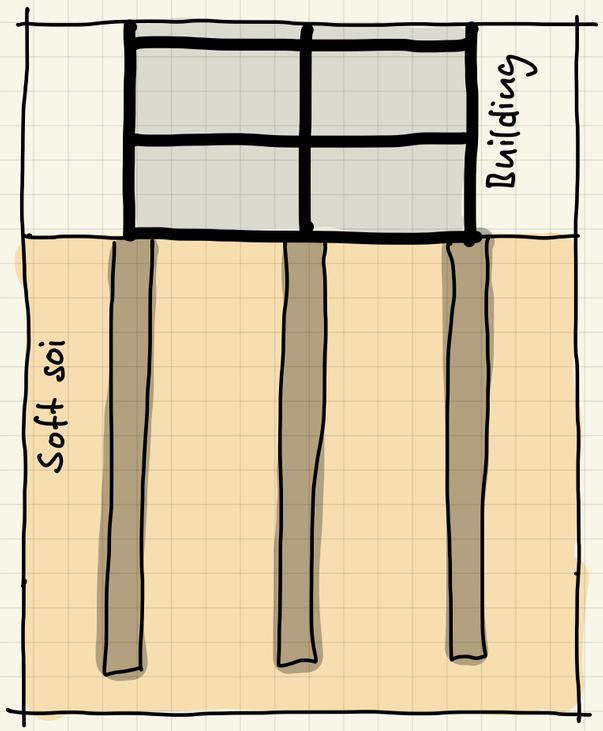
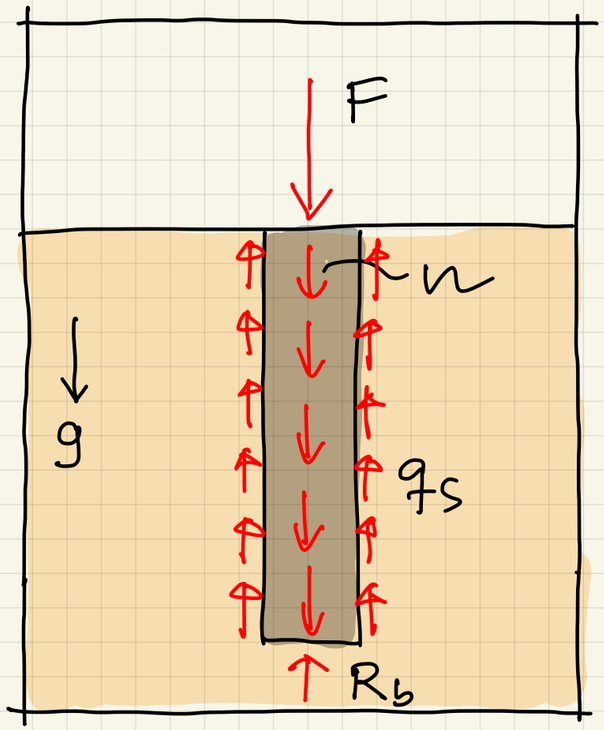
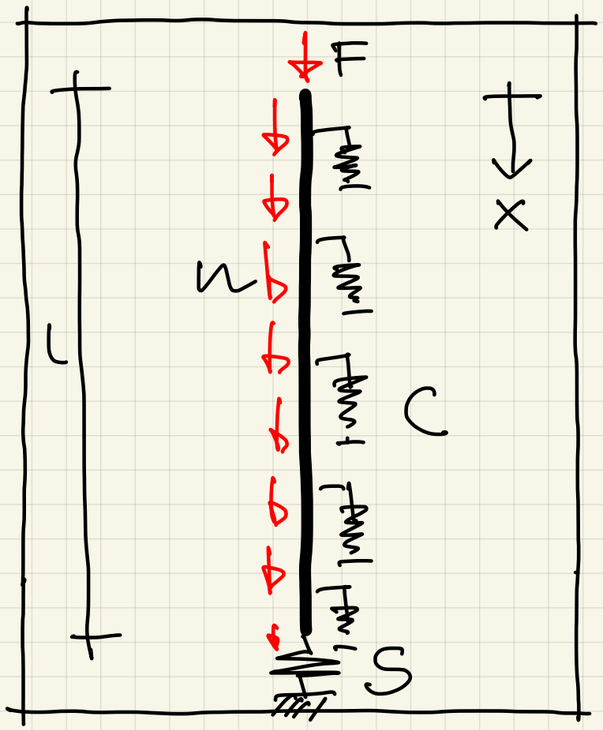
FEM for 1D problems

Strong form (boundary value problem)

Mechanical model







Loads and resistances

F Imposed load of building

n Dead weight of pile $n = g \cdot \rho \cdot A$

R_b Tip resistance

q_s Mantle resistance

Mechanical model

$u(x)$ Vertical displacement function

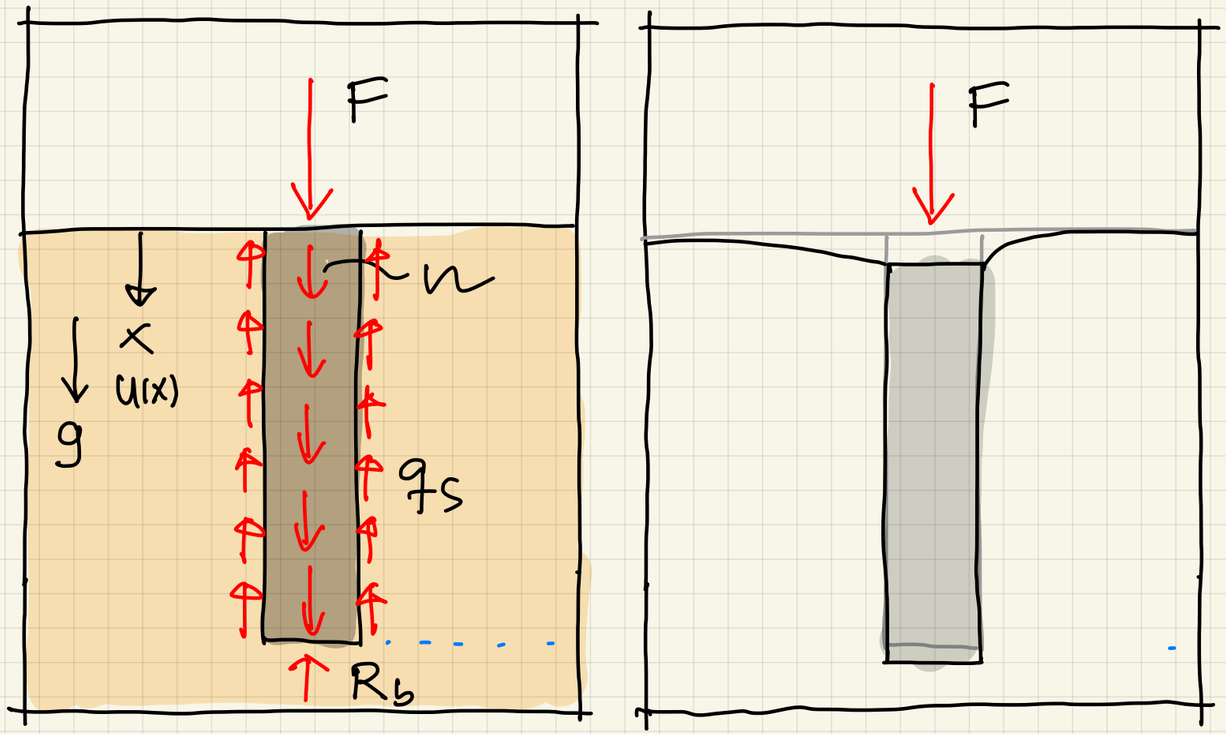
$N(x)$ Axial force $N(x) = EA \cdot u'(x)$

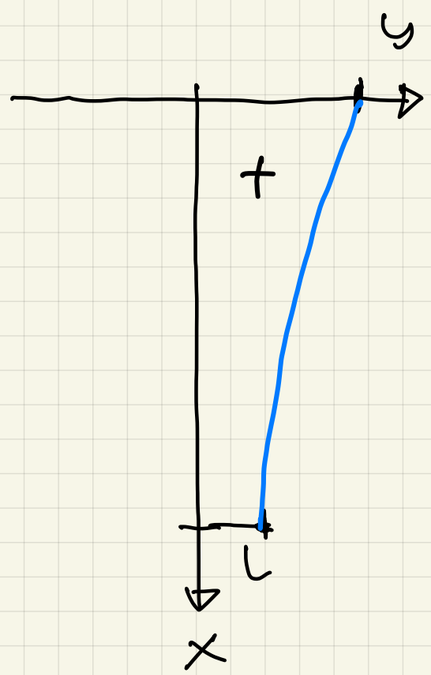
S Spring at tip $R_b = S \cdot u(l)$

C Distributed spring

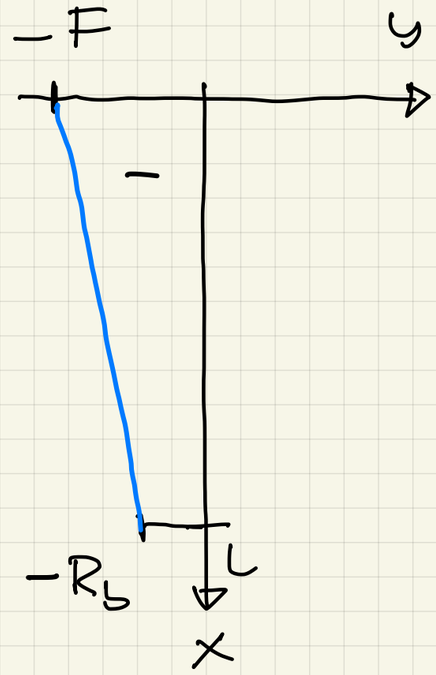
$q_s(x) = C \cdot u(x)$

Expected result





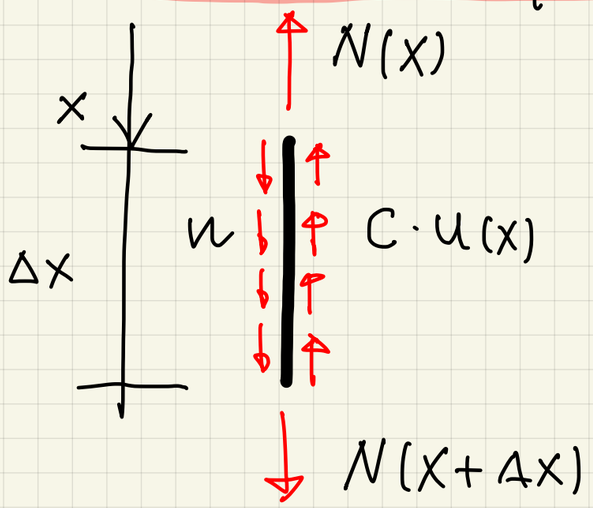
Displacement $u(x)$



Axial force $N(x)$

Mathematical model

Differential equation



$$\sum \downarrow: \quad N(x+\Delta x) - N(x) + n\cdot\Delta x - C\cdot u(x)\cdot\Delta x = 0 \quad | :\Delta x$$

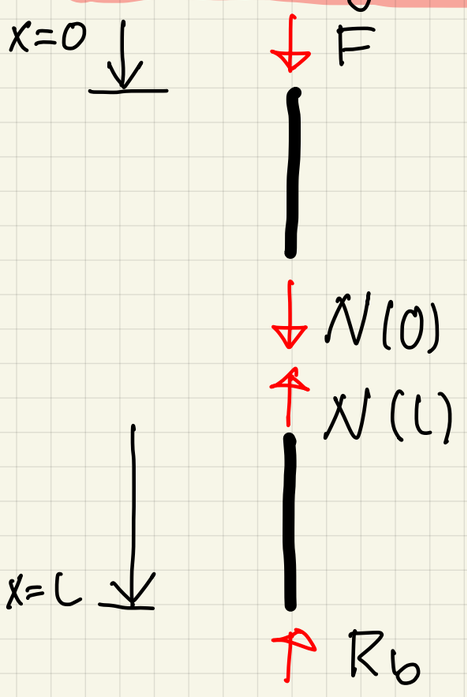
$$\frac{N(x+\Delta x)-N(x)}{\Delta x} + n - C\cdot u(x) = 0 \quad | \lim_{\Delta x\to 0}$$

$$N'(x) + n - C\cdot u(x) = 0$$

$$N(x) = EA\cdot u'(x)$$

$$\boxed{EAu''(x) - C\cdot u(x) = -n}$$

Boundary condition



$$\sum \downarrow: \quad F + N(0) = 0 \qquad \text{with} \quad N(0) = EAu'(0)$$

$$\boxed{EAu'(0) = -F}$$

$$\sum \uparrow: \quad R_b + N(l) = 0 \qquad \text{with} \quad R_b = S\cdot u(l)$$

$$\boxed{EAu'(l) + S\cdot u(l) = 0}$$

Strong form of pile foundation problem

Boundary value problem (D): Find a function $u : [0, L] \to \mathbb{R}$ which satisfies the differential equation

$$EAu''(x) - C \cdot u(x) = -n$$

and the boundary conditions

$$EAu'(0) = -F \qquad EAu'(l) + S \cdot u(l) = 0$$

Classification of boundary conditions

$$u(x_0) = c$$

Function value – Dirichlet BC

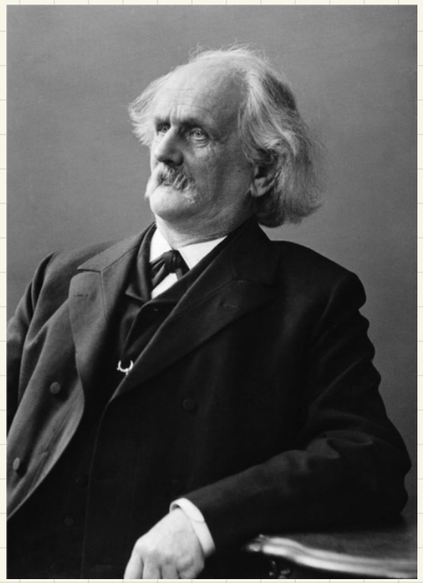
$$u'(x_0) = c$$

Derivative – Neumann BC

$$u(x_0) + b \cdot u'(x_0) = c$$

Mixed – Robin BC

Dirichlet



Neumann