CMT117 Problem Sheet 1
Autumn semester 2022

ANSWER ALL PARTS OF BOTH QUESTIONS. Each question is worth 25 marks and the number of marks available for each question part is indicated.

Question 1: Propositional Logic

(a) Write down exactly one propositional contradiction such that the only propositional variable appearing in the sentence is p, and the only logical connectives are $\neg$ and $\vee$. (Note your answer may include p and $\neg$ and $\vee$ as many times as you like). [1]

(b) Let $L = \{p, q\}$ and let v be the valuation such that $v(p) = F$ and $v(q) = T$. For each of the following sentences in SL, state whether v satisfies that sentence.

(i) $(p \vee \neg q) \wedge (\neg p \vee q)$ [1]

(ii) $\neg p \leftrightarrow q$ [1]

(c) Suppose we know the following facts about four people *Michael*, *Jenny*, *James* and *Sarah*:

- *If Michael is rich, then either Jenny is not young or James is not tall*
- *If Jenny is young, then Sarah is hungry*
- *If Sarah is hungry and James is tall, then Michael is rich*
- *James is tall*

Using an appropriate choice of propositional variables, write each of these four facts as a sentence in propositional logic. [5]

(d) For any set S of propositional sentences and any propositional sentence A, a *maximal subset of S that doesn't entail A* is any set X of propositional sentences that satisfies all the following three conditions:

1. $X \subseteq S$
2. $A \notin Cn(X)$
3. For any $Y \subseteq S$: if $X \subseteq Y$ and $Y \nsubseteq X$ then $A \in Cn(Y)$

Let $L = \{p, q, r, s\}$ and let

$$S_1 = \{\neg r, s, ((\neg r \vee q) \to p), q\}.$$

Write down all the maximal subsets of S_1 that don't entail p. Justify your answer. [6]

(e) Let $L = \{s, t, u\}$. Determine whether there exists a derivation of $u \vee t$ from $s \vee u$ using the rules of Natural Deduction. Justify your answer. [5]

(f) Assume $L = \{p, q, r\}$. Consider the sentence $A = p \to (q \leftrightarrow \neg r)$.

(i) Establish whether there exists a Horn sentence in SL that is logically equivalent to A. Justify your answer, stating in full any statement or result from the lecture notes that you rely on in your justification. [4]

(ii) Use your answer to part *(i)* to briefly illustrate one advantage or disadvantage of using Horn logic rather than propositional logic as a logic for knowledge representation. [2]

Question 2: Nonmonotonic Reasoning & Belief Revision

(a) Consider the *Monotonicity* rule for inference relations $\mid\!\sim$:

$$\frac{A \mid\!\sim C}{A \wedge B \mid\!\sim C}$$

(i) Assume $L = \{p, q\}$. Show that *Monotonicity* fails for some rational consequence relation, and some specific choice of sentences $A, B, C \in SL$. State clearly any Theorem from the module's lecture notes that you rely on in your answer. [5]

Consider now the following rule for inference relations $\mid\!\sim$, which we call *Chain*:

$$\frac{X \mid\!\sim Y \qquad Y \mid\!\sim Z}{X \mid\!\sim Z}\ (\mathit{Chain})$$

(ii) Show how *Monotonicity* can be derived from the set of rules given by the KLM rules **plus** *Chain*. [3]

(b) Assume $L = \{p, q\}$ and consider the following ranked model $R = (V, \preceq)$ with $V = \{TT, TF, FT, FF\}$ and $\preceq$ given in tabular form as follows:

FT	TF FF	TT

(each valuation is represented as a pair denoting the truth-values of p, q respectively, and the further to the left a valuation appears in the above table, the more normal it is deemed to be.) State clearly whether the following conditionals hold in R. Justify your answers in each case.

(i) $p \mid\!\sim_R \neg q$ [3]

(ii) $q \to p \mid\!\sim_R p$ [3]

(c) Switching now to belief revision, let $L = \{p, q, r\}$ and let $\preceq$ be the following plausibility order over the set of valuations:

FFT TTT TTF	TFT	FTT TFF	FFF	FTF

(each valuation is represented as triple denoting the truth-values of p, q, r respectively, and the further to the left a valuation appears in the above table, the more plausible it is deemed to be.)

(i) What is the belief set $K(\preceq)$ associated to this order? (Your answer should be of the form $Cn(A)$ for some suitable propositional sentence A). [1]

Recall that $*_L$ denotes lexicographic revision.

(ii) Write down $\preceq^{*_L}_{\neg(p\to q)\vee(\neg p\wedge r)}$ in tabular form and give the belief set $K(\preceq^{*_L}_{\neg(p\to q)\vee(\neg p\wedge r)})$ associated to this new order (again in the form $Cn(A)$ for some suitable sentence A). [4]

(d) Let us consider a brand new operator $*_T$ for revising plausibility orders such that, given any initial ordering $\preceq$ and revision input sentence A, the revised ordering $\preceq^{*_T}_A$ is formally defined as follows (for any valuations v_1, v_2):

$$v_1 \preceq^{*_T}_A v_2 \text{ iff } \begin{array}{ll} \textit{either} & v_1 \in Mod(A) \text{ and } v_2 \in Mod(\neg A) \\ \textit{or} & v_1, v_2 \in Mod(A) \text{ and } v_1 \preceq v_2 \\ \textit{or} & v_1, v_2 \in Mod(\neg A) \end{array}$$

One of the postulates **(P1)**, **(P2)**, **(P3)** for iterated revision is *not* satisfied by this operator $*_T$. Which one is it? Justify your answer with the use of an example involving specific choices for $\preceq$ and A (you may assume $L = \{p, q\}$). Any results from the lecture notes that you rely on in your justification must be clearly stated. [6]