heat-diffusion

https://github.com/acciptris/heat-diffusion

1 Equations

1.1 Real cell governing equation

Diffusion equation is given as,

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

Governing equation can be found by,

$$\iiint_{\Omega} \frac{\partial T}{\partial t} \, d\Omega = \iiint_{\Omega} \alpha \nabla^2 T \, d\Omega$$

which when discretized using an explicit scheme comes out to be,

$$\frac{T^{n+1} - T^{n}}{\Delta t} = \alpha \left(\frac{T^n_{i+1,j} + T^n_{i-1,j} - 2T^n_{i,j}}{(\Delta x)^2} + \frac{T^n_{i,j+1} + T^n_{i,j-1} - 2T^n_{i,j}}{(\Delta y)^2} \right)$$

$$T^{n+1} = T^{n} + \alpha \Delta t \left(\frac{T^n_{i+1,j} + T^n_{i-1,j} - 2T^n_{i,j}}{(\Delta x)^2} + \frac{T^n_{i,j+1} + T^n_{i,j-1} - 2T^n_{i,j}}{(\Delta y)^2} \right) \tag{1}$$

Equation 1 is applicable in all the real cells in the domain.

1.2 Fictitious cell governing equation

T_R and T_F are the average value of temperature in a real cell and an adjoining fictitious cell respectively. T_b is the temperature at the common face of the real and fictitious cell which is defined by the boundary condition. T_R, T_F and T_b can be approximated as,

$$\frac{T_F + T_R}{2} = T_b$$

$$T_F = 2T_b - T_R \tag{2}$$

Equation 2 can be used to update the temperature in the fictitious cells.

1.3 Steady State Convergence Residual

T_{rms} is calculated after every time step to find the change in solutions. A very low value of T_{rms} suggests that steady state has been achieved. T_{rms} is defined as,

$$T_{rms} = \sqrt{\frac{\sum_{i,j} \left(T^{n+1}_{i,j} - T^{n}_{i,j}\right)^2}{N^2}} \tag{3}$$

2 Numerical Algorithm

Algorithm 1 Numerical Algorithm

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Choose the number of divisions of each side of the domain (N).
Choose Δx = 1/N, Δy = 1/N and Δt.
Create arrays of size (N + 2) x (N + 2) to store the following values.
    • Temperature at n^th time level (T^n)
    • Temperature at (n + 1)^th time level (T^{n+1})
Initialize the values in real cells with some initial value (T_initial), i.e. for all
    2 ≤ i ≤ N + 1 and 2 ≤ j ≤ N + 1, T^n_{i,j} = T_initial.
Initialize the values in fictitious cells using equation 2.
Initialize T_rms with a value greater than convergence criteria (C).
while T_rms > C do
    Find T^{n+1} for all real cells using equation 1.
    Find T^{n+1} for all fictitious cells using equation 2 and T^{n+1} at real cells.
    Calculate T_rms using values of all the cells and equation 3.
    Replace T^n with T^{n+1}.
end while
return T^{n+1}
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