Stochastic Processes, Long Quiz, 2025 Fall

Solution and Grading

- Duration: 60 minutes
- Weight: 10% of final grade
- Closed material, No calculator

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- Write legibly.
- Justification is necessary unless stated otherwise.

#1. State the definition of memoryless property by providing a mathematical expression. [5pts]

For a random variable X, X is memoryless if $P(X > s + t|X > t) = P(X > s)$ for all $s, t \geq 0$.

Difficulty: Medium
Amount of work: 50%

#2. Let $X \sim exp(\lambda)$. Find $\mathbb{E}X^2$. `[5pts]`

$$\begin{aligned}
\mathbb{E}X^2 &= \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{\infty} x^2 \lambda e^{-\lambda x}\,dx = \lambda \int_0^{\infty} x^2 e^{-\lambda x}\,dx \\
&= \lambda\left(x^2 \cdot \frac{-1}{\lambda} e^{-\lambda x}\Big|_0^{\infty} - \int_0^{\infty} 2x \cdot \frac{-1}{\lambda} e^{-\lambda x}\,dx\right) \\
&= \lambda\left((0-0) + \frac{2}{\lambda}\int_0^{\infty} x e^{-\lambda x}\,dx\right) \\
&= 2\int_0^{\infty} x e^{-\lambda x}\,dx \\
&= 2\left[x \cdot \frac{-1}{\lambda} e^{-\lambda x}\Big|_0^{\infty} - \int_0^{\infty} \frac{-1}{\lambda} e^{-\lambda x}\,dx\right] \\
&= 2\left[\infty \cdot \frac{-1}{\lambda} e^{-\lambda\infty} - \left(0 \cdot \frac{-1}{\lambda} e^{-\lambda\cdot 0}\right) + \frac{1}{\lambda}\left[\frac{-1}{\lambda} e^{-\lambda x}\right]_0^{\infty}\right] \\
&= 2\left[0 - 0 + \frac{1}{\lambda}\left[\frac{-1}{\lambda} e^{-\lambda\infty} + \frac{1}{\lambda} e^{-\lambda\cdot 0}\right]\right] \\
&= 2 \cdot \frac{1}{\lambda}\left(0 + \frac{1}{\lambda}\right) = \frac{2}{\lambda^2}
\end{aligned} \tag{1}$$

Difficulty: Medium
Amount of work: 50%