CS161: FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE

Spring 2020
Assignment 7. Due Friday, May 22, 2020, 11:55pm

Please submit your solutions on gradescope in a well-formatted PDF file named *hw7.pdf*. You may also submit a scanned PDF of a handwritten solution, but please ensure that the scanned file is clearly legible.

1. Prove the following identity using induction:

$$Pr(\alpha_1, \ldots, \alpha_n \mid \beta) = Pr(\alpha_1 \mid \alpha_2, \ldots, \alpha_n, \beta) Pr(\alpha_2 \mid \alpha_3, \ldots, \alpha_n, \beta) \ldots Pr(\alpha_n \mid \beta).$$

2. A well is being drilled on a farm. Based on what has happened to similar farms, we judge the probability of oil being present to be 0.5, the probability of natural gas being present to be 0.2, and the probability of neither being present to be 0.3. Oil and natural gas cannot be present at the same time. If oil is present, a geological test will give a positive result with probability 0.9; if natural gas is present, it will give a positive result with probability 0.3; and if neither are present, the test will be positive with probability 0.1. Suppose the test comes back positive. What's the probability that oil is present?

3. We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 40%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3. A bell will ring "on" if all coins flips come out the same. Draw the Bayesian network corresponding to this setup and define the necessary CPTs (Conditional Probability Tables).

4. Consider the DAG in Figure 1:
   (a) List the Markovian assumptions asserted by the DAG.
   (b) True or false? Why?
      - $d\_separated(A, F, E)$
      - $d\_separated(G, B, E)$
      - $d\_separated(AB, CDE, GH)$
   (c) Express $Pr(a, b, c, d, e, f, g, h)$ in factored form using the chain rule for Bayesian networks.
   (d) Compute $Pr(A = 1, B = 1)$ and $Pr(E = 0 \mid A = 0)$. Justify your answers.

$Pr(A = 0)$	$Pr(A = 1)$
.8	.2

$Pr(B = 0)$	$Pr(B = 1)$
.3	.7

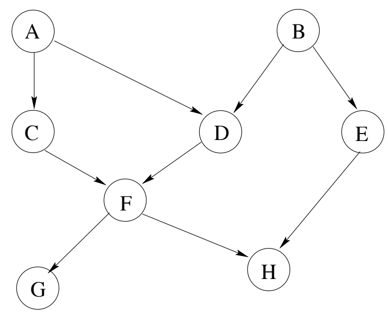

Figure 1: The DAG of a Bayesian network.

	$Pr(E = 0 \mid B)$	$Pr(E = 1 \mid B)$
$B = 0$	.1	.9
$B = 1$	.9	.1

	$Pr(D = 0 \mid A, B)$	$Pr(D = 1 \mid A, B)$
$A = 0, B = 0$	.2	.8
$A = 0, B = 1$	.9	.1
$A = 1, B = 0$	.4	.6
$A = 1, B = 1$	.5	.5

5. Consider the joint probability distribution in Table 1 and the propositional sentence $\alpha : A \Rightarrow B$.

   (a) List the models of α.

   (b) Compute the probability $Pr(\alpha)$.

   (c) Compute the conditional probability distribution $Pr(A, B \mid \alpha)$ as in Table 1.

   (d) Compute the probability $Pr(A \Rightarrow \neg B \mid \alpha)$.

	A	B	$Pr(A, B)$
w_0	T	T	0.3
w_1	T	F	0.2
w_2	F	T	0.1
w_3	F	F	0.4

Table 1: A joint probability distribution.