ITM426, Quiz 1, 2023 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 23F
- Sep 18, 2023
- Duration: 60 minutes
- Weights: 10% or 20% depending on other quiz scores
- 5 Questions

- Name: ____________________
- Student ID: ____________________
- E-mail: ______________ `@seoultech.ac.kr`

- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	10
2	10
3	10
4	15
5	15
Total	60

#1. Complete the following statement for the definition of linear independence. `[10pt]`

> "Consider a set of k vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_k}$. If (*complete here*), then we say the vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_k}$ are **linearly independent**"

Difficulty: Easy
Amount of work: 20 %
Suggested answer:
"If any vector cannot be expressed as a linear combination of the other $k-1$ vectors".

#2. Prove the following statement. `[10pt]`

- For a 2×2 matrix A, if its row vectors are independent, then its column vectors are independent.

Difficulty: Medium
Amount of work: 20 %
Suggested answer:
Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. We have that its row vectors are independent. Now, consider a matrix whose column vectors are (a, b) and (c, d), i.e. $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$. Since the column vectors are row vectors of matrix A, they are independent and its determinant $ad - bc$ is nonzero. Now, consider the determinant of original matrix A, which is $ad - bc$, which is nonzero, thus the column vectors of the matrix A are independent.

#3. Let $\mathbf{y} = [2 \quad 4]$ and $\mathbf{u} = [6 \quad 2]$. Compute the vector $\mathbf{z}$ such that $\mathbf{z} = \frac{\mathbf{y} \bullet \mathbf{u}}{\mathbf{u} \bullet \mathbf{u}} \mathbf{u}$, where $\bullet$ is the dot-product operator. `[10pt]`

Difficulty: Medium
Amount of work: 20%
Solution: $\mathbf{z} = [3 \quad 1]$.

#4. For a $n \times n$ matrix A, the followings are all equivalent. Complete the other three bullet points. `[each 5pt]`

- $A\mathbf{x} = \mathbf{b}$ has a unique solution.
-
-
-

Difficulty: Medium
Amount of work: 20%
Solution:
a) Its column vectors are linearly independent. b) The matrix A has zero-determinant. c) The matrix A is non-singular. d) A is invertible.

#5. Prove that the vectors $\mathbf{v}_1 = (0, 1, -1)$, $\mathbf{v}_2 = (1, 0, 1)$, and $\mathbf{v}_3 = (-1, 1, 0)$ span a 3-dimensional vector space. `[15pt]`

Difficulty: Hard
Amount of work: 20%
Solution:
Let $\mathbf{v} = (x, y, z)$ be an arbitrary real-numbered 3-dimensional vector. We claim that $\mathbf{v}$ can be expressed as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ such as $\mathbf{v} = (x, y, z) = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$, where a, b, c are all real numbers.
After some work, it can be shown that $(x, y, z) = \frac{x+y-z}{2}(0, 1, -1) + \frac{x+y+z}{2}(1, 0, 1) + \frac{-x+y+z}{2}(-1, 1, 0)$. And all coefficients are real numbers. This proves the claim. $\square$

(blank)

Write your name before detaching this page. Your Name: ______________________________