SEHH1069 Calculus and Linear Algebra
Individual Assignment

☠ Late / E-mail / Re-submissions will **NOT** be accepted ☠

Intended Learning Outcomes

- Describe the concepts of limit and continuity of functions, derivatives of functions, indefinite and definite integrals
- Evaluate limits, derivatives and integrals of functions in a single variable
- Implement techniques of solving systems of linear equations
- Perform techniques of solving systems of linear equations
- Apply methods of calculus and linear algebra to solve a range of practical problems

Instructions

1. This assignment should be completed individually and neatly in the given **answer booklet**. All answers and solutions must be **hand-written**. Computer-typed works are NOT accepted.
2. All numerical answers should be **EXACT**.
3. If you apply any results / theorems which are NOT in the lecture notes, you are required to prove them from scratch.
4. *You are required to show all the details when applying elementary row / column operations in each question.*
5. Plagiarism will be penalised severely. Marks will be deducted for assignments that are plagiarised in whole or in part, regardless of the sources. **Extensive copy-pasting from AI-generated content without citation is also considered plagiarism.**
6. Please submit a **hard copy** of your completed assignment to your lecturer **before / during the period specified below**. Submissions after this period will be counted 'LATE'. *You are also required to make a photocopy of the assignment for your record.*

Class	Lecturer	Submission Period
A01	Chi-honn CHENG	Tutorials on Thursday 7 November 2024
A02	Chi-honn CHENG	Lecture on Tuesday 5 November 2024
A03	Maggie LAM	Tutorials on Monday 4 November 2024

7. Late / E-mail / Re-submissions are **NOT** accepted and / or may be subject to penalty.
8. **Write down your name and student number on the first page of the answer booklet.**

<u>IMPORTANT</u> Each student is required to complete this assignment individually using his/her student ID number. The method is shown below.

***R* = the 8th digit of your student ID**

Example: If your student ID number is 24345678A, then *R* = 8.

<u>Question 1 (10 marks)</u>

Use Gauss-Jordan elimination to solve the system of linear equations

$$\begin{cases} x_1 - x_2 + 2x_3 + 2x_5 = R, \\ 2x_1 - 3x_2 + 3x_3 + 4x_4 + 2x_5 = R+1, \\ 3x_1 - 4x_2 + 5x_3 + x_4 + 4x_5 = R+4. \end{cases}$$

<u>Question 2 (10 marks)</u>

Consider the system of linear equations

$$\begin{bmatrix} 1 & 1 & p+1 \\ p & 1 & p+1 \\ 1 & p & p^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} q \\ p \\ 2 \end{bmatrix},$$

where p and q are constants. Use elementary row operations to find all the value(s) of p and q, if any, such that this system has

(a) a unique solution.

(b) infinitely many solutions.

(c) no solution.

Question 3 (12 marks)

Let

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -2 \\ 2 & 0 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -6 & 8 & 3 \\ 2 & 4 & 4 \end{bmatrix} \text{ and } \mathbf{C} = \begin{bmatrix} 1 & 4 \\ 3 & R+1 \end{bmatrix}.$$

Find each of the following.

(a) $2\mathbf{A}\mathbf{B}^{\mathrm{T}} - 4\mathbf{I}_2$

(b) The values of h and k for which $\mathbf{M} := \mathbf{C}\begin{bmatrix} 2 & h \\ k & 1 \end{bmatrix}$ is symmetric and $\mathrm{tr}(\mathbf{M}) = R+1$

(c) The matrix $\mathbf{X}$ such that $\mathbf{CX} = 6\mathbf{X} + \mathbf{A}$

Question 4 (22 marks)

Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & -2 \\ 1 & c & 1 \\ 1 & 0 & c \end{bmatrix}$, where c is a real constant.

(a) Find the inverse of $\mathbf{A}$ by using

(i) the inversion algorithm.

(ii) the adjoint method.

(b) Use the result of (a) to find the adjoint of $2\mathbf{A}^{\mathrm{T}}$.

(c) Use Cramer's rule to solve the system of linear equations $\mathbf{Ax} = \begin{bmatrix} 0 \\ R+1 \\ 1 \end{bmatrix}$.

(Note: The answers of (a) – (c) are in terms of c.)

Question 5 (14 marks)

Find all the value(s) of x for which each of the following matrices is non-singular (i.e., invertible).

(a) $\mathbf{M}=\begin{bmatrix} x+1 & 9 & 4 \\ 9 & x+4 & 1 \\ 4 & 1 & x+9 \end{bmatrix}$

(b) $\mathbf{M}=\begin{bmatrix} -2 & 4 & 6 & 6 \\ 0 & 0 & 5 & x \\ 2 & -3 & 0 & 4 \\ x & 2 & 3 & 3 \end{bmatrix}$

Question 6 (10 marks)

Let $f(x)=\ln\left(x+\sqrt{1+x^2}\right)$.

(a) Show that f is well-defined for all $x\in\mathbb{R}$.

(b) Find f^{-1}.

(c) Verify that $\left(f^{-1}\circ f\right)(x)=x$ for every $x\in\mathbb{R}$.

Question 7 (12 marks)

Find $\dfrac{dy}{dx}$ for each of the following.

(a) $y=(x+2)^{\sin^{-1}\left(e^{3x}\right)}$

(b) $\tan\left(x-2y^2\right)=x^4 y$

(c) $y=\left(x^3+3\right)^{\tan^{-1}x}$ (Note: You are required to express the answer in terms of x **ONLY**)

Question 8 (10 marks)

Evaluate each of the following limits, if it exists. Otherwise, explain why the limit does not exist.

(a) $\lim\limits_{x\to-\infty}\left(\sqrt{2x^2+1}-\sqrt{2x^2-(R+1)x+3}\right)$

(b) $\lim\limits_{x\to 0}\dfrac{\left(e^{|x|}-1\right)^2}{\cos x-1}$

Question 9 (*NOT to be handed in*)

Find, if any,

(i) the interval(s) on which the function f is strictly increasing or strictly decreasing.

(ii) the interval(s) on which the function f is convex or concave.

(iii) all the relative extreme point(s) and point(s) of inflexion of f.

(iv) all the vertical asymptote(s) and horizontal asymptote(s), if any, of the graph of $y = f(x)$.

(a) $f(x)=\dfrac{x-1}{(x+3)^3},\ \ x\neq -3$

(b) $f(x)=(x-2)e^{-3x}$

Question 10 (*NOT to be handed in*)

Evaluate the following integrals.

(a) $\displaystyle\int\frac{x^2}{(x+3)(2x-3)^2}\,dx$

(b) $\displaystyle\int\frac{2x+9}{(2x+3)(x^2+9)}\,dx$

(c) $\displaystyle\int\frac{\ln x}{x^4}\,dx$

Question 10 (Continued)

(d) $\int \sin^{-1}(2x)\,dx$

(e) $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^5 x\, dx$

(f) $\int \frac{1}{x^2}\sin\frac{1}{x}\,dx$

(g) $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$ (Hint: Let $u=\sqrt{x^2+1}$)

(h) $\int_1^{\sqrt{3}} \frac{1}{x^2\left(1+x^2\right)^{\frac{1}{2}}}\,dx$ (Hint: Let $x=\tan\theta$ for $0<\theta<\frac{\pi}{2}$)

– END OF THE ASSIGNMENT –