ME41 FALL 2020

Analysis of a Roll Forge

Project 2

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Contents

Introduction

In this report, we will analyze the loading conditions of a simplified roll-forge model. To do so, we will approximate the rollers as two sets of simply supported beams. Using a given applied load, we will determine the reaction forces of the bearings and assess the roller model for static and fatigue failure by finding the factor of safety for both conditions. This will help us understand how the roll forge works and if the initial model used is viable for a roll-forge. We will further this process by measuring the peak deflection and slope of the rollers to gain a physical representation of the impact of the load on the roller structure. Outside of the roller structure, we will identify the train value and fatigue on the gears to quantify the gear system and to gain insight into the longevity and wear resistance of the gears. Finally, we will enrich our understanding by investigating the roll-forging forces on the workpiece, rather than simply focusing on the rollers. We will calculate true strain and average flow stress to gain a better understanding of the torque and contact length between the rollers and the workpiece.

Assumptions

Given

→ Assume longitudinal roll-forging with semi-cylindrical dies (180°)

→ Assume the workpiece is a pair of connecting rod preforms made of hot-worked AISI 1055 steel, with given properties provided below:

Material Property	Assumed Value
Yield strength, S_y	560 MPa
Ultimate strength, S_{ut}	660 MPa
Elastic modulus, E	200 GPa
% elongation	10%
Strain hardening exponent, n	0.042

→ A single preform cycle takes 10 seconds (360 preforms per hour) and requires 4 passes

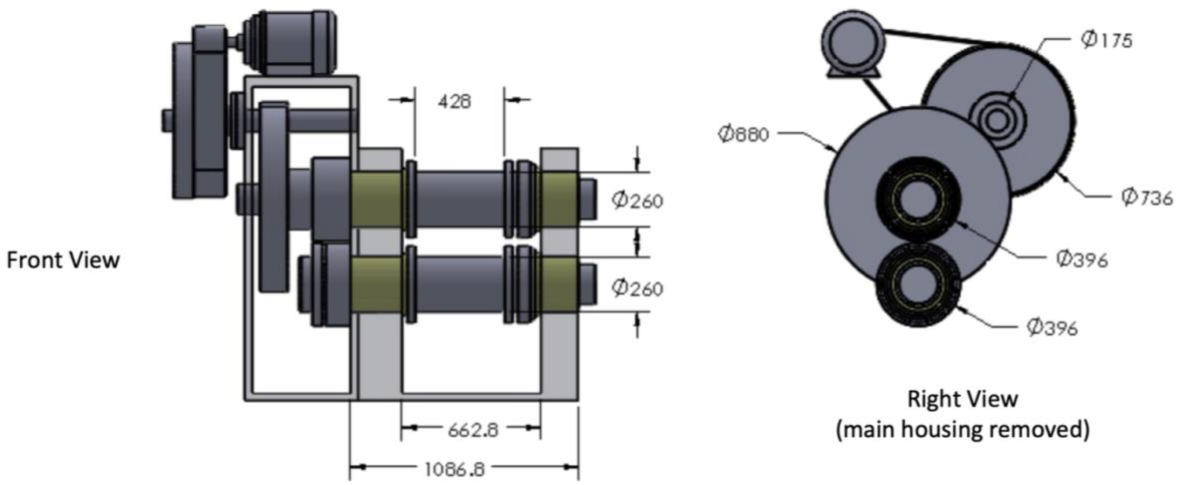
→ The roll-forge is run an average of 8 hours each day of production, and production will last 260 days:

Production Duration	Given Value
Passes per cycle	4
Cycles per hour	360
Hours per production day	8
Duration of production	260 days

→ Assume each pass requires the same peak torque and force of 25.7 kN*m and 1.41 MN, respectively, at a rotational speed of 0.5 rev/sec:

Peak Applied Load	Given Value
Peak torque, T_{max}	25700 N*m
Peak force, F_{max}	1410000 N
Rotational speed	0.5 rev/sec

→ Assume the following dimensions:



Relevant Dimensions	Given Value
Length of rollers, L	0.6628 m
Diameter of rollers, d	0.26 m

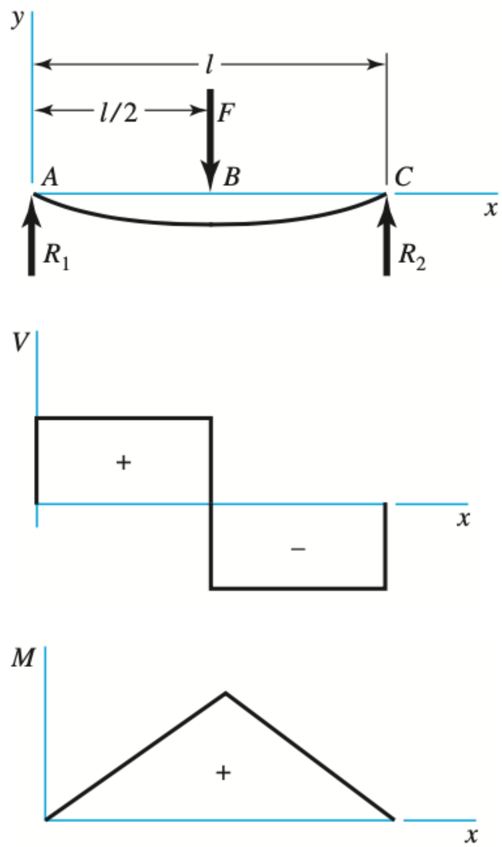
Simplifications

→ The system can be simplified to the top roller, the bottom roller, and four identical bearings
→ Assume the rollers and bearings have no friction
→ Assume any movement or rotation of the rollers or bearings in the x- or z- directions is negligible
→ Assume the mass of the rollers and bearings are negligible compared to the radial force
→ Assume the radial force acts in the center of the rollers
→ Assume the rollers are made from cold-drawn AISI 1020 Steel, with given properties provided below:

Material Property	Assumed Value
Tensile strength, S_y	390 MPa
Ultimate strength, S_{ut}	470 MPa
Elastic modulus, E	205 GPa
Brinell Hardness	131

→ Assume the rollers can each be modeled by Table A-9: 5 Simple Supports–Center Load from *Shigley's Mechanical Engineering Design, 10th Edition*, provided below:

5 Simple supports—center load



$$R_1 = R_2 = \frac{F}{2}$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fx}{2} \qquad M_{BC} = \frac{F}{2}(l - x)$$

$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$y_{\max} = -\frac{Fl^3}{48EI}$$

What are the reactions at the 4 bearings (shown in yellow)?

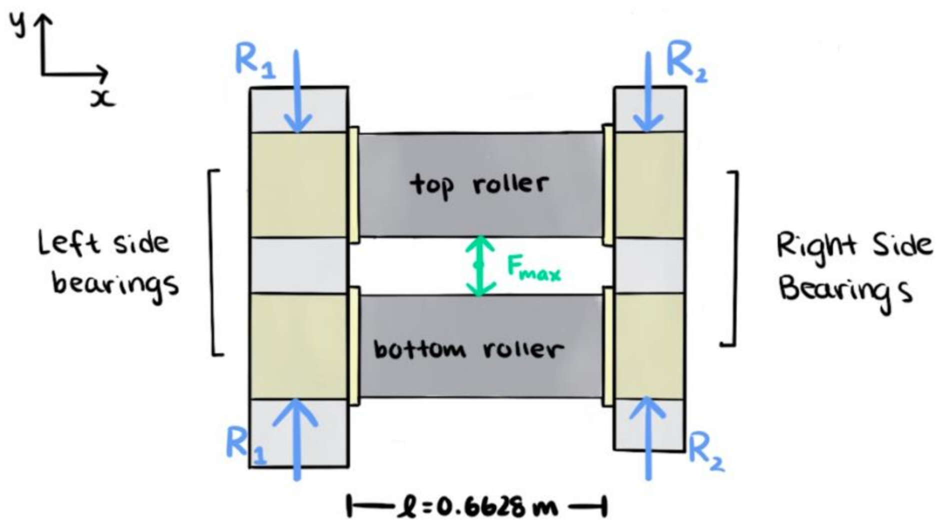
Responsibility: Maia Taffe, Allison Moore, Rebecca Shen, Emily Kostolansky

Given

Relevant Variable	Given Value
Peak force, F_{max}	1410000 N

Free-Body Diagram

Assuming the rollers can be modeled by Table A-9: 5 Simple Supports–Center Load and are symmetric, the free-body diagram of the top roller, bottom roller, and four identical bearings is illustrated below:



Reactions at the Bearings

Applying the reaction force equations from Table A-9: 5 Simple Supports–Center Load, the reaction forces at each of the four identical bearings was calculated as 705 kN.

$$R_1 = R_2 = \frac{F_{max}}{2} = \frac{1410000}{2} = 705\ kN$$

What is the factor of safety guarding against static failure of the rollers?

Responsibility: Allison Moore

Assumptions

→ The cross section of the roller is a continuous circle of diameter 260 mm

Given

Relevant Variables	Given Values
Peak force, F_{max}	1410000 N
Peak forque, T_{max}	25700 N*m
Length of rollers, L	0.6628 m
Diameter of rollers, d	0.260 m

Moment of Inertia

Applying the moment of inertia equation for a beam with a circular cross-section, the moment of inertia of each roller was calculated as 0.0002243 m^4.

$$I = \frac{\pi d^4}{64} = \frac{\pi (0.26)^4}{64} = 0.0002243\ m^4$$

Alternating Bending Stress

The bending stress alternates as the shaft rotates, and different elements in the shaft experience maximum and minimum bending stress depending where they are in the rotation cycle.

$$M_{max} = \frac{F_{max} L}{2} = \frac{1410000(0.6628)}{2} = 467.3\ kN * m$$

$$c = 0.5d = 0.5(0.26) = 0.13\ m$$

$$\sigma_x = \frac{Mc}{I} = \frac{467274(.13)}{0.0002243} = 270.8\ MPa$$

$$\sigma_y = 0\ \mathrm{MPa}$$

The maximum and minimum bending stresses were calculated as 270.8 and 0 MPa, respectively.

Constant Torsional Shear

Torsional shear acts across the roller and is constant throughout each cycle.

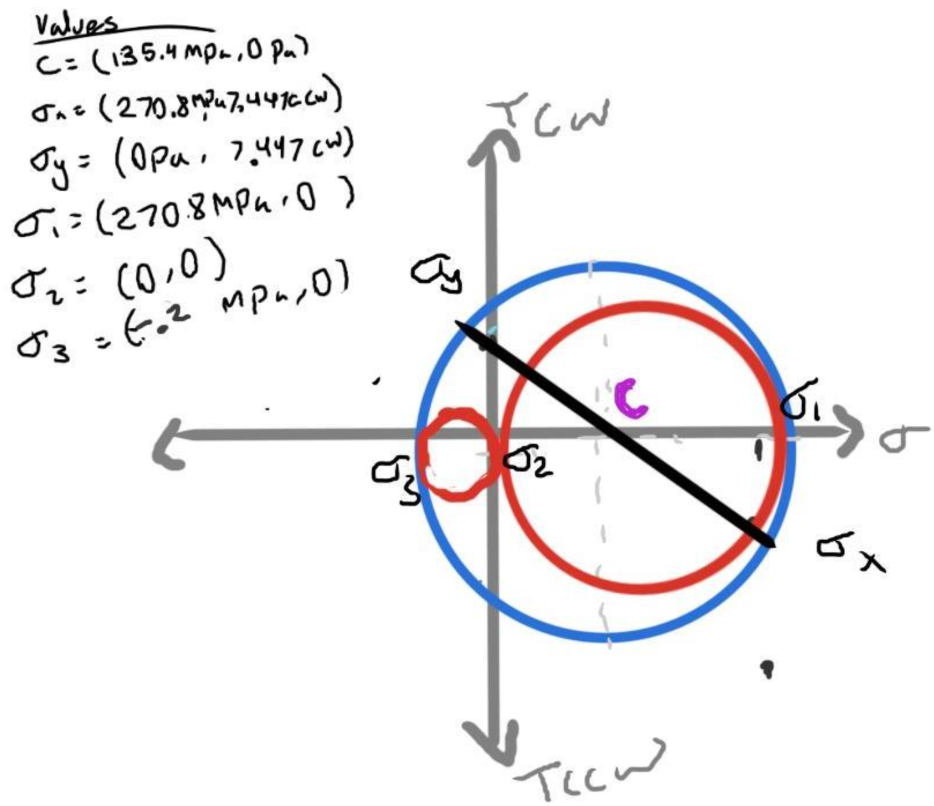
$$J = \frac{\pi d^4}{32} = \frac{\pi (0.26)^4}{32} = 0.0004486$$

$$\tau_{max} = \frac{T_{max} \mathrm{r}}{J} = \frac{25700(0.13)}{0.0004486} = 7.447\ MPa$$

The constant torsional shear was calculated as 7.447 MPa.

3D Mohr's Circle

Since triaxial stress is applied across and through the roller, 3D Mohr's circle must be used to find the principal stresses. σ_y can be assumed as zero, since bending only occurs along the x-axis. Stress is thus maximized on one side of the shaft and no stress is applied along the other side, indicating that one of the three principal stresses must be zero.



$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{270.8 + 0}{2} = 135.4\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{max}^2} = \sqrt{\left(\frac{270.8 - 0}{2}\right)^2 + (7.447)^2} = 135.6\ MPa$$

$$\sigma_1 = -C + R = 135.6 + 135.4 = 271\ MPa$$

$$\sigma_2 = \sigma_y = 0\ Pa$$

$$\sigma_3 = C - R = -135.6 + 135.4 = -0.2\ MPa$$

Given that one of the three principal stresses must be zero, the maximum and minimum principal stresses were calculated as 271 MPa and –0.2 MPa.

Distortion Energy Criteria

We applied the Distortion Energy Criteria to calculate the static factor of safety of the rollers, since mild steels like AISI 1020 are ductile and the tensile and compressive yield strengths of AISI 1020 steel are equivalent. While Distortion Energy Criteria does not provide the most conservative estimate of static failure, it may yield a more accurate factor of safety than a more conservative metric would produce.

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} = \sqrt{\frac{(271\ - 0)^2 + (0 + 0.2)^2 + (-0.2 - 271)^2}{2}}$$

$$= 271.1MPa$$

$$\eta_y = \frac{S_y}{\sigma'} = \frac{390}{271.1} = 1.43$$

Static failure is not predicted, since the Distortion Energy factor of safety 1.43 is greater than 1.25.

What is the peak deflection of the rollers and the slope of the rollers at the bearings?

Responsibility: Rebecca Shen

Assumptions

→ Peak deflection occurs at the center of each roller, since the deflection-causing load is applied at the center of the rollers

→ The slope at the midspan of the beam is zero, since the rollers and loads are symmetric relative to the midspan

→ The reaction forces and slopes at either bearing are equal due to symmetry

Given

Relevant Variables	Given Values
Peak force, F_{max}	1410000 N
Length of rollers, L	0.6628 m
Diameter of rollers, d	0.260 m
Elastic modulus, E	205 GPa

Peak Deflection:

$$\delta_{max} = -\frac{Fl^3}{48EI} = \frac{-(1410000N)(0.6628m)^3}{(48)(205 \times 10^9 Pa)\left(\frac{\pi}{64}\right)(0.26m)^4} = -1.85998 \times 10^{-4}\ m$$

Applying the maximum deflection equation from Table A-9: 5 Simple Supports–Center Load, the peak deflection was calculated as 1.86*10^{-4} m downwards at the center of the roller, 0.3314 m.

Slope of Bearings:

For the roller's left half, from x = 0 m to x = L/2 m:

$$\theta = \frac{dy}{dx} = \frac{d}{dx}(y_{AB}) = \frac{d}{dx}\left(\frac{Fx^2}{48EI}(4x^2 - 3L^2)\right)$$
$$= \frac{F(4x^2 - 3L^2)}{48EI} + \frac{8Fx^2}{48EI} = \frac{4Fx^2 - 3FL^2 + 8Fx^2}{48EI}$$

Evaluating at x = 0 m:

$$\theta = \frac{4(1410000)(0)^2 - 3(1410000)\left(\frac{0.6628}{2}\right)^2 + 8(1410000)(0)^2}{48(205 * 10^9)(0.26)}$$
$$= -1.8157 * 10^{-7}\ radians = -1.0403 * 10^{-5}\ degrees$$

Due to symmetry, the slopes of the bearings are equal with a leftmost slope of $\pm$ 1.0403*10^{-5} degrees and a rightmost slope of $\mp$ .0403*10^{-5} degrees.

What is the factor of safety guarding against fatigue failure of the rollers?

Responsibility: Emily Kostolansky

Assumptions

→ The rollers undergo combined loading of bending and torsion
→ The loading fluctuates and is therefore not fully reversed

Given

Relevant Variables	Given Values
Passes per cycle	4
Cycles per hour	360
Hours per production day	8
Duration of production	260 days
Peak force, F_{max}	1410000 N
Length of rollers, L	0.6628 m
Diameter of rollers, d	0.260 m
Tensile strength, S_y	390 MPa
Ultimate strength, S_{ut}	470 MPa
Elastic modulus, E	205 GPa
Maximum principal stress, σ_1	271 MPa
Minimum principal stress, σ_3	–0.2 MPa

Duration of Production Run

$$N = \left(4\frac{passes}{preform}\right)\left(360\frac{preforms}{hour}\right)\left(8\frac{hours}{day}\right)\left(260\frac{days}{production}\right) = 2.995 * 10^6\ cycles$$

The rollers undergo 2.995*10^6 cycles during production, indicating that the Stress-Life method (RR Moore/Marin Factors) should be used to evaluate the endurance limit.

Endurance Limit

$$S_e' = 0.5S_{ut} = 0.5(470) = 235\ MPa$$

$k_a = aS_{ut}^b = 4.51(235)^{-0.265} = 1.0613$ for cold-drawn steel

$k_b = 1.51d^{-0.157} = 1.51(260)^{-0.157} = 0.6307$ assuming the size factor equation applies past 254 mm for 260 mm

$k_c = 1$ for combined loading

$k_d = 1$ assuming minimal heat transfer between the rollers and the connecting rod

$k_e = 0.702$ for 99.99% reliability

$k_f = 1$ assuming negligible miscellaneous effects

$$S_e = k_a k_b k_c k_d k_e k_f S_{ut} = 1.0613(0.6307)(1)(1)(0.702)(1)(235) = 110.42\ MPa$$

Applying the Stress-Life method (RR Moore/Marin Factors), the endurance limit of the rollers was calculated to be 110.42 MPa.

Alternating and Mean Stresses

$$\sigma_a = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\sigma_1 + \sigma_3}{2} = \frac{271.0 - 0.196}{2} = 135.4\ MPa$$

$$\sigma_m = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_1 - \sigma_3}{2} = \frac{271 + 0.196}{2} = 135.6\ MPa$$

The fluctuating loads yield fluctuating stresses. Using the maximum and minimum 3D Mohr's Circle principal stresses, the alternating and mean stresses were calculated as 135.4 and 135.6 MPa, respectively.

Goodman Fatigue Criteria

$$\eta_{Goodman} = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{1}{\frac{135.4}{110.42} + \frac{135.6}{470}} = 0.66$$

$$\eta_{Goodman} < 1.25$$

The Goodman Fatigue Criteria is used to calculate the factor of safety guarding against fatigue failure of the rollers, since it is the most conservative evaluation of brittle fatigue. Failure in fatigue is predicted, since the Goodman factor of safety 0.66 is less than 1.25.

Revised Design

To guard against failure in fatigue, the diameter of the rollers is increased to 360 mm; the relevant calculations are revised below:

Revised Endurance Limit

$k_b = 1.51d^{-0.157} = 1.51(360)^{-0.157} = 0.599$ assuming the size factor equation applies past 254 mm for 360 mm

$$S_e = k_a k_b k_c k_d k_e k_f S_{ut} = 1.0613(0.599)(1)(1)(0.702)(1)(235) = 104.87\ MPa$$

The revised endurance limit of the rollers is 104.87 MPa.

Revised Stress and Shear

$$\sigma_x = \frac{M_{max}c}{I} = \frac{(0.5F_{max}L)(0.5d)}{\frac{\pi}{64}d^4} = \frac{16F_{max}L}{\pi d^2} = \frac{16(1410000)(0.6628)}{\pi(0.36)^3} = 102.0\ MPa$$

$$\tau_{max} = \frac{T_{max}r}{J} = \frac{T_{max}(0.5d)}{\frac{\pi}{32}d^4} = \frac{16T_{max}}{\pi d^3} = \frac{16(25700)}{\pi(0.36)^3} = 2.805\ MPa$$

The revised bending stress and torsional shear are 102.0 MPa and 2.805 MPa, respectively.

Revised 3D Mohr's Circle

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{102.0 + 0}{2} = 51.0\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{max}^2} = \sqrt{\left(\frac{10.20 - 0}{2}\right)^2 + (2.805)^2} = 51.08\ MPa$$

$$\sigma_1 = C + R = 51.0 + 51.08 = 102.08\ MPa$$

$$\sigma_3 = C - R = 51.0 - 51.08 = -0.08\ MPa$$

The revised maximum and minimum 3D Mohr's Circle principal stresses are 102.08 MPa and –0.08 MPa, respectively.

Revised Alternating and Mean Stresses

$$\sigma_a = \frac{\sigma_1 + \sigma_3}{2} = \frac{102.08 - 0.08}{2} = 51.0\ MPa$$

$$\sigma_m = \frac{\sigma_1 - \sigma_3}{2} = \frac{102.08 - 0.08}{2} = 51.08\ MPa$$

The revised alternating and mean stresses are 51.0 MPa and 51.08 MPa, respectively.

Revised Goodman Fatigue Criteria

$$\eta_{Goodman} = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{1}{\frac{51.0}{104.87} + \frac{51.08}{470}} = 1.68$$

$$\eta_{Goodman} > 1.25$$

Increasing the diameter of the rollers to 360 mm yields a Goodman factor of safety of 1.68; failure in fatigue is no longer predicted, since the revised Goodman factor of safety is greater than 1.25.

Revised Distortion Energy Failure Criteria

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} = \sqrt{\frac{(102.08 - 0)^2 + (0 + 0.08)^2 + (-0.08 - 102.08)^2}{2}}$$
$$= 102.12\ MPa$$

$$\eta_y = \frac{S_y}{\sigma'} = \frac{390}{102.12} = 3.82$$

Using the Distortion Energy Failure Criteria, the increased diameter of 360 mm yields a revised static factor of safety of 3.82.

Revised Peak Deflection

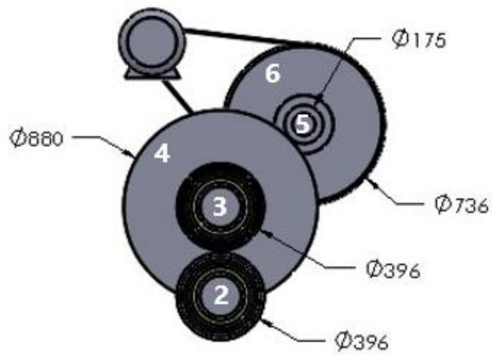
$$\delta_{max} = -\frac{F_{max}L^3}{48EI} = -\frac{F_{max}L^3}{48E\left(\frac{\pi}{64}d^4\right)} = -\frac{F4_{max}L^3}{3\pi E d^4} = -\frac{4(1410000)(0.6628)^3}{3\pi(205 * 10^9)(0.36)^4} = -5.06 * 10^{-5}\ m$$

Applying the maximum deflection equation from Table A-9: 5 Simple Supports–Center Load, the increased diameter of 360 mm yields a revised peak deflection of $-5.06*10^{-5}$ m.

Tabulate the speeds and torques experienced by each of the gears; identify overall train value; evaluate the wear and fatigue potential for the gear tooth surfaces

Responsibility: Maia Taffe, Rebecca Shen, Emily Kostolansky, Allison Moore

Gear System Table and Diagram



Gear Number	Description	Diameter (mm)	Speed (rpm)	Torque (N*m)
6	Compound Gear	736	150.857	5110.80
5	Compound Gear	175	150.857	5110.80
4	Top Roller Driver	880	30	25700
3	Top Roller	396	30	25700
2	Bottom Roller	396	30	25700

Speed

$$Roller\ Speed = \frac{0.5\ rev}{sec}\left(\frac{60\ sec}{1\ min}\right) = 30\ {}^{rev}/_{min}$$

Given the top roller's rotational speed of 0.5 rev/sec, the rotational speed of the top roller, bottom roller, and top roller driver were calculated as 30 RPM.

$$Compound\ Gear\ Speed = n_6\left(\frac{N_4}{N_5}n_4\right)n_3\left(\frac{N_2}{N_3}n_2\right) = \frac{N_4}{N_5}\frac{N_2}{N_3}n_2 = \frac{880}{175}\frac{396}{396}(30) = 150.8571429\ rpm$$

Applying gear train analysis to the compound gears, the rotational speed of the compound gears was calculated as 150.857 RPM.

Torque

$$\frac{Top\ Roller\ Driver\ diameter}{Top\ Roller\ Driver\ torque} = \frac{Compound\ Gear\ diameter}{Compound\ Gear\ torque}$$

$$\frac{880\ mm}{25700\ Nm} = \frac{175\ mm}{x}$$

$$x = 5110.795\ N * m$$

Given the torque of the top roller and applying proportional analysis, the torque of the compound gears was calculated as 5110.795 N*m.

Overall Gear Train Value

$$e = \frac{Product\ of\ number\ of\ driving\ teeth}{Product\ of\ number\ of\ driven\ teeth} = \frac{\prod N_{drive}}{\prod N_{drive}} = \frac{n_{driven}}{n_{drive}}$$

$$e = \frac{30}{150.857} = 0.19886$$

The overall gear train value was calculated as 0.19886 by analyzing the ratio of the product of the number of driving teeth over the product of the number of driven teeth.

Wear and Fatigue Potential of the Gear System

Assumptions

→ Assume pinion (compound gear 5) is a spur gear made of Grade 1 steel through induction-hardening

Wear and Fatigue Factors

For C_H (hardness-ratio factor), assume $C_H = 1$ because the pinion is being evaluated.

For C_F (surface condition factor): Assume unity since no information is given on surface condition:

$$C_F = 1$$

For C_p (elastic coefficient): Using Table 14-8 for a steel pinion and gear,

$$C_p = 191\ MPa$$

For F (face width): Using the Front View diagram, the 428 mm length was used to visually approximate the face width of gears 2 and 3, which were estimated as a third of the length:

$$F = 142.67\ mm$$

For I (pitting resistance geometry factor): Assuming a common pressure angle for spur gears $\emptyset_t = 20°$, load sharing ratio for spur gears $m_N = 1$, and speed ratio between gears 2 and 3 $m_G = 1$,

$$I = \frac{cos\emptyset_t sin\emptyset_t}{2m_N}\frac{m_G}{m_G + 1} = \frac{\cos(20°)\sin(20°)}{2(1)}\frac{1}{1+1} = 0.080$$

For d_p (pitch diameter) and P_d (diametral pitch): Given the outer diameter of the pinion is 175 mm, from https://www.mcmaster.com/gears/component~gear/pitch-diameter~6-67/, assume that the pinion has 40 gear teeth with a pitch diameter of 169.5 mm (6.67 inches):

$$N = 40\ teeth,\ \ d_p = 6.67\ in = 169.5\ mm$$

$$P_d = \frac{25.4\frac{mm}{in}}{N/d_p} = \frac{25.4\frac{mm}{in}}{40\ teeth/6.67\ in} = 4.233\ teeth/mm$$

For J (geometry factor): From Figure 14-6, given that the pinion mates with the Top Roller Driver of outer diameter 880 mm, assume the mating gear has at least 170 gear teeth:

$$J = 0.43$$

For K_B (rim-thickness factor): Assume $K_B = 1$ since no information is given on the rim-thickness of the gear system.

For K_m (load-distribution factor): Assuming uncrowned teeth

$$K_m = C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right)$$

$$C_{mc} = 1\ for\ uncrowned\ teeth$$

$$C_{pf} = \frac{F}{10d_p} - 0.0375 + 0.0125F\ for\ 1 < F \leq 17in$$

$$C_{pf} = \frac{5.617}{10(0.04)} - 0.037(5.617\ in) + 0.0125(5.617\ in) = 14.0752$$

$$C_{pm} = 1.1\ for\ straddle - mounted\ pinion\ with\ {}^{S_1}/_{S} \geq 0.175$$

$$C_{ma} = A + BF + CF^2$$

Empirical constant A, B, and C for C_{ma} from Table 14-9 for commercial, enclosed units:

$$A = 0.127, \qquad B = 0.0158, \qquad C = -0.93 * 10^{-4}$$

$$C_{ma} = 0.127 + 0.0158(5.617) - 0.930(10^{-4})(5.617)^2 = 0.212814$$

$$C_e = 1\ for\ conditions\ other\ than\ gear\ adjustment\ at\ assembly\ or\ lapping$$

$$K_m = C_{mf} = 1 + 1((14.0752)(1.1) + (0.212814)(1)) = 16.695534$$

For K_O (overload factor): From Figure 14-17, assume uniform power source with moderate shock:

$$K_O = 1.25$$

For K_R (reliability factor): Using Table 14-10, assume a reliability of 99%:

$$K_R = 1$$

For K_s (size factor): Assume $K_S = 1$ since no information is given on any lack of uniformity due to the gear system size.

For K_T (temperature factor): Assume $K_T = 1$ because heat transfer between the gears is minimal.

For S_c (allowable contact stress): From Table 14-6,

$$\mathrm{S_c} = 175000\ psi = 1.206583 * 10^9\ Pa$$

For S_f (bending fatigue factor): Assume $S_f = 1$ since no information is given on bending fatigue.

For S_H (safety factor), assume $S_H = 1$ because the gear system will not fail before production finishes.

For S_t (allowable bending stress): Assuming induction hardened steel yields the same allowable bending stress as through-hardened steel and given a Brinell Hardness factor of 131, from Figure 14-2,

$$S_t = 0.533H_B + 88.3 = 0.533(131) + 88.3 = 158123\ Pa$$

For V (velocity in m/s):

$$V = 37.7\frac{m}{min} = 0.628\frac{m}{s}$$

For V_m (velocity in mm/s):

$$V_m = \pi dn = \pi(396mm)(30rpm) = 37322.12\ mm/min$$

For K_V (dynamic factor): Assuming quality number $Q_v = 5$ for a commercial quality gear,

$$B = 0.25(12-5)^{2/3} = 0.915$$

$$A = 50 + 56(1-B) = 50 + 56(1-0.915) = 54.78$$

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B = \left(\frac{54.78+\sqrt{200(0.628)}}{54.78}\right)^{0.915} = 1.186$$

For Y_N (stress cycle factor): From Figure 14-14, given a total of $2.995*10^6$ cycles,

$$Y_N = 1.6831N^{-0.0323} = 1.6831(2.995*10^6)^{-0.0323} = 1.0397$$

For Z_N (stress cycle life factor): From Figure 14-15,

$$\mathrm{Z_N} = 2.466\mathrm{N}^{-0.056} = 2.466(2.995*10^6)^{-0.056} = 1.0698$$

Power Transmission Limit in Wear

$$\sigma_c^{allow} = \frac{S_c Z_N C_H}{S_H K_T K_R} = \frac{(1206583000)(1.0698)(1)}{(1)(1)(1)} = 1.291\ GPa$$

$$W_c^t = \left[\frac{\sigma_c^{allow}}{C_p}\right]^2 \frac{FD_pI}{K_oK_VK_sK_mC_F} = \left[\frac{1290802493}{191000000}\right]^2 \frac{(142.67)(169.5)(0.080)}{1.25(1.186)(1)(16.695534)(1)} = 3.569\ kN$$

$$H_c = \frac{W_c^tV_m}{63025} = \frac{3569(37322.12)}{63025} = 2.113\ kW$$

The allowable wear stress of the pinion was calculated as 1.291 GPa, the transmission load limit in wear was calculated as 3.569 kN, and the transmission power limit in bending was calculated as 3.569 kN.

Power Transmission Limit in Fatigue

$$\sigma_b^{allow} = \frac{S_TY_N}{S_fK_TK_R} = \frac{158123(1.0397)}{1(1)(1)} = 164.4\ kPa$$

$$W_b^t = \frac{\sigma_{bend}^{allow}FJ}{K_oK_vK_sp_dK_mK_B} = \frac{164400(142.67)(0.43)}{1.25(1.186)(1)(4.233)(16.69)(1)} = 96.295\ kN$$

$$H_b = \frac{W_{bend}^tV_m}{63025} = \frac{96295(37322.12)}{63025} = 57.024\ kW$$

The allowable bending stress of the pinion was calculated as 164.4 kPa, the transmission load limit in bending was calculated as 96.295 kN, and the transmission power limit in bending was calculated as 57.024 kW.

Evaluation of Power Transmission Limits

$$H_c < H_b$$

$$2.113\ kW < 57.024\ kW$$

The failure mechanism of the gear system is likely wear, since the transmission power limit in wear 2.113 kW is significantly less than the transmission power limit in bending 57.024 kW.

Additional Questions

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What is true strain and average flow stress experienced by the workpiece in rolling?

Assumptions

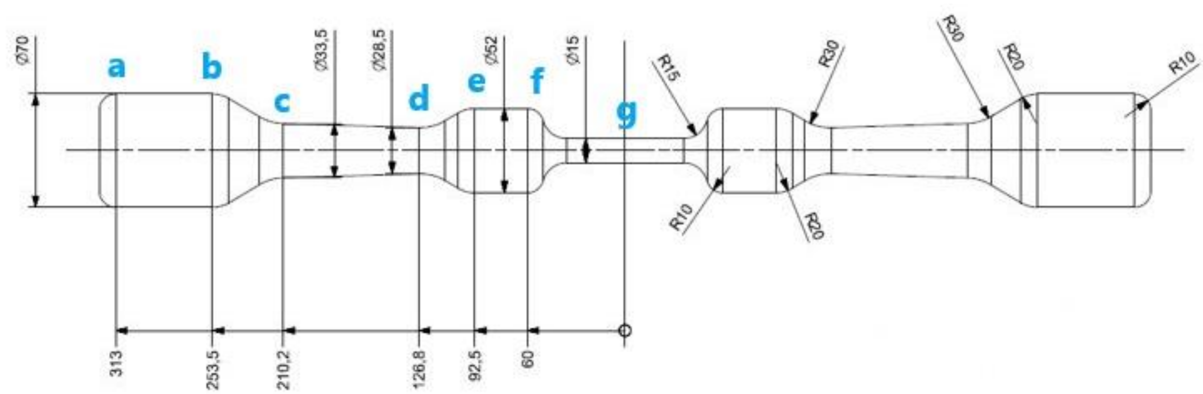
→ Assume that flat-rolling equations apply to roll-forging

Given

Material Property	Assumed Value
Yield strength, S_y	560 MPa
Ultimate strength, S_{ut}	660 MPa
Elastic modulus, E	200 GPa
% elongation	10%
Strain hardening exponent, n	0.042

Diagram

The connecting rod was split into sections to adjust for varying thickness:



Strength Coefficient

$$S_y = K(\frac{n}{e})^n$$

$$K = \frac{S_y}{\left(\frac{n}{e}\right)^n} = \frac{660 * 10^6}{\left(\frac{0.042}{e}\right)^{0.042}} = 786335410\ Pa = 786\ MPa$$

The strength coefficient of the workpiece was calculated as 786 MPa.

True Strain and Average Flow Stress by Section

Section a-b

$$\varepsilon' = ln\frac{t_0}{t_f} = ln\frac{70}{70} = 0$$

$$\bar{Y}_f = \frac{K\varepsilon'^n}{1+n} = \frac{K0^{0.042}}{1+0.042} = 0$$

Section a-c

$$\varepsilon' = ln\frac{t_0}{t_f} = ln\frac{70}{33.5} = 0.7369498032$$

$$\bar{Y}_f = \frac{K\varepsilon'^n}{1+n} = \frac{(786 * 10^6 Pa)0.737^{0.042}}{1+0.042} = 745027248.7 \text{ Pa} = 745 \text{ MPa}$$

Section a-d

$$\varepsilon' = ln\frac{t_0}{t_f} = ln\frac{70}{28.5} = 0.8985911548$$

$$\bar{Y}_f = \frac{K\varepsilon'^n}{1+n} = \frac{(786 * 10^6 Pa)0.899^{0.042}}{1+0.042} = 751259063 \text{ Pa} = 751 \text{ MPa}$$

Section a-e

$$\varepsilon' = ln\frac{t_0}{t_f} = ln\frac{70}{52} = 0.2972515235$$

$$\bar{Y}_f = \frac{K\varepsilon'^n}{1+n} = \frac{(786 * 10^6 Pa)(0.297^{0.042})}{1+0.042} = 717152185.7 \text{ Pa} = 717 \text{ MPa}$$

Section a-f

$$\varepsilon' = ln\frac{t_0}{t_f} = ln\frac{70}{52} = 0.2972515235$$

$$\bar{Y}_f = \frac{K\varepsilon'^n}{1+n} = \frac{(786 * 10^6 Pa)(0.297^{0.042})}{1+0.042} = 717152185.7 \text{ Pa} = 717 \text{ MPa}$$

Section a-g

$$\varepsilon' = ln\frac{t_0}{t_f} = ln\frac{70}{15} = 1.540445041$$

$$\bar{Y}_f = \frac{K\varepsilon'^n}{1+n} = \frac{(786 * 10^6 Pa)(1.540^{0.042})}{1+0.042} = 768459980 \text{ Pa} = 768 \text{ MPa}$$

Section a-g, in which the thickness decreased from 70 mm to 15 mm, sustained the greatest true strain and greatest average flow stress. Applying the true strain and true stress equations, section a-g of the workpiece experienced a true stress of 1.54 and an average flow stress of 768 MPa.

What is the length of contact between the rollers and the workpiece?

Section a-g (calculated using a true strain value of 1.54)

$$L = \sqrt{R(t_0 - t_f)} = \sqrt{\frac{260mm}{2}(70mm - 15mm)} = 84.557 \text{ mm}$$

The length of contact between the rollers and the workpiece was calculated as 84.557 mm.

What is the force required to maintain separation between the two rollers?

Section a-g (calculated using a true strain value of 1.54)

$$F = \int_0^L p\, dL \approx \bar{Y}_f \cdot w_0 \cdot \mathrm{L} = (768459980Pa)(0.07m)(0.084557m) = 4548543.122\ \mathrm{N} = 4.55\ \mathrm{MN}$$

Assuming that the initial width is the same as the initial thickness, 70 mm, the force required to maintain the separation between the two rollers was calculated as 4.55 MN.

What is the torque required to drive each roller?

Section a-g (calculated using a true strain value of 1.54)

$$T = 0.5 \cdot F \cdot L = 0.5(4548543.122N)(0.084557m) = 192307.1101\ N * m = 192.31\ kN * m$$

The torque required to drive each roller was calculated as 192.31 kN*m.

Conclusion

In this report, we analyzed the loading conditions of a simplified roll-forge model. To do so, we approximated the rollers as two sets of simply supported beams. Using a given applied load of 1.41 MN, we determined the reaction force of the bearings as 705 kN. We assessed the roller model for static and fatigue failure, calculating the static factor of safety as 1.43 using Distortion Energy Criteria and a fatigue factor of safety as 0.66 using Goodman Fatigue Criteria. To gain a physical representation of the impact of the load on the roller structure, we calculated the peak deflection and slope of the rollers as $-1.86*10^{-4}$ m and $-1.0403*10^{-5}$ degrees, respectively. We then revised the roller design to withstand fatigue failure by increasing the diameter to 360 mm; this yielded a revised static factor of safety of 3.82 using Distortion Energy Criteria, a revised fatigue factor of safety of 1.68 using Goodman Fatigue Criteria, and a revised peak deflection of $-5.06*10^{-5}$ m. To quantify the gear system, we tabulated the speed and torque of each gear and identified a train value of 0.199. We calculated the power transmission limit in wear and in bending as 2.113 kW and 57.024 kW, respectively, and predicted the failure mechanism of the gear system as wear. Finally, we enriched our understanding of the roll-forging process by investigating the roll-forging forces on the workpiece. We calculated the maximum true strain and average flow stress of the workpiece as 1.54 and 768 MPa, respectively. We then evaluated the contact length between the rollers and the workpiece as 84.56 mm, the force required to maintain separation between the two rollers as 4.55 MN, and the torque required to drive each roller as 192.31 kN*m.