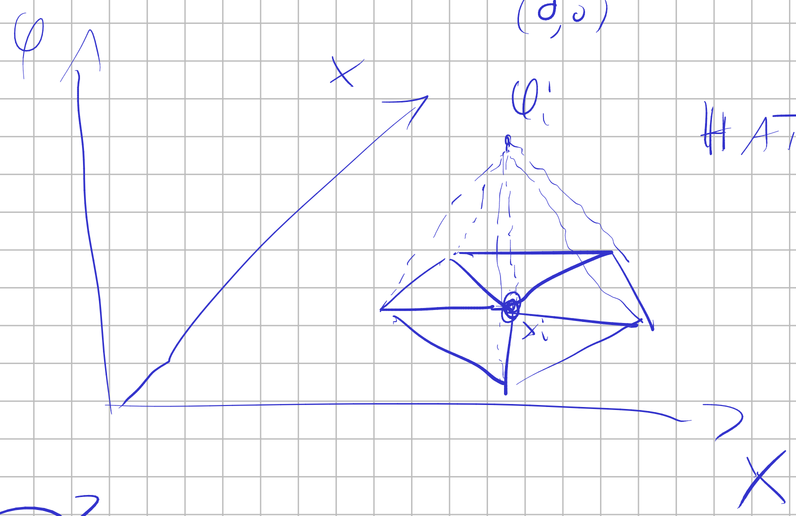
- $\varphi_1(x,y) = 1 - x - y$
- $\varphi_2(x,y) = x$
- $\varphi_3(x,y) = y$

$\varphi_1(0,0) = 1$

$\varphi_1(0,1) = 0$

$\varphi_1(1,0) = 0$





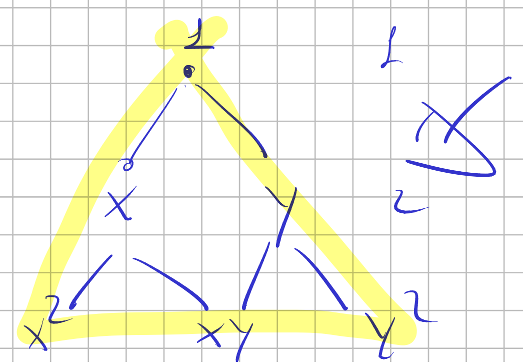
HAT FUNCTION

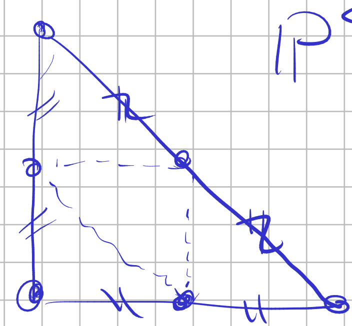
$\mathbb{P}^2 \ni 1, x, y, x^2, xy, y^2 \Rightarrow \dim \mathbb{P}^2 = 6$

$\mathbb{P}^0$

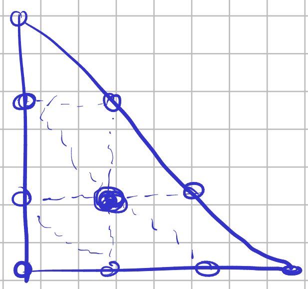
$\mathbb{P}^1$

$\mathbb{P}^2$





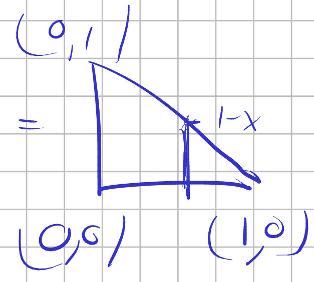
$\mathbb{P}^3$ $\quad x^3 \quad x^2y \quad xy^2 \quad y^3 \quad p+1$ $\qquad \mathbb{P}^3$



#TUN8

$$\dim(\mathbb{P}^p) = \frac{(p+1)(p+2)}{2}$$

$$\sum_{i=1}^{p+1} i = \frac{(p+1)(p+2)}{2}$$

$\hat{K} =$ 

$$\hat{a}_{ij} = \int_{\hat{K}} \nabla\hat{\varphi}_i \cdot \nabla\hat{\varphi}_j \, dx\,dy$$

$$= \int_0^1 \int_0^{1-x} \nabla\hat{\varphi}_i(x,y) \, \nabla\hat{\varphi}_j(x,y) \, dx$$

$\hat{K} = \{x, y :$
$x > 0$
$y > 0$
$y \leq 1-x \}$

$$a_{ij} = \int_\Omega \nabla\varphi_i \cdot \nabla\varphi_j \, d\underline{x}$$

$$= \sum_{K \in \mathcal{R}_h} \int_K \nabla\varphi_i \, \nabla\varphi_j \, d\underline{x}$$

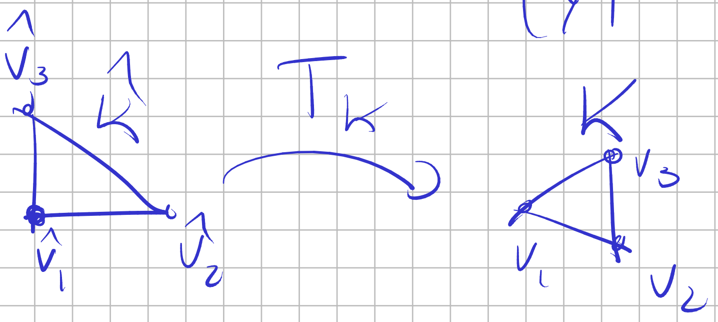
$\mathbb{R}^2 \ni \underline{b}$

$\underline{A} \in \mathbb{R}^{2\times 2}$

$T_K : \hat{K} \to K$

$$\begin{pmatrix} x \\ y \end{pmatrix} = T\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \underline{A}\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} + \underline{b}$$

4 $\qquad$ 2



6 equazioni

sono

$\forall$ vertice $\begin{pmatrix} x_i \\ y_i \end{pmatrix} = v_i$ $\qquad T\begin{pmatrix} \hat{x}_i \\ \hat{y}_i \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$ 2 eq.

$\forall \; i = 1 \_ 3$

$\Rightarrow$ TOTALE 6 equazioni

$x \in \Omega$ fisico $\quad \hat{x} \in \hat{K}$ riferimento

$$\int_K \underbrace{\nabla_x \varphi_i}_{} \cdot \nabla_x \varphi_j \, dx =$$

$$= \int_{\hat{K}} \overbrace{\nabla_x T^{-1}(x) \cdot \nabla_{\hat{x}} \hat{\varphi}_i(\hat{x})} \cdot$$

$$\cdot \nabla_x T^{-1}(x) \cdot \nabla_{\hat{x}} \hat{\varphi}_j(\hat{x})$$

$$\cdot \det(\nabla_{\hat{x}} T) \, d\hat{x}$$

1D

$\partial_x \varphi_i$

$$= \boxed{\frac{\partial \hat{x}}{\partial x}} \cdot \frac{\partial \hat{\varphi}_i}{\partial \hat{x}}$$

$$\parallel$$

$$\frac{1}{\Delta x}$$

Δx

$T(\hat{x}) = A\hat{x} + b \qquad T^{-1}(x) = A^{-1}x - A^{-1}b$

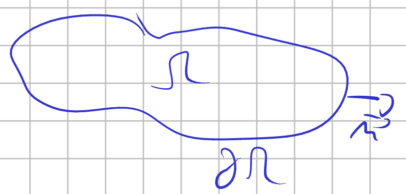
$\nabla_{\hat{x}} T = A$

$\nabla_x T^{-1}(x) = A^{-1}$

$$= \int_{\hat{K}} \underbrace{A^{-1}}_{\text{COST}} \cdot \nabla_{\hat{x}} \hat{\varphi}_i(\hat{x}) \cdot \underbrace{A^{-1}}_{\text{COSTANTE}} \nabla_{\hat{x}} \hat{\varphi}_j(\hat{x}) \underbrace{\det(A)}_{\text{COSTANTE}} \cdot d\hat{x}$$

$a \nabla u \cdot \nabla v$

$a(x,y)$



$$\int_\Omega -\Delta u \cdot v = +\int_\Omega \nabla u \cdot \nabla v \, dx - \int_{\partial\Omega} \underbrace{\nabla u \cdot \vec{n}}_{\substack{\parallel \\ g}} \cdot v \cdot ds$$

$\nabla u \cdot n = g$ su Γ_N