ITM426, Quiz 1, 2024 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 24F
- Sep 20, 2024
- Duration: 60 minutes
- Weights: 20%
- 5 Questions

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- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	10
2	10
3	10
4	10
5	10
Total	50

#1. We have a matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $E = \begin{bmatrix} -3 & & \\ 2 & 2 & \\ & 1 & -1 \end{bmatrix}$. Let us call each row vector of matrix A as $(R1)$, $(R2)$, and $(R3)$. Describe each row of a matrix EA in terms of rows of A. For example, you can write such as "The first row of EA is $-2 \times (R1) + 5 \times (R2)$". [10pt]

1. The first row of EA is ________________
2. The second row of EA is ________________
3. The third row of EA is ________________

Difficulty: Medium
Amount of work: 20%
Solution:
$-3(R1)$; $2(R2) + 2(R1)$; $-(R3) + (R2)$

#2. Consider a set of k vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_k}$. What does it mean by *this set of vectors is linearly independent*? Provide a clear definition. `[10pt]`

Difficulty: Easy
Amount of work: 20 %
Suggested answer:
"If any vector cannot be expressed as a linear combination of the other $k - 1$ vectors, then this set of vectors are said to be linearly independent.".

#3. Suppose we have a $n \times n$ matrix A that its determinant of A is zero. Among the following statements, select those are true. `[10pt]`

- (a) $A\mathbf{x} = \mathbf{b}$ has a unique solution.
- (b) $A\mathbf{x} = \mathbf{b}$ does not have a unique solution.
- (c) A is non-singular.
- (d) A is singular.
- (e) A^{-1} does not exist.
- (f) A^{-1} exists.
- (g) A set of column vectors in A is linearly dependent.
- (h) A set of column vectors in A is linearly independent.

Difficulty: Easy
Amount of work: 20 %
Suggested answer:
(b),(d),(e),(g)

#4. What is the value of c that makes the determinant of A to be zero?

$$A = \begin{bmatrix} c & 0 & 3 \\ 1 & 2 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

The following theorem may be useful. `[10pt]`

> "For a square matrix, its determinant is nonzero if and only if its column vectors are linearly independent."

Difficulty: Medium
Amount of work: 20 %
Suggested answer:
To have zero determinant, its column vectors must be linearly dependent. Thus, $(c\ 1\ 0)$ must be linear combination of $(0\ 2\ -1)$ and $(3\ 2\ 1)$. Setting $(c\ 1\ 0) = x(0\ 2\ -1) + y(3\ 2\ 1)$ gives $x = y = 1/4$. Thus, $c = 3/4$.

#5. Suppose that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a basis of 3-dimensional vector space. Carefully show that $\{\mathbf{x}, \mathbf{x}+\mathbf{y}, \mathbf{x}-\mathbf{y}+2\mathbf{z}\}$ is a basis of 3-dimensional vector space as well. `[10pt]`

Difficulty: Medium-Hard
Amount of work: 20%
Solution: The proof is very similar to the one to Problem 11 in the prenote.