

Adam Cole

UID: [REDACTED]

HW6

1. a.  $P \Rightarrow Q, \neg Q \Rightarrow \neg P$

P	Q	$P \Rightarrow Q$	$\neg Q \Rightarrow \neg P$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$\neg P \vee Q$

$Q \vee \neg P$

same statements  $\therefore$  equivalent

b.  $P \Leftrightarrow \neg Q, ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$P \Rightarrow \neg Q$	$\neg Q \Rightarrow P$	$P \Leftrightarrow \neg Q$
T	T	F	T	F
T	F	T	T	T
F	T	T	T	T
F	F	T	F	F
		$P \wedge \neg Q$	$\neg P \wedge Q$	$(P \wedge \neg Q) \vee (\neg P \wedge Q)$
T	T	F	F	F
T	F	T	F	T
F	T	F	T	T
F	F	F	F	F

$$2.a. (Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$$

$$\neg(\neg Smoke \vee Fire) \vee (Smoke \vee \neg Fire)$$

Smoke	Fire	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
T	T	T
T	F	T
F	T	F
F	F	T

$\therefore$  satisfiable, not valid.

$$b. (Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$$

$$\neg(\neg Smoke \vee Fire) \vee (\neg(Smoke \vee Heat) \vee Fire)$$

Smoke	Heat	Fire	
T	T	T	T
T	T	F	T
F	T	F	F

$\therefore$  satisfiable, not valid

$$c. ((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$$

$$\neg(\neg(S \wedge H) \vee F) \Leftrightarrow ((\neg S \vee F) \vee (\neg H \vee F))$$

F	H	S	$(\neg(S \wedge H) \vee F)$	$((\neg S \vee F) \vee (\neg H \vee F))$	out
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

$\therefore$  satisfiable , valid .

3. a.

$(\text{mythical} \rightarrow \neg \text{mortal}) \wedge (\neg \text{mythical} \rightarrow \text{mortal} \wedge \text{mammal})$   
 $\neg \text{mortal} \vee \text{mammal} \rightarrow \text{horned}$   
 $\text{horned} \rightarrow \text{magical}$

b.  $(\neg \text{myth} \vee \neg \text{mort}) \wedge (\text{myth} \vee (\text{mort} \wedge \text{mamm}))$   
 $\neg(\neg \text{mort} \vee \text{mamm}) \vee \text{horned}$   
 $\neg \text{horned} \vee \text{magical}$

c. mythical = F (Prove by Contradiction)  
 given statement #1,  $\rightarrow$  mort = T , mamm = T  
 given statement #2,  $\rightarrow$  horned = T  
 given statement #3,  $\rightarrow$  magical = T

$\therefore$  no contradiction found

$\therefore$  cant be proven that unicorns are always mythical

$\text{magical} = F$  (Prove by Contradiction)  
 given statement #3,  $\rightarrow \text{horned} = F$   
 given statement #2,  $\rightarrow (\text{mortal} = T, \text{mamm} = F)$   
 given statement #1,  $\rightarrow (\text{myth} = F, \text{mortal} = T, \text{mamm} = T)$

$\therefore$  contradiction, mammal must be T and F to be a unicorn.

$\therefore$  can be proven unicorns must be magical.

$\text{horned} = F$   
 given statement #3  $\rightarrow$  nothing  
 given statement #2  $\rightarrow (\text{mortal} = T, \text{mamm} = F)$   
 given statement #1  $\rightarrow (\text{myth} = F, \text{mortal} = T, \text{mamm} = T)$

$\therefore$  contradiction, mammal must be T and F to be a unicorn.

$\therefore$  can be proven that unicorns must be horned.

4. Figure 1:

Decomposable: T

- all "ands" have no submodules  
 Smooth with shared variables  
 (left = A, B, right = C, D)

Deterministic: T

- all "or's" have at most 1 high input

Smooth: F

- middle two "or" gates use an unequal # of arguments in Left/Right



Figure 2 :

Decomposable : T

because all and gates have subcircuits with no shared variables.

Deterministic : T

because all "or" gates have at most one "high" input.

Smooth : T

all or gates take in equivalent variables on the left and right side.

5. a.

	A	B	$(\neg A \wedge B) \vee (\neg B \wedge A)$
	T	T	F
*	T	F	T
*	F	T	T
	F	F	F

$$\omega(A) * \omega(\neg B) = 0.2 * 0.6 = 0.12$$

$$\omega(\neg A) * \omega(B) = 0.8 * 0.4 = 0.32$$

$$\boxed{0.44}$$

$$\begin{aligned} \text{b.} &= (\omega(\neg A) * \omega(B)) + (\omega(\neg B) * \omega(A)) \\ &= (0.8 * 0.4) + (0.2 * 0.6) \\ &= 0.12 + 0.32 \\ &= \boxed{0.44} \end{aligned}$$

relationship : equivalent

$$\begin{aligned}
 C. \quad \omega(A) &= 0.2 & \omega(\neg A) &= 0.8 \\
 \omega(B) &= 0.4 & \omega(\neg B) &= 0.6 \\
 \omega(C) &= 0.6 & \omega(\neg C) &= 0.4 \\
 \omega(D) &= 0.8 & \omega(\neg D) &= 0.2
 \end{aligned}$$

$$\begin{aligned}
 AND1 &= 0.8 * 0.4 (.32) & AND5 &= 0.6 * 0.2 (.12) \\
 AND2 &= 0.8 * 0.6 (.48) & AND6 &= 0.6 * 0.8 (.48) \\
 AND3 &= 0.4 * 0.2 (.08) & AND7 &= 0.4 * 0.2 (.08) \\
 AND4 &= 0.6 * 0.2 (.12) & AND8 &= 0.4 * 0.8 (.32)
 \end{aligned}$$

$$\begin{aligned}
 OR1 &= AND1 + AND4 = .32 + .12 = (.44) \\
 OR2 &= AND6 + AND7 = .48 + .08 = (.56) \\
 OR3 &= AND2 + AND3 = .48 + .08 = (.56) \\
 OR4 &= AND5 + AND8 = .12 + .32 = (.44)
 \end{aligned}$$

$$\begin{aligned}
 AND9 &= OR1 * OR2 = .44 * .56 = (0.2464) \\
 AND10 &= OR3 * OR4 = .56 * .44 = (0.2464)
 \end{aligned}$$

$$\begin{aligned}
 OR5 &= AND9 + AND10 = 0.2464 + 0.2464 \\
 &= \boxed{0.4928}
 \end{aligned}$$