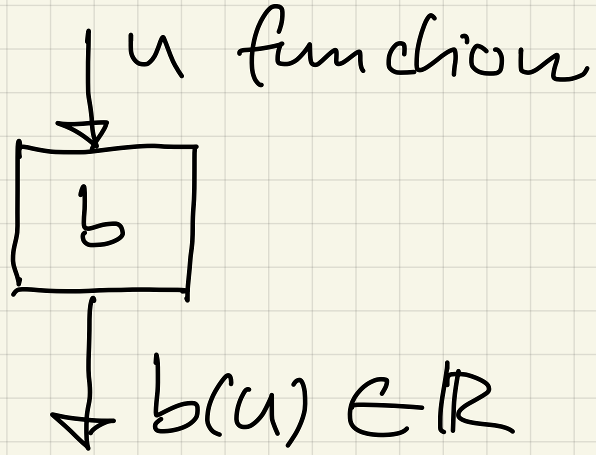
FEM for 1D problems

Properties of

$$a(u,\delta u) = EA\int_0^l u' \cdot \delta u' \, dx + C \cdot \int_0^l u \cdot \delta u \, dx + S \cdot u(l) \cdot \delta u(l)$$

$$b(\delta u) = n \cdot \int_0^l \delta u \, dx + F \cdot \delta u(0)$$

Properties of b



$$\boxed{b(u+v)} = n \cdot \int_0^1 (u+v)\,dx + F \cdot (u(0) + v(0))$$

$$= \underbrace{n \cdot \int_0^1 u\,dx + F \cdot u(0)}_{b(u)} + \underbrace{n \cdot \int_0^1 v\,dx + F \cdot v(0)}_{b(v)}$$

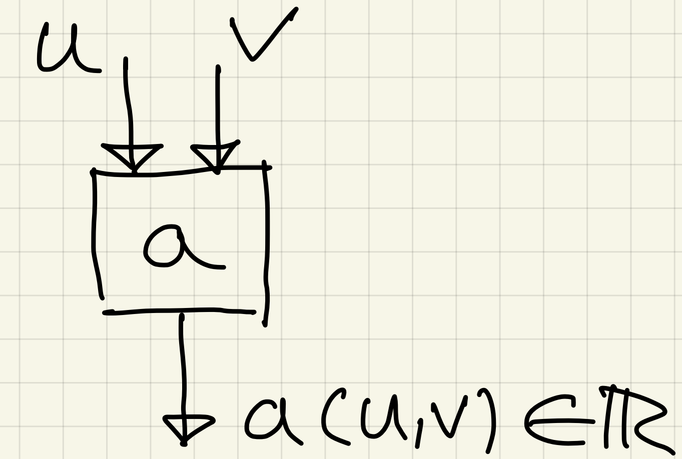
$$= b(u) + b(v) = \boxed{b(u)} + \boxed{b(v)}$$

$$\boxed{b(\alpha \cdot u)} = n \cdot \int_0^1 \alpha\, u\,dx + F \cdot \alpha \cdot u(0)$$

$$= \alpha \cdot \left(n \cdot \int_0^1 u\,dx + F \cdot u(0) \right) = \boxed{\alpha \cdot b(u)}$$

Definition (linear form): A mapping $V \to \mathbb{R}$ with the above properties is called **linear form**.

Properties of a



$$a(u+v, w) = a(u,w) + a(v,w)$$

$$a(\alpha \cdot u, v) = \alpha \cdot a(u,v)$$

$$a(u, v+w) = a(u,v) + a(u,w)$$

$$a(u, \alpha \cdot v) = \alpha \cdot a(u,v)$$

Definition (bilinear form): A mapping $V \times V \to \mathbb{R}$ with the above properties is called **bilinear form**

Definition (positive definite): A bilinear form $a : V \times V \to \mathbb{R}$ with

$$a(u,u) \geq 0 \quad \text{for all } u \in V$$

$$a(u,u) = 0 \iff u = 0$$

is called *positive definite*.

Definition (symmetric): A bilinear form $a : V \times V \to \mathbb{R}$ with

$$a(u,v) = a(v,u)$$

is called *symmetric*.

Definition (scalar product): A bilinear form which is positive definite and symmetric is called a *scalar product*.

The mapping $a : V \times V \to \mathbb{R}$ from the pile foundation problem is a scalar product.