Stochastic Processes, Mid-term, 2025 Fall

- Duration: 90 minutes
- Weight: 30% of final grade
- Closed material, No calculator

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- Write legibly.
- Justification is necessary unless stated otherwise.

#1. The selling price of lettuce salad is $6, the buying price of one unit of lettuce is $1. Of course, leftover lettuce of a day cannot be used for future salad and you have to pay 50 cents per unit of lettuce for disposal. The demand for the lettuce salad is given as discrete uniform distribution between 21 and 30.

(a) What is the optimal order quantity for lettuce salad? [5pts]

(b) What is the expected profit if 25 units of lettuce salad is ordered? [5pts]

#2. A small bank is staffed by a single server. It has been observed that, during a normal business day, the inter-arrival times of customers to the bank are iid having exponential distribution with mean 3 minutes. Also, the processing times of customers are iid having the following distribution (in minutes):

$$P(X = 1) = \frac{1}{4}, \quad P(X = 2) = \frac{1}{2}, \quad P(X = 3) = \frac{1}{4}.$$

An arrival finding the server busy joins the queue. The waiting space is infinite.

(a) What is long-run average waiting time of each customer in the queue? `[5pts]`

(b) What is long-run average number of customers in the bank? `[5pts]`

#3. A company operates under an (S, s) inventory policy with parameters $S = 5$ and $s = 2$. That is, at the end of each day, the inventory level is checked. If the inventory is less than or equal to 2 units, an order is placed to raise the stock level to 5 units. Orders are replenished before the beginning of the next day. The daily demand follows a discrete uniform distribution between 1 and 4 units (inclusive).

(a) Any item of inventory at the end of the day counts toward the holding cost. If the holding cost is \$10 per unit per day, what is the average daily holding cost? `[5pts]`

(b) Over a 30-day month, how many times is an order placed on average? `[5pts]`