ITM426, Quiz 1, 2022 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 22F
- Sep 23, 2022
- Duration: 60 minutes
- Weights: 10% or 20% depending on other quiz scores
- 5 Questions

- Name: ______________________
- Student ID: ______________________
- E-mail: ______________ @seoultech.ac.kr

- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	15
2	20
3	15
4	15
5	15
Total	80

#1. Show that the set of the following vectors are linearly dependent. `[15pt]`

$$(2, 3, 0),\ (0, 2, -1),\ (4, 8, -1)$$

Difficulty: Easy
Amount of work: 20 %
Suggested answer:
With the three vectors notated as $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$, then $\mathbf{z} = 2\mathbf{x} + \mathbf{y}$. Hence, dependent.

#2. Complete the following theorem. [20pt]

For a $n \times n$ matrix A, the followings are all equivalents.

- (invertibility) The matrix A is invertible.
- (determinant)
- (solution of $A\mathbf{x} = \mathbf{b}$)
- (singularity)
- (column vectors)

Difficulty: Easy
Amount of work: 20 %
Suggested answer:
(determinant) The determinant is not zero. ($A\mathbf{x} = \mathbf{b}$) The solution is unique (singularity) A is singular (column vectors) The set of column vectors are linear independent.

#3. Prove the following statement. `[15pt]`

- For a 2×2 matrix A, if its column vectors are independent, then its row vectors are independent.

Difficulty: Medium
Amount of work: 20 %
Suggested answer:
Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Since its column vectors are independent, the determinant $ad - bc \neq 0$. The row vectors of A are (a, b) and (c, d). Setting these two vectors into a column vector of a matrix, i.e. $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$. This matrix has non-zero determinant $ad - cb \neq 0$. Thus, the column vectors of this matrix, (a, b) and (c, d), are independent. In other words, the row vectors of A are independent.

#4. Write the matrix formular for the following system of linear equation. Find the inverse of the coefficient matrix. Find the solution to the system of linear equation in vector form. `[15pt]`

$$\begin{aligned} 2x + 3y &= 13 \\ 4x + 2y &= 14 \end{aligned}$$

Difficulty: Medium
Amount of work: 20 %
Suggested answer:
After writing $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \end{bmatrix}$, the solution is $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8} \cdot \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 13 \\ 14 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

#5. Suppose that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a basis of 3-dimensional vector space. Carefully show that $\{\mathbf{x}, \mathbf{x}-\mathbf{y}, \mathbf{x}+\mathbf{y}-\mathbf{z}\}$ is a basis of 3-dimensional vector space as well. `[15pt]`

Difficulty: Medium-Hard
Amount of work: 20%
Solution: The proof is very similar to the one to Problem 11 in the prenote.

(blank)

Write your name before detaching this page. Your Name: ______________________

(blank)