ITM426, Final, 2023 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 23F
- Dec 18, 2023
- Duration: 90 minutes
- Weights: 30%
- 5 Questions

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- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1,2	10
3	10
4	10
5	10
6	10
Total	50

#1. Fill in the blank to complete the definition. `[5pt]`

> A function $g : \mathbb{R} \to \mathbb{R}$ is ( ) on a set $S \subseteq \mathbb{R}$, if there is a constant γ, with $0 < \gamma < 1$, such that $||g(x) - g(z)|| \leq \gamma ||x - z||$ for all $x, z \in S$.

Difficulty: Medium
Amount of work: 10 %
Suggested answer: contractive

#2. Mark True or False regarding the following statement. No justification is necessary. `[5pt]`

> Consider applying fixed point iteration method to solve an equation $f(x) = 0$. One must have the derivative form of $f(x)$ analytically available.

(`TRUE` / `FALSE`)

Difficulty: Easy
Amount of work: 10 %
Suggested answer: FALSE. As long as evaluating $f(x)$ is possible, the fixed point method is applicable.

#3. Consider solving an equation $f(x) = 9x - 7 = 0$ by the interval bisection method. Assume the initial interval is given as $[0, 1]$ with a tolerence of 0.2. Carefully perform the interval bisection method and report the final interval. `[10pt]`

Difficulty: Medium-Hard
Amount of work: 20 %
Suggested answer: Students are expected to follow the procedure and the interval changes as the following: $[0, 1] \rightarrow [0.5, 1] \rightarrow [0.75, 1] \rightarrow [0.75, 0.875]$.

#4. From an experiment, four observations of two variables are collected: $(1,2),(2,6),(3,4)$, and $(4,8)$. Set up a normal equation for a linear regression. `[10pt]`

Difficulty: Medium
Amount of work: 20 %
Suggested answer:
Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 6 \\ 4 \\ 8 \end{bmatrix}$, then the normal equation is $A^T A\mathbf{x} = A^T\mathbf{b}$. It leads to its least-squre solution of $\mathbf{x} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 1 \\ 8/5 \end{bmatrix}$

#5. The given set of vectors is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.`[10pt]`

$$\begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix}$$

Difficulty: Medium
Amount of work: 20 %
Suggested answer:

Set $\mathbf{v}_1 = \mathbf{x}_1$ and compute that $\mathbf{v}_2 = \mathbf{x}_2 - \frac{\mathbf{x}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 = \mathbf{x}_2 - (-3)\mathbf{v}_1 = \begin{bmatrix} 4 \\ 6 \\ -3 \\ 0 \end{bmatrix}$. Thus an orthogonal basis for W is $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -3 \\ 0 \end{bmatrix} \right\}$.

Then, the orthonormal basis is obtained by applying constants of $1/\sqrt{3^2 + (-1)^2 + 2^2 + (-1)^2} = 1/\sqrt{15}$ and $1/\sqrt{4^2 + 6^2 + (-3)^2 + 0^2} = 1/\sqrt{61}$, respectively.

#6. Prove the following statement or disprove it by a counterexample. `[10pt]`

For a square matrix A, vectors in $Col\, A$ are orthogonal to vectors in $Nul\, A$.

Difficulty: Medium
Amount of work: 20 %
Suggested answer:
False. Counterexample: $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(blank)

Write your name before detaching this page. Your Name: ______________________________