# ITM529. Stochastic Processes

Short Quiz 1, 2025F, Weighting of 5%

Solution

**Name: [ ]**

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**#1.** For $X \sim \text{Poi}(\lambda)$, show that $\text{Var}(X) = \lambda$. `[5pt]`

$$
\begin{aligned}
\mathbb{E}[X^2] &= \sum_{x=-\infty}^{\infty} x^2 p(x) = \sum_{x=0}^{\infty} x^2 \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} x^2 \frac{\lambda^x}{x!} \\
&= e^{-\lambda} \left( 0^2 \frac{\lambda^0}{0!} + \sum_{x=1}^{\infty} x^2 \frac{\lambda^x}{x!} \right) = e^{-\lambda} \left( \sum_{x=1}^{\infty} x \frac{\lambda^x}{(x-1)!} \right) \\
&= e^{-\lambda} \left( \sum_{x=1}^{\infty} (x-1+1) \frac{\lambda^x}{(x-1)!} \right) \\
&= e^{-\lambda} \left[ \sum_{x=1}^{\infty} (x-1) \frac{\lambda^x}{(x-1)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} \right] \\
&= e^{-\lambda} \left[ \sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} \right]
\end{aligned}
$$

Change of variables: $y := x - 2$, $z := x - 1$:

$$
\begin{aligned}
&= e^{-\lambda} \left[ \lambda^2 \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} + \lambda \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} \right] \\
&= e^{-\lambda} \left[ \lambda^2 e^{\lambda} + \lambda e^{\lambda} \right] = \lambda^2 + \lambda
\end{aligned}
$$

Finally,

$$
\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = (\lambda^2 + \lambda) - \lambda^2 = \boxed{\lambda}.
$$

**Difficulty: Easy**

**Amount of work: 100%**