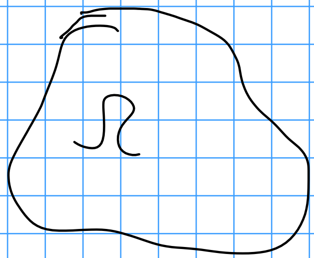


$\Omega \subset \mathbb{R}^d$

$u : \Omega \to \mathbb{R}^s \qquad s \in \mathbb{N}$

PDE STAZIONARIA DI ORDINE K

$$F(x, u, \nabla u, \nabla^{(2)} u, \dots, \nabla^{(k)} u, \underbrace{g}_{\text{DATI ESTERNI}}) = 0$$

$$\nabla u \to \begin{pmatrix} \partial_{x_1} u \\ \partial_{x_2} u \\ \vdots \\ \partial_{x_d} u \end{pmatrix} \qquad \nabla^{(2)} u \to \begin{pmatrix} \partial_{x_1}\partial_{x_1} u & \partial_{x_1}\partial_{x_2} u & \dots & \partial_{x_1}\partial_{x_d} u \\ \vdots & & & \\ \partial_{x_d}\partial_{x_1} u & \dots & & \partial_{x_d}\partial_{x_d} u \end{pmatrix}$$

$$P(u, g) = F\left(x, u, \frac{\partial}{\partial x_1} u, \dots, \frac{\partial}{\partial x_d} u, \frac{\partial^2}{\partial x_1 \partial x_1} u, \dots\right)$$

$$\frac{\partial^{p_1 + \dots + p_d}}{\partial^{p_1}_{x_1} \cdots \partial^{p_d}_{x_d}} u \; : \; p_1 + \dots + p_d \le K$$

$u \in C^K(\Omega)$

PDE NON STAZIONARIE t

$$P(u, g) = F\left(x, t, u, \frac{\partial u}{\partial t}, \dots, \frac{\partial^2 u}{\partial t \partial x}, \dots\right)$$

$$\frac{\partial^{p_0 + p_1 + \dots + p_d}}{\partial^{p_0} t \, \partial^{p_1} x_1 \cdots \partial^{p_d} x_d} u \qquad p_0 + \dots + p_d \le K$$

SOLUZIONE PDE $P(u, g)$ $\quad u \in C^K(\Omega \times [0, T])$

PRIMO ORDINE LINEARE

EQUAZIONE DEL TRASPORTO

?• $\partial_t u + \partial_x u = 0$

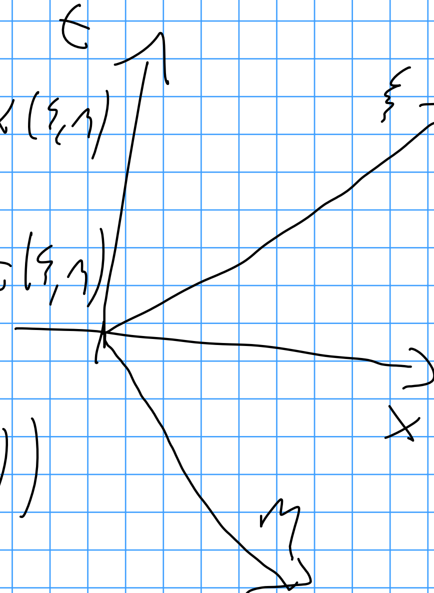
$\Omega = \mathbb{R}$

$t \in [0, T]$

$u : \mathbb{R} \times [0,T] \to \mathbb{R}$

$(x, t) \to (\xi, \eta)$

$\xi(x,t) = \underline{x+t}$ $\qquad \eta(x,t) = x - t$



$$\frac{\xi+\eta}{2} = \frac{x+t+x-t}{2} = \frac{2x}{2} = x(\xi,\eta)$$

$$\frac{\xi-\eta}{2} = \frac{x+t-x+t}{2} = \frac{2t}{2} = t(\xi,\eta)$$

$$V(\xi,\eta) := u\big(x(\xi,\eta), t(\xi,\eta)\big)$$

$$u_x = V_\xi \cdot \xi_x + V_\eta \cdot \eta_x$$

$$u_t = V_\xi \cdot \xi_t + V_\eta \cdot \eta_t$$

$$\begin{cases} \xi_x = \dfrac{\partial(x+t)}{\partial x} = 1 \\ \xi_t = 1 \\ \eta_x = \dfrac{\partial(x-t)}{\partial x} = 1 \\ \eta_t = \dfrac{\partial(x-t)}{\partial t} = -1 \end{cases}$$

$$\begin{cases} u_x = V_\xi + V_\eta \\ u_t = V_\xi - V_\eta \end{cases}$$

$$u_t + u_x = 0$$

$$V_\xi + \cancel{V_\eta} + V_\xi - \cancel{V_\eta} = 0$$

$2V_\xi = 0 \Rightarrow$ ODE !

$\Downarrow$

$V =$ COSTANTE DIPENDE DA η

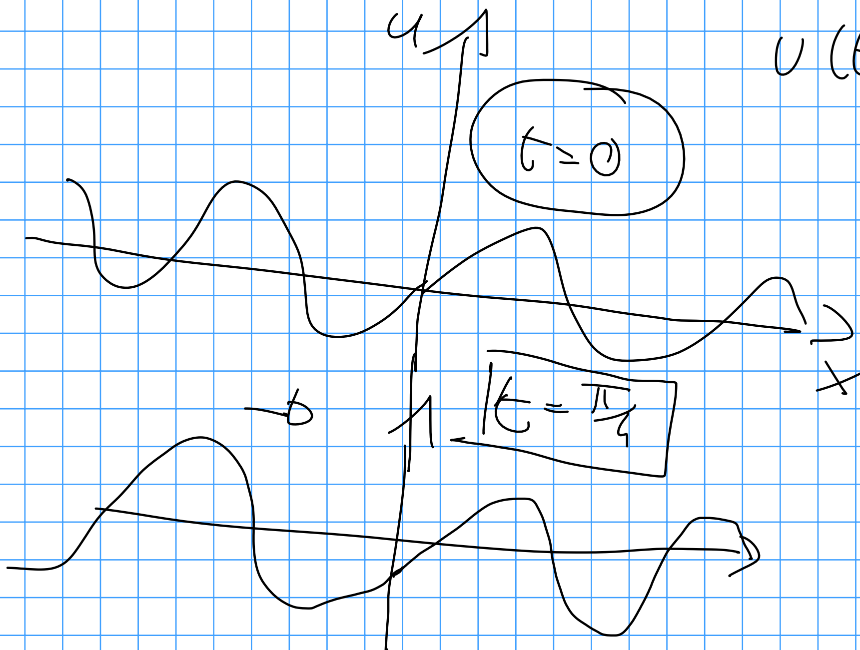
$V(\xi,\eta) = f(\eta) \qquad f \in C^1(\mathbb{R})$

$u(x,t) = V(\xi(x,t), \eta(x,t)) = f(\eta) = f(\underline{x-t})$

$\boxed{u_t + u_x = 0}$

$u(t=0, x) = \sin(x) = f(x-0) = f(x)$

$u(t=\frac{\pi}{4}, x) = \sin(x-\frac{\pi}{4})$

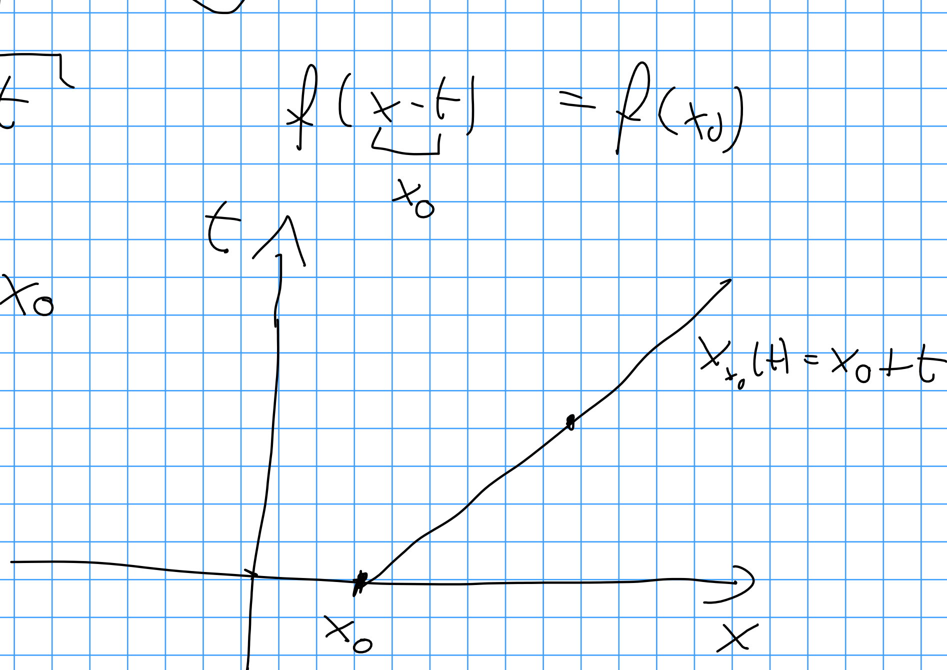


$\boxed{X_{x_0}(t) = \underline{x_0} + t}$

$\Downarrow$

$X_{x_0}(t) - t = x_0$

$f(\underbrace{x-t}_{x_0}) = f(x_0)$



$$u_t + u_x = 0 \to v_\xi = 0$$

$$a(t,x)\, u_t + b(t,x)\, u_x + c(x,t)\, u = g(t,x)$$

$$u : \mathbb{R} \times [0,T] \to \mathbb{R}$$

$$(t,x) \to (\xi, \eta)$$

$$\left| \frac{\partial(\xi,\eta)}{\partial(t,x)} \right| = \begin{vmatrix} \xi_t & \xi_x \\ \eta_t & \eta_x \end{vmatrix} = \xi_t \eta_x - \xi_x \eta_t \neq 0$$

$$u_t = v_\xi \xi_t + v_\eta \eta_t \, ; \quad u_x = v_\xi \xi_x + v_\eta \eta_x$$

$$a(t,x)\left(v_\xi \xi_t + \overbrace{v_\eta \eta_t} \right) + b(t,x)\left(v_\xi \xi_x + \overbrace{v_\eta \eta_x} \right)$$
$$+ c(t,x) \cdot v = g(t,x)$$

$t(\xi,\eta)$
$x(\xi,\eta)$

$$a(t,x)\, v_\eta \eta_t + b(t,x)\, v_\eta \eta_x = 0$$

$$a v_\xi \cdot \xi_t + b v_\xi \xi_x + c \cdot v = g$$

$$v_\xi (a \xi_t + b \xi_x) + c v = g$$

$$v_\xi + \frac{c \cdot v}{a \xi_t + b \xi_x} = \frac{g}{a \xi_t + b \xi_x}$$

$$\boxed{a \eta_t + b \eta_x = 0} \quad (A)$$

$$\eta(t, x(t)) = \eta_0$$

$$0 = \frac{d \eta(t, x(t))}{dt} = \eta_t + \eta_x \cdot \frac{dx}{dt} = 0$$

$$\boxed{\frac{\eta_t}{\eta_x} = - \frac{dx}{dt}}$$

(A) $$a \frac{\eta_t}{\eta_x} = -b$$

$$-\frac{dx}{dt} = -\frac{b}{a}$$

$$\frac{dx}{dt} = \frac{b}{a} \Rightarrow x(t) = t\frac{b}{a} + \eta_0$$

$$\Rightarrow \eta_0 \Rightarrow \eta = x - t\frac{b}{a}$$