

ITM426, Quiz 1, 2022 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 22F
 - Sep 23, 2022
 - Duration: 60 minutes
 - Weights: 10% or 20% depending on other quiz scores
 - 5 Questions
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- Name: _____
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- Write legibly.
 - Justification is necessary unless stated otherwise.
 - Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	15
2	20
3	15
4	15
5	15
Total	80

#1. Show that the set of the following vectors are linearly dependent. [15pt]

$$(2, 3, 0), (0, 2, -1), (4, 8, -1)$$

Difficulty: Easy

Amount of work: 20 %

Suggested answer:

With the three vectors notated as \mathbf{x} , \mathbf{y} , and \mathbf{z} , then $\mathbf{z} = 2\mathbf{x} + \mathbf{y}$. Hence, dependent.

#2. Complete the following theorem. [20pt]

For a $n \times n$ matrix A , the followings are all equivalents.

- (invertibility) The matrix A is invertible.
- (determinant)
- (solution of $A\mathbf{x} = \mathbf{b}$)
- (singularity)
- (column vectors)

Difficulty: Easy

Amount of work: 20 %

Suggested answer:

- (invertibility) The matrix A is invertible.
- (determinant) The determinant is not zero.
- (solution of $A\mathbf{x} = \mathbf{b}$) The solution is unique.
- (singularity) A is non-singular.
- (column vectors) The set of column vectors are linear independent.

#3. Prove the following statement. [15pt]

- For a 2×2 matrix A , if its column vectors are independent, then its row vectors are independent.

Difficulty: Medium

Amount of work: 20 %

Suggested answer:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Since its column vectors are independent, the determinant $ad - bc \neq 0$. The row vectors of A are (a, b) and (c, d) . Setting these two vectors into a column vector of a matrix, i.e. $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$. This matrix has non-zero determinant $ad - cb \neq 0$. Thus, the column vectors of this matrix, (a, b) and (c, d) , are independent. In other words, the row vectors of A are independent.

#4. Write the matrix formular for the following system of linear equation. Find the inverse of the coefficient matrix. Find the solution to the system of linear equation in vector form.[15pt]

$$\begin{aligned}2x + 3y &= 13 \\4x + 2y &= 14\end{aligned}$$

Difficulty: Medium

Amount of work: 20 %

Suggested answer:

After writing $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \end{bmatrix}$, the solution is $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8} \cdot \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 13 \\ 14 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

#5. Suppose that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a basis of 3-dimensional vector space. Carefully show that $\{\mathbf{x}, \mathbf{x} - \mathbf{y}, \mathbf{x} + \mathbf{y} - \mathbf{z}\}$ is a basis of 3-dimensional vector space as well. [15pt]

Difficulty: Medium-Hard

Amount of work: 20%

Solution: The proof is very similar to the one to Problem 11 in the prenote.

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Write your name before detaching this page. Your Name: _____

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