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UID: [REDACTED]

HW9

1. Find attribute w/ lowest entropy:

$H(A)$

<u>=T</u>	
x_1 - Yes	(x 1)
x_2 - Yes	(x 6)
x_3 - No	(x 3)
x_4 - No	(x 1)

$$\begin{aligned} \therefore &= -\left(\frac{7}{11} \log \frac{7}{11} + \frac{4}{11} \log \frac{4}{11}\right) \\ &= -\frac{7}{11} \log \frac{7}{11} - \frac{4}{11} \log \frac{4}{11} \\ &= 0.9456 \end{aligned}$$

=F

x_5 - Yes	(x 1)
x_6 - No	(x 6)
x_7 - Yes	(x 2)
x_8 - No	(x 3)

$$\begin{aligned} &= -\left(\frac{3}{12} \log \frac{3}{12} + \frac{9}{12} \log \frac{9}{12}\right) \\ &= -\frac{1}{4} \log \frac{1}{4} - \frac{3}{4} \log \frac{3}{4} \\ &= 0.8112 \end{aligned}$$

compute weighted average:

$$H(A) = 0.9456\left(\frac{11}{23}\right) + 0.8112\left(\frac{12}{23}\right) = \underline{\underline{0.8754}}$$

$H(B)$

<u>=T</u>	
x_1 - Yes	(x 1)
x_2 - Yes	(x 6)
x_5 - Yes	(x 1)
x_6 - No	(x 6)

$$\begin{aligned} &= -\left(\frac{8}{14} \log \frac{8}{14} + \frac{6}{14} \log \frac{6}{14}\right) \\ &= 0.9852 \end{aligned}$$

=F

x_3 - No	(x 3)
x_4 - No	(x 1)
x_7 - Yes	(x 2)
x_8 - No	(x 3)

$$\begin{aligned} &= -\left(\frac{2}{9} \log \left(\frac{2}{9}\right) + \frac{7}{9} \log \left(\frac{7}{9}\right)\right) \\ &= 0.7642 \end{aligned}$$

$$H(B) = 0.9852\left(\frac{14}{23}\right) + 0.7642\left(\frac{9}{23}\right) = \underline{\underline{0.8987}}$$

$H(C)$

<u>=T</u>	
x_1 - Yes	(x 1)
x_3 - No	(x 3)
x_5 - Yes	(x 1)
x_7 - Yes	(x 2)

$$= -\left(\frac{4}{7} \log \frac{4}{7} + \frac{3}{7} \log \frac{3}{7}\right) = 0.9852$$

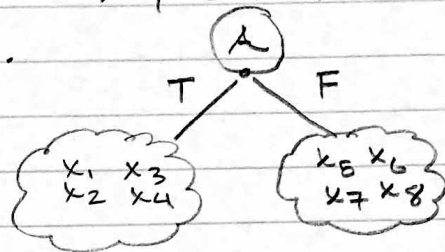
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x_2 - Yes	(x 6)
x_4 - No	(x 1)
x_6 - No	(x 6)
x_8 - No	(x 3)

$$= -\left(\frac{6}{16} \log \frac{6}{16} + \frac{10}{16} \log \frac{10}{16}\right) = 0.9544$$

$$H(C) = 0.9852\left(\frac{7}{23}\right) + 0.9544\left(\frac{16}{23}\right) = \underline{\underline{0.9637}}$$

Since $H(A) < H(B) < H(C)$, our first split will be on attribute A.



Recalculate for next.

Left Branch

$H(B)$

$$\begin{aligned}
 & \begin{array}{l} \text{=T|} \quad \overbrace{\quad\quad\quad}^7 \\ x_1 - \text{Yes } (x_1) \\ x_2 - \text{Yes } (x_2) \end{array} \quad \begin{array}{l} \text{=F|} \quad \overbrace{\quad\quad\quad}^4 \\ x_3 - \text{No } (x_3) \\ x_4 - \text{No } (x_4) \end{array} \\
 & = - \left(\frac{7}{7} \log \frac{7}{7} + \frac{0}{0} \log \frac{0}{0} \right) \quad = - \left(\frac{0}{0} \log \frac{0}{0} + \frac{4}{4} \log \frac{4}{4} \right) \\
 & = - (1 \log 1 + 0) = 0 \quad = 0 \\
 & H(B) = 0 \left(\frac{7}{11} \right) + 0 \left(\frac{4}{11} \right) = \underline{\underline{0}}
 \end{aligned}$$

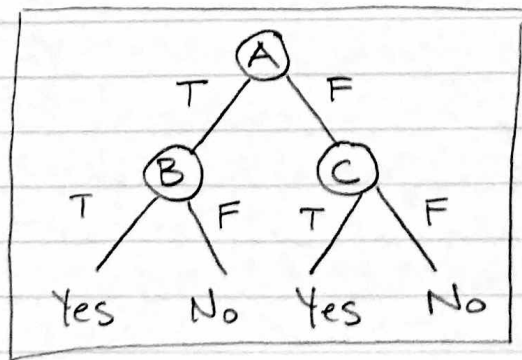
Since $H(B) = 0 = \text{minimum entropy}$, we don't have to check $H(C)$. Choose B as next split.

Right Branch

$H(C)$

$$\begin{aligned}
 & \begin{array}{l} \text{=T|} \quad \overbrace{\quad\quad\quad}^3 \\ x_5 - \text{Yes } (x_1) \\ x_7 - \text{Yes } (x_2) \end{array} \quad \begin{array}{l} \text{=F|} \quad \overbrace{\quad\quad\quad}^9 \\ x_6 - \text{No } (x_6) + x_8 \\ x_8 - \text{No } (x_3) \end{array} \\
 & = - \left(\frac{3}{3} \log \frac{3}{3} + \frac{0}{0} \log \frac{0}{0} \right) \quad = - \left(\frac{0}{0} \log \frac{0}{0} + \frac{9}{9} \log \frac{9}{9} \right) \\
 & = 0 \quad = 0 \\
 & H(C) = 0 \left(\frac{3}{12} \right) + 0 \left(\frac{9}{12} \right) = 0
 \end{aligned}$$

Since $H(C) = 0 = \text{minimum entropy}$, choose C as split.

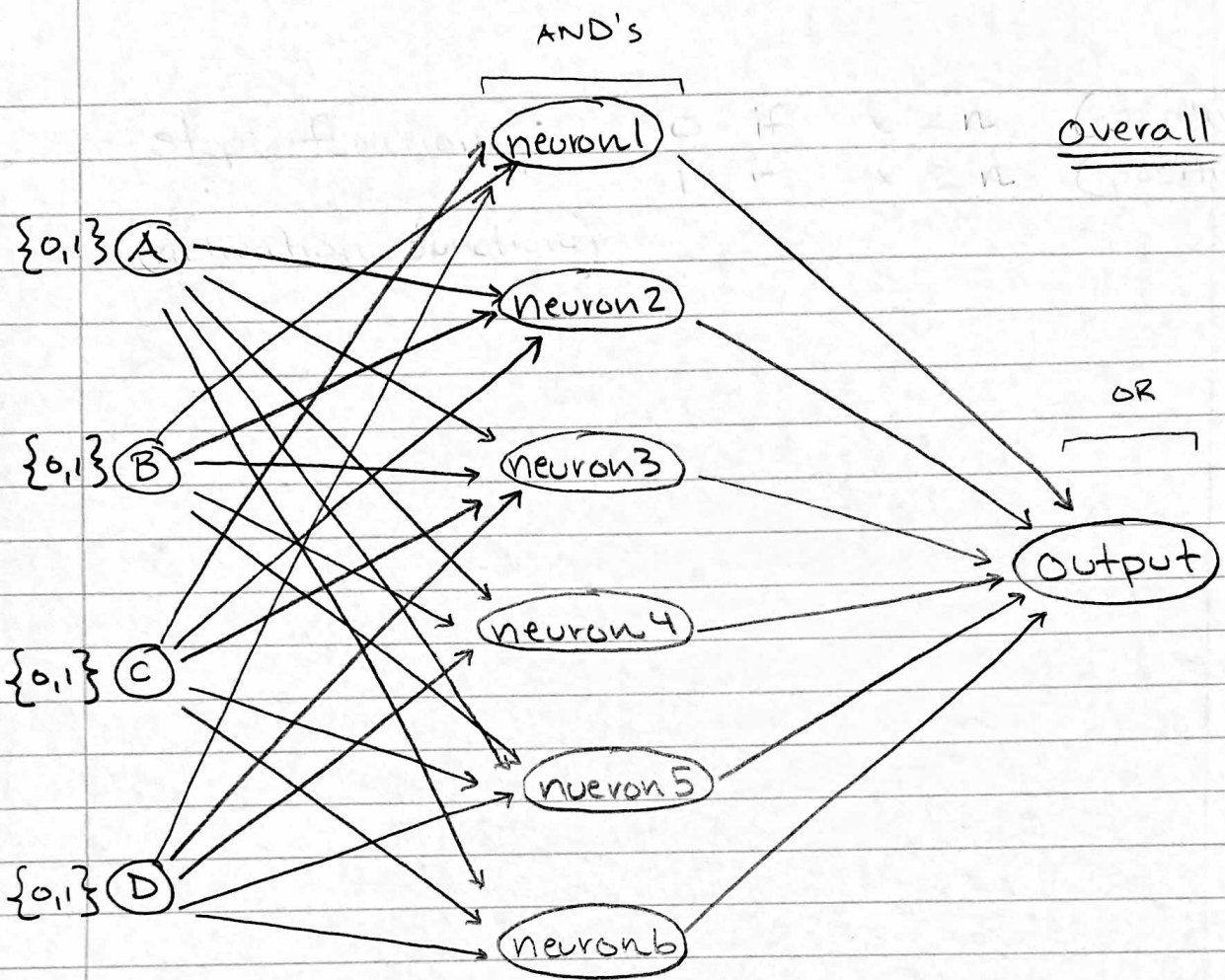


2. create $(A \vee \neg B) \oplus (\neg C \vee D)$

A	B	C	D	$A \vee \neg B$	$\neg C \vee D$	output
0	0	0	0	1	1	0
0	0	0	1	1	1	0
0	0	1	0	1	0	1 ✗
0	0	1	1	1	1	0
0	1	0	0	0	1	1 ✗
0	1	0	1	0	1	1 ✗
0	1	1	0	0	0	0
0	1	1	1	0	1	1 ✗
1	0	0	0	1	1	0
1	0	0	1	1	1	0
1	0	1	0	1	0	1 ✗
1	0	1	1	1	1	0
1	1	0	0	1	1	0
1	1	0	1	1	1	0
1	1	1	0	1	0	1 ✗
1	1	1	1	1	1	0

	A	B	C	D
neuron 1:	✗	0	1	0
neuron 2:	0	1	0	✗
neuron 3:	0	1	0	1
neuron 4:	0	1	✗	1
neuron 5:	1	0	1	0
neuron 6:	1	✗	1	0

have a hidden neuron for each case.



(broken down:)

$\begin{array}{l} \textcircled{B} \xrightarrow{-1} \\ \textcircled{C} \xrightarrow{5} \\ \textcircled{D} \xrightarrow{-1} \end{array} \rightarrow \text{neuron1}$	$f = -B + 5C - D$	$\begin{array}{l} f \geq 5 = 1 \\ f < 5 = 0 \end{array}$
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$\begin{array}{l} \textcircled{A} \xrightarrow{-1} \\ \textcircled{B} \xrightarrow{5} \\ \textcircled{C} \xrightarrow{-1} \end{array} \rightarrow \text{neuron2}$	$f = -A + 5B - C$	$\begin{array}{l} f \geq 5 = 1 \\ f < 5 = 0 \end{array}$
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$\begin{array}{l} \textcircled{A} \xrightarrow{-1} \\ \textcircled{B} \xrightarrow{5} \\ \textcircled{C} \xrightarrow{-1} \\ \textcircled{D} \xrightarrow{5} \end{array} \rightarrow \text{neuron3}$	$f = -A + 5B - C + 5D$	$\begin{array}{l} f \geq 10 = 1 \\ f < 10 = 0 \end{array}$
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