Mit

$$a(T,\delta T) = C\cdot\int_0^l \dot{T}\cdot\delta T\,dx + C\cdot v\cdot\int_0^l T'\cdot\delta T\,dx + \alpha\cdot T(0,t)\cdot\delta T(0)$$

mit

$$A_{ij} \overset{C\cdot V}{=} \int \varphi_i\,\varphi_j'\,dx + \alpha\cdot S_{ij}$$

im Anschluss vorerst ignoriert weil nur Diagonale ($A_{11}, \dots$) beeinflusst

Matrix Construction revised

$C\cdot\int_0^l \dot{T}\cdot\delta T\,dx$ Transient (time-dependent)

$C\cdot v\cdot\int_0^l T'\cdot\delta T\,dx$ Convection (spatial-dependent)

$\alpha\cdot T(0,t)\cdot\delta T(0)$ Boundary Condition ($x=0$) Penalty Method

Fehler zuvor:

Guess: Symetrical Matrix A allowed system to form Q-Equilibrium

```python
def compute_A(n, l, C, v, alpha):
    A = np.zeros((n, n))
    h = l / (n - 1)
    for i in range(n):
        for j in range(n):
            if i == j:
                A[i, j] = C * v / h + (alpha if i == 0 and j == 0 else 0)
            if abs(i - j) == 1:
                A[i, j] = -C * v / h / 2
    return A
```

→ creates sym. Matrix A

A

However, directional nature of convection should lead to asymetrical Matrix

Correction Idea:

- $A_{ij} \neq 0$ with "downstream" influence ($j > i$)
- use foreward / backward differences (depending on discretization)
  ↳ to reflect directional convection

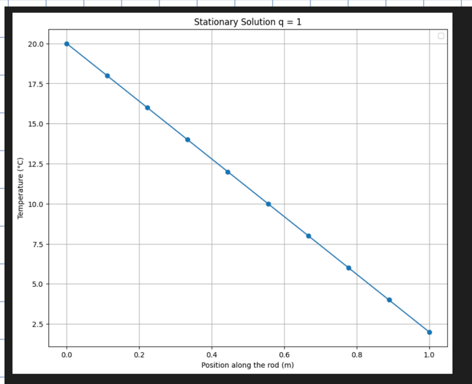
⇒ A_{ij} to be constructed with distinction to account for directionality

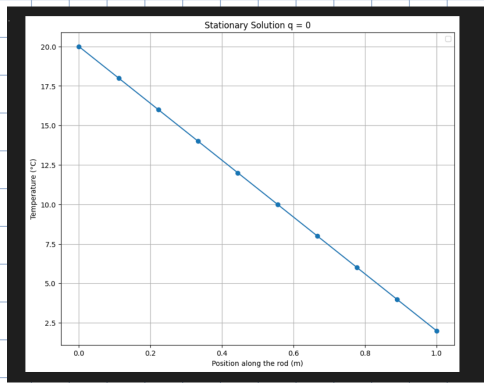
$$A_{ij} \neq A_{ji} \quad \text{for} \quad i \neq j$$

Updated Function:

```python
def compute_A_asymmetric(n, l, C, v, alpha):
    A = np.zeros((n, n))
    h = l / (n - 1)
    for i in range(n):
        for j in range(n):
            if i == j:
                A[i, j] = C * v / h + (alpha if i == 0 else 0)
            elif i == j + 1:
                A[i, j] = -C * v / h  # Forward difference (i > j)
            elif i == j - 1:
                A[i, j] = 0  # No backward difference (asymmetry)
    return A
```

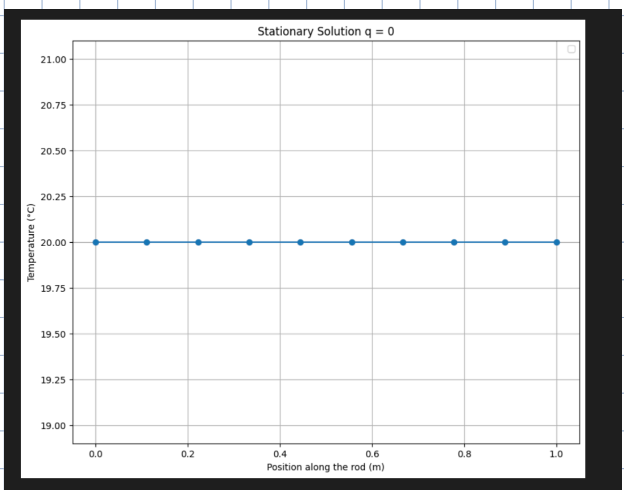
Instationary results before correction





$\rightarrow$ same solution regardless of q

Instationary results after correction

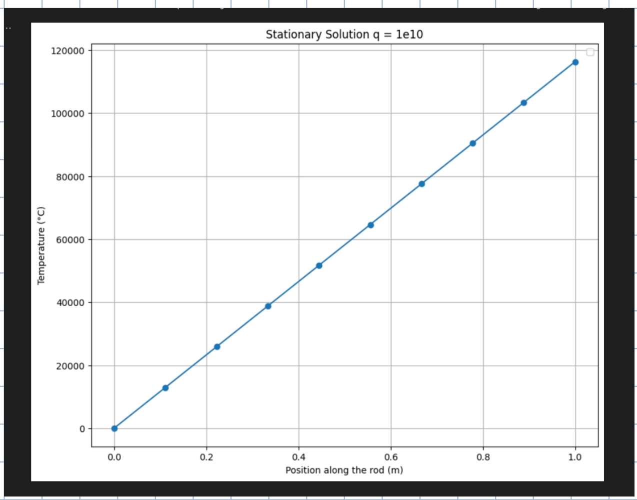


T =

# index	# 0
	Missing: 0 (0%)
	Distinct: 1 (10%)
	1 Distinct values
0	19.999914021569616
1	19.999914021569616
2	19.999914021569616
3	19.999914021569616
4	19.999914021569616
5	19.999914021569616
6	19.999914021569616
7	19.999914021569616
8	19.999914021569616
9	19.999914021569616

Penalty

$\alpha = 1e10$



T =

Export as file   Refresh data   Repo

# index	# 0
	Missing: 0 (0%)
	Distinct: 10 (100%)
	Min 20.0554693... Max 116327.79644...
0	20.0554693382962
1	12943.137800228136
2	25866.220131117974
3	38789.302462007814
4	51712.384792897654
5	64635.467123787486
6	77558.54945467731
7	90481.63178556715
8	103404.71411645699
9	116327.79644734682

Trials to understand system behaviour with different α Penalty Factor

Reminder: Initial condition $T(x=0) = 20\,°C$ else $T(x \neq 0) = 0$

Stationary Solution

$q = 0$

	A	B	C	D	E	F	G	H	I	J	K
1	alpha	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_10
2	1.00E+00	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
3	1.00E+01	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047
4	1.00E+02	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464
5	1.00E+03	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547
6	1.00E+04	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743
7	1.00E+05	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871
8	1.00E+06	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757
9	1.00E+07	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144
10	1.00E+08	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914
11	1.00E+09	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991
12	1.00E+10	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999
13	1.00E+11	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000

$q = 1$

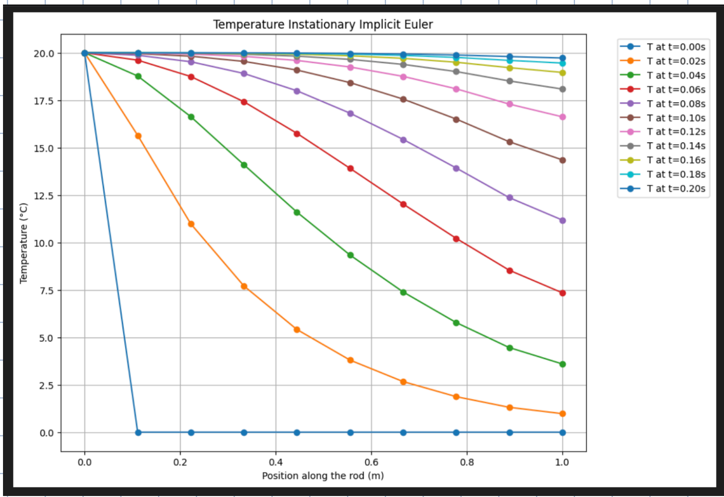
	A	B	C	D	E	F	G	H	I	J	K
1	alpha	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_10
2	1.00E+00	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
3	1.00E+01	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047
4	1.00E+02	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464
5	1.00E+03	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547
6	1.00E+04	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743
7	1.00E+05	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871
8	1.00E+06	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757
9	1.00E+07	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144
10	1.00E+08	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914
11	1.00E+09	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991
12	1.00E+10	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999
13	1.00E+11	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000

$q = 1e10$

	A	B	C	D	E	F	G	H	I	J	K
1	alpha	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_10
2	1.00E+00	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
3	1.00E+01	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047
4	1.00E+02	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464	0.0464
5	1.00E+03	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547	0.4547
6	1.00E+04	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743	3.7743
7	1.00E+05	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871	13.9871
8	1.00E+06	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757	19.1757
9	1.00E+07	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144	19.9144
10	1.00E+08	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914	19.9914
11	1.00E+09	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991	19.9991
12	1.00E+10	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999	19.9999
13	1.00E+11	20.0550	12943.1380	25866.2200	38789.3020	51712.3850	64635.4670	77558.5490	90481.6320	103404.7140	116327.7960

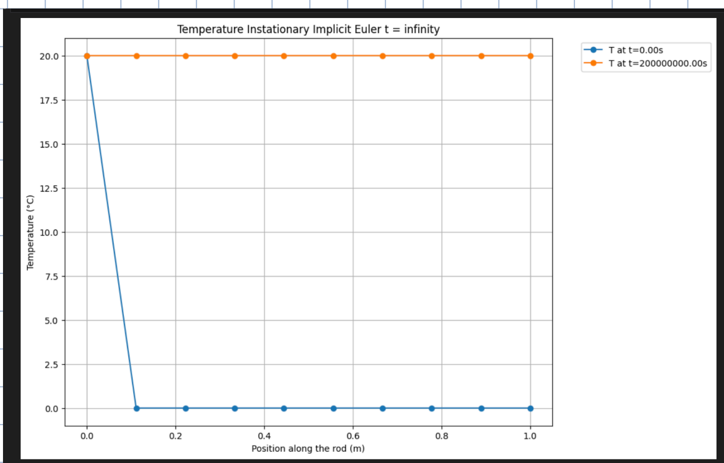
Kurz gesagt: hoher Penalty Faktor notwendig um Initial Condition zu erzwingen. Hoher Penalty Faktor führt aber auch dazu, dass System sehr hohes q benötigt um sich aufzuwärmen, weil sonst Einfluss von $T(x = 0)$ "zu stark"

Instationary Solutions: with: $q = 0$ $\alpha = 1e10$



Time = 0,2 s

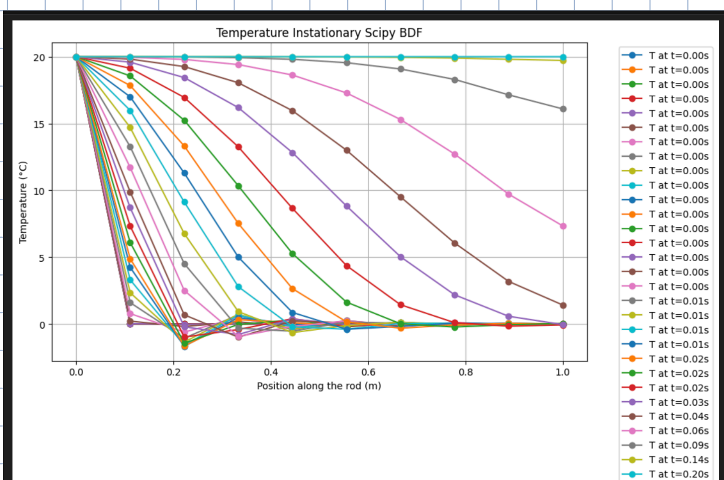
dt = 0,02 s



Time = 2e8 s

dt = 2e8 s

→ validating stationary solution



Proof A = asym.

Assume : Rod with 3 nodes $\rightarrow \varphi_1(x), \varphi_2(x), \varphi_3(x)$

Linear shape functions at...

... Node 1 (x_1):

$$\varphi_1' = -\frac{1}{h} \qquad \varphi_2' = \frac{1}{h} \qquad \varphi_3' = 0$$

... Node 2 (x_2):

$$\varphi_1' = 0 \qquad \varphi_2' = -\frac{1}{h} \qquad \varphi_3' = \frac{1}{h}$$

... Node 3 (x_3):

$$\varphi_1' = 0 \qquad \varphi_2' = 0 \qquad \varphi_3' = -\frac{1}{h}$$

Example A_{12}/A_{21}

$$A_{12} = C \cdot v \cdot \int_0^{\ell} \varphi_2' \cdot \varphi_1 \, dx = -C \cdot v \cdot \frac{1}{h} \int_0^{h} \varphi_1 \, dx$$

$$A_{21} = C \cdot v \cdot \int_0^{\ell} \varphi_1' \cdot \varphi_2 \, dx = C \cdot v \cdot \frac{1}{h} \int_h^{2h} \varphi_2 \, dx$$

$A_{12} \neq A_{21}$ therefore A not symetrical