Long Quiz

Sim, Min Kyu, Ph.D., mksim@seoultech.ac.kr

- Name: ______________________________
- Student ID: ______________________________
- E-mail: __________________ `@seoultech.ac.kr`

- ITM426, Engineering Math, Long Quiz, 2019 Fall
- Sep 26, 2019
- Time for Quiz: 1 hour
- Weighting 10 %

- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

Problem 1

Let $\mathtt{x} = (3, 4)$, $\mathtt{y} = (1, 1)$, and $\mathtt{z} = (2, 7)$. Find real numbers a_1 and a_2 that solves the following equation. $\mathtt{z} = a_1\mathtt{x} + a_2\mathtt{y}$. **[10pts]** (No justification is necessary)

Problem 2

Investigate whether the following sets of vectors are linearly independent or dependent: $(4, 1, 0)$, $(0, 2, -1)$, $(3, 2, 0)$. **[10pts]**

Problem 3

Complete the following proof that the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ span 3−dimensional vector space. (In other words, the set of the three vectors is a basis of 3−dimensional vector space.) `[10pts]`

- $\mathbf{v}_1 = (0, 1, 1)$
- $\mathbf{v}_2 = (1, 0, 1)$
- $\mathbf{v}_3 = (1, 1, 0)$

Proof: Let $\mathbf{v} = (x, y, z)$ be an arbitrary 3D vector. I shall show that $\mathbf{v}$ can be expressed as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ such as

$$\mathbf{v} = (x, y, z) = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$$

Problem 4

Based on your answer of **Problem 2** above, tell me whether the determinant of the following matrix (i.e. $|A|$) is zero or non−zero. `[5pts]`

$$A = \begin{bmatrix} 4 & 0 & 3 \\ 1 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix}$$

Problem 5

Carry out the following matrix multiplication. Missing elements are all zeros. **[5pts]**

$$\begin{bmatrix} 1 & & \\ & 2 & \\ -1 & & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Problem 6

Find the inverse matrix of the following. Make sure your answer is correct by checking $AA^{-1} = A^{-1}A = I$. `[10pts]`

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$$

Problem 7

Express following system of linear equations using matrix notation and find a solution. **[10pts]**

$$\begin{aligned} 3x + 2y &= 3 \\ 2x + 2y &= 1 \end{aligned}$$

Problem 8

For a $n \times n$ matrix. TFAE (The followings are all equivalent). Complete the other three bullet points. **[each 5pts]**

- $A\mathrm{x} = \mathrm{b}$ does not have a unique solution.
-
-
-

Problem 9

We have $L = \begin{bmatrix} 1 & \\ 2 & 2 \end{bmatrix}$ and $U = \begin{bmatrix} 3 & 1 \\ & 2 \end{bmatrix}$.
Find the matrix A, where $A = LU$. **[5pts]**

In upcoming lectures⋯

- L is called a Lower triangular matrix. (No non−zero element in upper diagonal)
- U is called a Upper triangular matrix. (No non−zero element in lower diagonal)
- $A = LU$ is called a LU−decomposition, or a LU−factorization of matrix A.

Problem 10

We have

$$A = \begin{bmatrix} 2 & & -2 \\ 1 & 1 & -2 \\ & & 1 \end{bmatrix}, \ \mathrm{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

.

1. Find $A\mathrm{x}$. **[5pts]**

2. Find $A^2\mathrm{x}$. **[5pts]** (Hint: Remind that $A^2\mathrm{x} = AA\mathrm{x} = A(A\mathrm{x})$. That is, using your answer above will help you avoid doing $A \times A$ by hand.)

3. Find $A^3\mathbf{x}$. **[5pts]** (Hint: Remind that $A^3\mathbf{x} = A(A^2\mathbf{x})$. That is, using your answer above will help you avoid doing $A \times A \times A$ by hand.)

4. Find $A^n\mathbf{x}$. **[5pts]** (Hint: You already have good answers for $n = 1, 2$, and 3 from above. Now can you generalize your findings?)

```
"Linear Algebra"
```

```
## [1] "Linear Algebra"
```

Write your name before detaching this page.
Your Name: ______________________________

Write your name before detaching this page.
Your Name: ______________________________