ITM426, Quiz 2, 2022 Fall

Solution and Grading

- ITM 426 Engineering Mathematics
- Oct 28, 2022
- Duration: 90 minutes
- Weights: 25% or 30% depending on other quiz scores
- 6 Questions

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- Write legibly.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	25
2	15
3	15
4	15
5	15
6	15
Total	100

#1. Mark True or False. No justification is necessary. `[Each 5pt]`

- Every matrix is row equivalent to a unique matrix in echelon form. (`TRUE` / `FALSE`)
- Any system of n linear equations in n variables has at most n solutions. (`TRUE` / `FALSE`)
- If a system of linear equations has no free variables, then it has a unique solution. (`TRUE` / `FALSE`)
- If A is an $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$. (`TRUE` / `FALSE`)
- If matrix A is an $n \times n$ and the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then A has fewer than n pivot positions.
  (`TRUE` / `FALSE`)

Difficulty: Hard
Amount of work: 25 %
Suggested answer:

- False. The word "reduced" is missing.
- False. False. Counterexample: Let A be any $n \times n$ matrix with fewer than n pivot columns. Then the equation $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions. Theorem 2 in Section 1.2 says that a system has either zero, one, or infinitely many solutions, but it does not say that a system with infinitely many solutions exists.
- False. Some systems may have no free variables and no solution.
- False. Only if the matrix A is invertible.
- True. By invertible matrix theorem.

#2. Find all solutions to the system of the homogeneous equations of A. [15pt]

$$A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 15 %
Suggested answer:

Solve $A\mathbf{x} = \mathbf{0}$. $\begin{bmatrix} 1 & 3 & 9 & 2 & 0 \\ 1 & 0 & 3 & -4 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ -2 & 3 & 0 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 9 & 2 & 0 \\ 0 & -3 & -6 & -6 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 9 & 18 & 9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 9 & 2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & -6 & 0 \\ 0 & 9 & 18 & 9 & 0 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 3 & 9 & 2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -18 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 9 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{ccccccc} x_1 & & + & 3x_3 & & = & 0 \\ & x_2 & + & 2x_3 & & = & 0 \\ & & & & x_4 & = & 0 \end{array} \qquad \begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 \\ x_3 \text{ is free} \\ x_4 = 0 \end{cases} \qquad \mathbf{x} = \begin{bmatrix} -3x_3 \\ -2x_3 \\ x_3 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

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#3. Find an LU factorization of the following matrix `[15pt]`

$$A = \begin{bmatrix} 2 & -2 & 4 \\ 1 & -3 & 1 \\ 3 & 7 & 5 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 15 %
Suggested answer:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & -6 \end{bmatrix} \tag{1}$$

(blank)

#4. Find an inverse of the following matrix `[15pt]`

$$A = \begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 1 \end{bmatrix}$$

Difficulty: Medium
Amount of work: 15 %
Suggested answer:

$$A^{-1} = \begin{bmatrix} -7 & -19/3 & -8/3 \\ -4 & -11/3 & -4/3 \\ 3 & 3 & 1 \end{bmatrix}$$

Students are highly encouraged to perform the final check if AA^{-1} indeed produces the identity matrix.

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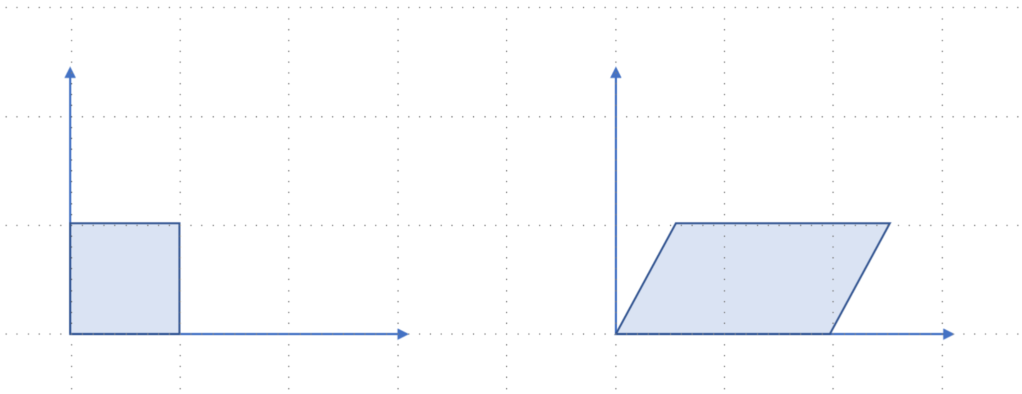
#5. Suppose A and B are $n \times n$, B is invertible, and AB is invertible. Show that A is invertible. [15pt]

Difficulty: Medium
Amount of work: 15 %
Suggested answer:
Let $C = AB$. Then $CB^{-1} = ABB^{-1}$, so $CB^{-1} = AI = A$. This shows that A is the product of invertible matrices, hence invertible.

#6. The unit square on the left becomes the parellogram on the right by a linear transformation. What would be the standard matrix for this linear transformation? Justification is necessary. `[15pt]`



Difficulty: Medium
Amount of work: 15 %
Suggested answer:
It is by shears of $k = 0.25$ with respect to x-axis, then doubling x-axis. Thus, the standard matrix is $\begin{bmatrix} 2 & 0.5 \\ 0 & 1 \end{bmatrix}$

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Write your name before detaching this page. Your Name: ______________________________