ITM426, Long Quiz 1, 2025 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 25F
- Duration: 60 minutes
- Weights: 10%
- 2 Questions

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- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

#1. Let $\mathbf{y} = (2, 4)$ and $\mathbf{u} = (6, 2)$.

a) Compute the vector $\mathbf{z}$ such that

$$\mathbf{z} = \frac{\mathbf{y} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}} \mathbf{u},$$

where $\cdot$ is the dot-product operator. `[2.5pt]`

b) Draw the vector y, u, and z in a two-dimensional space as precisely as possible. `[2.5pt]`

Difficulty: Easy
Amount of work: 50%
Solution:
$\mathbf{z} = (3, 1)$. Students are expected to mark the vectors in 2D grid, where $\mathbf{y}$ and $\mathbf{z}$ are overlapped.

#2. We have a matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Let us call each column vector of matrix A as $\mathbf{v_1}, \mathbf{v_2}$, and $\mathbf{v_3}$. Prove that the set of these three vectors span a 3-dimensional vector space.[5pt]

Difficulty: Medium
Amount of work: 50%
Solution:
Let $\mathbf{v} = (x, y, z)$ be an arbitrary three dimensional vector. Identifying the relationship $(x, y, z) = \frac{-x+y+z}{2}(0, 1, 1) + \frac{x-y+z}{2}(1, 0, 1) + \frac{x+y-z}{2}(1, 1, 0)$ is a key to the proof. For the full version of the proof, please refer to the prenote.