ITM429 - Quiz 3 - Fall 2019

Name (이름): ( )

#1. Let $\mathbf{u_1} = [2\ 5\ \ -1]^t$ and $\mathbf{u_2} = [-2\ 1\ 1]^t$. Since $\mathbf{u_1}$ and $\mathbf{u_2}$ are orthogonal, $\{\mathbf{u_1}, \mathbf{u_2}\}$ is an orthogonal basis for $W = Span\{\mathbf{u_1}, \mathbf{u_2}\}$. In other words, $Span\{\mathbf{u}_1, \mathbf{u_2}\}$ is the space spanned by its orthogonal basis $\{\mathbf{u_1}, \mathbf{u_2}\}$. Let $\mathbf{y} = [1\ 2\ 3]^t$, then the orthogonal projection of $\mathbf{y}$ onto W can be obtained by using the orthogonal basis of W, namely, $\{\mathbf{u_1}, \mathbf{u_2}\}$.

(a) Specifically, the orthogonal projection of $\mathbf{y}$ onto W can be obtained by

$$\hat{\mathbf{y}} = \frac{\mathbf{y} \cdot \mathbf{u_1}}{\mathbf{u_1} \cdot \mathbf{u_1}}\mathbf{u_1} + \frac{\mathbf{y} \cdot \mathbf{u_2}}{\mathbf{u_2} \cdot \mathbf{u_2}}\mathbf{u_2} \tag{1}$$

Use the above formula to find $\hat{\mathbf{y}}$.

(b) (**This is optional, NOT counted for quiz score**) Using your answer in (a) above, find $\mathbf{y} - \hat{\mathbf{y}}$. This vector is orthogonal to W (i.e. $(\mathbf{y} - \hat{\mathbf{y}}) \perp W$), thus this vector $\mathbf{y} - \hat{\mathbf{y}}$ is orthogonal to all vectors in W, including $\mathbf{u_1}$ and $\mathbf{u_2}$. Check if i) $(\mathbf{y} - \hat{\mathbf{y}}) \perp \mathbf{u_1}$ and ii) $(\mathbf{y} - \hat{\mathbf{y}}) \perp \mathbf{u_2}$. If not, you should go back to the problem (a) and find $\mathbf{y}$ again.