ITM426, Mid-term, 2019 Fall

Code	ITM 426
Title	Engineering Math.
Time for Exam	2 hours
Questions	8
Weighting	20 %

- Name: ______________________
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- Closed book, closed notes, no calculator.
- Only writing instrments are allowed on the desk.
- Absolutely no phone on the desk.
- Do not remove the original staple.
- You may detach the last three sheets of this packet.
- Write legibly.

"Exams are formidable even to the best prepared, for the greatest fool may ask more than the wisest man can answer". - Charles Caleb Colton

1	10
2	10
3	5
4	10
5	5
6	5
7	5
8	10
Total	60

#1. For each of the following statements, write either True or False. If True, then explain your reasoning. If False, then provide a **counter-example**.[each 2pts]

- (a) If A is a 3×3, then $|5A| = 5|A|$.
- (b) IF A is a 2×2 matrix with a zero determinant, then all elements of A are equal to zero.
- (c) Every square matrix is a product of elementary matrices.
- (d) If A and B are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of AB.
- (e) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on the columns of the $n \times n$ identity matrix.

#2. Find the general solutions of the systems whose augmented matrices are given as follows: [`10pts`]

(a)

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & 5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

#3. Suppose the solution set of a certain system of linear equations can be described as $x_1 = 3x_4, x_2 = 8 + x_4, x_3 = 2 - 5x_4$, with x_4 free. Use vectors to describe this set as a "line" in $\mathbb{R}^4$. [5pts]

#4. Consider the following vectors

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, \mathbf{c}_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}.$$

(a) Find a linear transformation matrix that translate $\mathbf{b}_1$ into $\mathbf{c}_1$ and $\mathbf{b}_2$ into $\mathbf{c}_2$. [5pts]

(b) What would the above linear transformation translate a vector $z = [2, -8]^t$ into?[5pts]

#5. Find the inverse of the following matrix.[`5pts`]

$$\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

#6. Note that followings are regarding block matrices. Suppose A_{11} is invertible, Find X and Y such that

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} I & O \\ X & I \end{bmatrix} \begin{bmatrix} A_{11} & O \\ O & S \end{bmatrix} \begin{bmatrix} I & Y \\ O & S \end{bmatrix},$$

where $S = A_{22} - A_{21}A_{11}^{-1}A_{12}$. The matrix S is called the **Schur complement**.[`5pts`]

#7. Find an LU factorization of the following matrix.[5pts]

$$\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$

#8. Find the determinants of the following matrices.[5pts]
(a)

$$\begin{bmatrix} 1 & 5 & -4 \\ -1 & -4 & 5 \\ -2 & -8 & 7 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix}$$

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