

A Dual Bounding Framework through Cost Splitting for Binary Quadratic Optimization

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This document describes the data of the paper for adjacent quadratic semi-assignment problem (AQSAP), quadratic semi-assignment problem (QSAP) and multi object tracking (MOT) problem as well as the detailed results. The files contain the data used in the numerical experiments of the paper.

1 AQSAP

For this problem we provide 41 instances where we have 41 data files for linear cost, c , and 41 data files for quadratic cost, q . Files' names are in the following formats:

“c- $\{n\}$ - $\{h\}$.dat”

“q- $\{n\}$ - $\{h\}$.dat”

- n : The number of clients

- h : The number of servers

These text files (provided in the folder “AdjacentSemiAssignmentData”) contain data related to both c and q which are stored in one dimension, where the delimiter “,” is chosen. Linear and quadratic costs are needed to be reshaped to c_{ij} and q_{ijkl} in the shape of (n, h) and (n, h, n, h) respectively.

As an example, linear cost can be read to a Numpy array in Python as following:

```
import numpy as np
c_data = np.genfromtxt('c_10_3.dat', delimiter=',')
c = c_data.reshape(10,3)
```

2 QSAP

For this problem we provide 41 instances for each experiment of adding 10%, 15%, 20% and 25% of non-adjacent quadratic cost. Data files' names are in the following formats:

"c- $\{n\}$ - $\{h\}$ - $\{percent\}$.dat"

"q- $\{n\}$ - $\{h\}$ - $\{percent\}$.dat"

- **n**: The number of clients
- **h**: The number of servers
- **percent**: Percentage of non-adjacent quadratic cost

The text files are provided in the folder "SemiAssignmentData"

Reading the data files are exactly the same as reading the data related to the AQSAP.

3 MOT

There are 27 instances tested for MOT problem in the paper. Thus, 27 text files related to the linear cost and 27 text files for the quadratic cost are provided in the folder "MOTData". The names of the data files are in the following formats:

"c- $\{T\}$ - $\{h\}$ - $\{d\}$ - $\{detection\}$.dat"

"q- $\{T\}$ - $\{h\}$ - $\{d\}$ - $\{detection\}$.dat"

- **T**: The number of frames in the video
- **h**: The maximum number of tracks
- **d**: The maximum number of considered adjacent frames
- **detection**: This is not a parameter for the instance. It specifies the number of detections in the data instance. We embed it in the name of the instance file to be used when we want to read the data from each file.

Both c and q are saved in the text files in one dimension, where the delimiter "," is chosen. They need to be reshaped to c_i and q_{ij} . The size of linear cost matrix is n and the size of quadratic cost matrix is (n, n) , where n is the number of detections stated in the name of the data file.

4 Results

We have reported the detailed results of our experiments in the following tables (also shown in the paper):

Table 1: AQSAP – Comparing different methods – At least one of the methods stops within the time limit

Instance (n-h)	BFS	BQP		CG+BQPPricing		CG+HeuristicBQPPricing		CG+HeuristicSLTPricing		SLT		CG+SLTPricing		RLT		OuterApproximation	
		LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time
10-3	3038.8	1	0	1	2	1	8	1	6	1	1	1	2	1	1	1	0.1
10-4	2829.3	1	0	1	4	1	11	1	8	1	2	1	1	1	2	1	0.1
15-3	32202.4	1	52	1	56	1	18	1	14	1	40	1	16	1	30	1	6
15-4	21336.7	1	524	1	57	1	28	1	28	1	314	1	17	1	593	1	120
15-6	4472.7	1	327	1	41	1	25	1	16	1	233	1	10	1	1395	1	95
18-3	8168.7	1	43	1	298	1	35	1	24	1	25	1	142	1	61	1	13
18-4	6747.3	1	1852	1	142	1	5	1	8	1	909	1	127	1	1910	1	535
18-6	5906.2	0.82	10800	1	96	1	53	1	43	0.81	10800	1	147	0.73	10800	0.82	10800
18-8	4916	0.85	10800	1	96	1	83	1	82	0.8	10800	1	160	0.74	10800	0.79	10800
20-3	15549.8	1	1099	1	430	1	10	1	25	1	345	1	531	1	422	1	68
20-4	13657.3	0.92	10800	1	333	1	58	1	25	0.94	10800	1	331	0.86	10800	1	7406
20-6	9781.5	0.72	10800	1	227	1	101	1	52	0.68	10800	1	293	0.55	10800	0.66	10800
20-8	8559.1	0.62	10800	1	105	1	10	1	31	0.62	10800	1	176	0.55	10800	0.62	10800
20-10	4810.8	0.83	10800	1	151	1	18	1	59	0.81	10800	1	195	0.61	10800	0.79	10800
22-3	42873.5	0.97	10800	1	1744	1	51	1	74	1	7360	1	1612	0.98	10800	1.00	4807
22-4	24085.5	0.87	10800	1	500	1	27	1	76	0.84	10800	1	590	0.75	10800	0.80	10800
22-6	18316.3	0.72	10800	1	237	1	28	1	66	0.63	10800	1	404	0.46	10800	0.52	10800
22-8	7394.2	0.74	10800	1	205	1	13	1	31	0.67	10800	1	293	0.61	10800	0.67	10800
22-10	6017.8	0.78	10800	1	188	1	95	1	49	0.73	10800	1	284	0.59	10800	0.73	10800
22-12	4519.2	0.83	10800	1	158	1	100	1	73	0.76	10800	1	197	0.56	10800	0.80	10800
25-3	42794.9	0.94	10800	1	3127	1	51	1	72	0.98	10800	1	2417	0.89	10800	0.92	10800
25-4	14012.7	0.9	10800	1	598	1	16	1	79	0.9	10800	1	656	0.82	10800	0.88	10800
25-6	9478.1	0.68	10800	1	1937	1	684	1	174	0.59	10800	1	1477	0.46	10800	0.54	10800
25-8	8807.1	0.68	10800	1	581	1	566	1	252	0.64	10800	1	614	0.47	10800	0.57	10800
25-10	6318.8	0.7	10800	1	975	1	98	1	99	0.61	10800	1	591	0.43	10800	0.56	10800
25-12	4744	0.61	10800	1	265	1	228	1	179	0.52	10800	1	348	0.38	10800	0.47	10800
30-3	48847.1	0.89	10800	1	1741	1	70	1	98	0.94	10800	1	1863	0.90	10800	1.00	8424
30-4	20369.8	0.74	10800	0.99	10800	1	3665	1	1143	0.73	10800	1	9636	0.62	10800	0.65	10800
30-6	14167	0.52	10800	1	10800	1	2001	1	2287	0.5	10800	1	10800	0.39	10800	0.45	10800
30-8	10322.5	0.58	10800	1	6378	1	2612	1	964	0.5	10800	1	2953	0.31	10800	0.45	10800
30-14	6680.8	0.47	10800	1	1875	1	214	1	831	0.41	10800	1	818	0.31	10800	0.38	10800
40-14	10365.5	0.4	10800	0.98	10800	1	10800	1	4062	0.34	10800	1	10800	0.21	10800	0.31	10800
50-4	49468.9	0.43	10800	1	10800	1	6895	1	10800	0.37	10800	0.97	10800	0.29	10800	0.35	10800
50-6	40831	0.67	10800	1	10800	1	4156	0.56	10800	0.25	10800	0.82	10800	0.17	10800	0.22	10800
50-8	34960.5	0.5	10800	1	10800	1	7146	0.46	10800	0.24	10800	0.78	10800	0.18	10800	0.22	10800

Table 2: AQSAP – Comparing different methods – None of the methods stop within the time limit

Instance (n-h)	BFS	BQP	CG+BQPPricing	CG+Heuristic BQPPricing	CG+Heuristic SLTPricing	SLT	CG+SLTPricing	RLT	Outer Approximation
		LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS
40-3	46075.8	0.66	0.94	1	1	0.58	0.93	0.51	0.58
40-4	37264	0.46	0.89	0.80	0.80	0.41	0.89	0.28	0.35
40-6	17322.9	0.36	0.86	0.86	0.86	0.28	0.88	0.20	0.26
40-8	16018.9	0.34	0.94	0.84	1	0.29	0.90	0.21	0.27
50-3	52448.4	0.61	0.99	1	1	0.50	0.94	0.47	0.50
50-14	11696.4	0.29	0.99	1	0.41	0.34	0.78	0.21	0.29

Table 3: QSAP – 10% out-of-star quadratic matrix density – Comparing different methods – At least one of the methods stops within the time limit

Instance (n-h)	BFS	BQP		CG+BQPPricing		CG+HeuristicBQPPricing		CG+HeuristicSLTPricing		SLT		CG+SLTPricing		RLT		OuterApproximation	
		LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time
10-3	3061.4	1	0.1	1	6	1	6	1	3	1	2	1	2	1	0.1	1	0.1
10-4	2829.3	1	1	1	5	1	5	1	5	1	1	1	2	1	0.1	1	0.1
15-3	33075.4	1	4	0.98	77	0.98	10	0.98	6	1	11	0.98	22	1	2	1	1
15-4	21665.4	1	21	0.99	71	0.99	18	0.99	14	1	348	0.99	17	1	10	1	5
15-6	4497.2	1	7	0.99	40	0.99	15	0.99	15	1	79	0.99	64	1	5	1	6
18-3	8383	1	4	0.99	164	0.99	41	0.99	32	1	13	0.99	180	1	1	1	2
18-4	6884.3	1	23	0.98	144	0.98	29	0.98	24	1	119	0.98	169	1	12	1	25
18-6	5957.5	1	248	0.99	123	0.99	47	0.99	39	1	4021	0.99	135	1	239	1	395
18-8	4956.1	1	4762	0.99	125	0.99	43	0.99	40	0.76	10800	0.99	129	1	3748	1	6587
20-3	16245.3	1	20	0.97	533	0.97	103	0.97	55	1	75	0.97	561	1	25	1	5
20-4	13942.2	1	492	0.98	459	0.98	53	0.98	25	1	1701	0.98	502	1	170	1	37
20-6	9919.2	1	4867	0.99	224	0.99	42	0.99	23	0.51	10800	0.99	270	1	9593	1	2166
20-8	8697.8	0.75	10800	0.98	156	0.98	76	0.98	31	0.53	10800	0.98	250	0.71	10800	0.76	10800
20-10	4878.3	0.88	10800	0.99	194	0.99	87	0.99	34	0.64	10800	0.99	280	0.74	10800	0.81	10800
22-3	44517.4	1	444	0.96	1092	0.96	34	0.96	21	1	1632	0.96	1432	1	328	1	183
22-4	25071.6	1	4806	0.96	589	0.96	89	0.96	36	1	9336	0.96	768	1	3453	1	2033
22-6	18872.5	0.79	10800	0.97	312	0.97	89	0.97	31	0.55	10800	0.97	391	0.76	10800	0.70	10800
22-8	7587.9	0.82	10800	0.97	322	0.97	109	0.97	59	0.62	10800	0.97	511	0.75	10800	0.80	10800
22-10	6121.4	0.84	10800	0.99	336	0.99	84	0.99	107	0.65	10800	0.99	432	0.72	10800	0.78	10800
22-12	4566.7	0.88	10800	0.99	353	0.99	111	0.99	126	0.62	10800	0.99	478	0.71	10800	0.90	10800
25-3	44875.1	1	206	0.96	3207	0.96	516	0.96	176	1	918	0.96	4427	1	181	1	196
25-4	14811.7	1	83	0.95	1256	0.95	122	0.95	66	1	567	0.95	1298	1	184	1	153
25-6	9887.9	0.79	10800	0.96	2361	0.96	376	0.96	195	0.47	10800	0.96	2629	0.74	10800	0.79	10800
25-8	8999.1	0.73	10800	0.98	2051	0.98	673	0.98	402	0.53	10800	0.98	1492	0.64	10800	0.67	10800
25-10	6509.8	0.74	10800	0.97	2698	0.97	367	0.97	543	0.39	10800	0.97	1822	0.55	10800	0.63	10800
25-12	4921.6	0.47	10800	0.96	1130	0.96	1024	0.96	916	0.35	10800	0.96	1712	0.45	10800	0.57	10800
30-3	53502.1	1	203	0.93	8255	0.94	412	0.94	79	1	2902	0.94	1863	1	361	1	208
30-4	21896.3	1	1359	0.93	10800	0.93	4229	0.93	4350	1	8366	0.93	9636	1	1101	1	2049
30-6	14736.8	0.73	10800	0.95	10800	0.96	2324	0.96	3722	0.43	10800	0.96	10800	0.65	10800	0.64	10800
30-8	10807.5	0.40	10800	0.95	8560	0.95	4120	0.95	2720	0.24	10800	0.95	8120	0.44	10800	0.47	10800
30-14	6978.9	0.36	10800	0.96	7894	0.96	1063	0.96	10800	0.26	10800	0.96	10800	0.33	10800	0.39	10800

Table 4: QSAP – 10% out-of-star quadratic matrix density – Comparing different methods – None of the methods stops within the time limit

Instance (m-n)	BFS	BQP LB/BFS	CG+BQPPricing LB/BFS	CG+Heuristic BQPPricing LB/BFS	CG+Heuristic SLTPricing LB/BFS	SLT LB/BFS	CG+SLTPricing LB/BFS	RLT LB/BFS	Outer Approximation LB/BFS
40-3	51693.8	0.75	0.83	0.68	0.87	0.63	0.83	0.68	0.72
40-4	41684.2	0.52	0.78	0.71	0.71	0.22	0.78	0.40	0.48
40-6	19765.3	0.31	0.76	0.18	0.18	0.17	0.68	0.24	0.27
40-8	18865.9	0.21	0.43	0.35	0.35	0.17	0.43	0.23	0.26
40-14	11586.1	0.25	0.51	0.51	0.38	0.21	0.51	0.25	0.28
50-3	60996.3	0.83	0.87	0.88	0.82	0.58	0.79	0.73	0.65
50-4	62141.8	0.46	0.79	0.81	0.76	0.26	0.73	0.44	0.53
50-6	51281.2	0.27	0.78	0.79	0.27	0.16	0.50	0.18	0.23
50-8	44208.2	0.18	0.77	0.79	0.33	0.15	0.45	0.15	0.19
50-14	14426.3	0.22	0.37	0.25	0.30	0.20	0.29	0.17	0.23

Table 5: QSAP – 15% out-of-star quadratic matrix density – Comparing different methods – At least one of the methods stops within the time limit

Instance (n-h)	BFS	BQP		CG+BQPPricing		CG+HeuristicBQPPricing		CG+HeuristicSLTPricing		SLT		CG+SLTPricing		RLT		OuterApproximation	
		LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time
10-3	3172.5	1	0	0.97	4	0.97	4	0.97	4	1	2	0.97	1	1	0	1	0
10-4	2881.7	1	1	0.99	5	0.99	5	0.99	4	1	1	0.99	1	1	0	1	0
15-3	33324.7	1	2	0.97	54	0.97	14	0.97	10800	1	8	0.97	18	1	1	1	1
15-4	22318.5	1	6	0.96	63	0.96	26	0.96	20	1	65	0.96	65	1	5	1	4
15-6	4568.5	1	4	0.98	41	0.98	16	0.98	21	1	38	0.98	56	1	3	1	4
18-3	8669.3	1	2	0.96	182	0.96	24	0.96	22	1	6	0.96	212	1	0	1	1
18-4	6978.7	1	10	0.97	194	0.97	29	0.97	26	1	38	0.97	193	1	4	1	13
18-6	6126.4	1	130	0.96	122	0.96	62	0.96	59	1	1544	0.96	137	1	75	1	136
18-8	5034.8	1	419	0.98	97	0.98	57	0.98	52	1	7584	0.98	122	1	500	1	840
20-3	16434.4	1	9	0.96	600	0.96	71	0.96	45	1	27	0.96	550	1	6	1	2
20-4	14529.7	1	97	0.94	349	0.94	32	0.94	19	1	788	0.94	444	1	76	1	19
20-6	10257.6	1	1707	0.95	206	0.95	57	0.95	25	0.85	10800	0.95	283	1	1724	1	391
20-8	9062.6	1	9703	0.95	227	0.95	84	0.95	36	0.6	10800	0.95	289	0.85	10800	0.85	10800
20-10	4965.2	1	4169	0.97	196	0.97	110	0.97	50	0.65	10800	0.97	315	1	10800	1	6248
22-3	45955.1	1	222	0.93	897	0.93	52	0.93	37	1	1037	0.93	1525	1	151	1	88
22-4	26366.2	1	899	0.91	464	0.91	121	0.91	44	1	4682	0.91	654	1	868	1	899
22-6	19512	1	8792	0.94	256	0.94	73	0.94	29	0.61	10800	0.94	433	0.89	10800	1	7364
22-8	8214.5	1	4946	0.9	525	0.90	106	0.90	78	0.64	10800	0.9	492	1	6424	1	8687
22-10	6349.8	0.85	10800	0.95	259	0.95	168	0.95	237	0.65	10800	0.95	368	0.85	10800	1	9966
22-12	4821.5	0.70	10800	0.93	433	0.93	160	0.93	131	0.65	10800	0.93	490	1	10800	1	8706
25-3	48713.8	1	117	0.89	4418	0.89	287	0.89	202	1	377	0.89	4186	1	117	1	129
25-4	16521.4	1	46	0.86	1685	0.86	121	0.86	75	1	241	0.86	1631	1	81	1	122
25-6	10827.9	1	7729	0.88	1932	0.88	384	0.88	191	0.51	10800	0.88	1895	1	7087	1	8188
25-8	9745.2	0.61	10800	0.9	4584	0.9	952	0.9	511	0.65	10800	0.9	2642	0.72	10800	0.76	10800
25-10	7063.3	0.59	10800	0.89	3336	0.89	942	0.89	840	0.39	10800	0.89	2130	0.61	10800	0.68	10800
25-12	4738.4	0.44	10800	0.85	1328	0.85	10156	0.85	10283	0.32	10800	0.85	10800	0.45	10800	0.58	10800
30-3	57082.6	1	93	0.88	10800	0.88	1751	0.88	264	1	1025	0.88	10800	1	103	1	118
30-4	24213	1	109	0.84	10800	0.85	2085	0.85	1132	1	2207	0.85	10800	1	398	1	322
30-6	18614.7	0.68	10800	0.77	10800	0.77	809	0.77	2739	0.41	10800	0.77	10800	0.69	10800	0.64	10800
30-8	14294	0.38	10800	0.78	5906	0.78	5366	0.78	3479	0.25	10800	0.78	8978	0.39	10800	0.44	10800
30-14	10284	0.31	10800	0.80	9593	0.80	1371	0.80	1365	0.23	10800	0.80	10800	0.29	10800	0.34	10800
40-14	17160.5	0.17	10800	0.36	10800	0.22	1540	0.19	10800	0.15	10800	0.37	10800	0.17	10800	0.2	10800

Table 6: QSAP – 15% out-of-star quadratic matrix density – Comparing different methods – None of the methods stops within the time limit

Instance (n-h)	BFS	BQP	CG+BQPPricing	CG+Heuristic BQPPricing	CG+Heuristic SLTPricing	SLT	CG+SLTPricing	RLT	Outer Approximation
		LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS
40-3	55376	0.85	0.75	0.62	0.82	0.65	0.76	0.78	0.76
40-4	47234.8	0.68	0.70	0.64	0.64	0.32	0.69	0.41	0.50
40-6	23518.1	0.29	0.62	0.22	0.22	0.15	0.60	0.25	0.27
40-8	27886.8	0.16	0.29	0.17	0.17	0.13	0.29	0.17	0.20
50-3	65385.1	0.91	0.81	0.82	0.77	0.66	0.76	0.83	0.80
50-4	70572.9	0.57	0.69	0.70	0.66	0.29	0.63	0.54	0.58
50-6	62755.8	0.24	0.63	0.65	0.23	0.14	0.51	0.17	0.25
50-8	53024.2	0.16	0.63	0.66	0.32	0.13	0.35	0.14	0.18
50-14	26288.9	0.12	0.17	0.03	0.16	0.12	0.16	0.10	0.14

Table 7: QSAP – 20% out-of-star quadratic matrix density – Comparing different methods – At least one of the methods stops within the time limit

Instance (n-h)	BFS	BQP		CG+BQPPricing		CG+ HeuristicBQPPricing		CG+ HeuristicSLTPricing		SLT		CG+SLTPricing		RLT		OuterApproximation	
		LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time
10-3	3213.4	1	0	0.96	5	0.96	3	0.96	3	1	1	0.96	1	1	0	1	0
10-4	3001.4	1	1	0.96	9	0.96	5	0.96	4	1	1	0.96	2	1	0	1	0
15-3	34726.1	1	2	0.93	60	0.93	11	0.93	10	1	6	0.93	21	1	1	1	1
15-4	23852.4	1	7	0.90	72	0.90	19	0.90	19	1	54	0.90	75	1	4	1	3
15-6	5132.6	1	5	0.87	52	0.87	21	0.87	21	1	33	0.87	66	1	2	1	4
18-3	9895.4	1	2	0.84	182	0.84	31	0.84	26	1	5	0.84	227	1	1	1	1
18-4	7357.1	1	6	0.92	174	0.92	44	0.92	35	1	19	0.92	174	1	2	1	5
18-6	6379.6	1	43	0.92	138	0.92	54	0.92	48	1	374	0.92	138	1	21	1	63
18-8	5446	1	228	0.90	116	0.90	55	0.90	48	1	3503	0.90	134	1	201	1	571
20-3	18140.5	1	8	0.87	533	0.87	189	0.87	13	1	39	0.87	627	1	4	1	1
20-4	15836	1	73	0.86	394	0.86	28	0.86	20	1	553	0.86	461	1	76	1	14
20-6	11201.2	1	1681	0.87	197	0.87	51	0.87	26	1	6065	0.87	286	1	1108	1	315
20-8	10102.4	1	2891	0.85	169	0.85	124	0.85	40	0.62	10800	0.85	219	1	6039	1	5047
20-10	5464.7	1	4997	0.88	242	0.88	167	0.88	71	0.69	10800	0.88	337	1	4995	1	5199
22-3	49327.2	1	248	0.87	1066	0.87	64	0.87	28	1	692	0.87	1693	1	153	1	81
22-4	28349	1	705	0.85	524	0.85	113	0.85	45	1	4661	0.85	659	1	832	1	510
22-6	21754.1	1	7202	0.84	214	0.84	115	0.84	46	0.61	10800	0.84	431	1	10800	1	6822
22-8	9714.9	0.71	10800	0.76	342	0.76	132	0.76	87	0.63	10800	0.76	470	1	4536	1	4872
22-10	7733.6	0.74	10800	0.78	322	0.78	243	0.78	263	0.61	10800	0.78	438	1	10800	1	7526
22-12	5884.6	0.66	10800	0.77	685	0.77	461	0.77	687	0.54	10800	0.77	818	0.78	10800	1	10800
25-3	51647	1	88	0.84	4549	0.84	218	0.84	149	1	321	0.84	4396	1	135	1	110
25-4	17712.6	1	21	0.80	1669	0.80	112	0.80	69	1	74	0.80	1812	1	51	1	71
25-6	12832.8	1	2871	0.74	2300	0.74	241	0.74	196	0.77	10800	0.74	2235	1	4243	1	6120
25-8	13192.7	0.55	10800	0.67	5744	0.67	542	0.67	384	0.41	10800	0.67	2463	0.68	10800	0.63	10800
25-10	8560.9	0.52	10800	0.74	2446	0.74	1284	0.74	957	0.31	10800	0.74	1967	0.57	10800	0.62	10800
25-12	7883.8	0.35	10800	0.60	1181	0.60	366	0.60	295	0.27	10800	0.60	1173	0.36	10800	0.44	10800
30-3	61853.4	1	48	0.81	10800	0.81	1105	0.81	237	1	257	0.81	10800	1	69	1	71
30-4	31188.4	1	222	0.65	10800	0.66	10800	0.66	1557	1	2317	0.66	10800	1	546	1	711
30-6	25956.1	0.52	10800	0.55	10800	0.55	2629	0.55	1965	0.34	10800	0.55	10800	0.59	10800	0.59	10800
30-8	19918.5	0.33	10800	0.52	10800	0.52	4722	0.52	2499	0.18	10800	0.52	7811	0.30	10800	0.38	10800
30-14	14062.6	0.19	10800	0.48	3044	0.48	1185	0.48	1191	0.15	10800	0.48	1461	0.17	10800	0.22	10800
50-3	71284.3	1	4427	0.75	10800	0.75	10800	0.69	10800	1	5015	0.37	10800	1	7429	0.86	10800

Table 8: QSAP – 20% out-of-star quadratic matrix density – Comparing different methods – None of the methods stops within the time limit

Instance (n-h)	BFS	BQP	CG+BQPPricing	CG+Heuristic BQPPricing	CG+Heuristic SLTPricing	SLT	CG+SLTPricing	RLT	Outer Approximation
		LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS
40-3	59514.1	0.85	0.71	0.65	0.76	0.74	0.71	0.82	0.80
40-4	53056.6	0.70	0.62	0.50	0.50	0.27	0.61	0.42	0.50
40-6	28760	0.27	0.53	0.29	0.29	0.14	0.50	0.24	0.26
40-8	39915	0.14	0.19	0.11	0.11	0.10	0.21	0.13	0.16
40-14	28785.3	0.10	0.18	0.17	0.10	0.10	0.19	0.10	0.12
50-4	77901.2	0.74	0.63	0.64	0.60	0.36	0.56	0.68	0.65
50-6	74931.2	0.29	0.53	0.55	0.22	0.14	0.33	0.18	0.23
50-8	70230.7	0.15	0.48	0.49	0.23	0.11	0.27	0.13	0.15
50-14	27759.7	0.12	0.17	0.01	0	0.12	0.15	0.11	0.14

Table 9: QSAP – 25% out-of-star quadratic matrix density – Comparing different methods – At least one of the methods stops within the time limit

Instance (n-h)	BFS	BQP		CG+BQPPricing		CG+HeuristicBQPPricing		CG+HeuristicSLTPricing		SLT		CG+SLTPricing		RLT		OuterApproximation	
		LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time	LB/BFS	Time
10-3	3456.3	1	1	0.89	7	0.89	5	0.89	4	1	1	0.89	1	1	0	1	0
10-4	3161	1	1	0.91	6	0.91	8	0.91	6	1	0	0.91	2	1	0	1	0
15-3	37682.9	1	2	0.86	66	0.86	11	0.86	8	1	6	0.86	20	1	1	1	1
15-4	25935.9	1	6	0.82	77	0.82	26	0.82	22	1	53	0.82	89	1	3	1	3
15-6	6285.4	1	3	0.71	52	0.71	24	0.71	23	1	50	0.71	65	1	2	1	4
18-3	10703.9	1	2	0.77	168	0.77	37	0.77	38	1	5	0.77	221	1	0	1	1
18-4	9346.3	1	7	0.72	203	0.72	32	0.72	29	1	23	0.72	222	1	3	1	10
18-6	7765.1	1	76	0.76	124	0.76	51	0.76	41	1	722	0.76	142	1	27	1	100
18-8	6158.9	1	187	0.80	116	0.80	80	0.80	71	1	2245	0.80	131	1	156	1	551
20-3	20576.4	1	10	0.77	519	0.77	52	0.77	16	1	33	0.77	704	1	4	1	2
20-4	16772.2	1	34	0.82	462	0.82	51	0.82	22	1	154	0.82	712	1	21	1	10
20-6	12269.6	1	960	0.80	239	0.80	77	0.80	27	1	4011	0.80	285	1	725	1	165
20-8	11915.7	0.75	10800	0.72	175	0.72	174	0.72	43	0.61	10800	0.72	264	1	6101	1	2875
20-10	6446.6	1	10800	0.75	238	0.75	140	0.75	68	0.66	10800	0.75	346	1	3108	1	8047
22-3	50813.7	1	126	0.84	1037	0.84	42	0.84	42	1	307	0.84	1644	1	79	1	60
22-4	30416.8	1	621	0.79	376	0.79	95	0.79	54	1	1807	0.79	729	1	545	1	542
22-6	24468.6	1	9098	0.75	249	0.75	98	0.75	59	0.70	10800	0.75	360	1	6559	1	10800
22-8	11778.1	0.73	10800	0.63	336	0.63	125	0.63	90	0.66	10800	0.63	443	1	4523	1	5447
22-10	10706.9	0.54	10800	0.56	334	0.56	257	0.56	164	0.50	10800	0.56	491	0.67	10800	0.77	10800
22-12	7346.2	0.59	10800	0.61	590	0.61	812	0.61	723	0.46	10800	0.61	973	0.76	10800	1	10800
25-3	54414.9	1	59	0.80	3991	0.80	368	0.80	283	1	150	0.80	4858	1	102	1	74
25-4	19097.4	1	22	0.75	2513	0.75	175	0.75	75	1	45	0.75	2209	1	39	1	59
25-6	15827.4	0.79	10800	0.60	3070	0.60	378	0.60	131	0.78	10800	0.60	1609	1	5183	1	3281
25-8	16652.1	0.47	10800	0.53	3526	0.53	969	0.53	475	0.37	10800	0.53	1659	0.60	10800	0.58	10800
25-10	13344.2	0.40	10800	0.47	1363	0.47	389	0.47	271	0.22	10800	0.47	1273	0.42	10800	0.46	10800
25-12	12815	0.21	10800	0.37	8340	0.37	251	0.37	195	0.19	10800	0.37	1505	0.24	10800	0.31	10800
30-3	65491.3	1	28	0.77	10800	0.77	904	0.77	272	1	190	0.77	10800	1	18	1	27
30-4	35820.9	1	95	0.57	10800	0.57	6451	0.57	6451	1	1277	0.57	10800	1	212	1	349
30-6	33203.3	0.51	10800	0.43	10800	0.43	7491	0.43	3041	0.29	10800	0.43	10800	0.60	10800	0.54	10800
30-8	27684.8	0.27	10800	0.37	8469	0.37	1600	0.37	2229	0.15	10800	0.37	6692	0.29	10800	0.31	10800
30-14	18797.5	0.15	10800	0.36	9217	0.36	1482	0.36	1422	0.12	10800	0.36	3186	0.14	10800	0.19	10800
40-14	43918.3	0.07	10800	0.11	10800	0.1	1540	0.07	10800	0.06	10800	0.11	10800	0.07	10800	0.09	10800
50-3	74787.6	1	236	0.71	10800	0.72	10800	0.71	10800	1	2195	0.66	10800	1	2001	0.94	10800
50-4	78766.4	1	6021	0.62	10800	0.63	10800	0.59	10800	0.44	10800	0.55	10800	0.79	10800	0.83	10800

Table 10: QSAP – 25% out-of-star quadratic matrix density – Comparing different methods – None of the methods stops within the time limit

Instance (n-h)	BFS	BQP	CG+BQPPricing	CG+Heuristic BQPPricing	CG+Heuristic SLTPricing	SLT	CG+SLTPricing	RLT	Outer Approximation
		LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS	LB/BFS
40-3	65784.3	0.96	0.66	0.58	0.69	0.86	0.64	0.85	0.82
40-4	59743.9	0.72	0.56	0.42	0.42	0.34	0.55	0.45	0.50
40-6	32330.7	0.3	0.47	0.26	0.26	0.15	0.44	0.25	0.31
40-8	47935.6	0.13	0.19	0.15	0.33	0.09	0.17	0.11	0.16
50-6	77643.1	0.3	0.51	0.52	0.23	0.17	0.39	0.22	0.26
50-8	74816.2	0.14	0.46	0.44	0.24	0.11	0.25	0.11	0.16
50-14	27759.7	0.13	0.16	0.01	0	0.13	0.13	0.12	0.16

Table 11: Data association on the MOT16-09 data set – Comparing GUROBI and CG – At least one of the methods stops within the time limit

Instance (T-h-d)	BFS	BQP			CG+BQPPricing			SLT			CG+SLTPricing			RLT			CG+RLTPricing		
		LB	LB/BFS	Time	LB	LB/BFS	Time	LB	LB/BFS	Time	LB	LB/BFS	Time	LB	LB/BFS	Time	LB	LB/BFS	Time
3-20-2	-266.4	-401.7	1.51	10800	-266.8	1	75	-266.4	1	1089	-266.8	1	64	-266.4	1	509	-266.8	1	12
4-25-2	-486.2	-1064.7	2.19	10800	-486.2	1	72	-486.2	1	6969	-486.2	1	950	-486.2	1	367	-486.2	1	70
4-25-3	-516.7	-1372.8	2.66	10800	-516.7	1	507	-551.9	1.07	10800	-538.2	1.04	10800	-516.7	1	5498	-516.7	1	78
5-25-2	-636.1	-1693.5	2.66	10800	-636.1	1	200	-665.3	1.05	10800	-636.1	1	4674	-636.1	1	5939	-636.1	1	466
5-25-3	-740.1	-1774.6	2.4	10800	-740.1	1	6522	-1477.4	2	10800	-740.1	1	6347	-754.8	1.02	10800	-740.1	1	555
5-25-4	-783.2	-2024.3	2.58	10800	-783.2	1	7291	-2177.8	2.78	10800	-783.2	1	5676	-806.7	1.03	10800	-783.2	1	400
6-25-3	-910.1	-2359	2.59	10800	-1365.8	1.5	10800	-2754.3	3.03	10800	-945.1	1.04	10800	-937.6	1.03	10800	-913.1	1	733
6-25-4	-1014.5	-2817	2.78	10800	-1898.7	1.87	10800	-3883	3.83	10800	-1070.2	1.05	10800	-1064.4	1.05	10800	-1014.5	1	822
6-25-5	-1110.1	-3087.7	2.78	10800	-2260.7	2.04	10800	-3493	3.15	10800	-2288.8	2.06	10800	-1137.2	1.02	10800	-1110.6	1	821
7-30-3	-1040.5	-2932.4	2.82	10800	-2509.7	2.41	10800	-3915.9	3.76	10800	-1135.1	1.09	10800	-1077.3	1.04	10800	-1040.8	1	1304
7-30-4	-1172.7	-3538.3	3.02	10800	-3038	2.59	10800	-4499	3.84	10800	-1267	1.08	10800	-1236.6	1.05	10800	-1172.8	1	1374
7-30-5	-1336.9	-3753	2.81	10800	-3496.2	2.62	10800	-5062.4	3.79	10800	-1456.3	1.09	10800	-1417.3	1.06	10800	-1336.9	1	1718
8-30-3	-1229.1	-3379.1	2.75	10800	-2261.9	1.84	10800	-4567.4	3.72	10800	-1741.7	1.42	10800	-1303.5	1.06	10800	-1229.7	1	4293
8-30-4	-1497.8	-4778.2	3.19	10800	-3806.2	2.54	10800	-5646.2	3.77	10800	-2002	1.34	10800	-1638.2	1.09	10800	-1497.8	1	4252
8-30-5	-1619.4	-5207.5	3.22	10800	-4373.9	2.7	10800	-6429.1	3.97	10800	-2902	1.79	10800	-1892.7	1.17	10800	-1632.4	1.01	5088

Table 12: Data association on the MOT16-09 data set – Comparing GUROBI and CG – None of the methods stops within the time limit

Instance (T-h-d)	BFS	BQP		CG+BQPPricing		SLT		CG+SLTPricing		RLT		CG+RLTPricing	
		LB	LB/BFS	LB	LB/BFS	LB	LB/BFS	LB	LB/BFS	LB	LB/BFS	LB	LB/BFS
9-35-4	-1790.8	-5601.5	3.13	-5377.9	3	-6569.6	3.67	-4230.2	2.36	-2026.3	1.13	-1792.1	1.00
9-35-5	-1974.7	-6624.1	3.35	NA	NA	-7740.1	3.92	-5085.4	2.58	-2358.4	1.19	-1988.9	1.01
9-35-6	-2065.4	-7647.6	3.7	NA	NA	-8519.1	4.12	-5134.7	2.49	-2535.4	1.23	-2150.4	1.04
10-35-4	-1971.7	-6472	3.28	NA	NA	-7357.6	3.73	-5469.1	2.77	-2288.2	1.16	-2073.7	1.05
10-35-5	-2150.6	-8116.3	3.77	NA	NA	-8663.3	4.03	-5700.6	2.65	-2665.1	1.24	-2346.1	1.09
10-35-6	-2405.1	-9273.5	3.86	NA	NA	-9792.8	4.07	-6824.3	2.84	-2988.5	1.24	-2621.2	1.09
10-35-7	-2569.7	-10134.5	3.94	NA	NA	-10681.3	4.16	-7544.9	2.94	-3252.3	1.27	-2833.3	1.10
11-35-4	-2167.5	-7494.8	3.46	-6969.1	3.22	-8306.2	3.83	-5996.4	2.77	-2650.4	1.22	-2412.8	1.11
11-35-6	-2687	-10773.8	4.01	-8849.4	3.29	-11098.4	4.13	-8436	3.14	-3440.9	1.28	-3074.5	1.14
11-35-9	-3141.3	-12752.7	4.06	-9512.7	3.03	-13306.3	4.24	-9496.8	3.02	-4107.9	1.31	-3540.1	1.13
12-40-4	-2467.1	-8257	3.35	-8606.6	3.49	-9107.1	3.69	-8168.3	3.31	-2915.7	1.18	-2862.4	1.16
12-40-6	-3044.7	-11709.1	3.85	-10676.2	3.51	-12016.3	3.95	-10281.8	3.38	-3762.6	1.24	-3520.2	1.16