ITM426, Quiz 2, 2023 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 23F
- Oct 30, 2023
- Duration: 90 minutes
- Weights: 25% or 30% depending on other quiz scores
- 5 Questions

- Name: ______________________
- Student ID: ______________________
- E-mail: ____________ @seoultech.ac.kr

- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	20
2	10
3	10
4	10
5	10
Total	60

#1. Mark True or False. No justification is necessary. `[Each 5pt]`

- If A is an $m \times n$ matrix and if the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b}$ in $\mathbb{R}^m$, then A cannot have a pivot position in every row. (`TRUE / FALSE`)
- Any linear combination of vectors can always be written in the form $A\mathbf{x}$ for a suitable matrix A and vector $\mathbf{x}$. (`TRUE / FALSE`)
- A homogeneous equation is always consistent. (`TRUE / FALSE`)
- If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent. (`TRUE / FALSE`)

Difficulty: Hard
Amount of work: 20 %
Suggested answer:

- True. See Theorem 4 in the Chapter 1.
- True. See Example 2 in the Section 1.4, for example.
- True. Because a homogeneous system always has a trivial solution, which makes the system consistent.
- False. For instance, a set consisting of $(1, 2, 3)$ and $(2, 4, 6)$ is linearly dependent.

#2. Describe all solutions of the homogeneous system in parametric vector form
Find all solutions to the system of the homogeneous equations of A. [10pt]

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 20 %
Suggested answer:

$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & -2 & 0 & 0 & 7 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 - 4x_2 \qquad\quad + 5x_6 &= 0 \\ x_3 \qquad - x_6 &= 0 \\ x_5 - 4x_6 &= 0 \\ 0 &= 0 \end{aligned}$$

. The basic variables are x_1, x_3, and x_5. The remaining variables are free. In particular, x_4 is free (and not zero as some may assume). The solution is $x_1 = 4x_2 - 5x_6$, $x_3 = x_6$, $x_5 = 4x_6$, with x_2, x_4, and x_6 free. In parametric vector form,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4x_2 - 5x_6 \\ x_2 \\ x_6 \\ x_4 \\ 4x_6 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4x_2 \\ x_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -5x_6 \\ 0 \\ x_6 \\ 0 \\ 4x_6 \\ x_6 \end{bmatrix} = x_2 \underset{\substack{\uparrow \\ \mathbf{u}}}{\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}} + x_4 \underset{\substack{\uparrow \\ \mathbf{v}}}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}} + x_6 \underset{\substack{\uparrow \\ \mathbf{w}}}{\begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}}$$

#3. Find an LU factorization of the following matrix `[10pt]`

$$A = \begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 20%
Suggested answer:

$$\begin{bmatrix} 1 & & \\ -2 & 1 & \\ -3 & -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & 3 & 4 \\ & -2 & -1 \\ & & 9 \end{bmatrix} \tag{1}$$

#4. Find an inverse of the following matrix [10pt]

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 1/2 & 1 & 0 \\ 1 & 0 & 1/2 & 1 \end{bmatrix}$$

Difficulty: Medium
Amount of work: 20 %
Suggested answer:

$$A^{-1} = \begin{bmatrix} 1 & & & \\ -1/2 & 1 & & \\ 1/4 & -1/2 & 1 & \\ -9/8 & 1/4 & -1/2 & 1 \end{bmatrix}$$

#5. After a linear transformation, SeoulTech's amblem has been transformed. Specifically, the emblem is horizontally reversed and its horizontal length is doubled. Suggest a standard matrix for this linear transformation and explain it. `[10pt]`

before	after
	

Difficulty: Hard
Amount of work: 25 %
Suggested answer:
Doubing in x-axis can be achieved by $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, and reflection through y-axis can be achieved by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Doing these consecutively means multiplication of these matrices, which leads to a standard matrix of $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$

(blank)

Write your name before detaching this page. Your Name: ______________________