Quiz 1

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Solution

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- ITM426, Engineering Math, 2020 F, Quiz 1
- Sep 25
- Duration: 60 minutes
- Weighting: 10 % or 20 % depending on other quiz scores.

- In on-line exam, start every problem in a new page.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

Problem 1

Write whether following statement is true of false. `[8pt]` (No justification is necessary)

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $det(5A) = 5\, det(A)$. **(True/False)**

Difficulty: Easy
Amount of work: 5%
Solution: False. $det(5A) = 5^2\, det(A)$

Problem 2

Write whether following statement is true of false. (Hint: Work on the Problem 6 of this quiz first, then make use of the theorem.) `[10pt]` (Justification is necessary)

For a 3×3 matrix $A = \begin{bmatrix} 4 & 0 & 3 \\ 1 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix}$, $det(A) = 0$. (**True/False**)

Difficulty: Hard

Amount of work: 10%

Solution: False, because the column vectors are independent. Students are expected to show that the column vectors are independent.

Problem 3

Consider the following matrix:

$$\begin{bmatrix} 5-\lambda & 3 \\ -1 & 3-\lambda \end{bmatrix}$$

Find the values of λ that makes the determinant of matrix to be zero. (Hint: You need to solve a quadratic equation (이차방정식)) `[10pt]`

Difficulty: Easy
Amount of work: 10%
Solution: $\lambda = 2$ or $\lambda = 6$

Problem 4

Carry out the matrix multiplication of the following: (All missing elements in matrices are zero) `[8pt]`

$$\begin{bmatrix} 1 & & \\ 2 & 2 & \\ 2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 2 \\ & 2 & 1 \\ & & 1 \end{bmatrix}$$

Difficulty: Easy

Amount of work: 5%

Solution: $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$

We have a matrix $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$ and want to multiply another matrix to the left of the matrix A to generate another matrix, A'. That is,

$$A' = \begin{bmatrix} 2 & & & \\ -1 & 1 & & \\ & & 3 & \\ & 2 & & 1 \end{bmatrix} \times A.$$

a) Write A' `[5pt]`

$$A' = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

b) Complete the description of the above operation by filling in the blanks. `[5pt]`

Let each row vector of matrix A as $(R1)$, $(R2)$, $(R3)$, and $(R4)$, respectively. Then, Each row vector of matrix A' can be expressed as following:

1. The first row of A' is $\underline{2 \times (R1)}$
2. The second row A' is $\underline{(R2) - (R1)}$
3. The third row of A' is ____________
4. The fourth row A' is ________

Difficulty: Medium

Amount of work: 15%

Solution:

a) $\begin{bmatrix} 2a & 2b & 2c & 2d \\ -a+e & -b+f & -c+g & -d+h \\ 3a & 3b & 3c & 3d \\ 2e+m & 2f+n & 2g+l & 2h+o \end{bmatrix}$

b) $3 \times (R3)$; $(R4) - 2(R2)$

Problem 6

For a $n \times n$ matrix. TFAE (The followings are all equivalent). Complete the other three bullet points. `[each 5pt]`

- $A\mathrm{x} = \mathrm{b}$ has a unique solution.
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-
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Difficulty: Medium
Amount of work: 5%
Solution:
a) Column vectors are independent. b) The matrix A has zero-determinant. c) The matrix A is non-singular. d) The matrix A is invertible.

Problem 7

Complete the following proof that the vectors v_1, v_2, and v_3 span 3-dimensional vector space. (In other words, the set of the three vectors is a basis of 3-dimensional vector space.) `[10pt]`

- $\mathrm{v}_1 = (0, 1, -1)$
- $\mathrm{v}_2 = (1, 0, 1)$
- $\mathrm{v}_3 = (-1, 1, 0)$

Proof: Let $\mathrm{v} = (x, y, z)$ be an arbitrary real-numbered 3-dimensional vector. We claim that v can be expressed as a linear combination of v_1, v_2, and v_3 such as

$$\mathrm{v} = (x, y, z) = a\mathrm{v}_1 + b\mathrm{v}_2 + c\mathrm{v}_3, \text{ where } a, b, c \text{ are all real numbers.}$$

Difficulty: Hard
Amount of work: 20%
Solution:
After some work, it can be shown that
$(x, y, z) = \frac{x+y-z}{2}(0, 1, -1) + \frac{x+y+z}{2}(1, 0, 1) + \frac{-x+y+z}{2}(-1, 1, 0)$. And all coefficients are real numbers. This proves the claim. $\square$

Problem 8

Express the following system of linear equations using matrix notation and find a solution using the inverse matrix method. `[8pt]`

$$\begin{aligned} 5x + 2y &= 2 \\ 3x + 4y &= 1 \end{aligned}$$

Difficulty: Easy

Amount of work: 10%

Solution:
$\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, It follows that
$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6/14 \\ -1/14 \end{bmatrix}$. Students are expected to plug in the obtained values to the original matrix to confirm.

Problem 9

We have

$$A = \begin{bmatrix} 2 & & -2 \\ 1 & 1 & -2 \\ & & 1 \end{bmatrix}, \ \mathrm{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

a) Find $A\mathrm{x}$. `[0pt]`

b) Find $A^2\mathrm{x}$. `[5pt]` (Hint: Since $(AB)C = A(BC)$, $(AA)\mathrm{x} = A(A\mathrm{x})$ holds as well.)

c) Find $A^n\mathrm{x}$ (the generalized expression). `[5pt]`

Difficulty: Medium
Amount of work: 15%
Solution: a) $[4 \quad 3 \quad 0]^t$; b) $[8 \quad 7 \quad 0]^t$; c) $[2^{n+1} \quad 2^{n+1} - 1 \quad 0]^t$

Problem 10

Let $y = [2 \quad 4]$ and $u = [6 \quad 2]$.

a) Compute the vector z such that

$$z = \frac{y \bullet u}{u \bullet u} u,$$

where $\bullet$ is the dot-product operator.`[5pt]`

b) Draw the vector y, u, and z in a two-dimensional space as precisely as possible.`[5pt]`

Difficulty: Medium
Amount of work: 15%
Solution: $z = [3 \quad 1]$. Students are expected to mark the vectors in 2D grid, where y and z are overlapped.

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"Engineering Math - Quiz 1"
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Write your name before detaching this page.

Your Name: ______________________________