Stochastic Processes, Final, 2025 Fall

- Duration: 120 minutes

- Weight: 35% of final grade

- Closed material, No calculator

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- Write legibly.

- Justification is necessary unless stated otherwise.

#1. Let $X = \{X(t) : t \geq 0\}$ be a continuous time Markov chain with state space $\{1, 2, 3\}$ with a rate matrix G of the following:

$$G = \begin{pmatrix} -4 & ? & 1 \\ 1 & ? & 2 \\ 1 & ? & -2 \end{pmatrix}$$

Let us assume we have computed e^G as follows:

$$e^G = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.3 & 0.4 & 0.4 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}.$$

(a) Find the stationary distribution. [5pts]

(b) Find $\mathbb{P}\big[X(1) = 2, \ X(3) = 3 \mid X(0) = 2\big]$. [5pts]

#2. Customers arrive at a two-server system according to a Poisson process with rate $\lambda = 10$ per hour. An arrival finding server 1 (John) free will begin his service with server 1. An arrival finding server 1 busy and server 2 (Mary) free will join server 2. An arrival finding both servers busy goes away. Once a customer is served by either server, he departs the system. The service times at both servers are exponential random variables. Assume that the service rate of the first server is 6 per hour and the service rate of the second server is 4 per hour. Describe a continuous-time Markov chain to model the system and give the rate transition diagram.[5pts]

#3. An AI data center receives queries arriving according to a Poisson process with an average rate of 2500 queries per hour. Processing each query consumes 4 Wh of energy.

(a) What is the expected hourly energy consumption for processing the queries? **[5pts]**

(b) Approximate the 95% quantile of the hourly energy consumption using a normal approximation. (Hint: What is the standard deviation for hourly energy consumption?)**[5pts]**

#4. Consider a DTMC with the following transition matrix.

$$P = \begin{pmatrix} 0.25 & 0.5 & 0.25 & 0 & 0 & 0 \\ 0.5 & 0.25 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0.7 \\ 0 & 0 & 0 & 0 & 0.7 & 0.3 \end{pmatrix}$$

What is P^{100}? [10pts]