ITM426, Quiz 3, 2023 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 23F
- Nov 27, 2023
- Duration: 90 minutes
- Weights: 25% or 30% depending on other quiz scores
- 5 Questions

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- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	20
2	10
3	10
4	10
5	10
Total	60

#1. Mark True or False. No justification is necessary. `[Each 5pt]`

- If $H = Span\{\mathbf{b}_1, ..., \mathbf{b}_p\}$, then $\{\mathbf{b}_1, ..., \mathbf{b}_p\}$ is a basis for H. (`TRUE / FALSE`)
- In some cases, the linear dependence relations among the columns of a matrix can be affected by certain elementary row operations on the matrix. (`TRUE / FALSE`)
- A $n \times n$ matrix is diagonizable if A has n eigenvalues. (`TRUE / FALSE`)
- A single vector itself is linearly dependent. (`TRUE / FALSE`)

Difficulty: Hard
Amount of work: 20 %
Suggested answer:

- False. The set must be independent.
- False. Elementary row operations do not affect the linear dependence relations.
- False. The n eigenvectors must be linearly independent.
- False. The zero vector itself is linearly dependent.

#2. Prove the following statement or disprove it by a counterexample. [10pt]

A $n \times n$ matrix A is both diagonalizable and invertible, then so is A^{-1}.

Difficulty: Medium
Amount of work: 20%
Suggested answer:
If A is diagonalizable, then $A = PDP^{-1}$ for some invertible P and diagonal matrix D. Since A is invertible, 0 is not an eigenvalue of A. So the diagonal entries in D (which are eigenvalues of A) are not zero, and D is invertible. By the theorem on the inverse of a product, $A = (PDP^{-1})^{-1} = (P^{-1})^{-1}D^{-1}P^{-1}$. Since D^{-1} is obviously diagonal, A^{-1} is diagonalizable.

#3. Find a nonzero 2×2 matrix that is diagonalizable but not invertible. `[10pt]`

Difficulty: Medium-Hard
Amount of work: 20%
Suggested answer:

Any 2×2 matrix with two distinct eigenvalues is diagonalizable, by Theorem 6. If one of those eigenvalues is zero, then the matrix will not be invertible. Any matrix of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ has the desired properties when a and b are nonzero. The number a must be nonzero to make the matrix diagonalizable; b must be nonzero to make the matrix not diagonal. Other solutions are $\begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix}$ and $\begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix}$.

#4. Assume that A is row equivalent to B. Find bases for $Nul\,A$ and $Col\,A$.`[Each 5pts]`

$$A=\begin{bmatrix}-2 & 4 & -2 & -4\\ 2 & -6 & -3 & 1\\ -3 & 8 & 2 & -3\end{bmatrix},\ B=\begin{bmatrix}1 & 0 & 6 & 5\\ 0 & 2 & 5 & 3\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Difficulty: Easy
Amount of work: 20%
Suggested answer:

Since B is a row echelon form of A, we see that the first and second columns of A are its pivot columns. Thus a basis for Col A is $\left\{\begin{bmatrix}-2\\2\\-3\end{bmatrix},\begin{bmatrix}4\\-6\\8\end{bmatrix}\right\}$.

To find a basis for Nul A, we find the general solution of $A\mathbf{x}=\mathbf{0}$ in terms of the free variables: $x_1=-6x_3-5x_4$, $x_2=(-5/2)x_3-(3/2)x_4$, with x_3 and x_4 free. So

$$\mathbf{x}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=x_3\begin{bmatrix}-6\\-5/2\\1\\0\end{bmatrix}+x_4\begin{bmatrix}-5\\-3/2\\0\\1\end{bmatrix}\text{, and a basis for Nul } A \text{ is } \left\{\begin{bmatrix}-6\\-5/2\\1\\0\end{bmatrix},\begin{bmatrix}-5\\-3/2\\0\\1\end{bmatrix}\right\}.$$

#5. Find the determinants of the following matrices.`[10pts]`

$$\begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 20%
Suggested answer: -10

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Write your name before detaching this page. Your Name: ______________________