

# FEM for 1D problems

Linear spaces

# Vector calculus

$$\underline{x}, \underline{y} \in \mathbb{R}^n, \alpha \in \mathbb{R}$$

Addition

$$\underline{z} = \underline{x} + \underline{y}, \quad \underline{z} \in \mathbb{R}^n$$

Multiplication with a number

$$\underline{z} = \alpha \cdot \underline{x}, \quad \underline{z} \in \mathbb{R}^n$$

# Function spaces

$$f, g: \mathbb{R}^n \rightarrow \mathbb{R}, \quad \alpha \in \mathbb{R}$$

Addition

$$h = f + g, \quad h(x) = f(x) + g(x)$$

Multiplication with a number

$$h = \alpha \cdot f, \quad h(x) = \alpha \cdot f(x)$$

**Definition (linear space):** A set of objects which we can add and multiply with a number is called a linear space (if the usual rules apply)

Central idea: Work with functions in a similar way as with vectors in  $\mathbb{R}^n$

We use

$X$  for any kind of linear space and  $V$  for a space of functions