ME 689 **HOMEWORK SET 9** **SPRING 2019**
OUT: 04/23/19 DUE: 05/03/19

PROBLEM #1 (150 points)

Consider the same linear motor system as in Problem 1 of HW#7.

A. Synthesize an **indirect adaptive robust controller (IARC)** that achieves a guaranteed transient and steady-state tracking performance in general, and asymptotic output tracking (i.e., $e_y(t) = y(t) - y_m(t)$ converges to zero asymptotically) for constant external disturbances of $d(t) = d_0, \forall t$. ***To receive full credit, you need to provide the detailed designs and proof as well.***

B. Let the reference output be the output of a reference model given by

$$y_m(t) = G_m(p)\left[u_c(t)\right], \qquad G_m(s) = \frac{\omega_m^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2}, \quad \omega_m = 15, \quad \zeta_m = 1.0 \qquad \text{(H4)}$$

in which the reference command input u_c is a square wave type reference command with a half period of 0.6 sec representing a back-forth movement of travel distance of 0.2m. Assuming a sampling rate of 2kHz and using the Euler's approximation algorithm, discretize the continuous adaptive control law in part A. The initial values for the controller and estimator parameters should be chosen according to the initial estimates of $\hat{M}_e(0) = 0.055$, $\hat{B}(0) = 0.225$, $\hat{A}_{sc}(0) = 0.125$, $\hat{A}_{cog1}(0) = 0.03$, $\hat{A}_{cog3}(0) = 0.03$ and $\hat{d}_0(0) = 0$. Simulate the ARC control law with the continuous plant (H1) for the following sets of actual values for the linear motor:

Case 1: $M_e = 0.025, \quad B = 0.1, \quad A_{sc} = 0.1, \quad A_{cog1} = 0.01, \quad A_{cog3} = 0.05, \quad d_0 = 1$

Case 2: $M_e = 0.085, \quad B = 0.35, \quad A_{sc} = 0.15, \quad A_{cog1} = 0.05, \quad A_{cog3} = 0.05, \quad d_0 = 1$

Case 3: $M_e = 0.025, B = 0.1, A_{sc} = 0.1, A_{cog1} = 0.01, A_{cog3} = 0.05, \quad d(t) = 1 + (-1)^{\mathrm{round}(10t\sin(20t))}$

Case 4: $M_e = 0.085, B = 0.35, A_{sc} = 0.15, A_{cog1} = 0.05, A_{cog3} = 0.05, \ d(t) = 1 + (-1)^{\mathrm{round}(10t\sin(20t))}$

The total simulation time is 4 seconds. Obtain the following time plots:

(i) A plot showing the reference command input u_c, the reference output y_m, and the actual output y.
(ii) A plot showing the output tracking error $e = y - y_m$.
(iii) A plot showing the control input u.
(iv) A plot showing the parameter estimates and their actual values.

To receive full credit, you need to provide the details of all the controller gains used in the simulation and how they are chosen as well.

Please attach your simulation program so that they can be run to check the correctness of your simulation results as well.

PROBLEM #2 (150 points)

Consider the same linear motor system as in Problem 1 of HW#7.

C. Synthesize an **integrated direct/indirect adaptive robust controller (DIARC)** that achieves a guaranteed transient and steady-state tracking performance in general, and asymptotic output tracking (i.e., $e_y(t) = y(t) - y_m(t)$ converges to zero asymptotically) for constant external

Problem 1

A. $M_e \ddot{y} = u - B\dot{y} - A_{sc} S(\dot{y}) + A_{cog1} \cdot \sin\left(\frac{2\pi}{P} y\right) + A_{cog3} \cdot \sin\left(\frac{6\pi}{P} y\right) + d(t)$

Let $e = y - y_d$, $\dot{e} = \dot{y} - \dot{y}_d$, $p = \dot{e} + k_1 e$, $\dot{p} = \ddot{e} + k_1 \dot{e}$

$$\therefore M_e \dot{p} = u - B\dot{y} - A_{sc} \cdot S(\dot{y}) + A_{cog1} \sin\left(\frac{2\pi}{P} y\right) + A_{cog3} \cdot \sin\left(\frac{6\pi}{P} y\right) + \underbrace{d(t)}_{d_0 + \tilde{d}(t)} - M_e\left(\ddot{y}_d - k_1 \dot{e}\right)$$

$= u - \varphi^T \theta + \tilde{d}(t)$ where $\tilde{d}(t) = d(t) - d_0$, d_0 is constant

$\varphi^T = [\ \dot{y} \quad S(\dot{y}) \quad -\sin\left(\frac{2\pi}{P} y\right) \quad -\sin\left(\frac{6\pi}{P} y\right) \quad -1 \quad (\ddot{y}_d - k_1 \dot{e})]$

$\theta = [\ B \quad A_{sc} \quad A_{cog1} \quad A_{cog3} \quad d_0 \quad M_e]^T$

$\tilde{\theta} = \hat{\theta} - \theta$

Let $u = u_a + u_s + u_p$

$u_p = -k_2 p$, $u_a = \varphi^T \hat{\theta}$, with $u_a, u_p \rightarrow M_e \dot{p} + k_2 p = u_s + \varphi^T \tilde{\theta} + \tilde{d}(t)$

choose $u_s = -\frac{h^2}{4\varepsilon} p$ to satisfy $\begin{cases} \text{i. } u_s \cdot p \leq 0 \\ \text{ii. } p \cdot \{u_s + [\varphi^T \tilde{\theta} + \tilde{d}(t)]\} \leq \varepsilon(t) \end{cases}$

$h \geq |\varphi| \cdot (\theta_{max} - \theta_{min}) + d_{max}$

To estimate θ, rewrite the system plant as follow

$-u = -B\dot{y} - A_{sc} S(\dot{y}) + A_{cog1} \cdot \sin\left(\frac{2\pi}{P} y\right) + A_{cog3} \cdot \sin\left(\frac{6\pi}{P} y\right) + d_0 + \tilde{d}(t) - M_e \ddot{y}$

$u = \varphi_u^T \theta - \tilde{d}(t)$

$\varphi_u^T = [\ +\dot{y} \quad +S(\dot{y}) \quad -\sin\left(\frac{2\pi}{P} y\right) \quad -\sin\left(\frac{6\pi}{P} y\right) \quad -1 \quad +\ddot{y}]$

$\theta = [\ B \quad A_{sc} \quad A_{cog1} \quad A_{cog3} \quad d_0 \quad M_e]^T$

Assuming $\tilde{d}(t) = 0$, $u = \varphi_u^T \theta$ passing through $\frac{1}{T_f s + 1}$ becomes

$u_f = \varphi_{uf}^T \theta - \cancel{\tilde{d}_f(t)}$

u_f and φ_{uf}^T can be obtained as follows

$$\begin{cases} \dot{u}_f = \frac{1}{T_f}(-u_f + u) & u_f = u_f + \dot{u}_f \cdot T \\ \ddot{y}_f = \frac{1}{T_f}(-\dot{y}_f + \dot{y}) & \dot{y}_f = \dot{y}_f + \ddot{y}_f \cdot T \end{cases}$$

For other $\varphi_{uf,j}$, $\dot{\varphi}_{uf,j} = \frac{1}{T_f}(-\varphi_{uf,j} + \varphi_{u,j})$, $\varphi_{uf,j} = \varphi_{uf,j} + \dot{\varphi}_{uf,j} \cdot T$

$$\dot{\Gamma}(t) = \begin{cases} D_\Gamma & \text{if } \lambda_M(\Gamma(t)) < \rho_M \\ & \text{or } V_M^T D_\Gamma V_M < 0 \\ 0 & \text{else} \end{cases}$$

Estimate $\hat{\theta}$ using the following algorithm

$$\dot{\hat{\theta}} = \text{Proj}(\Gamma \varphi_{uf} \cdot \epsilon) \quad , \quad \epsilon = u_f - \varphi_{uf}^T \cdot \hat{\theta}$$

$$D_\Gamma = \mu_1 \Gamma - \mu_2 \Gamma \varphi_{uf} \varphi_{uf}^T \Gamma \quad \leftarrow$$ This has to be replaced with the following since euler approximation is only good in linear functions, $\dot{\Gamma}$ in this case is quadratic function.

...

Let: $\dot{R} = -\mu_1 R + \mu_2 \varphi_{uf} \varphi_{uf}^T \quad , \Gamma = R^{-1}$

$$R(t_{k+1}) - R(t_k) = -\mu_1 T R(t_k) + \mu_2 T \varphi_{uf}(t_k) \varphi_{uf}^T(t_k)$$

$$R(t_k) = (1 - \mu_1 T) \cdot \left[R(t_{k-1}) + \frac{\mu_2 T}{1-\mu_1 T} \varphi_{uf}(t_{k-1}) \varphi_{uf}^T(t_{k-1}) \right]$$

Applying matrix inversion lemma with $A = R(t_{k-1})$, $C = I$, $D = \varphi_{uf}^T(t_{k-1})$

$$\begin{cases} \Gamma(t_k) = R^{-1}(t_k) = \frac{1}{1-\mu_1 T} \left\{ \Gamma(t_{k-1}) - \frac{\mu_2 T}{1-\mu_1 T} \Gamma(t_{k-1}) \varphi_{uf}(t_{k-1}) \left[I + \frac{\mu_2 T}{1-\mu_1 T} \varphi_{uf}^T(t_{k-1}) \Gamma(t_{k-1}) \varphi_{uf}(t_{k-1}) \right]^{-1} \varphi_{uf}^T(t_{k-1}) \Gamma(t_{k-1}) \right\} \\ \dot{\hat{\theta}} = \text{Proj}(\Gamma \varphi_{uf} \cdot \epsilon), \quad \epsilon = u_f - \varphi_{uf}^T \hat{\theta} \end{cases}$$

$0 \leq \mu_2 < 2$ to guarantee stability

Proof:

1. convergence and bounded error p, e in the presence of model structure uncertainty

Choose P.D.F as $V_s = \frac{1}{2} M_e p^2$

$$\therefore \dot{V}_s = p M_e \dot{p} = p \cdot \left[-k_2 p + u_s + \left(\varphi^T \tilde{\theta} + \tilde{d}(t) \right) \right]$$

$$= -k_2 p^2 + p \left[u_s + \left(\varphi^T \tilde{\theta} + \tilde{d}(t) \right) \right] \leq -k_2 p^2 + \varepsilon(t)$$

$$p^2 = \frac{2V_s}{M_e} \Rightarrow \dot{V}_s \leq -k_2 p^2 + \varepsilon(t) = -\frac{2k_2}{M_e} V_s + \varepsilon(t) \quad , \text{ Let } \lambda_V = \frac{2k_2}{M_e}$$

Thus: $V_s(t) \leq e^{-\lambda_V t} V_s(0) + \frac{\varepsilon}{\lambda_V} [1 - e^{-\lambda_V t}] \rightarrow V_s(\infty) \leq \frac{\varepsilon}{\lambda_V} \rightarrow |p(\infty)| \leq \sqrt{\frac{\varepsilon}{\lambda_V}}$

2. Asymptotic output tracking when $\tilde{d}(t) = 0$

When $\tilde{d}(t) = 0$

$$M_e \dot{p} + k_2 p + \frac{h^2}{4\varepsilon} p = +\varphi^T \tilde{\theta}(t) = M_e \dot{p} + k_{eq} p \quad , \quad k_{eq} = k_2 + \frac{h^2}{4\varepsilon}$$

Let V_a be a P.D.F as follow.

$$V_a = \frac{1}{2}\left(M e P + T_f \varphi_{uf}^T \breve{\theta}\right)^2$$

$$\dot{V}_a = \left(M e P + T_f \varphi_{uf}^T \breve{\theta}\right) \cdot \left[M e \dot{P} + T_f \frac{d}{dt}\left(\varphi_{uf}^T \breve{\theta}\right)\right]$$

$$= \left(M e P + T_f \varphi_{uf}^T \breve{\theta}\right) \cdot \left[-K_{eq} \cdot P + \varphi^T \breve{\theta} - \varphi_{uf}^T \breve{\theta} + \varphi_u^T \breve{\theta} + T_f \varphi_{uf}^T \dot{\hat{\theta}}\right]$$

$$= -M e K_{eq} P^2 - T_f \left(\varphi_{uf}^T \breve{\theta}\right)^2 + M e P \varphi^T \breve{\theta} - M e P \varphi_{uf}^T \breve{\theta} + M e P \varphi_u^T \breve{\theta}$$
$$+ M e P T_f \varphi_{uf}^T \dot{\hat{\theta}} - K_{eq} P \cdot T_f \varphi_{uf}^T \breve{\theta} + T_f \varphi_{uf}^T \breve{\theta} \cdot \varphi^T \breve{\theta} + T_f \varphi_{uf}^T \breve{\theta} \cdot \varphi_u^T \breve{\theta}$$
$$+ T_f^2 \varphi_{uf}^T \breve{\theta} \cdot \varphi_{uf}^T \dot{\hat{\theta}}$$

$$= -M e K_{eq} P^2 - T_f \left(\varphi_{uf}^T \breve{\theta}\right)^2 + \left(M e P + T_f \varphi_{uf}^T \breve{\theta}\right) \varphi^T \breve{\theta} + \left(M e P + T_f \varphi_{uf}^T \breve{\theta}\right) \cdot \varphi_u^T \breve{\theta}$$
$$- \left(M e P + K_{eq} P \cdot T_f\right) \varphi_{uf}^T \breve{\theta} + \left(M e P T_f + T_f^2 \varphi_{uf}^T \breve{\theta}\right) \cdot \varphi_{uf}^T \dot{\hat{\theta}}$$

$$\dot{V}_a = -M e K_{eq} P^2 - T_f \left(\varphi_{uf}^T \breve{\theta}\right)^2 - \left(M e P + K_{eq} T_f P - T_f \varphi^T \breve{\theta} - T_f \varphi_u^T \breve{\theta}\right) \varphi_{uf}^T \breve{\theta}$$
$$+ M e P \cdot \left(\varphi^T \breve{\theta} + \varphi_u^T \breve{\theta}\right) + \left(M e P T_f + T_f^2 \varphi_{uf}^T \breve{\theta}\right) \cdot \varphi_{uf}^T \dot{\hat{\theta}}$$

$$\dot{V}_a \leq -K_\varepsilon P^2 - K_{\theta 1} \left(\varphi_{uf}^T \breve{\theta}\right)^2 + K_{\theta 2} \left(\varphi_u^T \breve{\theta}\right)^2 + C_\theta \left(\varphi_{uf}^T \dot{\hat{\theta}}\right)^2$$

For any given time t,

$$\int_0^t P^2 \, d\upsilon \leq \frac{1}{K_{\varepsilon m}} \left\{ V_a(0) - \int_0^t K_{\theta 1} \left(\varphi_{uf}^T \breve{\theta}\right)^2 d\upsilon + \int_0^t K_{\theta 2} \left(\varphi_u^T \breve{\theta}\right)^2 d\upsilon + \int_0^t C_\theta \left(\varphi_{uf}^T \dot{\hat{\theta}}\right)^2 d\upsilon \right.$$

Since all the closed-loop signals have been shown to be uniformly bounded in the proof 1, $K_{\theta 1}$, $K_{\theta 2}$, C_θ, φ^T, φ_u^T, φ_{uf}^T are uniformly bounded.

$\epsilon = U_f - \varphi_{uf}^T \hat{\theta} = -\varphi_{uf}^T \breve{\theta}$ if $\tilde{d}(t) = 0$, From theorem 3.P2 of system ID, $\epsilon \in L_2$, $\dot{\hat{\theta}} \in L_2^P$

Thus $\epsilon = -\varphi_{uf}^T \hat{\theta}$ goes to zero when $t \to \infty$

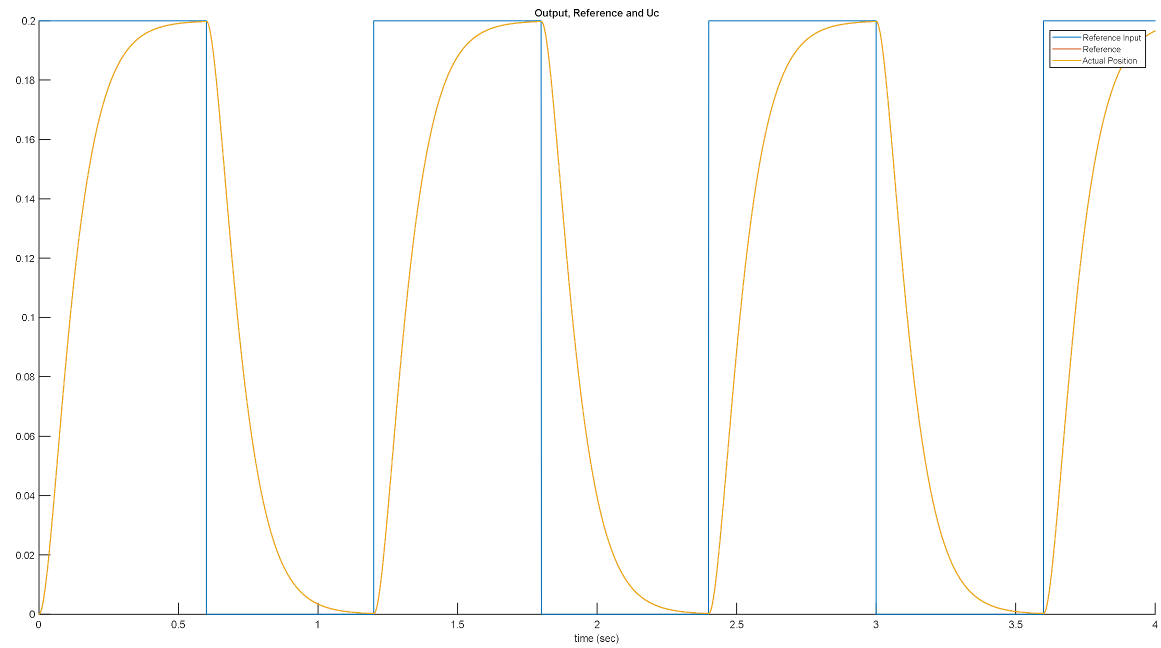
$\therefore$ The integrals on the right have finite limits as $t \to \infty$, which shows that $\int_0^t P^2 \, d\upsilon \leq \frac{1}{K_{\varepsilon m}} \left(V_a(0) + \text{finite value}\right) \Rightarrow P \in L_2$, $e \in L_2$

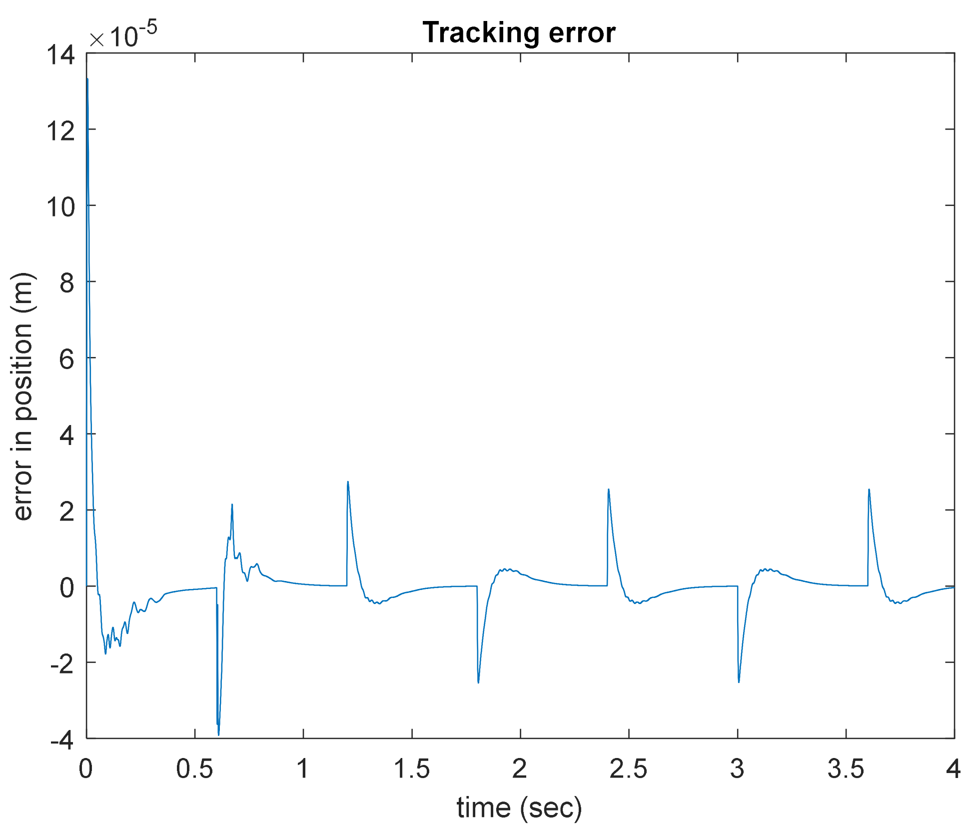
As $p \in L_\infty$ and $\tilde{\theta} \in L_\infty$. From $M_e \dot{p} + K_q p = \varphi^T \tilde{\theta}$, $\dot{p} \in L_\infty$ and thus p is uniformly continuous. Using Barbalat's Lemma,

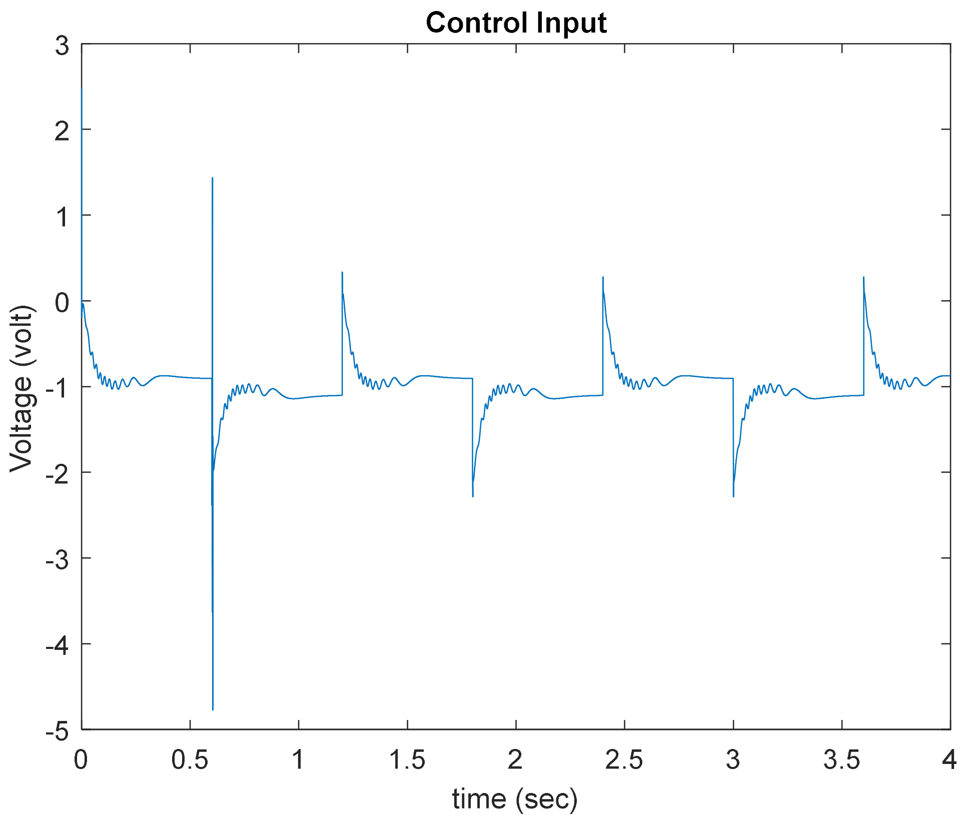
$p \to 0$ as $t \to \infty$ and $e \to 0$ as $t \to \infty$

Part B

Case 1: $M_e = 0.025, \quad B = 0.1, \quad A_{sc} = 0.1, \quad A_{cog1} = 0.01, \quad A_{cog3} = 0.05, \quad d_0 = 1$



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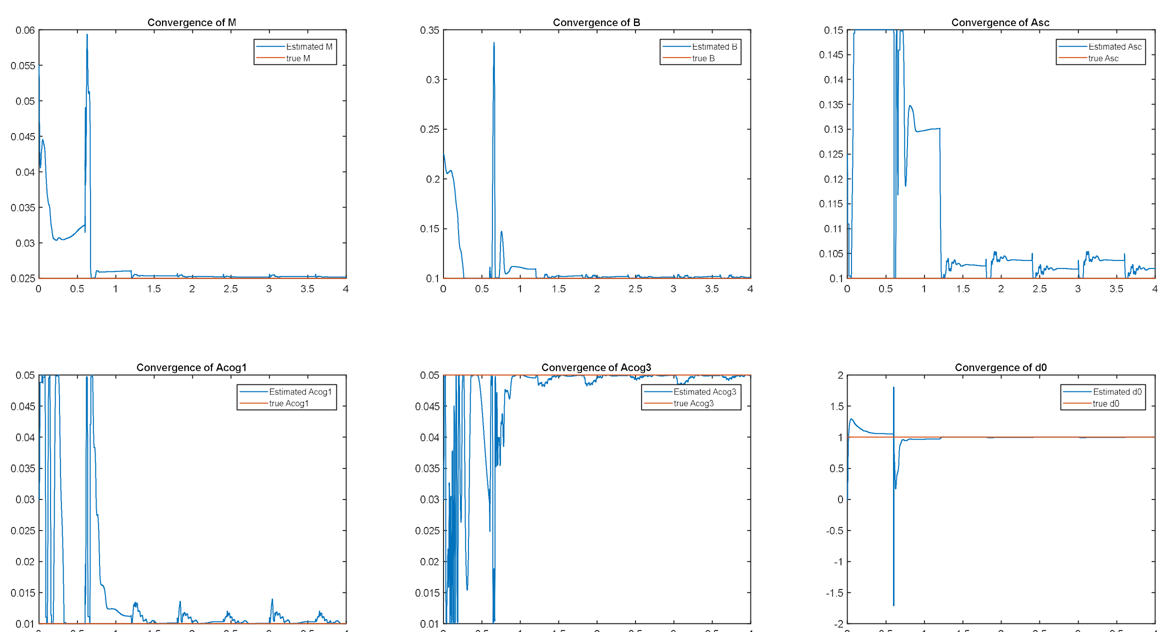
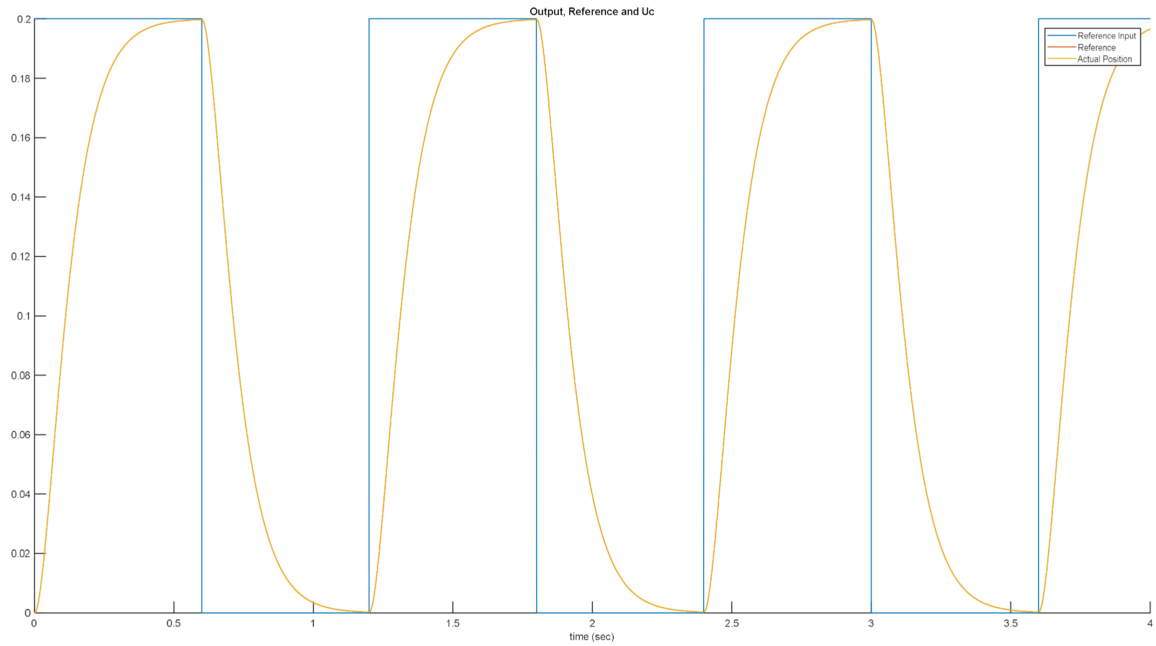
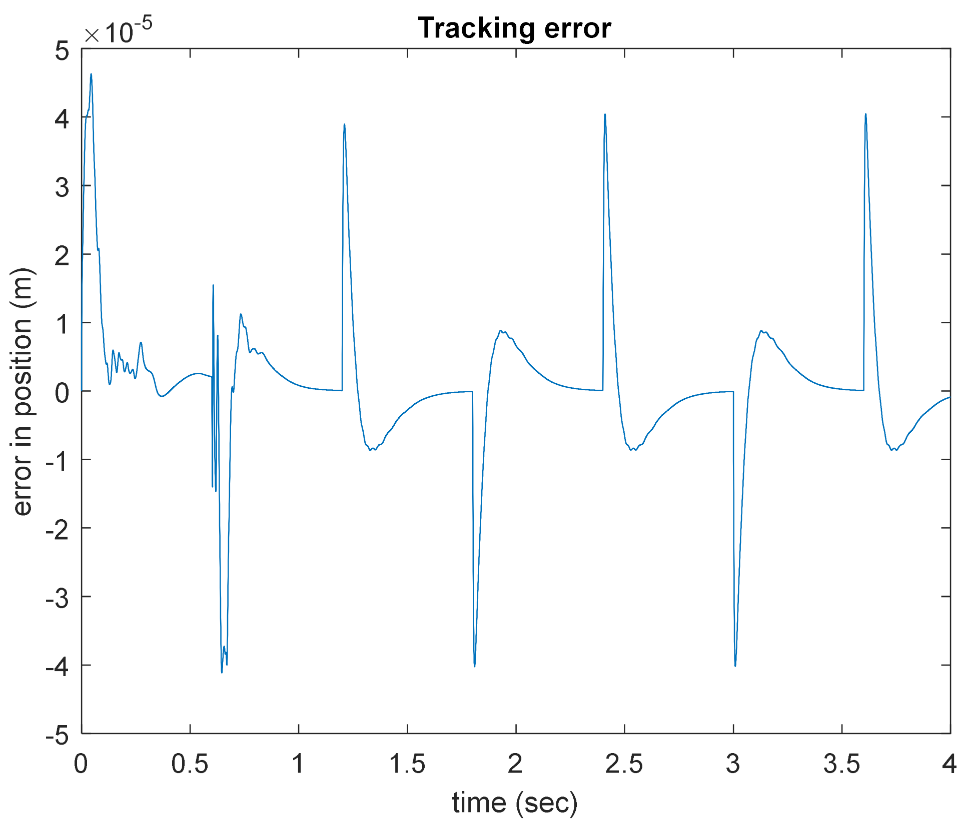
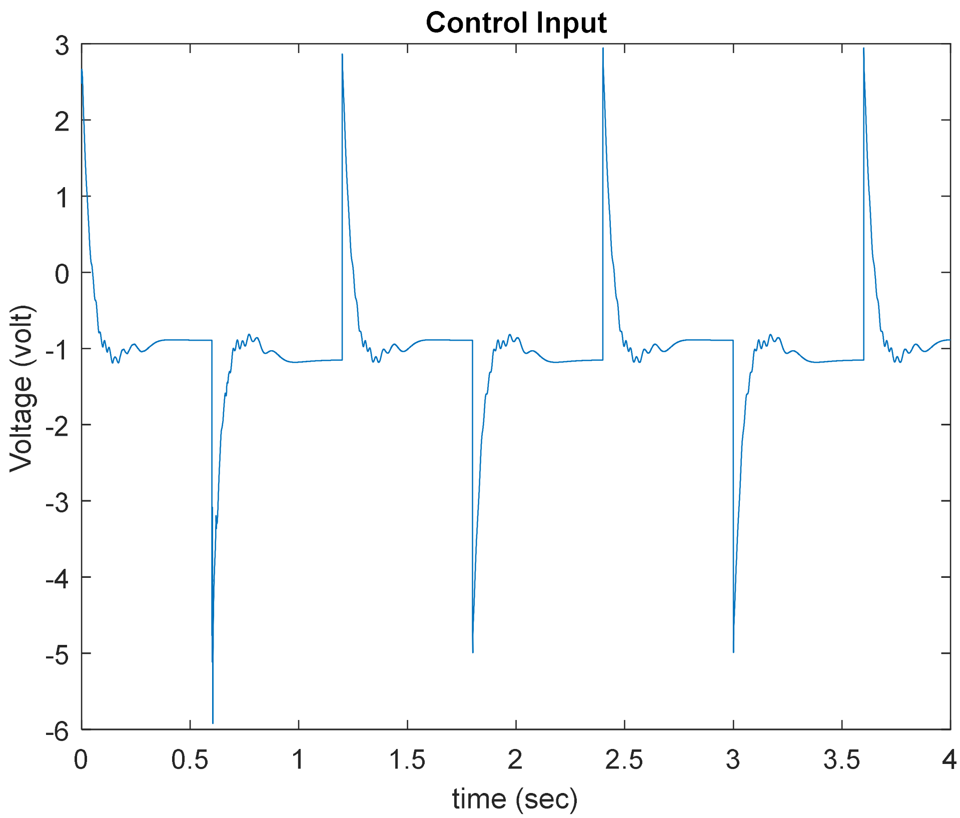
Voltage (volt)
time (sec)

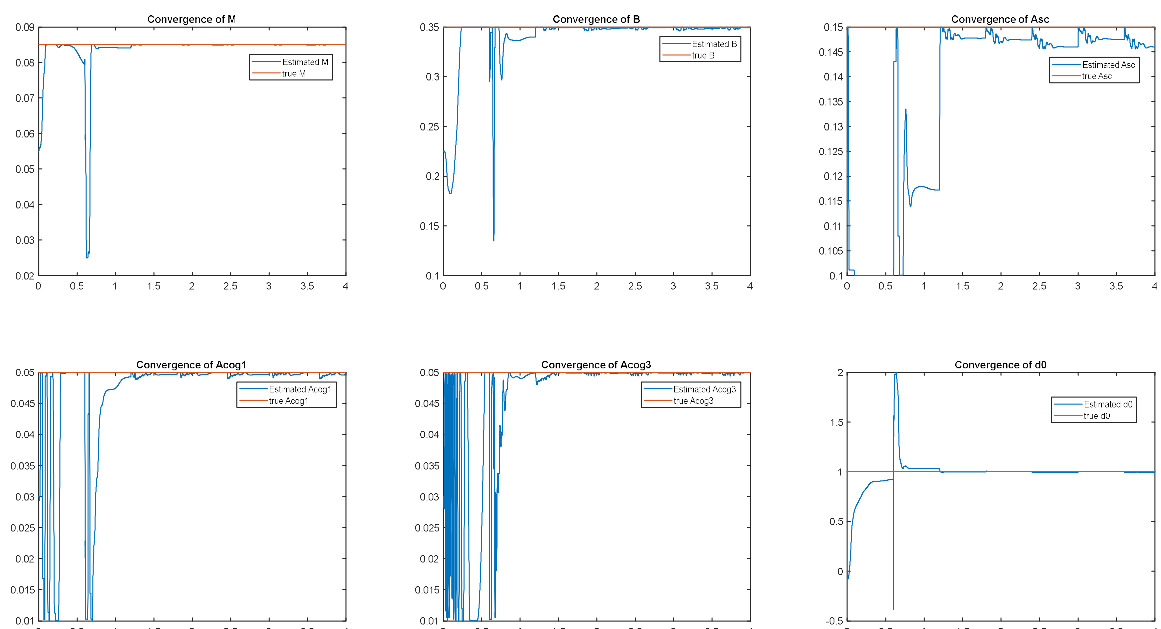

Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0

Case 2: $M_e = 0.085, \quad B = 0.35, \quad A_{sc} = 0.15, \quad A_{cog1} = 0.05, \quad A_{cog3} = 0.05, \quad d_0 = 1$

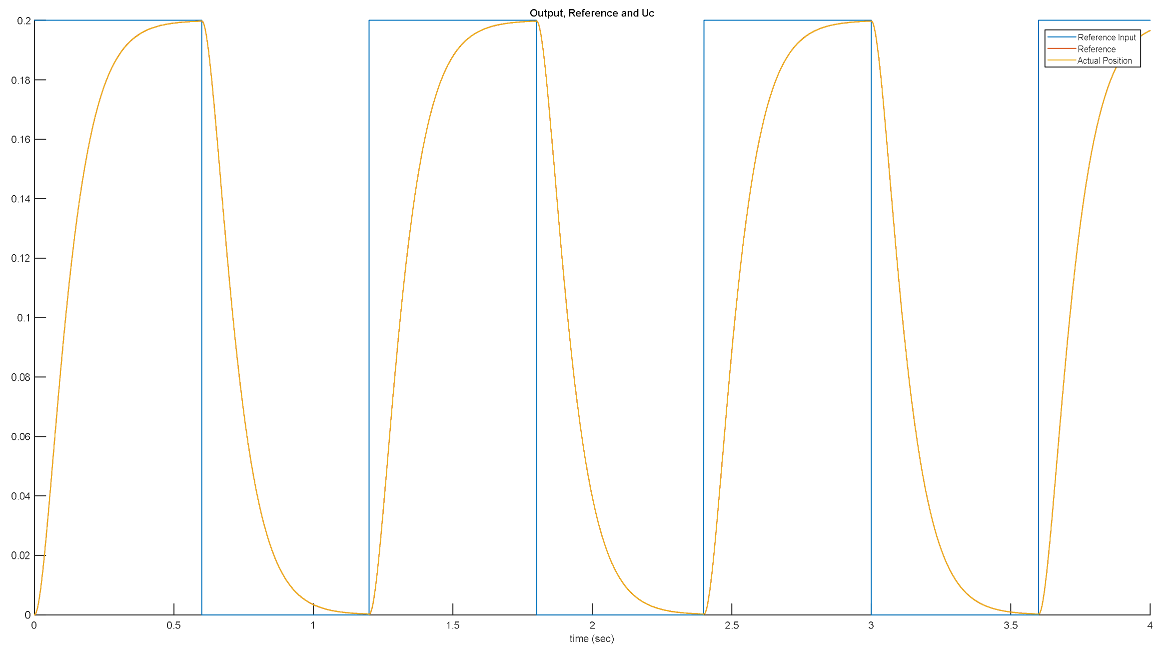


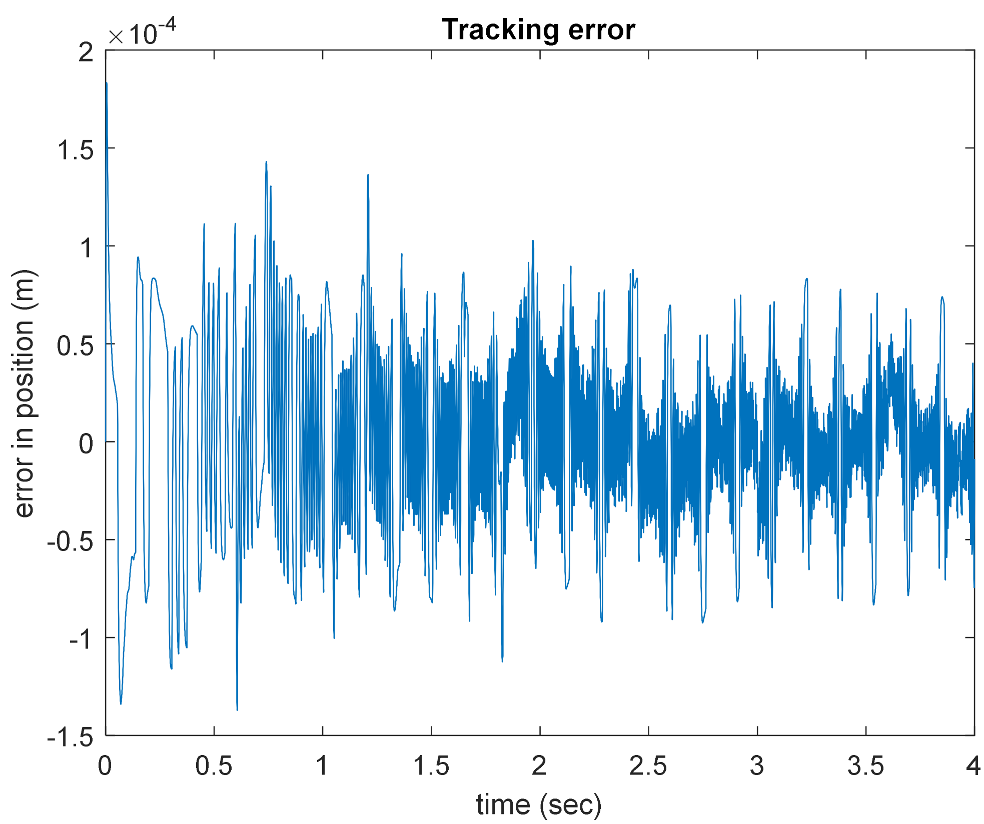


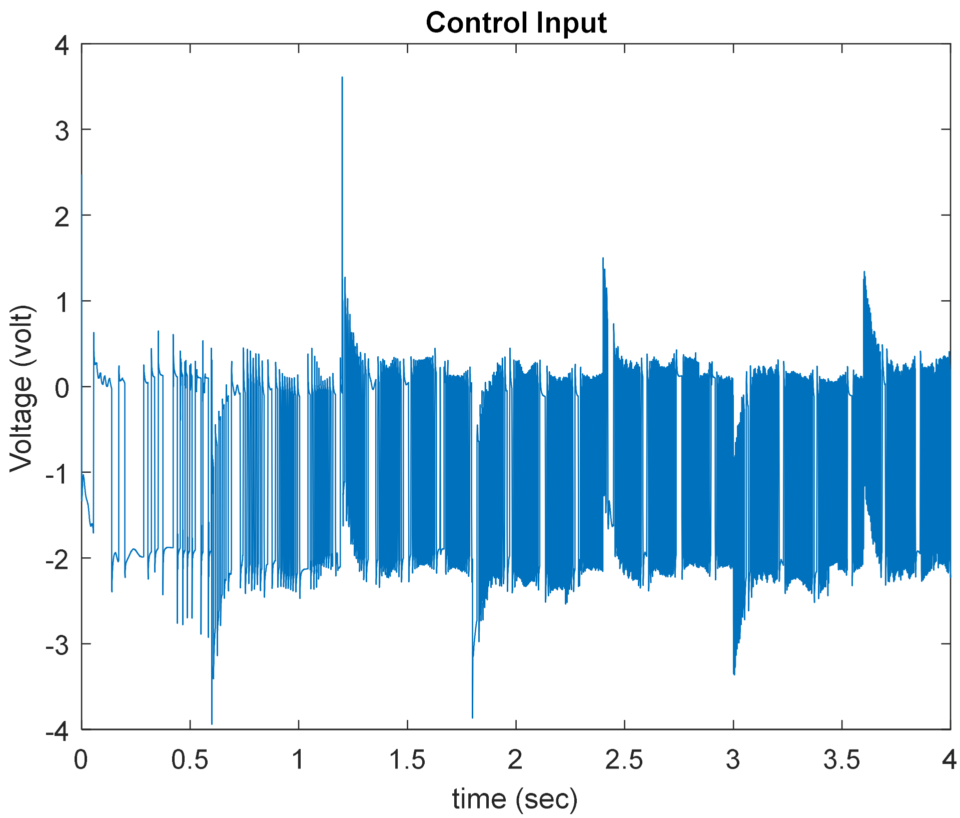

Control Input
Voltage (volt)
time (sec)

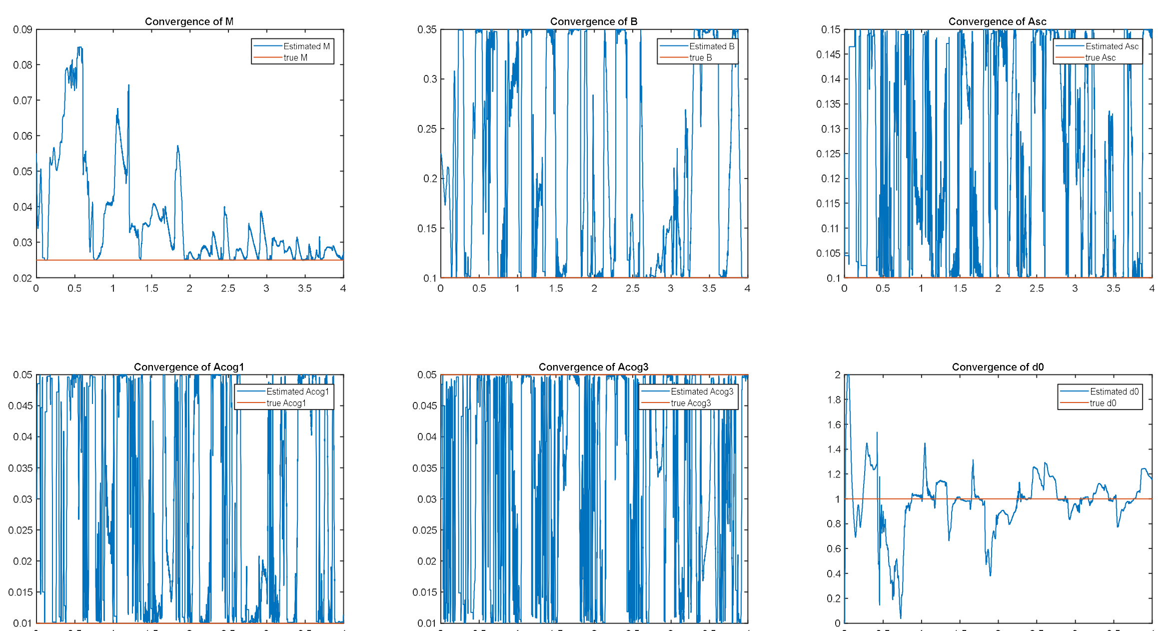

Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0

Case 3: $M_e = 0.025,\ B = 0.1,\ A_{sc} = 0.1,\ A_{cog1} = 0.01,\ A_{cog3} = 0.05, \quad d(t) = 1 + (-1)^{\mathrm{round}(10t \sin(20t))}$

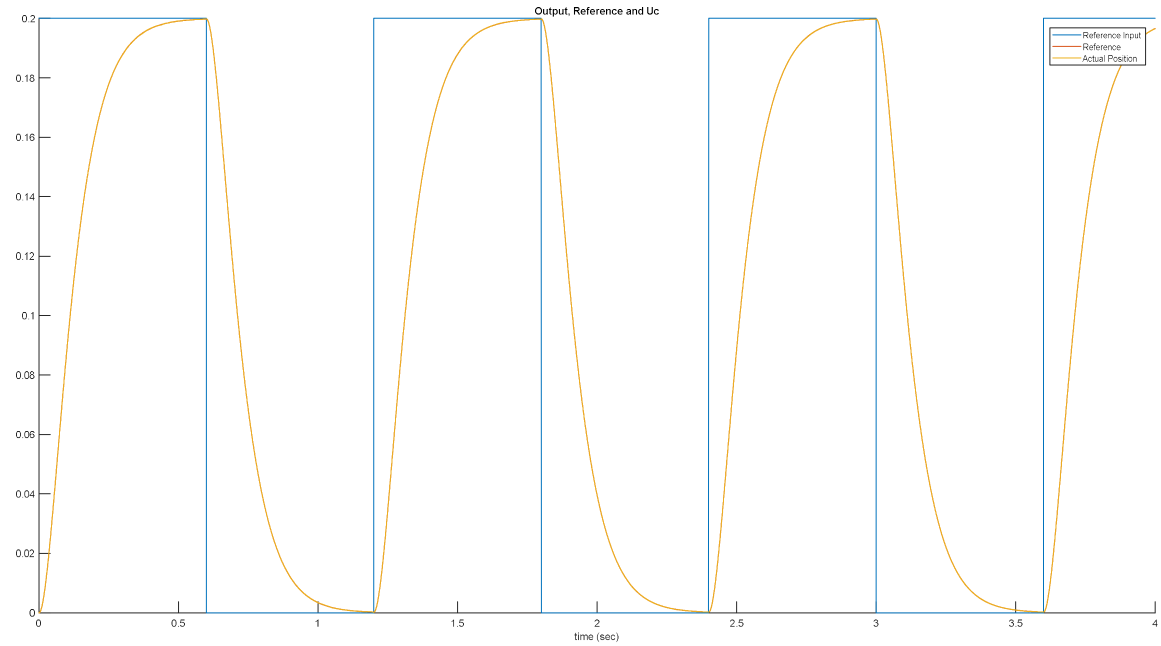


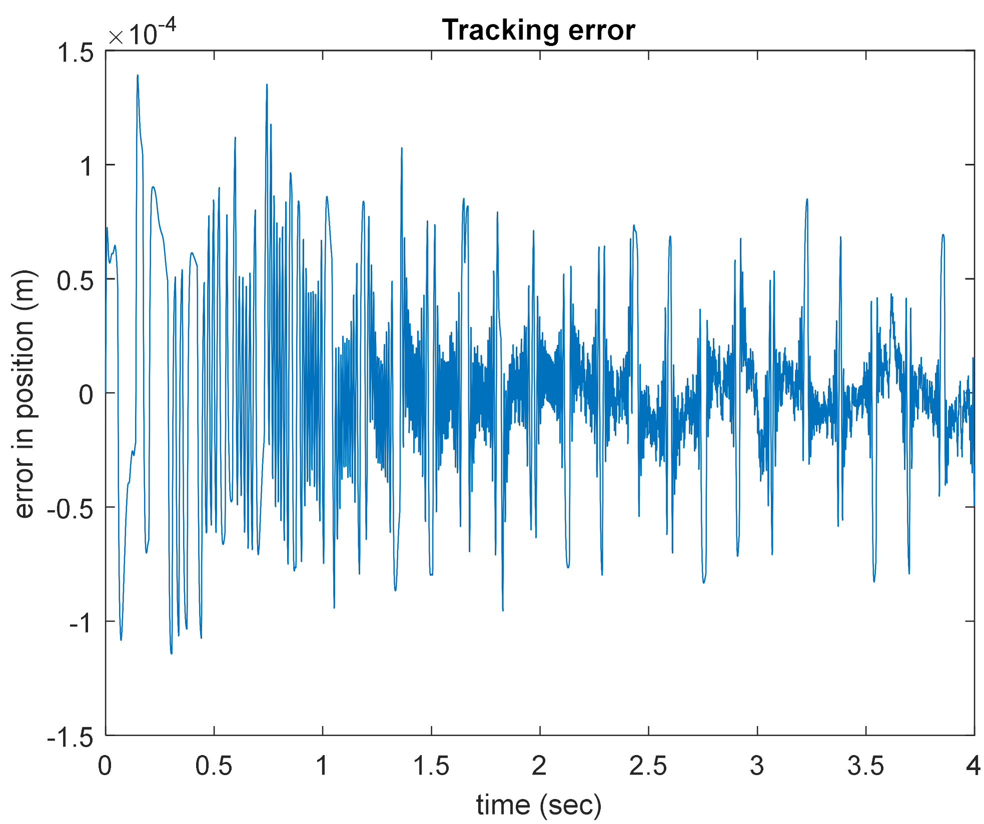


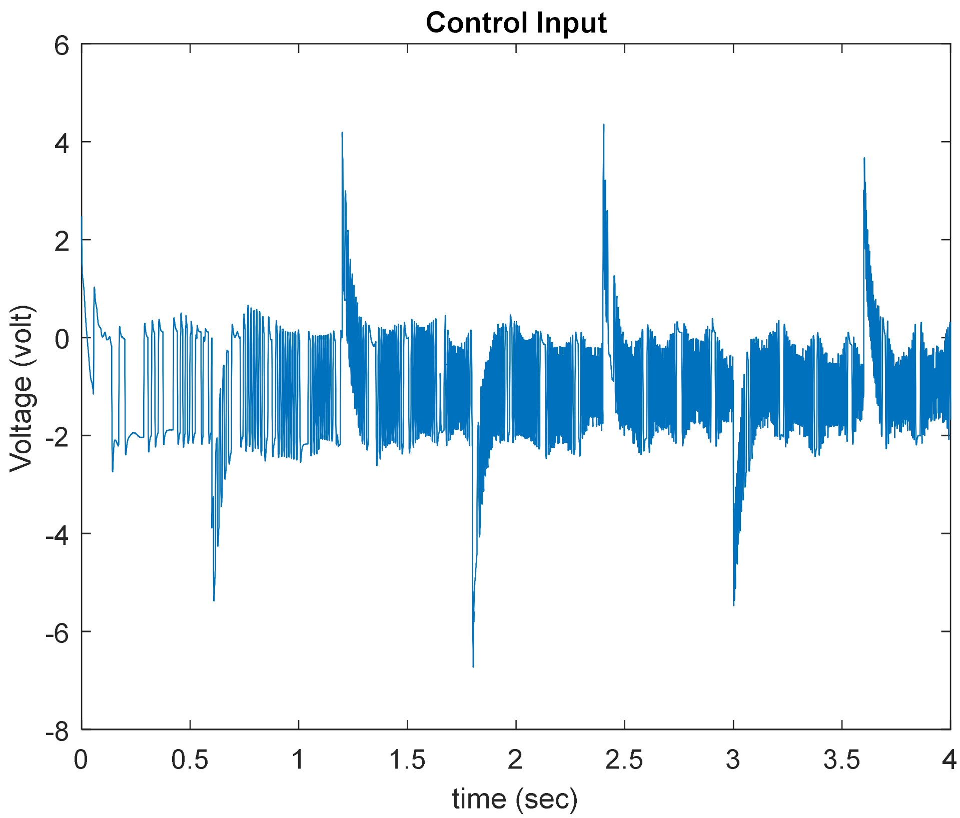

Control Input
Voltage (volt)
time (sec)


Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0

Case 4: $M_e = 0.085,\ B = 0.35,\ A_{sc} = 0.15,\ A_{cog1} = 0.05,\ A_{cog3} = 0.05,\ d(t) = 1 + (-1)^{\mathrm{round}(10t\sin(20t))}$






Control Input
Voltage (volt)
time (sec)
6
4
2
0
-2
-4
-6
-8
0
0.5
1
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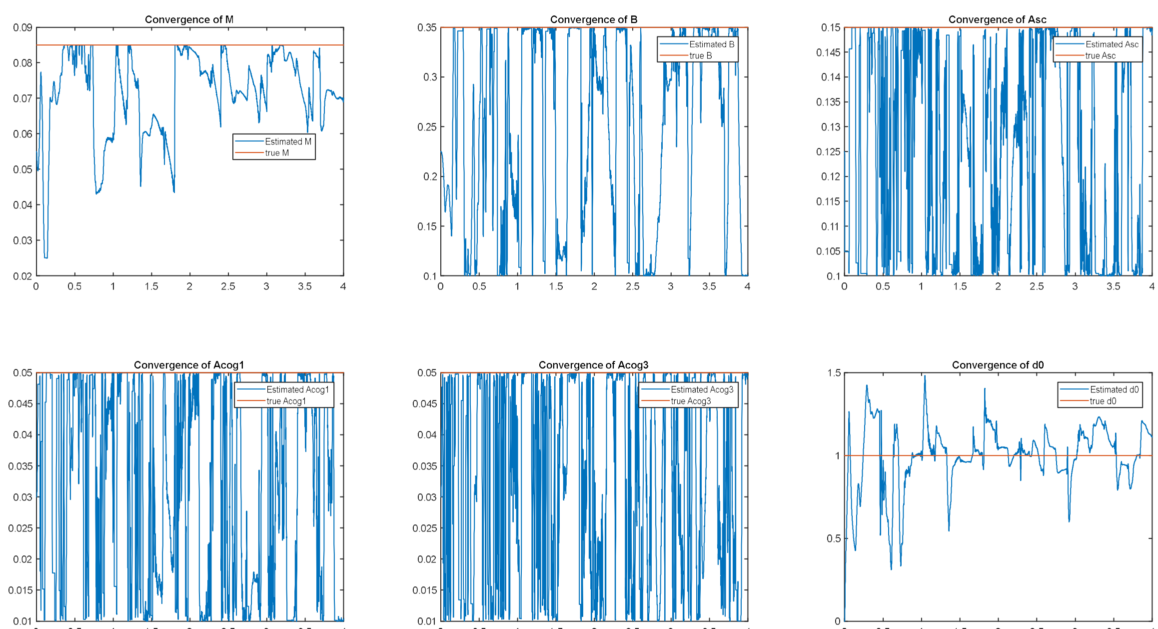
2
2.5
3
3.5
4


Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0

Problem 2

$$e = y - y_d$$
$$p = \dot{e} + k_1 e$$

C. Similar to problem 1A.

$$M_e \dot{p} = u - \varphi^T \theta + \tilde{d}(t) \qquad d(t) - d_0$$

$$\varphi^T = [\dot{y} \quad S(\dot{y}) \quad -\sin(\tfrac{2\pi}{p} y) \quad -\sin(\tfrac{6\pi}{p} y) \quad -1 \quad \ddot{y}_d - k_1 \dot{e}]$$

$$\theta = [B \quad A_{sc} \quad A_{cog1} \quad A_{cog3} \quad d_0 \quad M_e]^T$$

Then, Let $u = u_m + u_s$

$u_m = u_{m1} + u_{m2}$, $\underline{u_{m1} = \varphi^T \hat{\theta}}$ as model compensation

$u_s = u_{s1} + u_{s2}$, $\underline{u_{s1} = -k_2 p}$ to enhance stability of the C.L.

$$\boxed{\begin{aligned} \tilde{\theta} &= \hat{\theta} - \theta \\ \tilde{d}(t) &= d(t) - d_0 \end{aligned}}$$

$$\therefore M_e \dot{p} + k_2 p = u_{s2} + u_{m2} + \varphi^T \tilde{\theta} + \tilde{d}(t)$$

Define $d_c + \Delta^*(t) = \varphi^T \tilde{\theta} + \tilde{d}(t) \Rightarrow M_e \dot{p} + k_2 p = u_{s2} + u_{m2} + d_c + \Delta^*(t)$

where $\Delta^*(t)$ is the high frequency components of the combination of model compensation error and the physical parameter estimation error, and d_c is the low frequency part, which can be compensated using a fast adaptation as follows.

$$\dot{\hat{d}}_c = \mathrm{Proj}(\gamma_d \cdot p) = \begin{cases} 0 & \text{if } |\hat{d}_c| = d_{cM} \text{ and } \hat{d}_c p > 0 \\ \gamma_d \cdot p & \text{else} \end{cases}$$

with $\gamma_d > 0$ and $|\hat{d}_c(0)| \le d_{cM}$, guarantees that $|\hat{d}_c(t)| \le d_{cM} \ \forall t$

Thus, using $\underline{u_{m2} = -\hat{d}_c}$ as compensation, we can have the following:

$$M_e \dot{p} + k_2 p = u_{s2} + \tilde{d}_c + \Delta^*(t) \qquad \text{where } \tilde{d}_c = d_c - \hat{d}_c$$

Due to the use of projection type adaptation law, all estimation errors are now bounded within known bounds. Thus it is possible to use the same robust feedback synthesis technique as in DARC.

Let $\underline{u_{s2} = -\frac{h^2}{4\varepsilon} p}$ to satisfy

$$\begin{cases} \text{i. } u_{s2} \cdot p \le 0 \\ \text{ii. } p \cdot \{u_{s2} + \tilde{d}_c + \Delta^*(t)\} \\ \quad = p \cdot \{u_{s2} - \hat{d}_c + \varphi^T \tilde{\theta} + \tilde{d}(t)\} \le \varepsilon(t) \end{cases}$$

To estimate $\overset{\vee}{\theta}$ as $\hat{\theta}$, follow the same procedure as in problem 1A

$$u = \varphi_u^T \theta - \tilde{d}(t) \qquad d(t) = \tilde{d}(t) + d_0$$

$$\varphi_u^T = \left[\dot{y} \quad s(\dot{y}) \quad -\sin\left(\frac{2\pi}{p} y\right) \quad -\sin\left(\frac{6\pi}{p} y\right) \quad -1 \quad \ddot{y} \right]$$

$$\theta = \left[B \quad A_{sc} \quad A_{cog1} \quad A_{cog3} \quad d_0 \quad M_e \right]^T$$

Assume $\tilde{d}(t) = 0$ to have a linear regression model $u = \varphi_u^T \theta$

passing through $\frac{1}{T_f s + 1}$ becomes $u_f = \varphi_{uf}^T \theta - \cancel{\tilde{d}_f(t)} \to 0$

u_f, φ_{uf}^T can be obtained as follow:

$$\dot{u}_f = \frac{1}{T_f} \cdot (-u_f + u), \quad u_f = u_f + \dot{u}_f \cdot T \qquad T \text{ is the sampling Period}$$

$$\ddot{y}_f = \frac{1}{T_f} \cdot (-\dot{y}_f + \dot{y}), \quad \dot{y}_f = \dot{y}_f + \ddot{y}_f \cdot T,$$ other $\varphi_{uf,j}$ can be obtained in the same way as in Problem 1A.

Then Estimate θ as $\hat{\theta}$ using the following algorithm.

$$\dot{\hat{\theta}} = \mathrm{Proj}(\Gamma \varphi_{uf} \cdot \epsilon) \qquad \epsilon = u_f - \varphi_{uf}^T \cdot \hat{\theta}$$

$$D_\Gamma = \mu_1 \Gamma - \mu_2 \cdot \Gamma \varphi_{uf} \cdot \varphi_{uf}^T \cdot \Gamma$$

$$\dot{\Gamma}(t) = \begin{cases} D_\Gamma & \text{if } \lambda_M(\Gamma(t)) < \rho_M \text{ or } V_M^T D_\Gamma V_M < 0 \\ 0 & \text{else.} \end{cases}$$

As mentioned in problem 1A. $\dot{\Gamma}(t) \cdot T$ need to be replace with recursion on $\Gamma(t_k)$ as follows.

$$\begin{cases} \Gamma(t_k) = R^{-1}(t_k) = \frac{1}{1-\mu_1 T}\left\{ \Gamma(t_{k-1}) - \frac{\mu_2 T}{1-\mu_1 T} \Gamma(t_{k-1}) \varphi_{uf}(t_{k-1}) \left[I + \frac{\mu_2}{1-\mu_1 T} \varphi_{uf}^T \Gamma \varphi_{uf}(t_{k-1}) \right]^{-1} \varphi_{uf}^T(t_{k-1}) \Gamma(t_{k-1}) \right\} \\ \dot{\hat{\theta}} = \mathrm{Proj}(\Gamma \varphi_{uf} \epsilon), \quad \epsilon = u_f - \varphi_{uf}^T \cdot \hat{\theta} \qquad 0 < \mu_2 < 2 \text{ to ensure stability.} \end{cases}$$

Proof :

1. Exponential convergence and bounded error p, e in the presence of model structure uncertainty.

choose P.D.F as $V_s = \frac{1}{2} M_e \cdot p^2$

$$\dot V_s = p\cdot M_e \dot p = p\cdot\left(-k_2 p + u_{s2} + \breve d_c + \Delta^*(t)\right)$$

$$= -k_2 p^2 + p\cdot\left\{u_{s2} + \breve d_c + \Delta^*(t)\right\} \le -k_2 p^2 + \varepsilon(t)$$

$$p^2 = \frac{2V_s}{M_e} \rightarrow \dot V_s \le -\frac{2k_2}{M_e} V_s + \varepsilon(t) \qquad \text{Let } \lambda_v = \frac{2k_2}{M_e}$$

Thus $V_s(t) \le e^{-\lambda_v t}\cdot V_s(0) + \frac{\varepsilon}{\lambda_v}\left[1 - e^{-\lambda_v t}\right] \rightarrow V_s(+\infty) \le \frac{\varepsilon}{\lambda_v} \rightarrow |p(\infty)| \le \sqrt{\frac{\varepsilon}{\lambda_v}}$

2. Asymptotic output tracking when $\breve d(t) = 0$

when $\breve d(t) = 0$

$$M_e \dot p + k_{eq} p = -\hat d_c + \varphi^T \breve\theta \qquad k_{eq} = k_2 + \frac{h^2}{4\varepsilon}$$

Let P.D.F $V_a = \frac{1}{2}\left(M_e p + T_f\cdot\varphi_{uf}^T\breve\theta\right)^2 + \mu\left(M_e p + T_f\varphi_{uf}^T\breve\theta\right)\hat d_c + \frac{1}{2\gamma_d}\hat d_c^2$.

$$= \frac{1}{2}\cdot\begin{bmatrix} M_e p + T_f\varphi_{uf}^T\breve\theta \\ \hat d_c \end{bmatrix}^T \begin{bmatrix} 1 & \mu \\ \mu & \frac{1}{\gamma_d} \end{bmatrix} \begin{bmatrix} M_e p + T_f\varphi_{uf}^T\breve\theta \\ \hat d_c \end{bmatrix}$$

V_a is non-negative by choosing μ small enough such that

positive constant

$$\mu < \min\left\{\frac{1}{\sqrt{\gamma_d}}, \frac{4k_{eq}}{4\gamma_d + k_{eq}^2}\right\}$$

Thus. $\dot V_a = \left(M_e p + T_f\cdot\varphi_{uf}^T\breve\theta\right)\left(M_e\dot p + T_f\cdot\frac{d}{dt}\left(\varphi_{uf}^T\breve\theta\right)\right)$

$+ \mu\left(M_e p + T_f\varphi_{uf}^T\breve\theta\right)\dot{\hat d}_c + \mu\left(M_e\dot p + T_f\frac{d}{dt}\left(\varphi_{uf}^T\breve\theta\right)\right)\hat d_c + \frac{1}{\gamma_d}\hat d\cdot\dot{\hat d}$

$= \left(M_e p + T_f\cdot\varphi_{uf}^T\breve\theta\right)\cdot\left(-k_{eq}p - \hat d_c + \varphi^T\breve\theta - \varphi_{uf}^T\breve\theta + \varphi_u^T\breve\theta + T_f\varphi_{uf}^T\dot{\breve\theta}\right) + \mu\left(M_e p + T_f\varphi_{uf}^T\breve\theta\right)\dot{\hat d}_c$

$+ \mu\left(-k_{eq}p - \hat d_c + \varphi^T\breve\theta - \varphi_{uf}^T\breve\theta + \varphi_u^T\breve\theta + T_f\varphi_{uf}^T\dot{\breve\theta}\right)\hat d_c + \frac{1}{\gamma_d}\hat d\dot{\hat d}$

$$\dot{V}_a = -M_e k_{eq} p^2 - T_f(\varphi_{uf}^T\breve{\theta})^2 - (M_e p + k_{eq}T_f p - T_f\varphi^T\breve{\theta} - T_f\varphi_u^T\breve{\theta} - T_f\hat{d}_c)\varphi_{uf}^T\dot{\breve{\theta}}$$
$$\left(\frac{1}{\gamma_d}\dot{\hat{d}}_c - M_e p\right)\cdot\hat{d}_c + M_e p(\varphi^T\dot{\breve{\theta}} + \varphi_u^T\dot{\breve{\theta}}) + (M_e p T_f + T_f^2\varphi_{uf}^T\breve{\theta})\cdot\varphi_{uf}^T\dot{\hat{\theta}}$$
$$+\mu(M_e p + T_f\varphi_{uf}^T\breve{\theta})\dot{\hat{d}}_c - \mu\hat{d}_c^2 - [\mu k_{eq} p + \mu\varphi_{uf}^T\breve{\theta} - \mu T_f\varphi_{uf}^T\dot{\hat{\theta}}]\cdot\hat{d}_c$$

$$\dot{V}_a \le -M_e k_{eq} p^2 - T_f(\varphi_{uf}^T\breve{\theta})^2 + (M_e + k_{eq}T_f)|p|\cdot|\varphi_{uf}^T\dot{\breve{\theta}}| + (T_f|\varphi^T\breve{\theta}| + T_f|\varphi_u^T\breve{\theta}| + T_f|\hat{d}_c|)|\varphi_{uf}^T\dot{\breve{\theta}}|$$
$$-\mu\hat{d}_c^2 + [\mu k_{eq}|p| + \mu|\varphi_{uf}^T\breve{\theta}| + \mu T_f|\varphi_{uf}^T\dot{\hat{\theta}}|]\cdot|\hat{d}_c| + \mu(M_e|p| + T_f|\varphi_{uf}^T\breve{\theta}|)\cdot$$
$$+\mu\gamma_d(|p| + T_f|\varphi_{uf}^T\breve{\theta}|)|p|$$
$$+M_e|p|\cdot(|\varphi^T\dot{\breve{\theta}}| + |\varphi_u^T\dot{\breve{\theta}}|)$$

Note: $|\dot{\hat{d}}_c| \le \gamma_d\cdot|p|$ due to the projection type adaptation law.

$$\therefore \dot{V}_a \le -k_2 p^2 - k_{\theta 1}(\varphi_{uf}^T\breve{\theta})^2 - k_c\hat{d}_c^2 + c_\theta(\varphi_{uf}^T\dot{\hat{\theta}})^2 + k_{\theta 2}\cdot(\varphi^T\dot{\breve{\theta}})^2$$

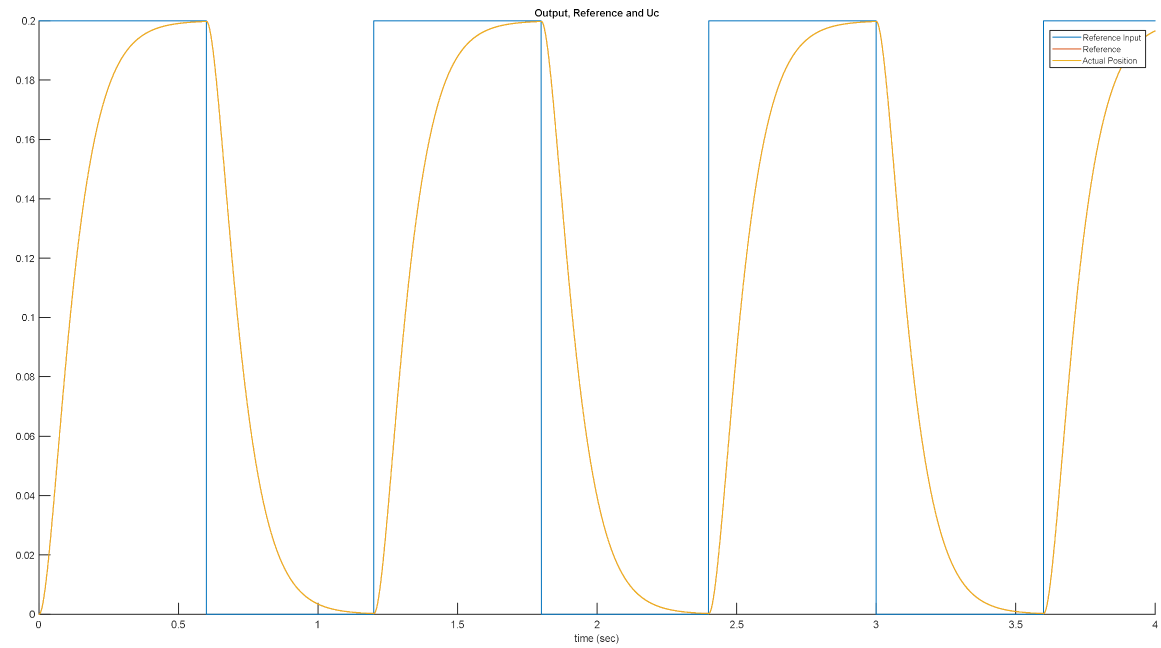
For any given time t,

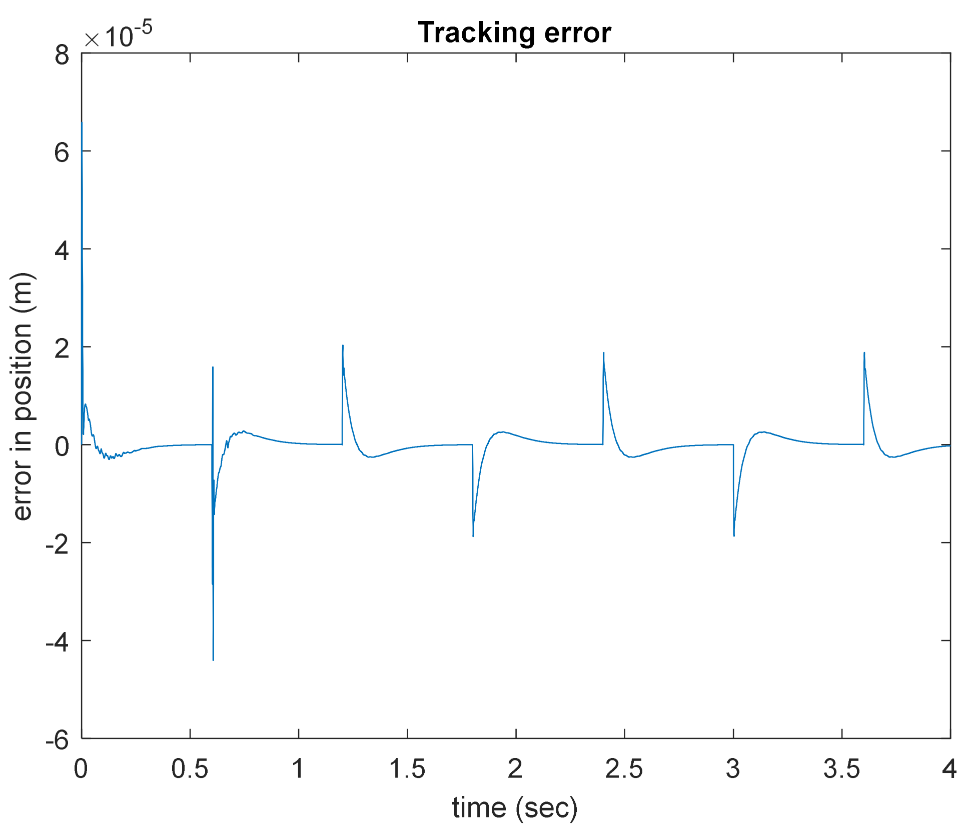
$$\int_0^t p^2 dv + \int_0^t k_c\cdot\hat{d}_c^2 \le \frac{1}{k_{2M}}\left\{V_a(0) - \int_0^t k_{\theta 1}(\varphi_{uf}^T\breve{\theta})^2 dv + \int_0^t c_\theta(\varphi_{uf}^T\dot{\hat{\theta}})^2 dv + \int_0^t k_{\theta 2}\cdot(\varphi^T\dot{\breve{\theta}})^2 dv\right\}$$

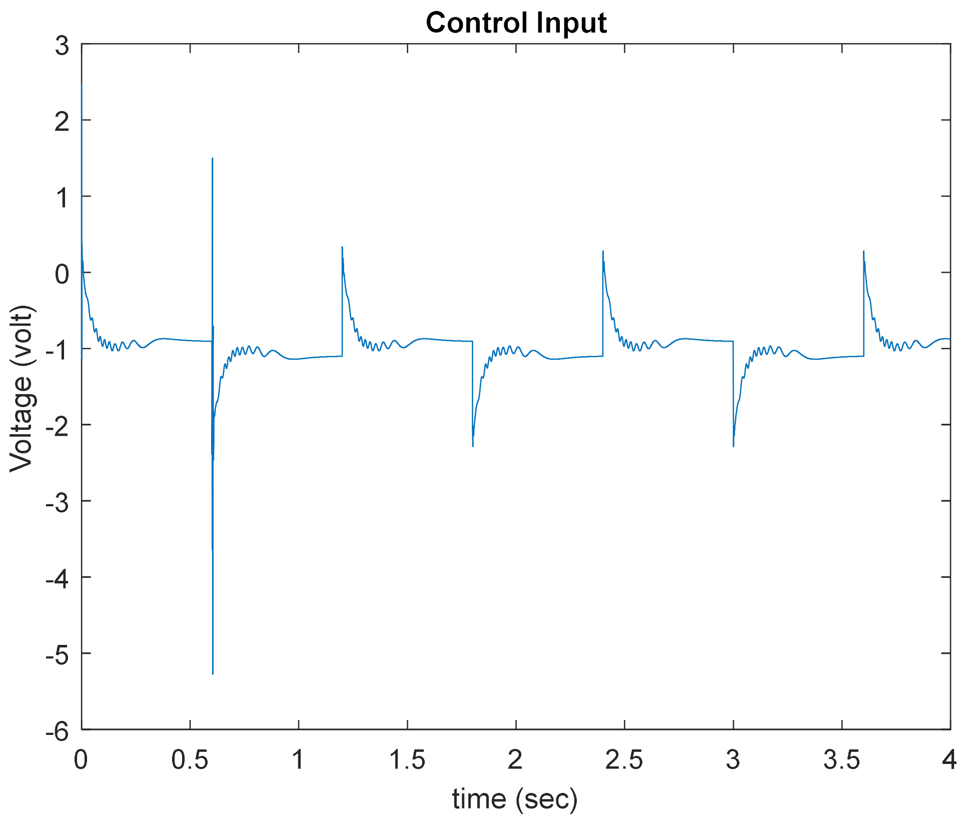
Since all the closed-loop signals have been shown to be uniformly bounded in Proof 1, $k_{\theta 1}, c_\theta, k_{\theta 2}, \varphi_{uf}^T, \varphi_u^T, \varphi^T$ are uniformly bounded.
As given in theorem 3.P2 of System ID, the PAA guarantees that $\varphi_{uf}^T\breve{\theta} \in L_2$ and $\dot{\hat{\theta}} \in L_2^P$. Thus, the integrals on the right hand have finite limit as $t \to \infty$, which shows that $p \in L_2, e \in L_2, \hat{d}_c \in L_2$.
As $p \in L_\infty$ and $\breve{\theta} \in L_\infty^P$. From $M_e\dot{p} + k_{eq}p = -d_c + \varphi^T\breve{\theta}$, $\dot{p} \in L_\infty$ and thus p is uniformly continuous. So using Barbalat's lemma, $p \to 0$ as $t \to \infty$ and thus $e \to 0$ as $t \to \infty$

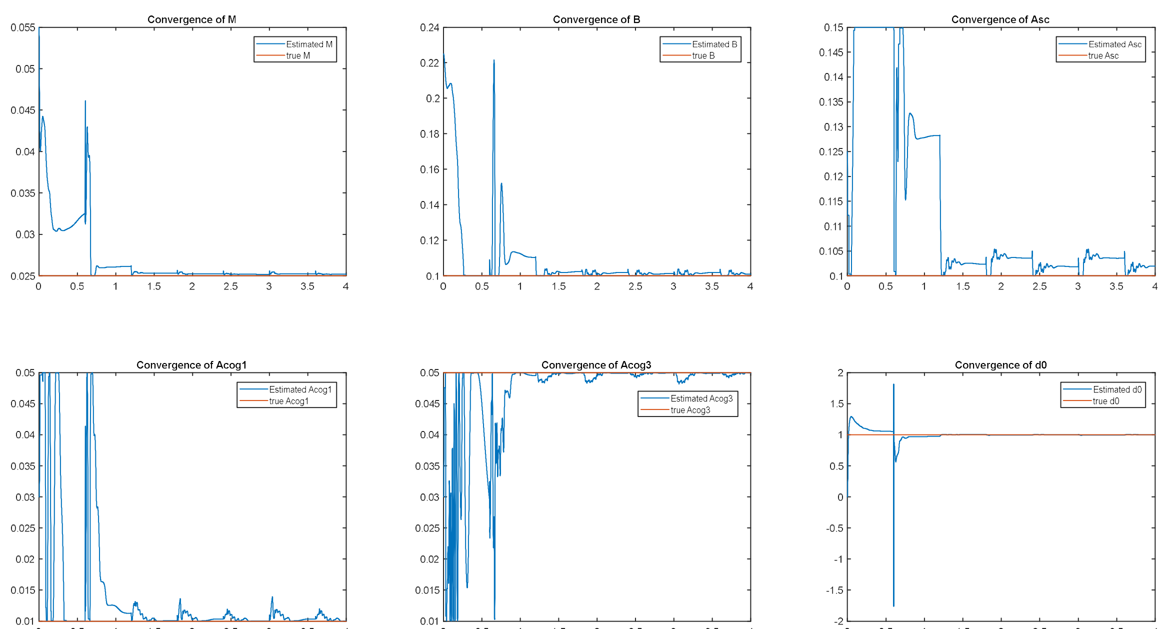
Part D

Case 1: $M_e = 0.025, \quad B = 0.1, \quad A_{sc} = 0.1, \quad A_{cog1} = 0.01, \quad A_{cog3} = 0.05, \quad d_0 = 1$

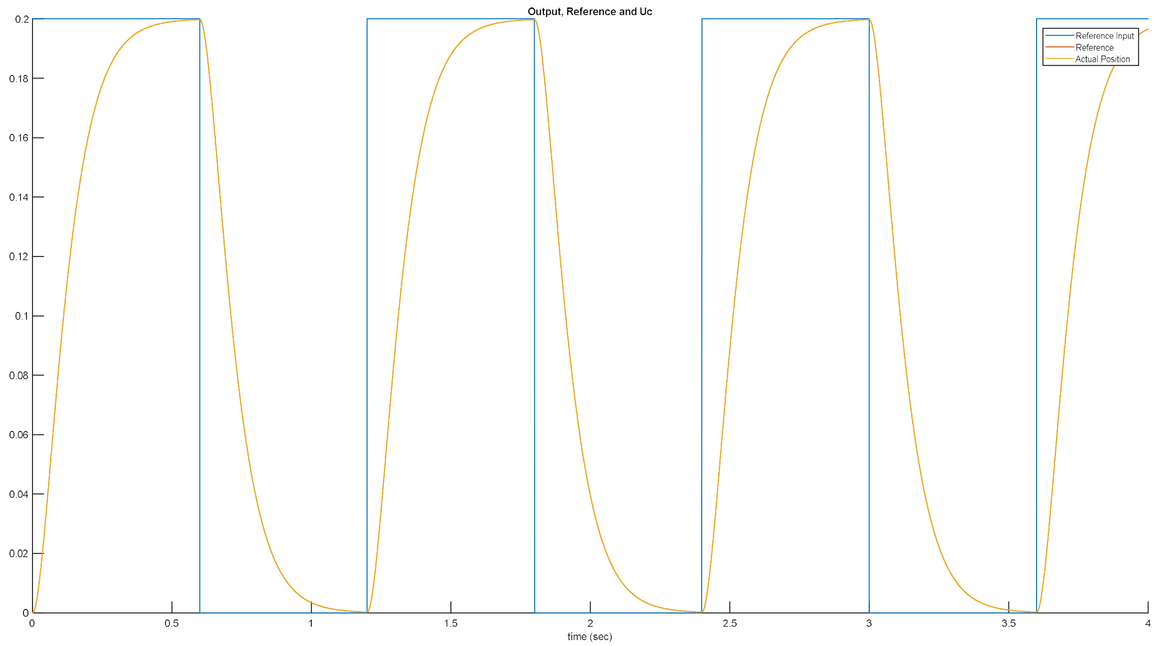


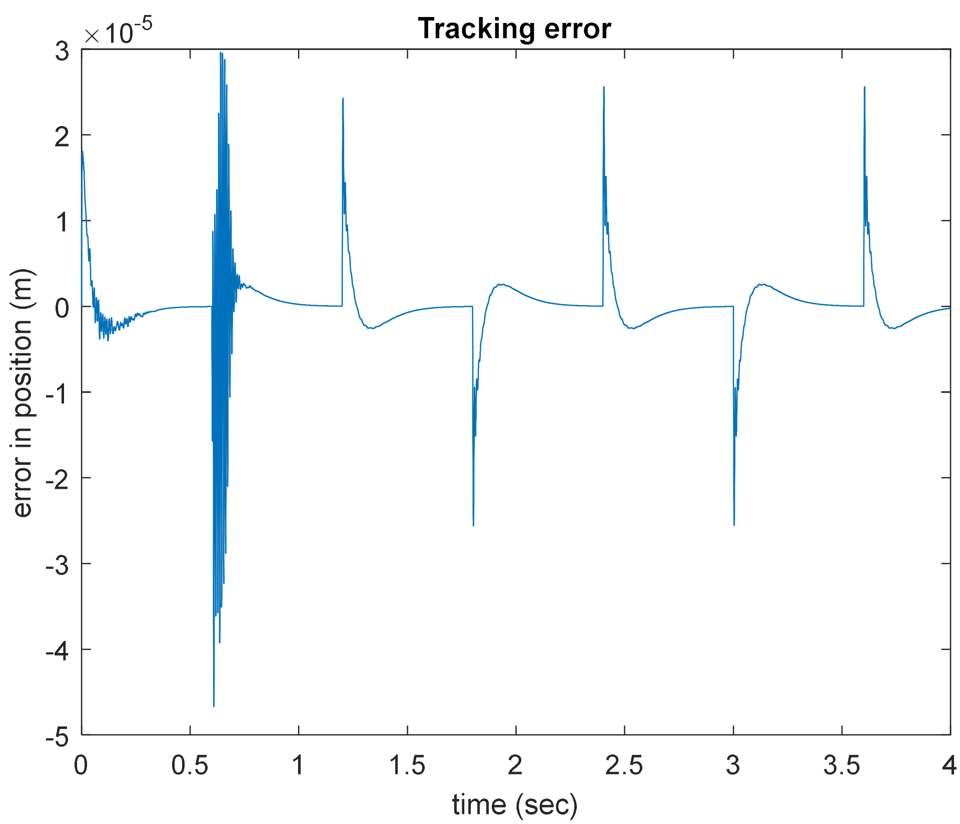


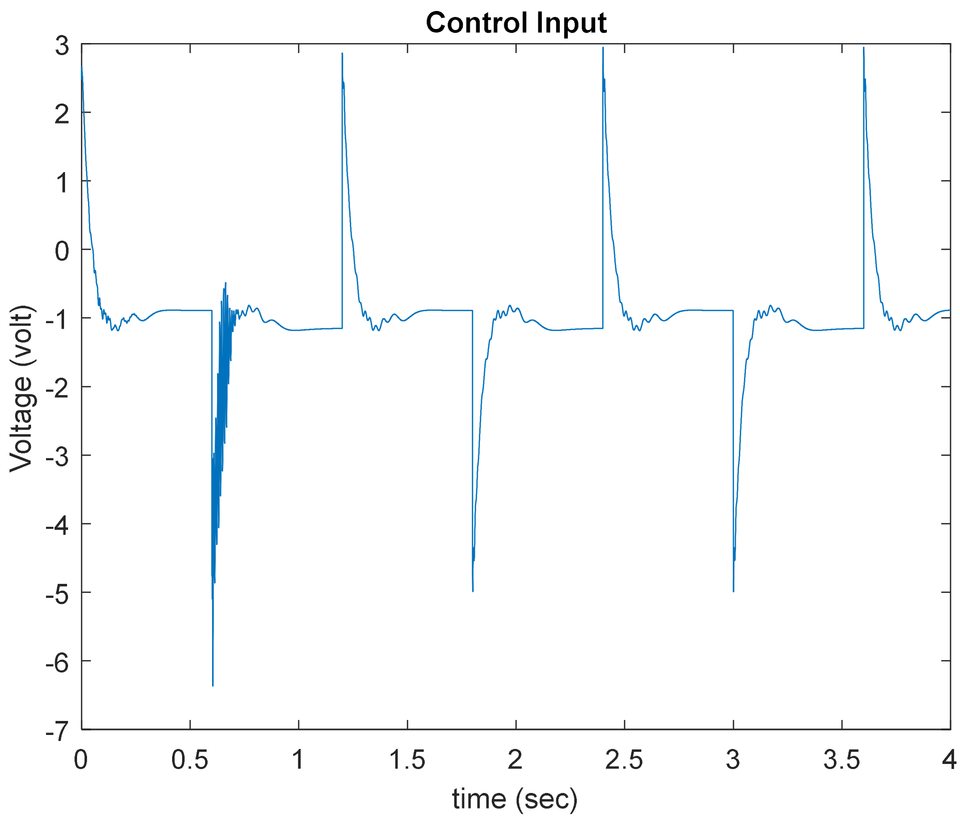

Control Input
Voltage (volt)
time (sec)

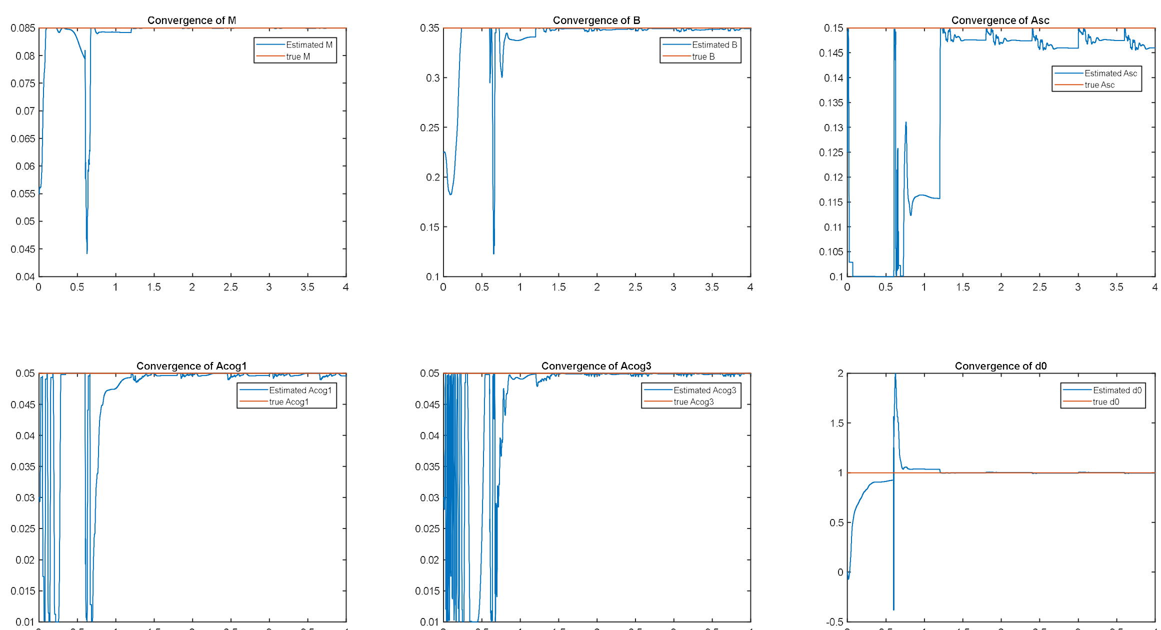

Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0

Case 2: $M_e = 0.085, \quad B = 0.35, \quad A_{sc} = 0.15, \quad A_{cog1} = 0.05, \quad A_{cog3} = 0.05, \quad d_0 = 1$

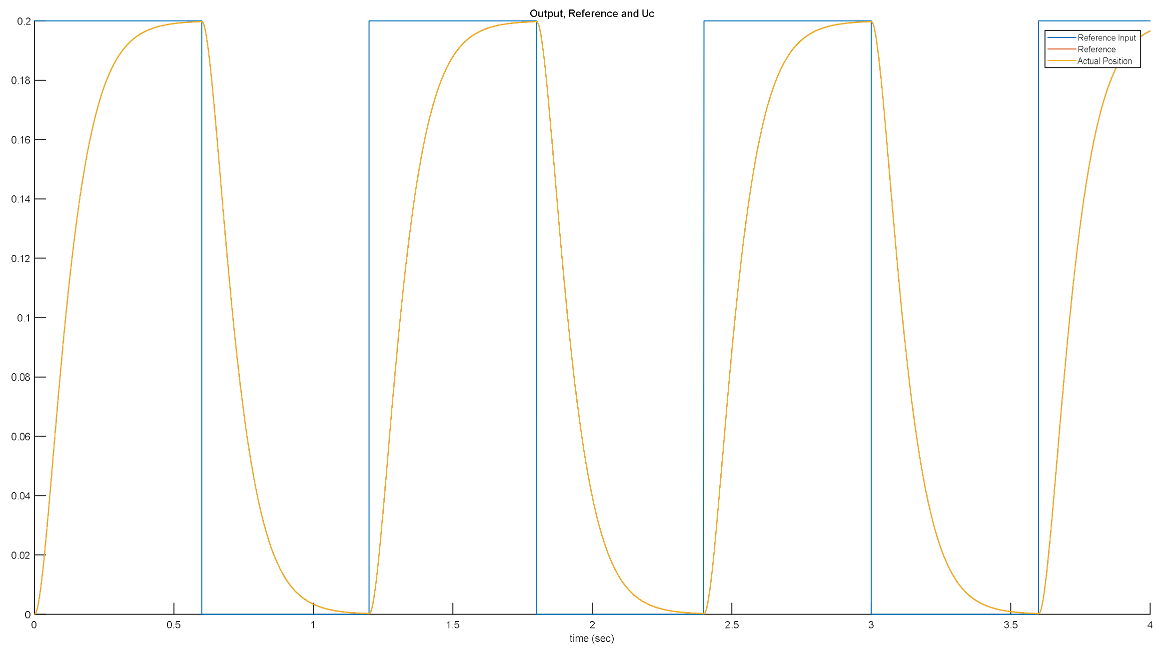


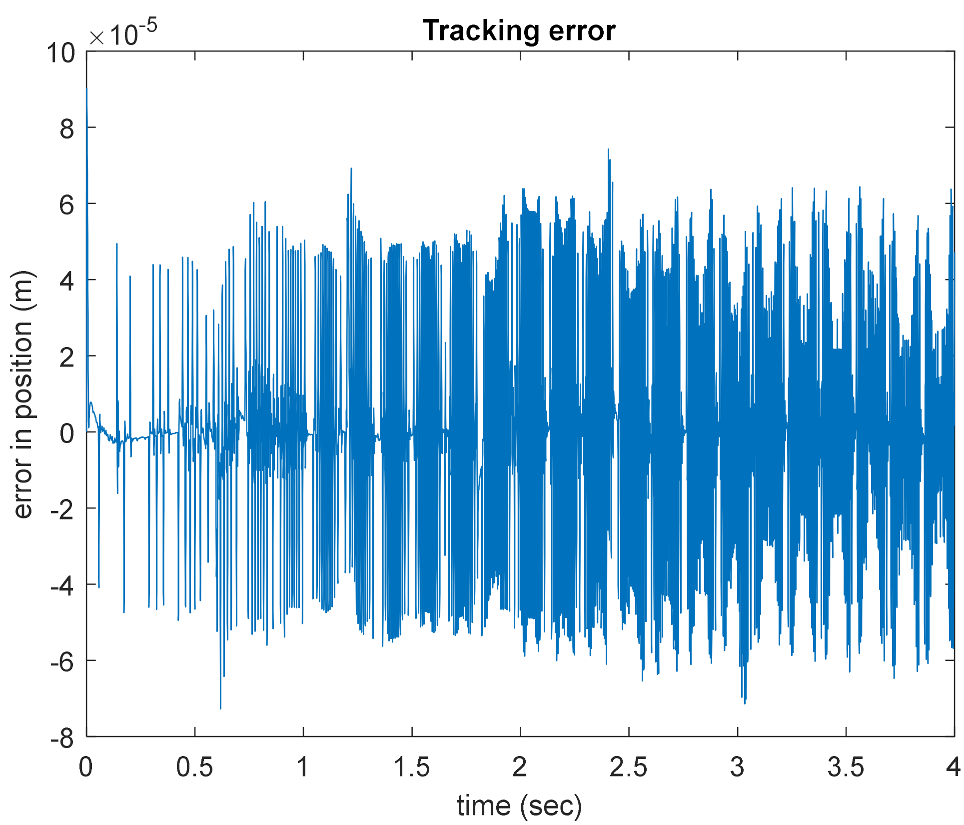


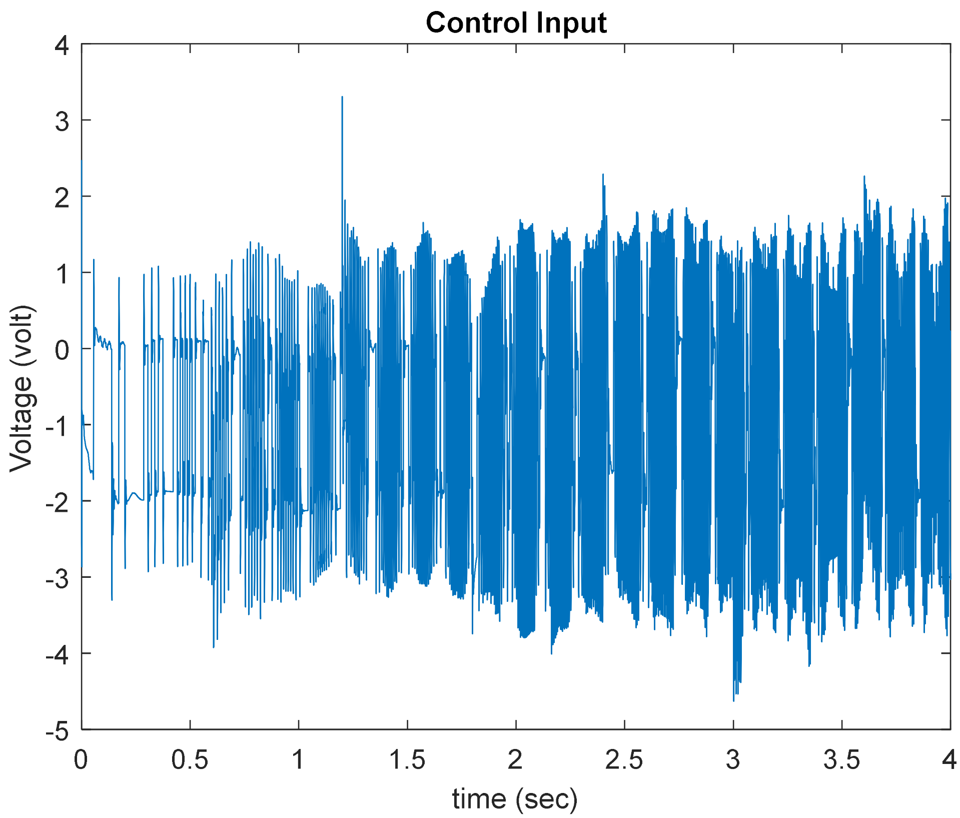

Control Input
Voltage (volt)
time (sec)

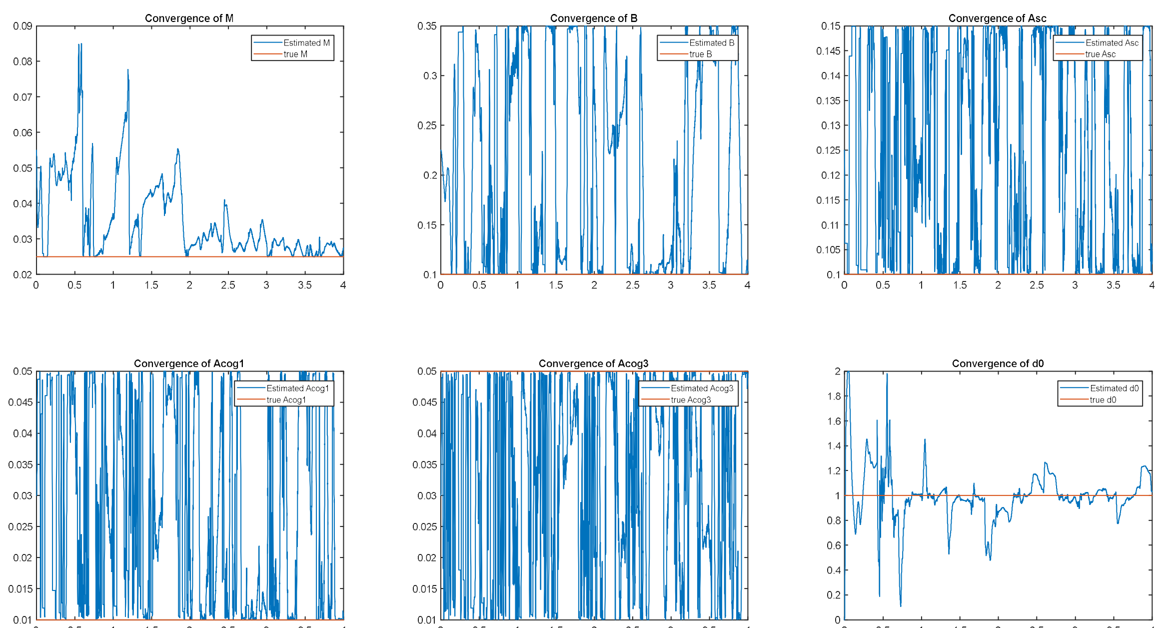

Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0

Case 3: $M_e = 0.025, B = 0.1, A_{sc} = 0.1, A_{cog1} = 0.01, A_{cog3} = 0.05, \quad d(t) = 1 + (-1)^{\text{round}(10t\sin(20t))}$

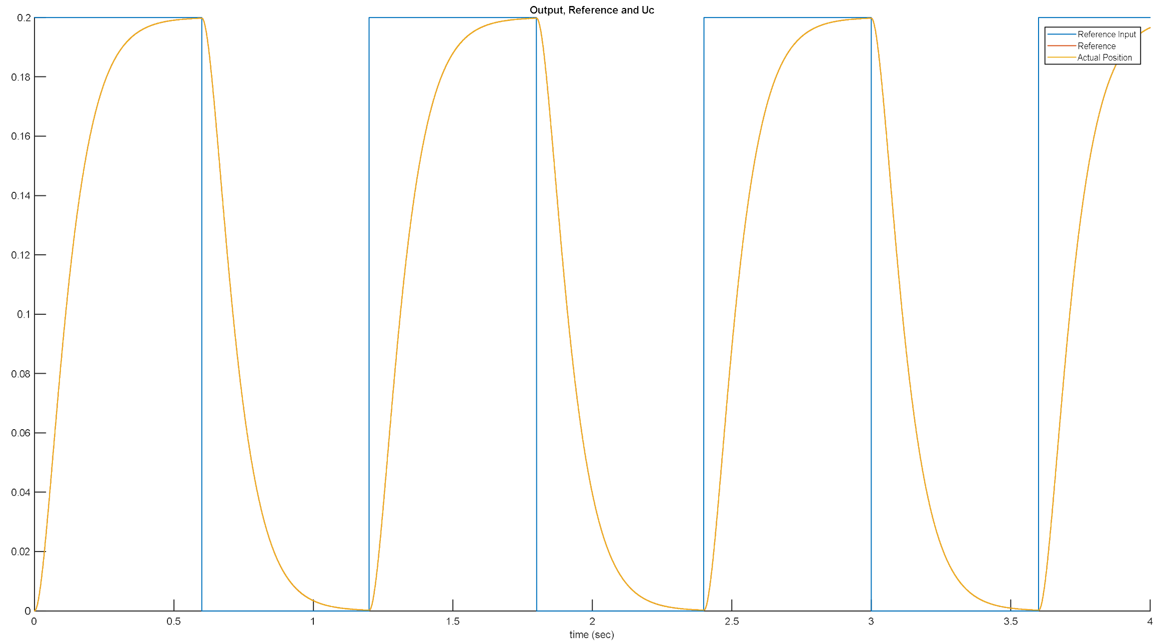


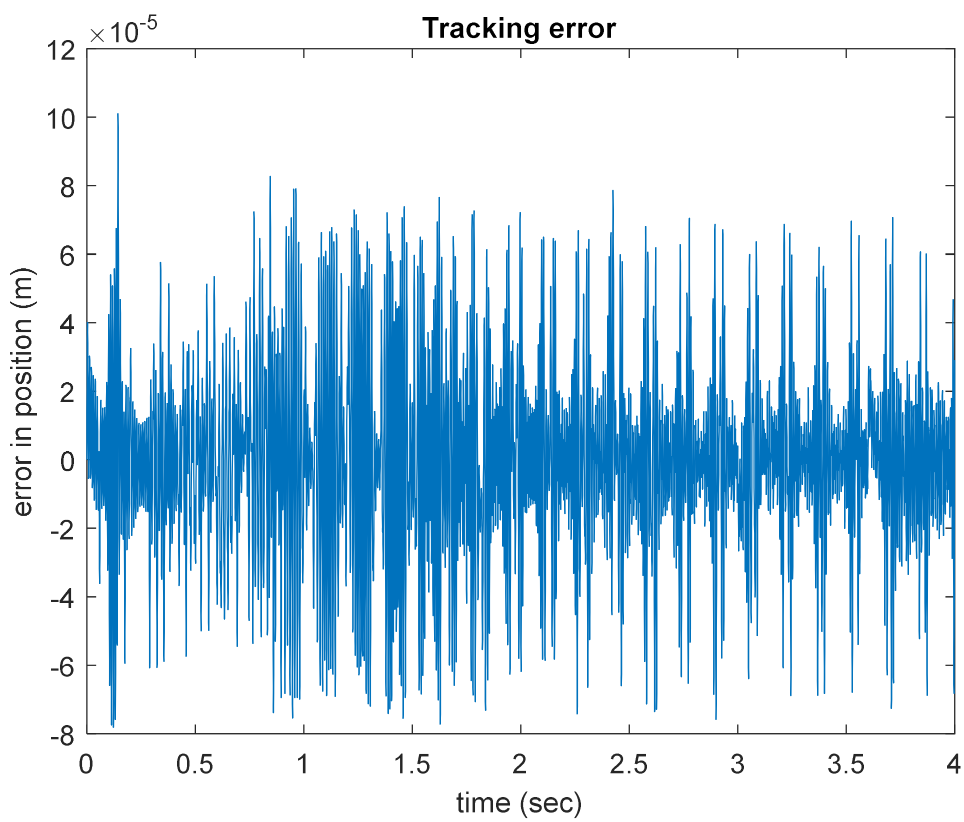


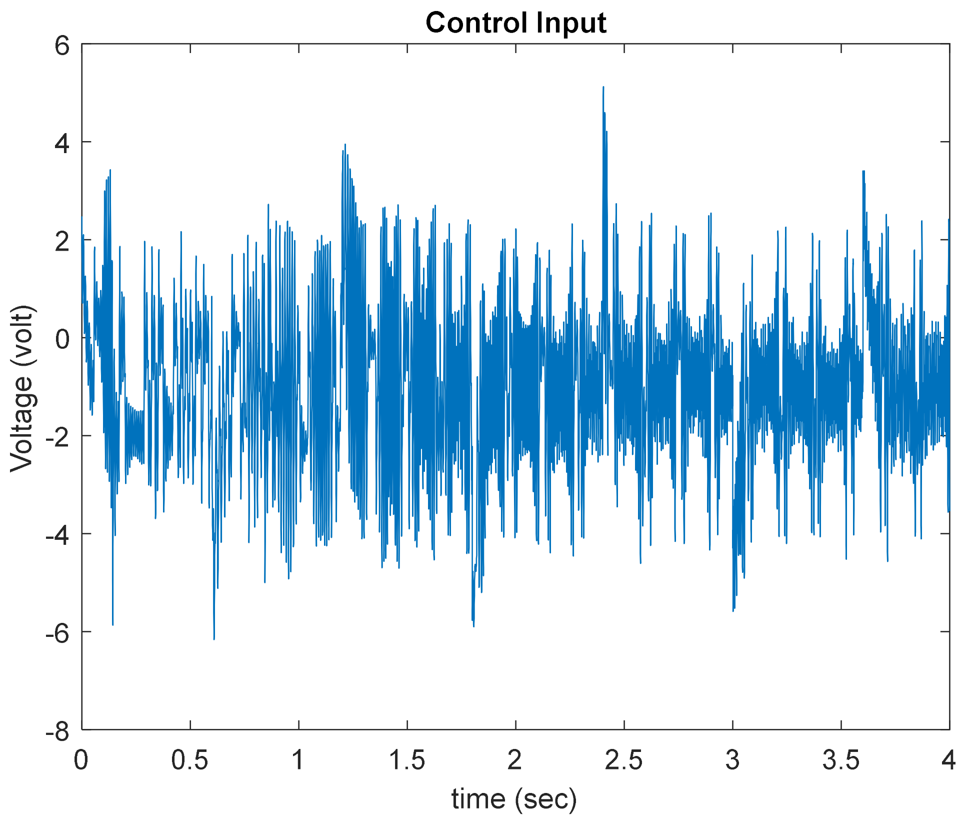

Control Input
Voltage (volt)
time (sec)

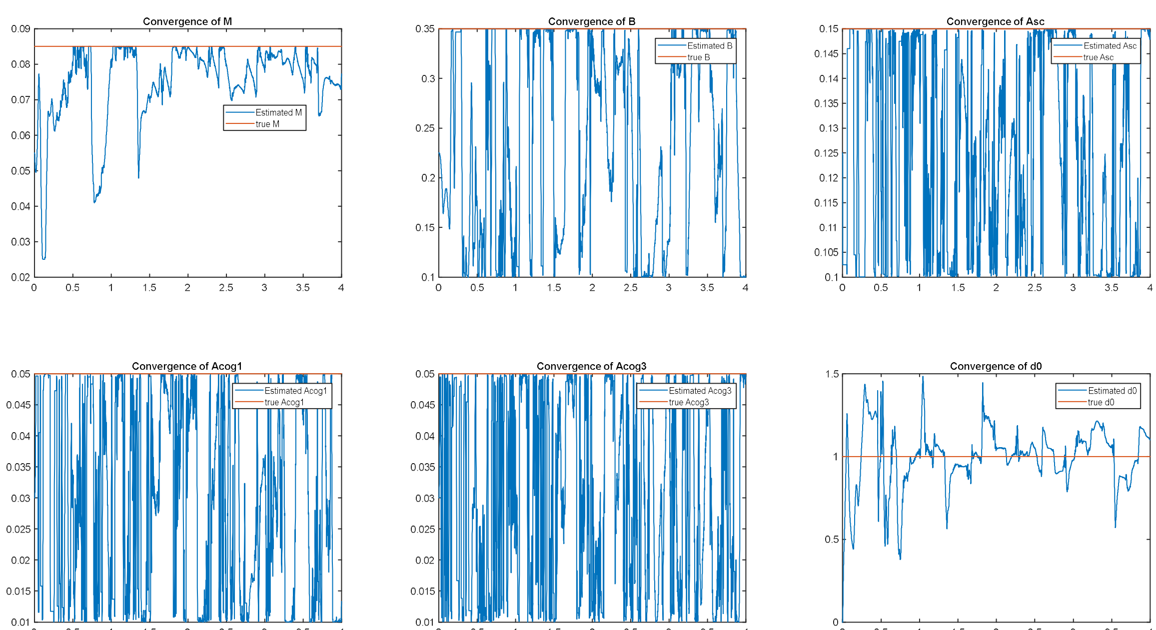

Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0

Case 4: $M_e = 0.085,\ B = 0.35,\ A_{sc} = 0.15,\ A_{cog1} = 0.05,\ A_{cog3} = 0.05,\ d(t) = 1 + (-1)^{\mathrm{round}(10t\sin(20t))}$






Control Input
Voltage (volt)
time (sec)
6
4
2
0
-2
-4
-6
-8
0
0.5
1
1.5
2
2.5
3
3.5
4


Convergence of M
Estimated M
true M
Convergence of B
Estimated B
true B
Convergence of Asc
Estimated Asc
true Asc
Convergence of Acog1
Estimated Acog1
true Acog1
Convergence of Acog3
Estimated Acog3
true Acog3
Convergence of d0
Estimated d0
true d0