ME41 FALL 2020

Design for Deflection

Aluminum Structure Design

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Contents

Executive Summary

This report outlines an approach to designing an aluminum structure that achieves a 0.5-inch deflection when a vertical load of 200 lbf is applied at a given loading point. After running finite element analysis (FEA) on a "C-shape" beam, an "S-shape" design was found to be the most optimal shape for our structure to achieve similar stability but with a smaller mass. SolidWorks Simulation and Castigliano's Theorem were utilized to predict the performance of the structure, along with several iterations to refine the performance following the results. A waterjet cutter was utilized to manufacture the final structure that was tested using an Instron machine.

Introduction

For Project 1, we designed an aluminum structure that underwent deflection using both numerical and analytical methods, specifically the strain energy method (Castigliano's Theorem) and Finite Element Analysis (FEA). Phase 1 of this project focused on first developing and practicing the analytical and numerical skills required by predicting the deflection of a known model. We were given a SolidWorks model of a preconstructed aluminum structure with the assumptions that it would be simply supported at the two holes on the bottom while a vertical force of 25 lbf was applied at the top right. This was to reflect the conditions of an Instron tensile test. To predict the vertical deflection at the point of load application, we first applied analytical methods discussed in class, mainly Castigliano's Theorem, to a simplified version of the structure. Then, we performed a numerical FEA on the CAD model using SolidWorks Simulation. Both results allowed us to practice forming a reasonable prediction of a real system's deflection using two forms of analysis. Additionally, through comparing and evaluating the results from both models, we not only gained a better grasp of deflection but are now able to justify our analytical calculations through simulation.

In Phase 2 of this project, we focused on designing an aluminum structure to withstand a vertical force of 200 lbf by using both numerical and analytical methods, specifically the strain energy method (Castigliano's Theorem) and Finite Element Analysis (FEA). We needed to achieve a deflection of 0.5 inches while minimizing weight and yielding. Our structure was then manufactured via waterjet cutting and tested using an Instron machine. We were able to apply key skills and concepts gained from Phase 1 of the project, and our analysis of the analytical, numerical, and experimental results are below.

Approach

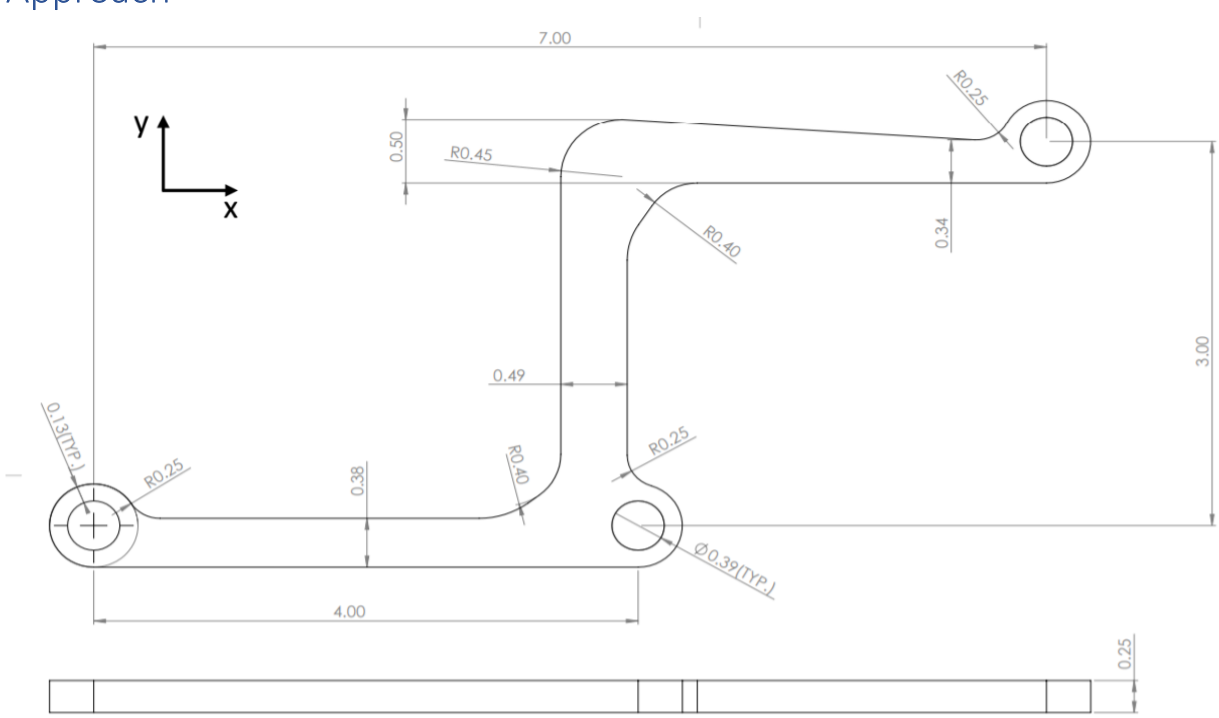

Figure 1. Structure for Analysis

Figure 1 shows the dimensioned geometry of our structure in inches. Our structure is modeled after an S-shape. The thickness of the entire structure is a constant 0.25 inches. Our total x-direction length from the center of the far-left hole to the center of the far-right hole is 7.0 inches; the total y-direction height from the center of the far-left hole to the center of the far-right hole is 3.0 inches. The x-direction length from the center of the bottom left hole to the center of the bottom right hole is 4.0 inches. The height of the horizontal bottom beam is a constant 0.38 inches; the height of horizontal top beam slopes from 0.5 inches to 0.34 inches for an average height of 0.42 inches. The width of the vertical center beam is a constant 0.42 inches. The top left corner of the vertical center beam has a fillet of radius 0.45 inches. The support gussets at the bottom left and top right corners of the vertical center beam have fillets of radius 0.40 inches. The three uniform circles at the bottom left, bottom right, and upper right corners of the S-shape have diameter 0.13 inches, as required to clamp the structure into the Instron machine. We maintained an outer diameter of 0.25 inches, such that the clamps of the Instron machine would not warp the material surrounding the holes. The sloping divots next to the holes have fillets of radius 0.25 inches.

Analysis

SolidWorks Analysis

An FEA model on SolidWorks was performed to accurately predict the deflection of the aluminum structure under load. A vertical force of 200 lbf was applied at the indicated load point, while the structure was simply supported at the two hole locations at the bottom – a vertical fixture was applied at the bottom right hole downwards, a cylindrical fixture was applied at the bottom left hole – and the material used for the structure is extruded aluminum alloy (6061-T6511), leading us to create a custom material inside SolidWorks. The modulus of elasticity and yield strength used were 10 x 10^6 psi and 46.50 ksi, respectively.

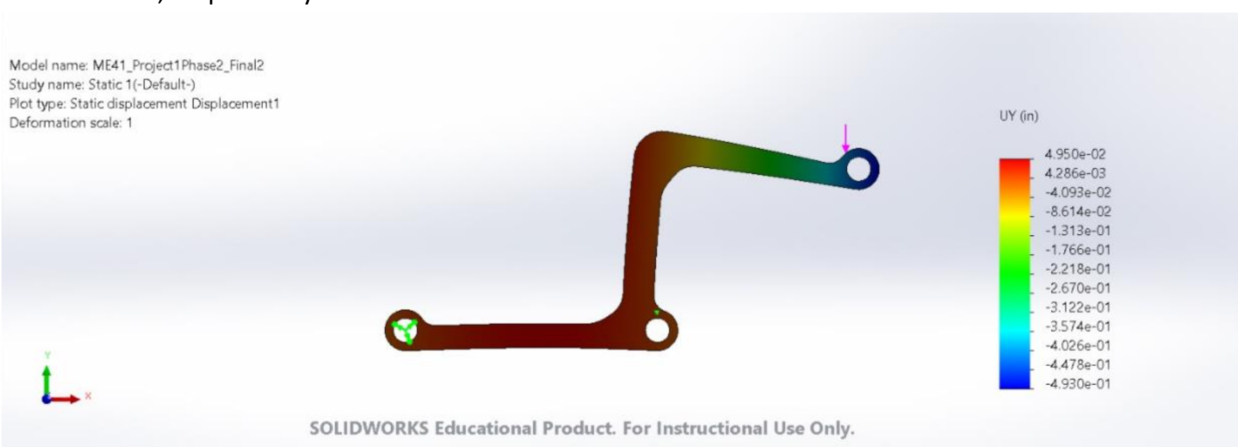

Figure 2. Deflection FEA Model

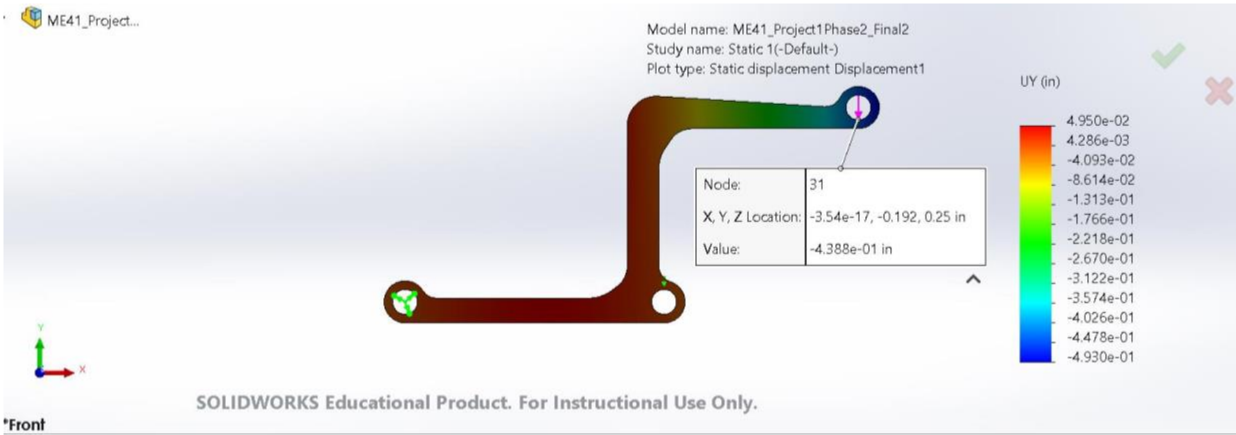

Figure 3. Maximum Deflection at Node

Figure 2 is an FEA model showing the deflection along the aluminum structure once a force of 200 lbf is applied to the indicated load point – the top right hole. Figure 3 shows the maximum deflection located at the load point, approximately 0.4388 inches.

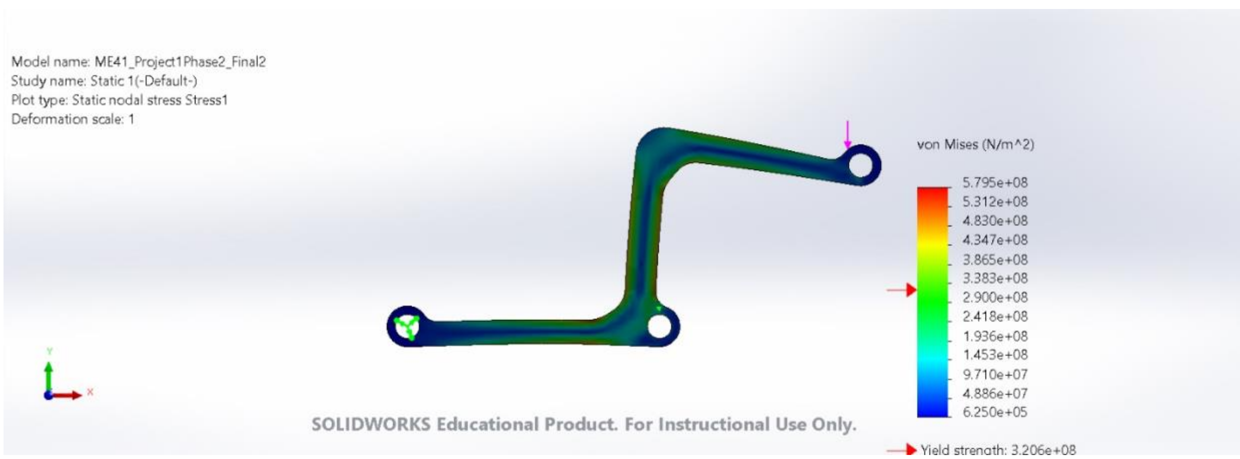

Figure 5. Yield FEA Model

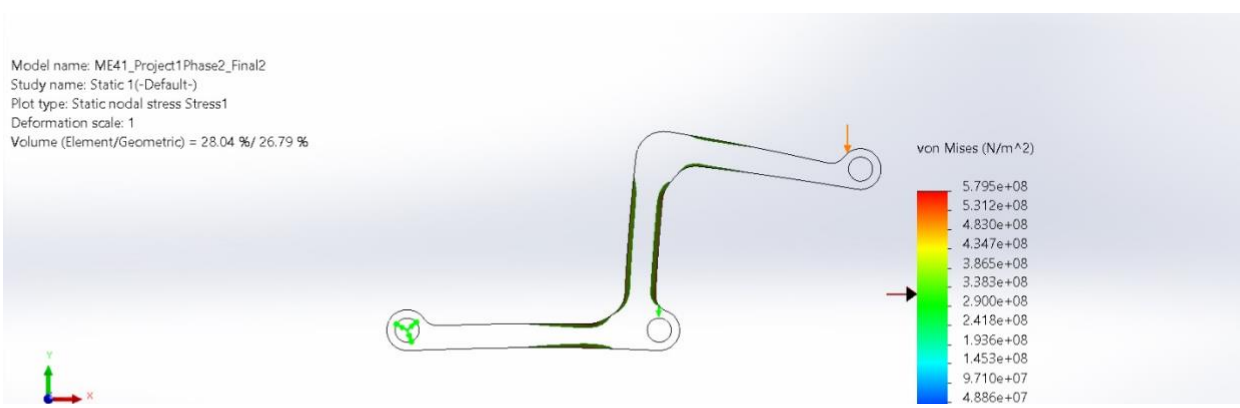

Figure 4. Stress Iso Clipping FEA Model

Figure 5 is an FEA model showing the stress along the aluminum structure once a force of 200 lbf is applied to the indicated load point. Figure 4 shows areas of yielding due to the stress being above the indicated yield strength of 46.50 ksi.

Mesh Analysis

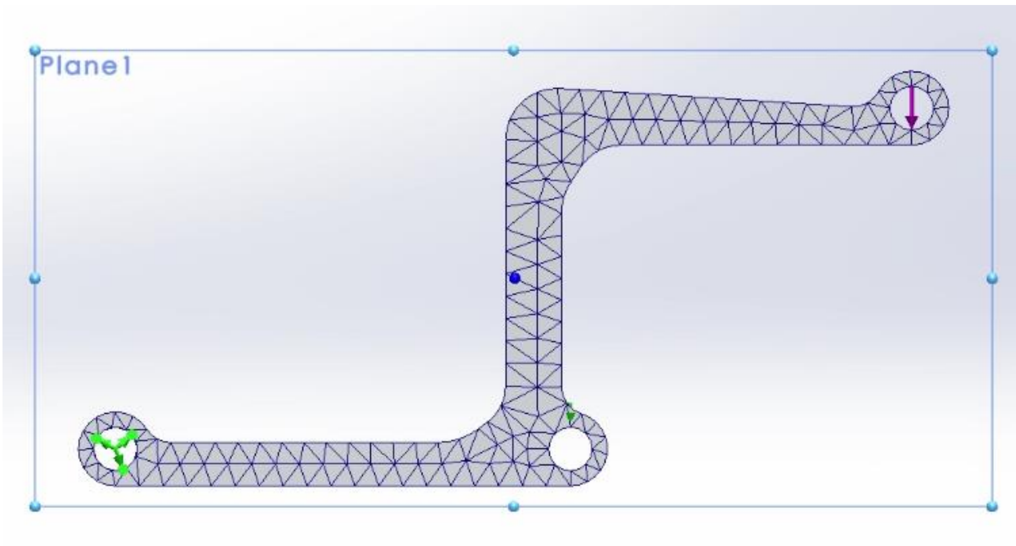

Figure 6. FEA Coarse Mesh (1456 elements in structure)

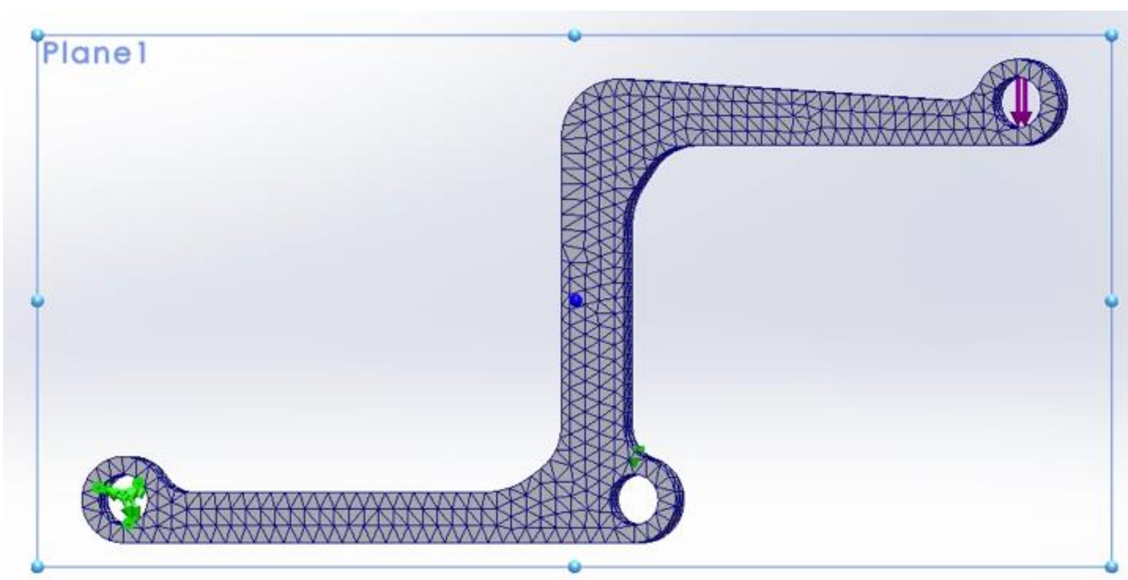

Figure 7. FEA Normal Mesh (5880 elements in structure)

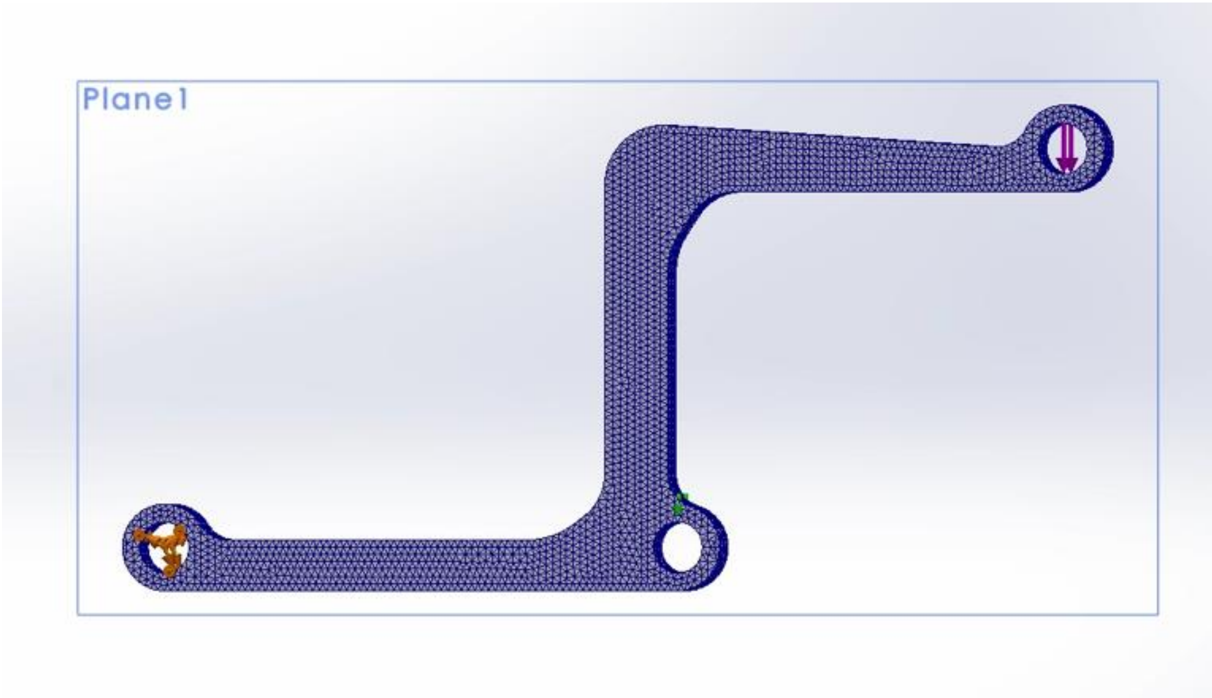

Figure 8. FEA Fine Mesh (57688 elements in structure)

Table 1. Deflection, Strain, and Element Findings for Mesh Analysis

	Coarse Mesh	**Normal Mesh**	**Fine Mesh**
Element Size (inches)	0.211548	0.116351	0.052887
Tolerance (inches)	0.0105774	0.00581757	0.00264435
Total Nodes	2932	10475	88839
Total Elements	1456	5880	57688
Time to Complete Mesh (hh:mm:ss)	00:00:01	00:00:01	00:00:03
Maximum Deflection (in)	0.492	0.4933	0.4950
Maximum Strain, von Mises (N/m^2)	5.718×10^8	5.844×10^8	6.120×10^8
Expected Yield Strength (N/m^2)	3.206×10^8	3.206×10^8	3.206×10^8

Figure 6 is an FEA mesh showing a coarse mesh with 1456 elements of 0.211548 inch in the structure. Figure 7 is an FEA mesh showing a normal mesh with 5880 elements of 0.116351 inch in the structure.

Figure 8 is an FEA mesh showing a fine mesh with 57688 elements of 0.052887 inch in the structure. Table 1 indicates the element size, tolerance, total nodes, total elements, computation time, deflection, strain, and yield strength for a coarse, normal, and fine mesh analysis.

We re-meshed our structure and ran three additional FEA models using coarse, normal, and fine meshes to assess convergence and accuracy. The coarse mesh had 2932 total elements, the normal mesh had 10475 total elements, and the fine mesh had 57688 total elements.

The mesh shapes for each element displayed a mostly triangular pattern however the size of the triangles used across the mesh varied significantly based on the “fine” or “coarse” rating of the mesh.

The three meshes produced similar results in terms of deflection and maximum strain. The meshes took similar times to render but the fine mesh study took much longer (around three minutes) compared to the other mesh scenarios. The range of deviation between the fine and coarse maximum strains is 4.02 x 10^7 (N/m^2). The range between the fine and coarse is 0.003 in. The results of the fine mesh were most like our measured results.

Castigliano’s Theorem

→ Assume constant thickness along each segment to simplify analysis

→ D acts downwards

$$+\circlearrowleft \sum M_D = 0:\ R_C(4) - 7F = 0$$

$$R_C = \frac{7}{4} F\ lbf$$

$$+\uparrow \sum F_y = 0:\ -R_D + R_C - F = 0$$

$$-R_D + \frac{7}{4}F - F = 0$$

$$R_D = \frac{3}{4} F\ lbf$$

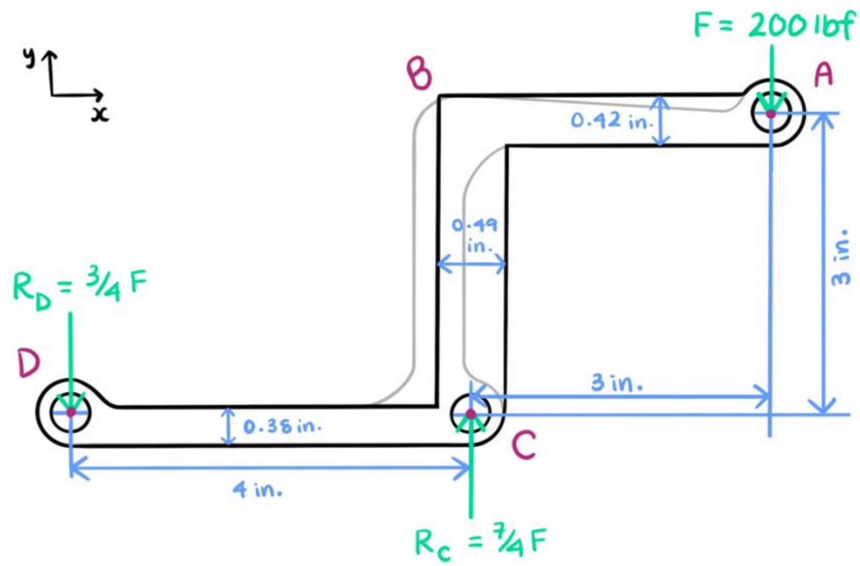

Figure 9. FBD of Simplified Structure

Element AB: Bending

$$+\uparrow \sum F_x = 0:\ -V_B - F = 0$$

$$V_B = F\ lbf$$

$$+\circlearrowleft \sum M_B = 0$$

$$-M_B - 3F = 0$$

$$M_B = -3F\ lbf \cdot in$$

$$M = -Fx$$

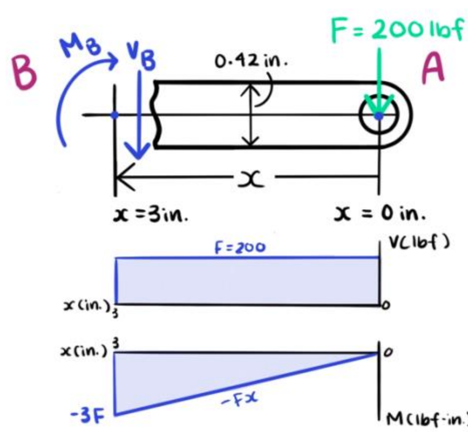

Figure 10. Method of Sections for Segment A-B

$$\frac{dM}{dx} = -x$$

$$\delta_{AB} = \int_A^B \frac{M\left(\partial M/_{\partial F}\right)}{EI_{AB}} dx = \frac{1}{EI_{AB}} \int_0^3 -Fx(-x)dx = \frac{F}{EI_{AB}} \int_0^3 x^2 dx$$

$$E = 10 \times 10^6 psi$$

$$I_{AB} = \frac{bh^3}{12} = \frac{(0.25)(0.42)^3}{12} = 0.001544 \; in^4$$

$$\delta_{AB} = \frac{200}{(10 \times 10^6)(0.001544)} \left[\frac{1}{3}(3)^3\right] = 0.1166 \; in$$

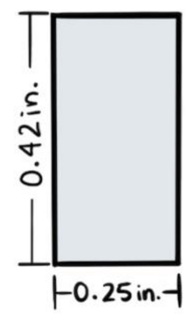

Figure 11. Cross Section of Segment A-B

Element BC: Compression, Bending

$$+\uparrow \sum F_y = 0: R_B = -V_B = -200 \; lbf$$

$$R_B = V_B = V = F$$

$$\frac{\partial V}{\partial F} = 1$$

$$+\circlearrowleft \sum M_C = 0 : M_C - M_B = 0$$

$$M_C = 3F \; lbf \cdot in$$

$$M = 3F$$

$$\frac{\partial M}{\partial F} = 3$$

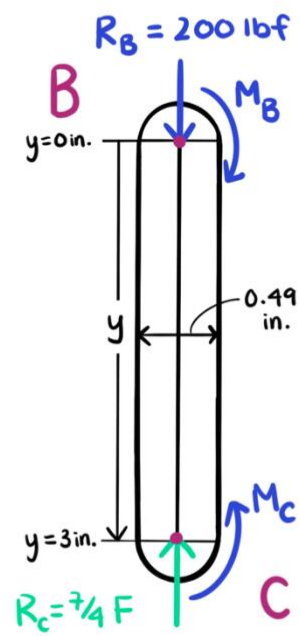

Figure 12. Method of Sections for Segment B-C

$$\delta_{BC,compression} = \int_B^C \frac{V\left(\partial V/_{\partial F}\right)}{EA_{BC}} dy = \frac{1}{EA_{BC}} \int_0^3 F(1)dy$$

$$A_{BC} = bh = (0.25)(0.49) = 0.1225 \; in^2$$

$$\delta_{BC,compression} = \frac{200}{(10 \times 10^6)(0.1225)}(3) = 4.898 \times 10^{-4} \; in$$

$$\delta_{BC,bending} = \int_B^C \frac{M\left(\partial M/_{\partial F}\right)}{EI_{BC}} dy = \frac{1}{EI_{BC}} \int_0^3 3F(3)dy = \frac{9F}{EI_{BC}} \int_0^3 1dy$$

$$I_{BC} = \frac{bh^3}{12} = \frac{(0.25)(0.49)^3}{12} = 0.002451 \; in^4$$

$$\delta_{BC,bending} = \frac{(9)(200)}{(10 \times 10^6)(0.002451)}(3) = 0.2203 \; in$$

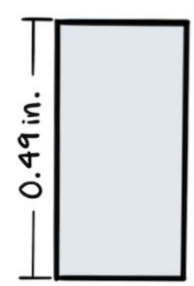

Figure 13. Cross Section for Segment B-C

$$\delta_{BC} = \delta_{BC,compression} + \delta_{BC,bending} = 4.898 \times 10^{-4} + 0.2203 = 0.2207\ in$$

Element CD: Bending

$$+\uparrow \sum F_y = 0: -R_D + V_C = 0$$

$$V_C = \frac{3}{4} F\ lbf$$

$$+\circlearrowleft \sum M_C = 0: R_D(4) - M_C = 0$$

$$\frac{3}{4} F(4) - 3F = 0$$

$$M = \frac{3}{4} Fx$$

$$\frac{\partial M}{\partial F} = \frac{3}{4} x$$

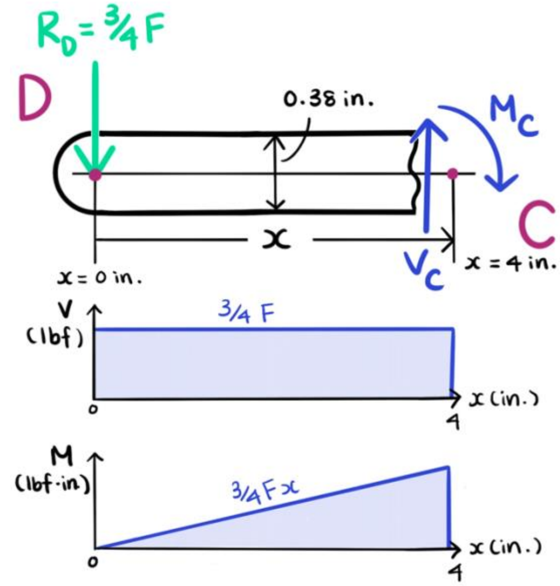

Figure 14. Method of Sections for Segment C-D

$$\delta_{CD} = \int_D^C \frac{M\left(\partial M/_{\partial F}\right)}{EI_{CD}} dx = \frac{1}{EI_{CD}} \int_0^4 \frac{3}{4} Fx \left(\frac{3}{4} x\right) dx = \frac{9F}{16EI_{CD}} \int_0^4 x^2 dx$$

$$I_{CD} = \frac{bh^3}{12} = \frac{(0.25)(0.38)^3}{12} = 0.001143\ in^4$$

$$\delta_{CD} = \frac{(9)(200)}{16(10 \times 10^6)(0.001143)} \left(\frac{1}{3}(4)^3\right) = 0.2099\ in$$

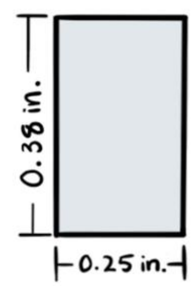

Figure 15. Cross Section for Segment C-D

Final Sum

$$\delta_{total} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 0.1166 + 0.2207 + 0.2099 = \mathbf{0.5472\ inches}$$

Comparison Between SolidWorks Analysis and Castigliano's Theorem

Castigliano's Theorem was used to calculate the maximum deflection of our simplified structure under an applied load of 200 lbf at the indicated point. SolidWorks was then used to design a more refined and complex structure. SolidWorks Simulation was conducted to create an FEA model indicating the deflection along the structure once the load was applied at two points: the indicated load point and at the end of the beam. The analytical value calculated while applying a simplified Castigliano's Theorem was 0.5472 inches for the maximum deflection. The numerical value from the FEA model for the maximum deflection at the load point indicated and at the end of the beam of the structure was 0.4388 inches and 0.4930 inches, respectively.

Error

Error calculated using the maximum deflection at the load point indicated on Figure 3:

$$\%_{error} = \left|\frac{analytical\ value - numerical\ value}{numerical\ value}\right| \times 100 = \left|\frac{0.5472 - 0.4388}{0.4388}\right| \times 100 = 24.70\%$$

Error calculated using the maximum deflection at the end of the top beam of the aluminum structure:

$$\%_{error} = \left|\frac{analytical\ value - numerical\ value}{numerical\ value}\right| \times 100 = \left|\frac{0.5472 - 0.4930}{0.4930}\right| \times 100 = 10.99\%$$

Using the maximum deflection at the indicated load, the error was calculated to be 24.70%; using the maximum deflection at the end of the top beam of the structure, the error was calculated to be 10.99%. The latter error is relatively small (< 15%), while the former error has more significance (~ 25%). However, we postulate that our analytical value still sufficiently agrees with the FEA model, since SolidWorks Simulations do not take yielding into account. There is a slight error due to our original assumptions – ignore fillets, holes, and forces from gravity – that allowed us to use Castigliano's Theorem to calculate the maximum deflection. Although these values seem reasonable, we will compare our analytical and numerical data to the experimental data provided by the Instron test to fully validate our results below.

Testing Results

Two Instron tests were performed to demonstrate the deflection of our initial and final structures under load. Our initial and final structures were manufactured from aluminum alloy (6061-T6511) using waterjet-cutting; the modulus of elasticity used was 10 x 10^6 psi and the yield strength was 46.50 ksi. The Instron machine applied a vertical force of 200 lbf at the indicated load point, while the structure was simply supported at the two holes located on either end of the bottom member.

Table 2. Spring Constant, Yielding, and Weight of each Instron-tested structure

Trial	Structure #	Spring Constant (lbf/in)	Yielding (estimated load at Start in lbf)	Amount of yielding	Weight (g)
1	4	418.63	120	Moderate	52
2	1e	377.75	125	Moderate	52

Table 2 indicates the spring constant, yielding, and weight of our initial and final structures, #4 and #1e respectively, provided by the Instron tests

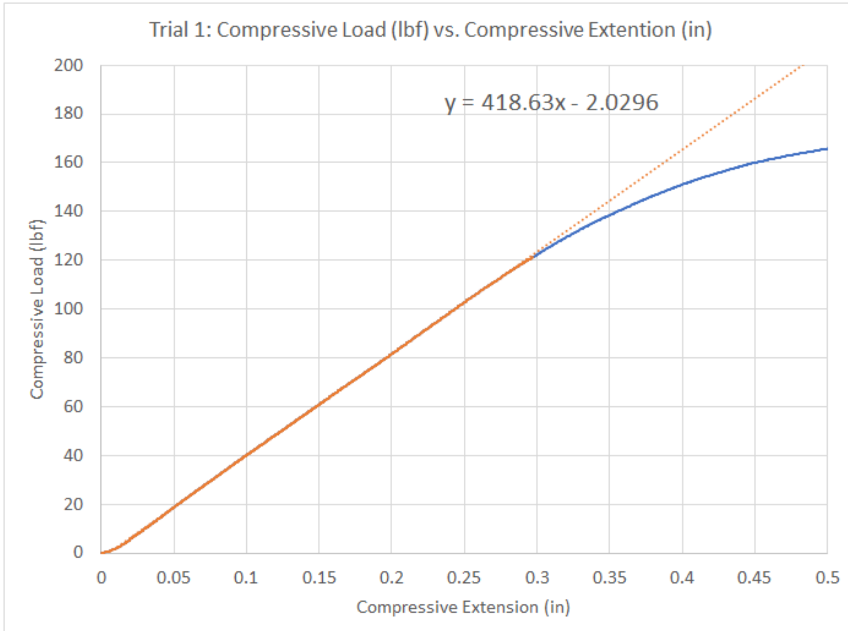

Figure 17. Spring Constant Ratio Plot (Trial 1)

Figure 17 depicts a plot of the spring constant ratio of compressive load to compressive extension for Trial 1 on initial structure #4 to demonstrate observed yielding. Initial structure #4 began yielding at a compressive load of 120 lbf and produced a moderate amount of yielding.

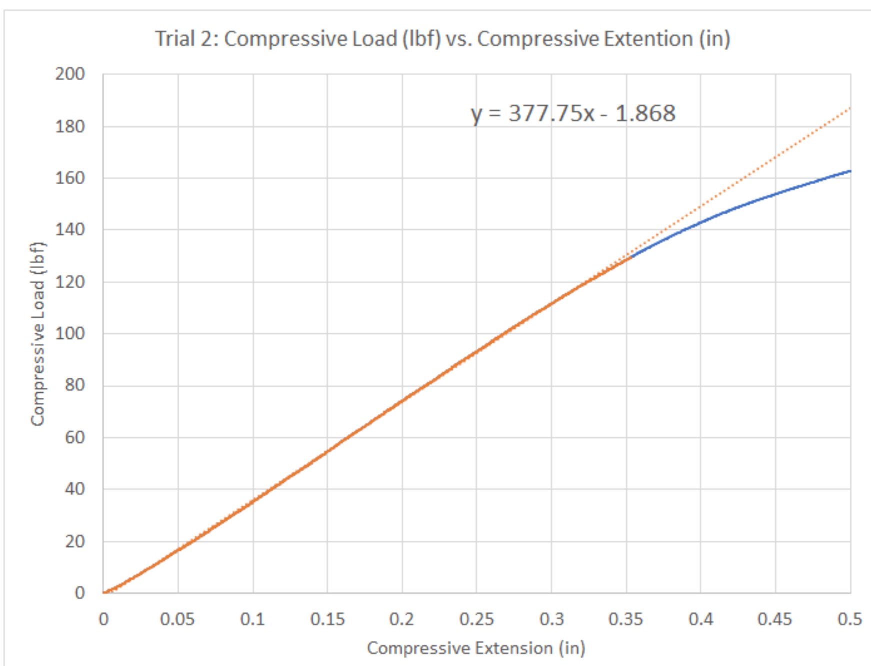

Figure 16. Spring Constant Ratio Plot (Trial 2)

Figure 16 depicts a plot of the spring constant ratio of compressive load to compressive extension for Trial 2 on final structure #1e to demonstrate observed yielding. Final structure #1e began yielding at a compressive load of 125 lbf and produced a moderate amount of yielding.

Error Between the Initial and Final Structures

Our initial and final structures, #4 and #1e respectively, differed in design: Initial structure #4 did not have a sloped horizontal top beam, but rather a constant horizontal top beam of 0.50 inches. Its vertical center beam also did not have support gussets in the bottom left and top right corners.

Our Instron tests demonstrate that our initial structure #4 produced a spring constant of 418.63 lbf/in and yielded a moderate amount at a compressive load of 120 lbf, while our final structure #1e produced a spring constant of 337.75 lbf/in and yielded a moderate amount at a compressive load of 125 lbf.

Given that the ideal spring constant is 400 lbf/in and yielding should only begin at or past 200 lbf, we calculated the error of our initial and final structures below:

Error of initial structure #4:

Spring constant error:

$$\%_{error} = \left|\frac{analytical\ value - numerical\ value}{numerical\ value}\right| \times 100 = \left|\frac{418.63 - 400}{400}\right| \times 100 = 4.66\%$$

Yielding start error:

$$\%_{error} = \left|\frac{analytical\ value - numerical\ value}{numerical\ value}\right| \times 100 = \left|\frac{120 - 200}{200}\right| \times 100 = 40.0\%$$

Error of final structure #1e:

Spring constant error:

$$\%_{error} = \left|\frac{analytical\ value - numerical\ value}{numerical\ value}\right| \times 100 = \left|\frac{337.75 - 400}{400}\right| \times 100 = 15.56\%$$

Yielding start error:

$$\%_{error} = \left|\frac{analytical\ value - numerical\ value}{numerical\ value}\right| \times 100 = \left|\frac{125 - 200}{200}\right| \times 100 = 37.50\%$$

Initial structure #4 demonstrated 4.66% error in the spring constant and 40.0% error in the yielding start; final structure #1e demonstrated 15.56% error in the spring constant and 37.50% error in the yielding start. We propose that our final structure performed worse in spring constant, but better in yielding because the additional support gussets prevented the final structure from yielding in excess, while also reducing the amount of deflection.

Discussion

The measured deflection for our manufactured Instron models was 0.585 inches in Trial 1 (initial structure #4) and 0.513 inches in Trial 2 (final structure #1e) with moderate yielding. In both cases, the weight was 52 g and yielding occurred at around 120 lbs to 125 lbs of applied force. The spring constant for Trial 1 was better than Trial 2, 418.63 lbf/in and 377.75 lbf/in respectively. Compared to our predictions, this yielding was similar to that predicted by Castigliano's at 0.5471 in and higher than that predicted by the SolidWorks Finite Element Method (FEM) of 0.493 in. Differences in these measured deflections between different versions may be due to the approximations made in our math to perform the Castigliano's, as well as difficulties in SolidWorks and Castigliano's ability to account for yielding.

In our model, we balanced for weight by revising our "C-shape" beam to an "S-shape" that could bear similar loading while requiring less weight because it removed significant length from the upper "C" arm. We tapered the ends of the S-shape to reduce weight while still allowing requisite thickness in high strain areas. While iterating on our design, we initially tried editing the original C-shape to maintain the "C" while reducing weight but shifted to the S-shape design since it allowed for similar stability and a much lower weight. The main drawback of the S-shape design is the stress concentration at the corners of the "S" where yielding is most likely to occur. Since a truss would have added more weight to the design, we attempted to compensate for this added corner strain by filleting the corners instead and tapering the ends of the "S" so that the support corners were sturdier

Table 3. Predicted and Observed Deflection Results for Castigliano, FEM and Instron testing methods

	Castigliano's	**FEM**	**Instron (Trial 1)**	**Instron (Trial 2)**
Deflection (in)	0.5471 in	0.493 in	0.585 in (yielding at 0.3 in)	0.513 in (yielding at 0.35 in)
Deviation from 0.5 in ideal deflection	9.42%	-1%	17%	2.6%

Deviations between our Instron test results and our predicted results using Castigliano's and FEM may have been partially due to weaknesses in the two predictive methods at accounting for yielding. Similarly, in our Castigliano's we simplified our model analysis by not accounting for factors like filleted corners or the tapered ends of the top horizontal beam. In the physical Instron test, there may have been environmental factors or variations on how the load was applied that varied from our idealized predictive model. Although there are differences between our predictive and Instron test models, these variations seem reasonable given the approximations we made in our predicative models and the weaknesses of predictive models in accounting for yielding.

The differences between our two test models are likely a result of the top beam fillet in the Trial 2 model that is not present in the Trial 1 model. Adding the fillet decreased our Spring constant by 41 lbf/in, but led to yielding at a slightly better load and resulted in a deflection of 0.513 in closer to the ideal deflection of 0.5 in.

Conclusion

In this project, we designed a structure to deflect 0.5 inches under an applied load of 200 lbs. We predicted the effectiveness of our proposed design using SolidWorks FEA and Castigliano's approach. We tested our final model using an Instron test of two different models, one without a tapered top edge in Trial 1 and one with a tapered top edge in Trial 2. The tapered model led to a more ideal deflection closer to 0.5 in, but a less ideal spring constant. Deviations between the Instron test models and the predicative Solid works and Castigliano's models may have been due to weaknesses in these predictive models in accounting for yielding. Similarly, in our Castigliano's approach, we did not consider the tapered top edge or gusseted corners of our model so that we could more effectively perform the requisite calculations. Overall, our measured values seem in line with our predictive models and match well with our target deflection. In future iterations, we would work toward achieving a model with less yielding since that was the primary weakness of our model relative to other groups. While we had a very good weight ratio for the yielding measured, this could be made better by constructing a more stable model.