FEM for 1D problems

Abstract variational problem

Variational problem (V): Find function $u:[0,L]\to\mathbb{R}$ such that

$$EA\cdot\int_0^L u'\cdot\delta u'\,dx + C\cdot\int_0^L u\cdot\delta u\,dx + S\cdot u(L)\cdot\delta u(L) = n\cdot\int_0^L \delta u\,dx + F\cdot\delta u(0)$$

for all (admissible) test functions δu.

We defined

$$a(u,\delta u) = EA\int_0^L u'\cdot\delta u'\,dx + C\cdot\int_0^L u\cdot\delta u\,dx + S\cdot u(L)\cdot\delta u(L)$$

$$b(\delta u) = n\cdot\int_0^L \delta u\,dx + F\cdot\delta u(0)$$

→ Write down in general form

Abstract variational problem (AV): Find function $u \in V$ such that

$$a(u, \delta u) = b(\delta u)$$

for all test functions $\delta u \in V$

A large class of problems can be expressed in this form

- Structural analysis
- Heat transfer
- Acoustics
- Electromagnetics
- Fluid mechanics

Basis for success of FEM: Method can be applied to a very large class of problems.