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HW #7

1. given conditional probability:

$$P(A|B) = P(A,B)/P(B)$$

base case: $\underline{n=2}$

LHS: $P(A_1, A_2 | \beta) = P(A_1, A_2, \beta)/P(\beta)$

RHS: $P(A_1 | A_2, \beta) = P(A_1, A_2, \beta)/P(A_2, \beta)$

$P(A_2 | \beta) = P(A_2, \beta)/P(\beta)$

$$\frac{P(A_1, A_2, \beta)}{P(\beta)} = \frac{P(A_1, A_2, \beta)}{\cancel{P(A_2, \beta)}} \cdot \frac{\cancel{P(A_2, \beta)}}{P(\beta)}$$

inductive case: $\underline{n = n+1}$

LHS: $P(a_1 \ldots a_{n+1} | \beta) = P(a_1 \ldots a_{n+1}, \beta)/P(\beta)$

RHS: $P(a_1 \ldots a_N | a_{n+1}, \beta) = P(a_1 \ldots a_{n+1}, \beta)/P(a_{n+1}, \beta)$

$P(a_{n+1} | \beta) = P(a_{n+1}, \beta)/P(\beta)$

$$\frac{P(a_1 \ldots a_{n+1}, \beta)}{P(\beta)} = \frac{P(a_1 \ldots a_{n+1}, \beta)}{\cancel{P(a_{n+1}, \beta)}} \cdot \frac{\cancel{P(a_{n+1}, \beta)}}{P(\beta)}$$

$\therefore$ for all n, the identity holds.

2. $P(oil) = 0.5$
$P(ng) = 0.2$
$P(neither) = 0.3$

find $P(oil \mid pos)$

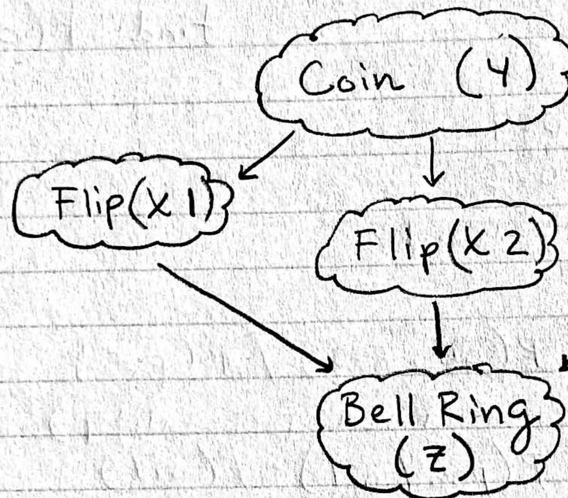
$P(pos \mid oil) = 0.9$
$P(pos / ng) = 0.3$
$P(pos / neither) = 0.1$

$P(oil / pos) = P(oil, pos) / P(pos)$
(conditional probability)
$P(oil, pos) = P(pos / oil) \cdot P(oil)$
$= 0.9 * 0.5 = 0.45$
(Product rule)
$\therefore P(oil \mid pos) = 0.45 / P(pos)$

$P(pos) = P(oil, pos) + P(ng, pos) + P(neither, pos)$
$= 0.45 + [P(pos / ng) P(ng)] + [P(pos / neither) P(neith)]$
$= 0.45 + (0.3 \cdot 0.2) + (0.1 \cdot 0.3)$
$= 0.45 + 0.06 + 0.03$
$= 0.54$
$\therefore P(oil \mid pos) = 0.45 / 0.54$
$= \boxed{0.83}$

3.

Network:



Y	P(Y)
a	1/3
b	1/3
c	1/3

Y	X1	P(X1 | Y)
a	h	0.2
a	t	0.8
b	h	0.4
b	t	0.6
c	h	0.8
c	t	0.2

same chart for X2, X3 also.

$P(h) = \frac{1}{3}(.2+.4+.8)$

$P(t) = \frac{1}{3}(.8+.6+.2)$

$P(h) = 0.46$

$P(t) = 0.54$

X1	X2	X3	Z	P(Z | X1, X2, X3)
t	t	t	Ring	0.157
t	t	h	None	0.134
t	h	t	None	0.134
t	h	h	None	0.114
h	t	t	None	0.134
h	t	h	None	0.114
h	h	t	None	0.114
h	h	h	Ring	0.097

4. a)

$(A, \emptyset, \{B, E\})$
$(B, \emptyset, \{A, C\})$
$(C, \{A\}, \{B, D, E\})$
$(D, \{A, B\}, \{C, E\})$
$(E, \{B\}, \{A, D, C, F, G\})$
$(F, \{C, D\}, \{A, B, E\})$
$(G, \{F\}, \{A, B, C, D, E, H\})$
$(H, \{E, F\}, \{B, D, A, C, G\})$

b) i) <u>false</u>, since a path between A and E exists that doesn't go through F.

ii) <u>false</u>, since a path between G and E exists that doesn't go through B.

iii) <u>true</u>, since CDE knowns block all paths from $\{A, B\}$ to $\{G, H\}$

c) $p(a, b, c, d, e, f, g, h) =$

1st layer → $P(A) \cdot P(B) \cdot$
2nd layer → $P(C \mid A) \cdot P(D \mid A, B) \cdot P(E \mid B) \cdot$
3rd layer → $P(F \mid C, D) \cdot$
4th layer of DAG → $P(G \mid F) \cdot P(H \mid F, E)$

d) i) $P(A=1, B=1) = P(A=1) \cdot P(B=1)$ *since linearly independant
$= 0.2 \cdot 0.7$
$= 0.14$

ii) $P(E=0 \mid A=0) = P(E=0)$
$= P(E=0 \mid B=0) P(B=0) + P(E=0 \mid B=1) P(B=1)$
$= 0.1 * 0.3 + 0.9 * 0.7$
$= 0.66$

5. a) $\alpha : A \Rightarrow B$

$: \neg A \vee B$

A	B	α
T	T	T
F	T	T
T	F	F
F	F	T

b)

P(A,B)	A	B	α	
0.3	T	T	T	✓
0.1	F	T	T	✓
0.2	T	F	F	X
0.4	F	F	T	✓

$0.3 + 0.1 + 0.4 = 0.8$

$P(\alpha) = 0.8 \ (T)$
$= 0.2 \ (F)$

c)

A	B	α	$P(A, B \mid \alpha)$
T	T	T	0.375
F	T	T	0.125
T	F	F	0
F	F	T	0.5

$= \frac{P(A B \alpha)}{P(\alpha)}$

d)

	A	B	α	$A \Rightarrow \neg B$
0.3	T	T	T	F
0.1	F	T	T	T
0.2	T	F	F	T
0.4	F	F	T	T

$P(A \Rightarrow \neg B \mid \alpha) = \frac{P(A \Rightarrow \neg B \wedge \alpha)}{P(\alpha)}$

$= \frac{0.1 + 0.4}{0.8}$

$= \frac{0.5}{0.8}$

$P(A \Rightarrow \neg B = T \mid \alpha) = 5/8$
$P(A \Rightarrow \neg B = F \mid \alpha) = 3/8$