$$C \cdot \sum_j \int_0^l \varphi_j \cdot \varphi_i \; dx \cdot \dot{\hat{T}}_j(t)$$

$$+ \; c \cdot v \cdot \sum_j \int_0^l \varphi_j' \cdot \varphi_i \; dx \cdot \hat{T}_j(t)$$

$$+ \; \alpha \cdot \varphi_i(0) \cdot \varphi_j(0) \cdot \hat{T}_j(t)$$

$$= \alpha \cdot T^* \cdot \varphi_i(0) + q \cdot \int_0^l \varphi_i \; dx$$

$$\underline{M}\,\dot{\hat{\underline{T}}} + \underline{A}\,\underline{T} = \underline{r}$$

Mit

$$M_{ij} \overset{c}{=} \int \varphi_i \varphi_j$$

$$A_{ij} \overset{c \cdot v}{=} \int \varphi_i \varphi_j' \, dx + \alpha \cdot S_{ij}, \qquad S_{ij} = \begin{cases} 1 & \text{für } i=j=1 \\ 0 & \text{sonst} \end{cases}$$

Matrix M_{ij}

$$T(t=0) = \begin{pmatrix} 20 \\ \vdots \\ 0 \end{pmatrix}$$

$$M_{ij} = c \cdot \int_0^l \varphi_i \cdot \varphi_j \; dx \qquad \text{mit} \quad \varphi_1(x) = 1 - x \qquad \varphi_2(x) = x$$

$$M_{11} = c \cdot \int_0^1 (1-x)^2 \, dx = c \cdot \left[x - 2 \cdot \frac{x^2}{2} + \frac{x^3}{3} \right]_0^1 = \frac{c}{3}$$

$$M_{12} = c \cdot \int_0^1 (1-x) \cdot x \, dx = c \cdot \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{c}{6}$$

$$M_{21} = M_{12} = \frac{c}{6}$$

$$M_{22} = c \cdot \int_0^1 x^2 \, dx = c \cdot \left[\frac{x^3}{3} \right]_0^1 = \frac{c}{3}$$

$$\Rightarrow M = \frac{c}{6} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

S_{ij}

$$S_{11} = 1 \qquad S_{12} = S_{21} = S_{22} = 0$$

A_{ij}

$$A_{ij} = C \cdot v \cdot \int_0^1 \varphi_i \cdot \varphi_j' \, dx + \alpha \cdot S_{ij}$$

$$A_{11} = C \cdot v \cdot \int_0^1 (1-x) \cdot (-1) \, dx + \alpha \cdot 1 = -C \cdot v \cdot \left[\frac{x^2}{2}\right]_0^1 + \alpha$$

$$= -\frac{C \cdot v}{2} + \alpha$$

$$A_{12} = C \cdot v \cdot \int_0^1 (1-x) \cdot 1 \, dx = C \cdot v \cdot \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{C \cdot v}{6}$$

$$A_{21} = C \cdot v \cdot \int_0^1 x \cdot (-1) \, dx = -C \cdot v \cdot \left[\frac{x^2}{2}\right]_0^1 = -\frac{C \cdot v}{2}$$

$$A_{22} = C \cdot v \cdot \int_0^1 x \cdot 1 \, dx = C \cdot v \cdot \left[\frac{x^2}{2}\right]_0^1 = \frac{C \cdot v}{2}$$

r_i

$$T^* = 20\,°C$$

$$r_i = \alpha \cdot T^* \cdot \varphi_i(0) + q \cdot \int_0^l \varphi_i \, dx$$

$$r_1 = \alpha \cdot 20 + q \cdot \int_0^1 (1-x) \, dx = \alpha \cdot 20 + q \cdot \left[x - \frac{x^2}{2}\right]_0^1$$

$$r_1 = 20\alpha + 0{,}5$$

$$r_2 = q \cdot \int_0^1 x \, dx = \frac{q}{2}$$