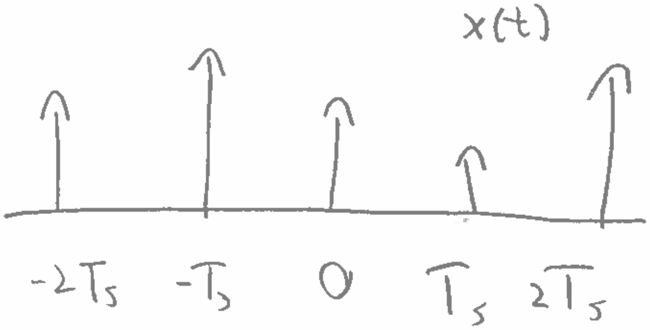
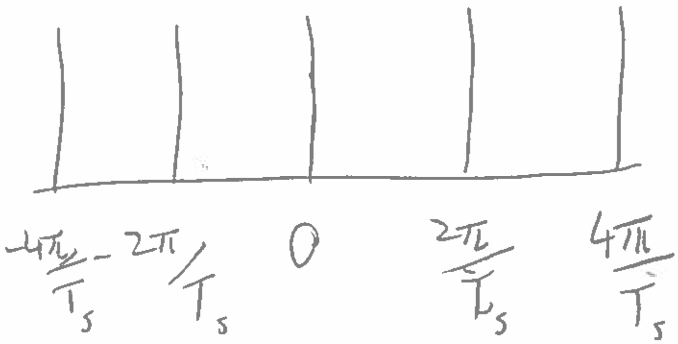
Sig Sys PS08

James Jang

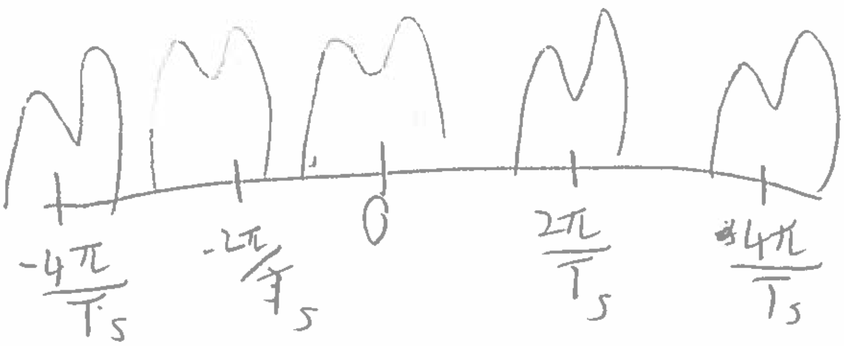
1a



b



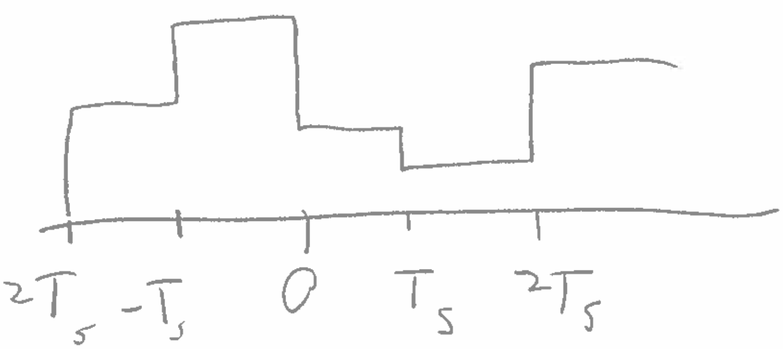
c



d. $w_m \le \frac{\pi}{T_s}$

e In order to recover $x(t)$ from $x_p(t)$ we have to band pass $x_p(t)$ b/w $-w_m$ and w_m then we scale by $\frac{T_s}{2\pi}$

g $z(t)$ $x_z(t) = x_p * z(t)$



h. Sketch $X_z(w)$

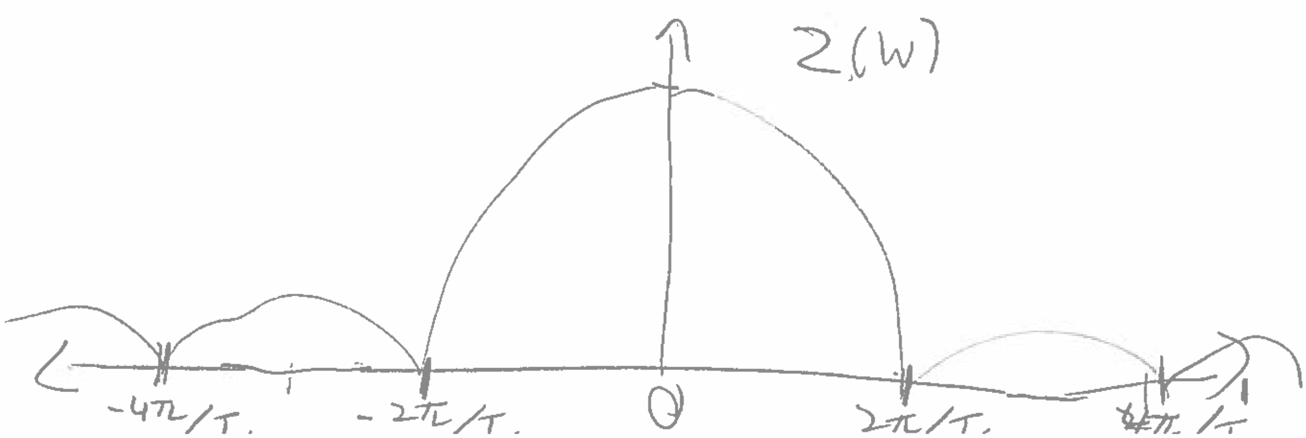
$z(t) = 1$ when $0 \le t \le T_s$
0 otherwise $\rightarrow$ $Z(w) = \int_{-\infty}^{\infty} z(t) e^{-jwt} dt$

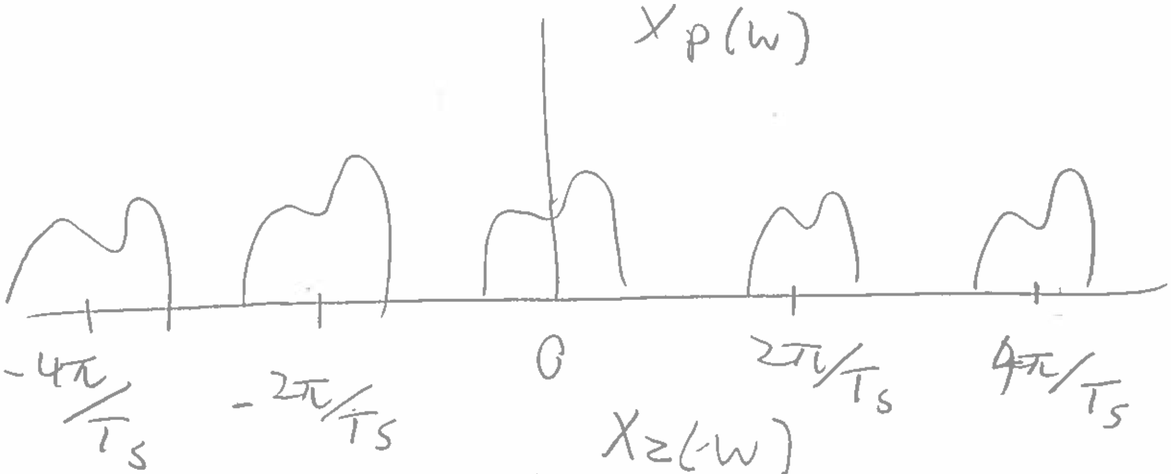
$= \int_0^{T_s} e^{-jwt} dt = \frac{e^{-jwt}}{-jw} \Big|_0^{T_s} = \frac{e^{-jwT_s}}{-jw} - \frac{1}{-jw}$

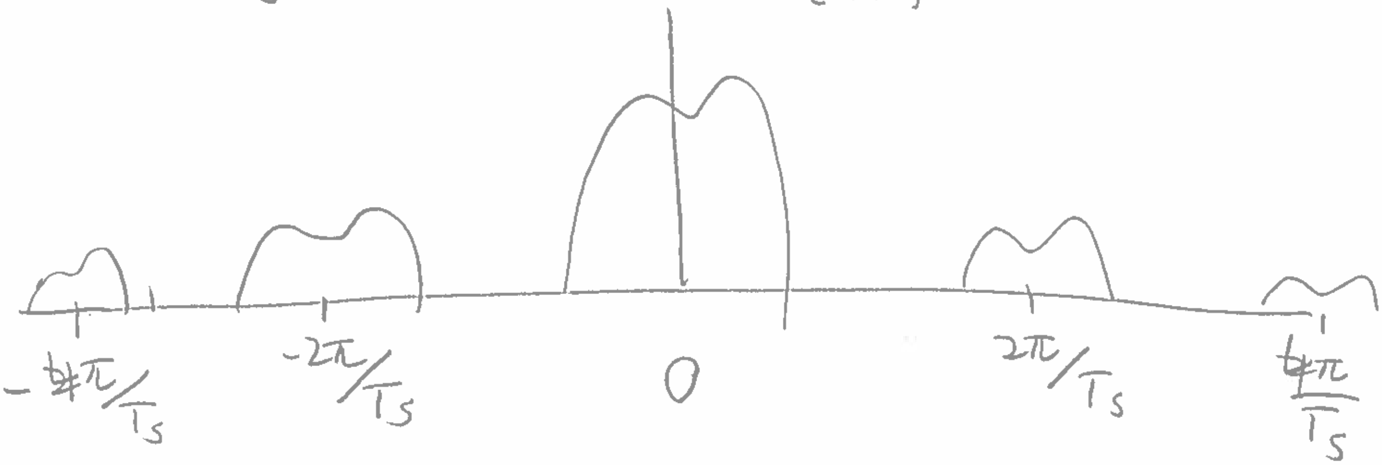
$= \frac{1 - e^{-jwT_s}}{jw}$

$Z(w) = e^{\frac{-jwT_s}{2}} \cdot T_s \operatorname{sinc}\left(\frac{wT_s}{2\pi}\right)$

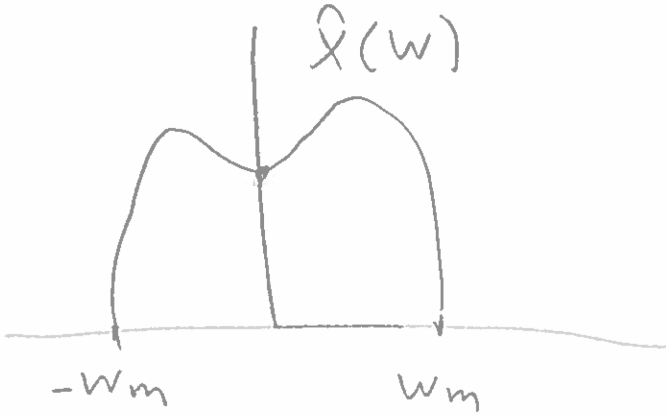
$X_z(w) = X_p(w)\, Z(w)$

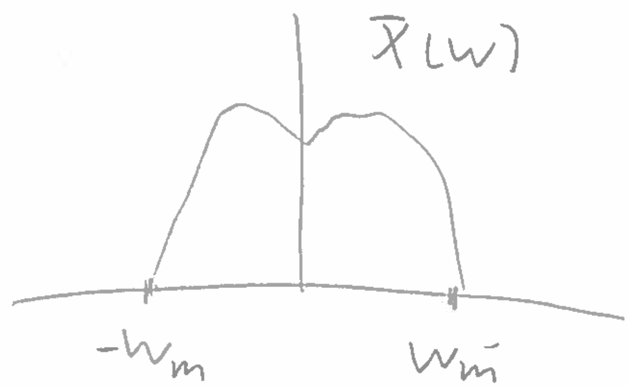






i) $\bar{F}(w) = X_z(w) H(w)$ and $\hat{X}(w) = X_p(w) H($





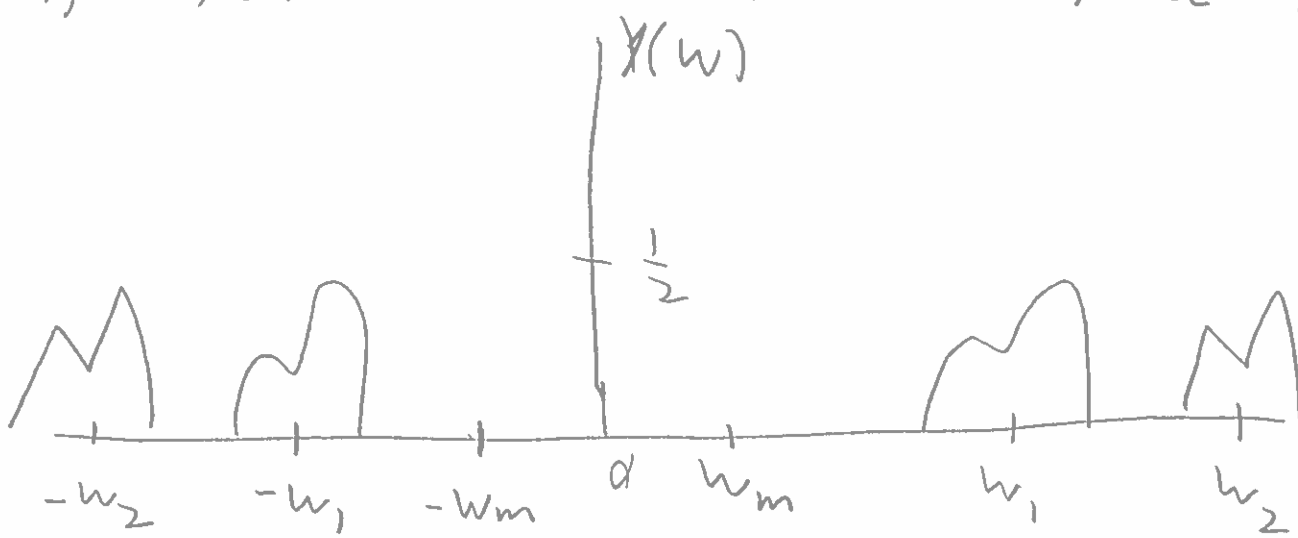
j) They have different amplitude. $\check{X}(w)$ is also partly multiplied by a sinc which means the frequencies decrease faster as you approach w_m from 0 k.) ratio of $\bar{X}(w_m)$ to $\hat{X}(w_m)$ $\quad w_m = \frac{\pi}{T_s}$

$$\hat{x}\left(\frac{\pi}{T_s}\right) = T_s \operatorname{sinc}\left(\frac{1}{2}\right) \rightarrow T_s \frac{\sin\left(\frac{\pi}{2}\right)}{\pi/2}$$

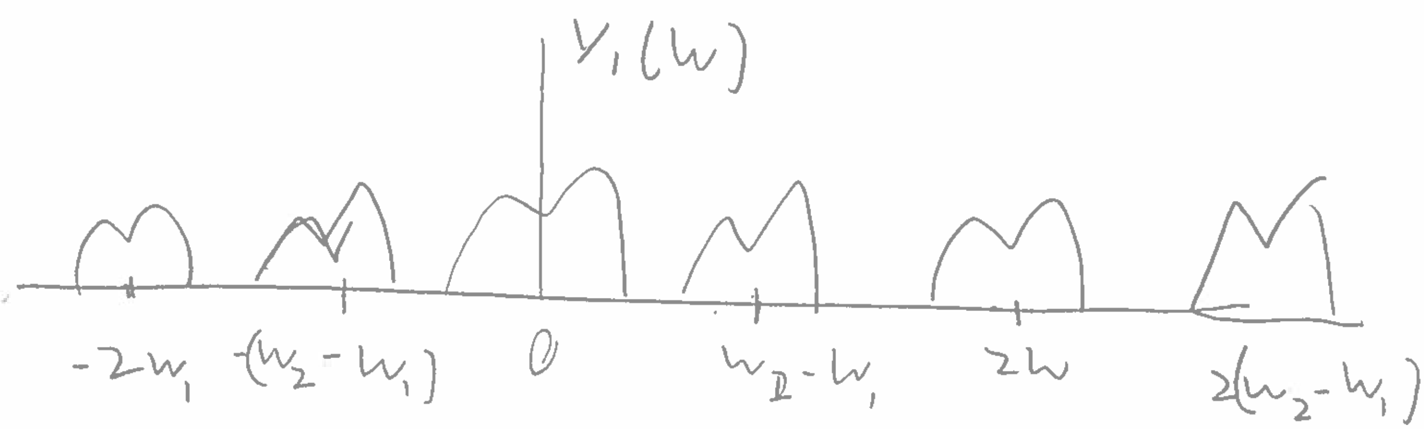
$$= \frac{2T_s}{\pi}$$

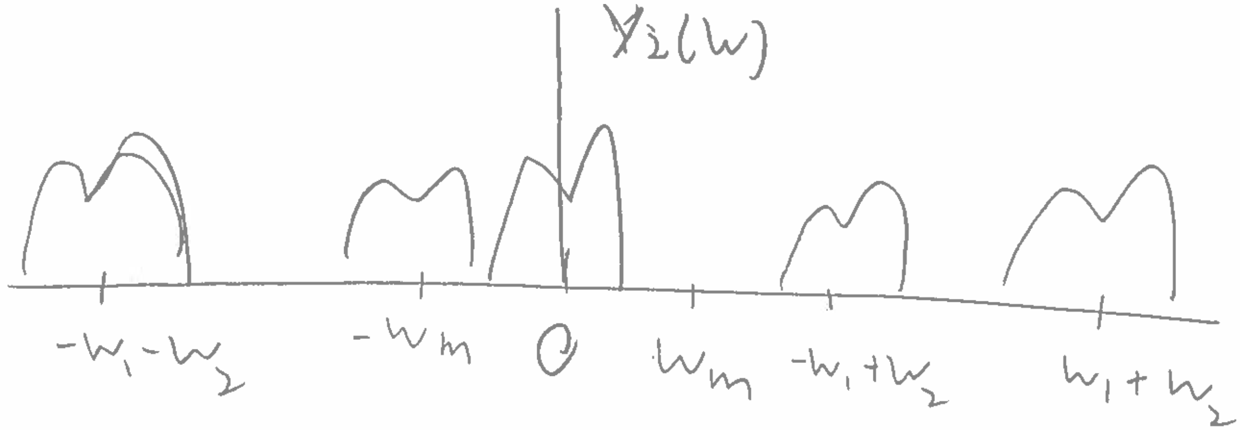
2.

a) $y(t) = x_1(t)\cos(w_1 t) + x_2(t)\cos(w_2 t)$



b. $y(t)\cos(w_1 t) \qquad y(t)\cos(w_2 t)$





C. To recover $y(t)$, apply a low pass filter to $y(t)\cos(\ldots$ with the cut off at W_m and multiply the amplitude by 2

For $x_2(t)$ repeat but with $\cos(w_2 t)$

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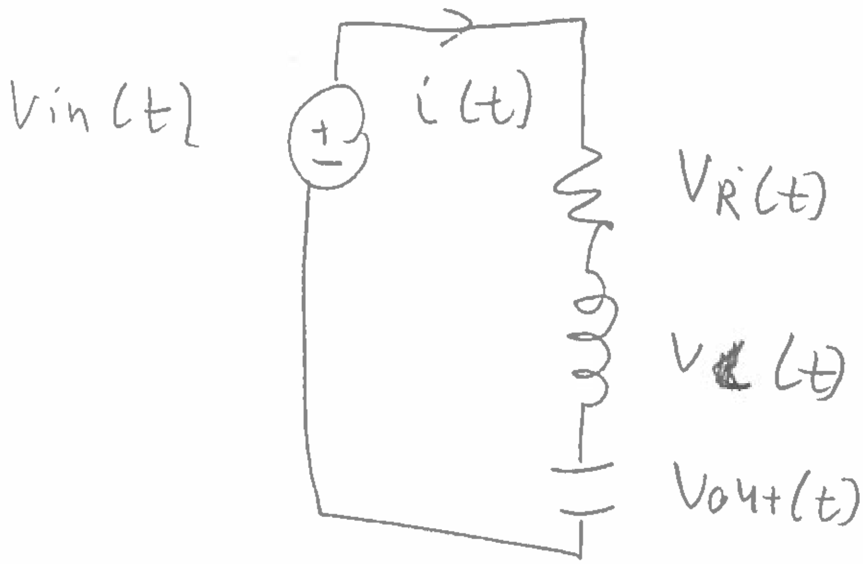


$i(t) = C\frac{d}{dt}V_{out}(t)$

$V_L(t) = L\frac{d}{dt}i(t)$

$V_R(t) = Ri(t)$

$V_L(t) = CL\frac{d^2}{dt^2}V_{out}$ $\quad V_R(t) = RC\frac{d}{dt}V_{out}(t)$

$V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t)$

$V_{in}(t) = RC\frac{d}{dt}V_o(t) + LC\frac{d^2}{dt^2}V_o(t) + V_o(t)$

b $V_i(t) = e^{jwt}$ $V_o(t) = H(w)e^{jwt}$

$$e^{jwt} = RCjw\,H(w)\,e^{jwt} + LC\frac{d^2}{dt^2}H(w)e^{jwt} + e^{jwt}$$

$$1 = RCjw\,H(w) + LCj^2w^2H(w) + H(w)$$

$$\boxed{H(w) = \frac{1}{RCjw - LCw^2 + 1}}$$

c) $$|H(w)| = \frac{1}{|jwRC + 1 - w^2LC|} = \frac{1}{\sqrt{(wRC)^2 + (1-w^2LC)^2}}$$

d. minimize $(wRC)^2 + (1-w^2LC)^2$ to maximi[ze] because $\frac{1}{\sqrt{(wRC)^2+(1-w^2LC)^2}}$ is always positive and only depends on that value

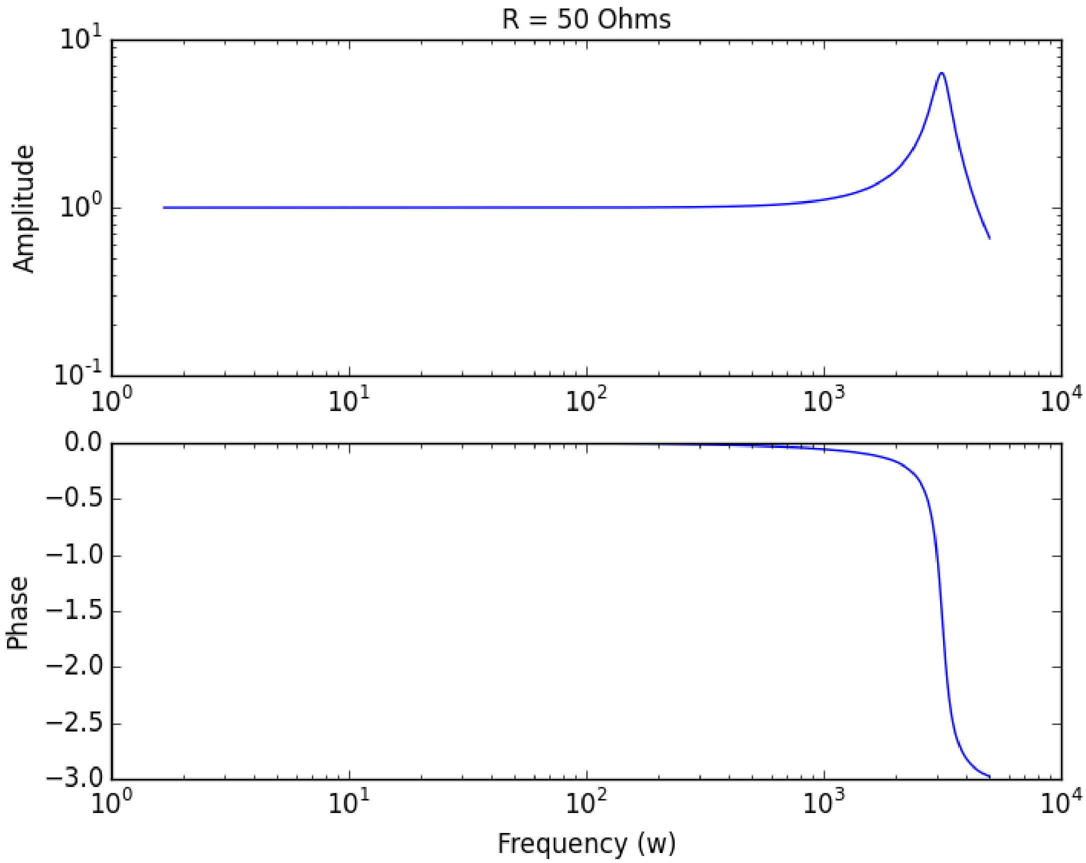
$$\frac{d}{dw}\left(w^2R^2C^2 + 1 - 2w^2LC + w^4(LC)^2\right)$$

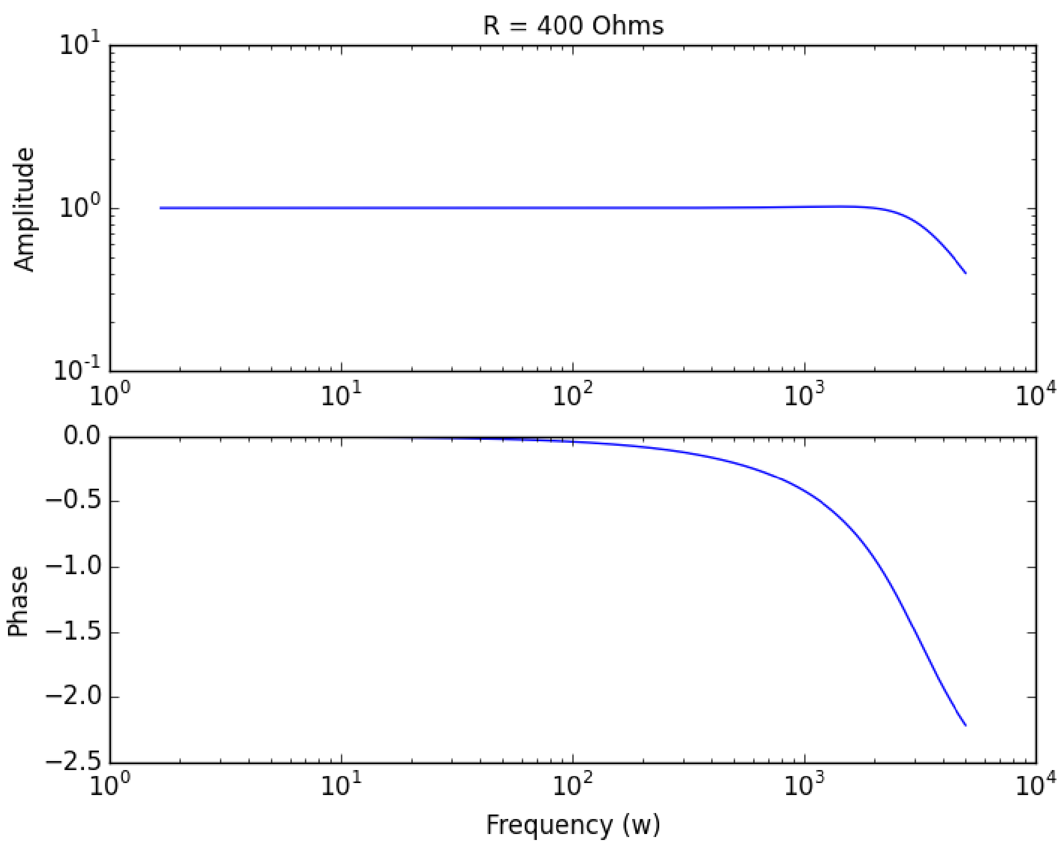
$$= 4L^2C^2w^3 - 4LCw + 2R^2C^2w = 0$$

$$w(4L^2C^2w^2 - 4LC + 2R^2C^2) = 0 \qquad w \neq 0$$

$$w^2 = \frac{4LC - 2R^2C^2}{4L^2C^2} = \frac{4LC}{4L^2C^2} - \frac{2R^2C^2}{4L^2C^2}$$

$$\boxed{w = \pm\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$


R = 50 Ohms
Amplitude
10^{1}
10^{0}
10^{-1}
10^{0} 10^{1} 10^{2} 10^{3} 10^{4}
Phase
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
10^{0} 10^{1} 10^{2} 10^{3} 10^{4}
Frequency (w)


R = 400 Ohms
Amplitude
10^{1}
10^{0}
10^{-1}
10^{0} 10^{1} 10^{2} 10^{3} 10^{4}
Phase
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
10^{0} 10^{1} 10^{2} 10^{3} 10^{4}
Frequency (w)