FVM-heat-diffusion

https://github.com/acciptris/heat-diffusion

1 Equations

1.1 Real cell governing equation

Diffusion equation is given as,

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

Governing equation can be found by,

$$\iiint_\Omega \frac{\partial T}{\partial t}\, d\Omega = \iiint_\Omega \alpha \nabla^2 T\, d\Omega$$

which when discretized using an explicit scheme comes out to be,

$$\frac{T^{n+1} - T^n}{\Delta t} = \alpha \left(\frac{T^n_{i+1,j} + T^n_{i-1,j} - 2T^n_{i,j}}{(\Delta x)^2} + \frac{T^n_{i,j+1} + T^n_{i,j-1} - 2T^n_{i,j}}{(\Delta y)^2} \right)$$

$$T^{n+1} = T^n + \alpha \Delta t \left(\frac{T^n_{i+1,j} + T^n_{i-1,j} - 2T^n_{i,j}}{(\Delta x)^2} + \frac{T^n_{i,j+1} + T^n_{i,j-1} - 2T^n_{i,j}}{(\Delta y)^2} \right) \tag{1}$$

Equation 1 is applicable in all the real cells in the domain.

1.2 Fictitious cell governing equation

T_R and T_F are the average value of temperature in a real cell and an adjoining fictitious cell respectively. T_b is the temperature at the common face of the real and fictitious cell which is defined by the boundary condition. T_R, T_F and T_b can be approximated as,

$$\frac{T_F + T_R}{2} = T_b$$

$$T_F = 2T_b - T_R \tag{2}$$

Equation 2 can be used to update the temperature in the fictitious cells.

1.3 Steady State Convergence Residual

T_{rms} is calculated after every time step to find the change in solutions. A very low value of T_{rms} suggests that steady state has been achieved. T_{rms} is defined as,

$$T_{rms} = \sqrt{\frac{\sum_{i,j} \left(T^{n+1}_{i,j} - T^n_{i,j}\right)^2}{N^2}} \tag{3}$$

2 Numerical Algorithm

Algorithm 1 Numerical Algorithm

Choose the number of divisions of each side of the domain (N).
Choose $\Delta x = 1/N$, $\Delta y = 1/N$ and Δt.
Create arrays of size $(N+2)\,\mathbf{x}\,(N+2)$ to store the following values.

- Temperature at n^{th} time level (T^n)
- Temperature at $(n+1)^{th}$ time level (T^{n+1})

Initialize the values in real cells with some initial value $(T_{initial})$, i.e. for all $2 \leq i \leq N+1$ and $2 \leq j \leq N+1$, $T^n_{i,j} = T_{initial}$.
Initialize the values in fictitious cells using equation 2.
Initialize T_{rms} with a value greater than convergence criteria (C).
while $T_{rms} > C$ **do**
    Find T^{n+1} for all real cells using equation 1.
    Find T^{n+1} for all fictitious cells using equation 2 and T^{n+1} at real cells.
    Calculate T_{rms} using values of all the cells and equation 3.
    Replace T^n with T^{n+1}.
end while
return T^{n+1}