CMT117 Problem Sheet 1 Autumn semester 2022

ANSWER ALL PARTS OF BOTH QUESTIONS. Each question is worth 25 marks and the number of marks available for each question part is indicated.

Question 1: Propositional Logic

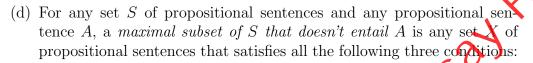
- (a) Write down exactly one propositional contradiction such that the only propositional variable appearing in the sentence is p, and the only logical connectives are \neg and \lor . (Note your answer may include p and \neg and \lor as many times as you like). [1]
- (b) Let $L = \{p,q\}$ and let v be the valuation such that v(p) = F and v(q) = T. For each of the following sentences in SL, state whether v satisfies that sentence.

$$(i) (p \lor \neg q) \land (\neg p \lor q)$$
 [1]

$$i \rightarrow p \rightarrow Q$$
 [1]

- Suppose we know the following facts about four people *Michael*, *Jenny*, *James* and *Sorah*:
 - (If Michael is rich, then either Jenny is not young or James is not tall
 - If Jenny is young, then Sarah is hungry
 - If Sarah is hungry and James is tall, then Michael is rich
 - James is tall

Using an appropriate choice of propositional variables, write each of these four facts as a sentence in propositional logic. [5]



- 1. $X \subseteq S$
- 2. $A \notin Cn(X)$
- 3. For any $Y \subseteq S$: if $X \subseteq Y$ and $Y \not\subseteq X$ then $A \in Cn(Y)$

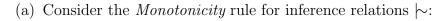
Let $L = \{p, q, r, s\}$ and let

$$S_1 = \{ \neg r, s, ((\neg r \lor q) \to p), q \}$$

Write down all the maximal subsets of S_1 that don't entail p. Justify your answer. [6]

- (e) Let $L = \{s, t, u\}$. Determine whether there exists a derivation of $u \vee t$ from $s \vee u$ using the rules of Natural Deduction. Justify your answer. [5]
- (f) Assume $L = \{p, q, r\}$. Consider the contence $A = p \to (q \leftrightarrow \neg r)$.
 - (i) Establish whether there exists a Horn sentence in SL that is logically equivalent to A Justify your answer, stating in full any statement or result from the lecture notes that you rely on in your xijustification. [4]
 - (ii) Use your apswer to part (i) to briefly illustrate one advantage or disadvantage of using Horn logic rather than propositional logic as a logic for knowledge representation. [2]

Question 2: Nonmonotonic Reasoning & Belief Revision



$$\frac{A \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} C}{A \wedge B \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} C}$$

(i) Assume $L = \{p, q\}$. Show that *Monotonicity* fails for some rational consequence relation, and some specific choice of sentences $A, B, C \in SL$. State clearly any Theorem from the module's leature notes that you rely on in your answer.

Consider now the following rule for inference relations \sim , which we call *Chain*:

$$\frac{X \triangleright Y \qquad Y \triangleright Z}{X \triangleright Z}$$
 (Chain)

(ii) Show how *Monotonicity* can be derived from the set of rules given by the KLM rules **plus** Chain. [3]

(b) Assume $L = \{p, q\}$ and consider the following ranked model $R = (V, \preceq)$ with $V = \{TT, TF, FT, FF\}$ and \preceq given in tabular form as follows:

(each valuation is represented as a pair denoting the truth-values of p, q respectively, and the further to the left a valuation appears in the above table, the more normal it is deemed to be.) State clearly whether the following conditionals hold in R. Justify your answers in each case.

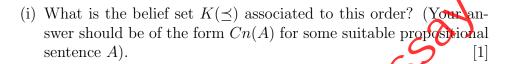
(i)
$$p \sim_R \neg g$$
 [3]

$$(i) \longrightarrow p \sim_R p$$
 [3]

c) Switching now to belief revision, let $L = \{p, q, r\}$ and let \leq be the following plausibility order over the set of valuations:

FFT	TFT	FTT	FFF	FTF
TTT		TFF		
TTF				

(each valuation is represented as triple denoting the truth-values of p, q, r respectively, and the further to the left a valuation appears in the above table, the more plausible it is deemed to be.)



Recall that $*_L$ denotes lexicographic revision.

- (ii) Write down $\preceq_{\neg(p\to q)\vee(\neg p\wedge r)}^{*L}$ in tabular form and give the belief set $K(\preceq_{\neg(p\to q)\vee(\neg p\wedge r)}^{*L})$ associated to this new order (again in the form Cn(A) for some suitable sentence A). [4]
- (d) Let us consider a brand new operator $*_T$ for revising plausibility orders such that, given any initial ordering \angle and revision input sentence A, the revised ordering $\preceq_A^{*_T}$ is formally defined as follows (for any valuations v_1, v_2):

$$v_1 \preceq_A^{*_T} v_2$$
 iff $v_1 = Mod(A)$ and $v_2 \in Mod(\neg A)$ $v_1 \neq Mod(A)$ and $v_1 \leq v_2 = Mod(\neg A)$

One of the postulates (P1) (P2), (P3) for iterated revision is *not* satisfied by this operator $*_T$. Which one is it? Justify your answer with the use of an example involving specific choices for \leq and A (you may assume $L = \{\rho, q\}$). Any results from the lecture notes that you rely on in your justification must be clearly stated. [6]