

in the official keras and tensorflow documentation, we further know

$$e^2 = w^2 e^3 (a^2 (1 - a^2)) = ((w^2)^T e^3) \cdot a^2 (1 - a^2)$$

$$e^2 = \begin{bmatrix} 0.128473 \\ 0.37641 \\ 0.019821 \end{bmatrix}$$

propagation update with SGD

$$w_{ij}^{(1)} = w_{ij}^{(1)} - \alpha * (d_j' * e_{2i}) \quad \text{for } i \text{ in range } (1, 3) \\ j \text{ in range } (1, 9)$$

$$w_{ij}^{(2)} = w_{ij}^{(2)} - \alpha * (d_j^2 * e_{3i}) \quad \text{for } i \text{ in range } (1, 10) \\ j \text{ in range } (1, 4)$$

W1

$$= \begin{bmatrix} 1.06 & 0.846 & 0.591 & -0.321 & -0.579 & -0.138 & -0.62 & 0.02 & 0.681 \\ -0.47 & -1.24 & 1.261 & 0.341 & 0.721 & 0.943 & 0.27 & 0.62 & 1.463 \\ 1.231 & 0.693 & -1.243 & 1.168 & -1.341 & 0.812 & -0.33 & -0.131 & 0.701 \end{bmatrix}$$

$$= \begin{bmatrix} 1.414 & 1.463 & 1.043 & -0.531 \\ -1.41 & 0.921 & -0.476 & 0.291 \\ -0.812 & -0.813 & -0.071 & 0.543 \\ 0.193 & -0.221 & 0.203 & 0.236 \\ -1.01 & -0.231 & 0.654 & 1.304 \\ -0.81 & -0.504 & -0.136 & 0.789 \\ -0.37 & -1.056 & 0.067 & 0.328 \\ 0.39 & 0.172 & 1.042 & -0.768 \\ -0.01 & -0.963 & 0.071 & 0.88 \\ 0.65 & -0.378 & -0.031 & -0.34 \end{bmatrix}$$

W2