Sig Sys PS 7

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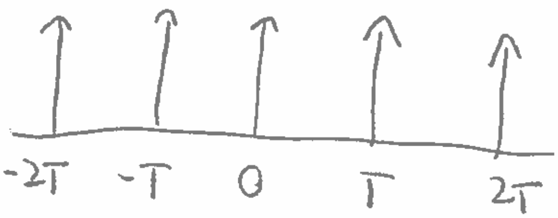
1 a)



b) $p(t) = \sum_{k=-\infty}^{\infty} C_k e^{\frac{2\pi k t j}{T}} dt$

$C_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j\frac{2\pi}{T}kt}\, dt = \frac{1}{T}\left(e^{-j\frac{2\pi}{T}0}\right) =$

$\frac{1}{T} \rightarrow \boxed{p(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j\frac{2\pi}{T}kt}}$

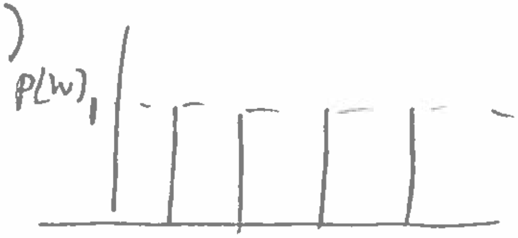
c) $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$

$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt \rightarrow C_k = \frac{1}{T} X(w)$

$\rightarrow \boxed{X(w) = T C_k} \rightarrow X(w) \int_{-\infty}^{\infty} x(t) e^{-jw_0 t} dt$

d) $P(w) = T C_k = \boxed{P(w) = 1}$

e)



A change in T will change the spacing between the impulses

As T $\uparrow$ p(t) $\downarrow$ and as T $\downarrow$ p(t) $\uparrow$

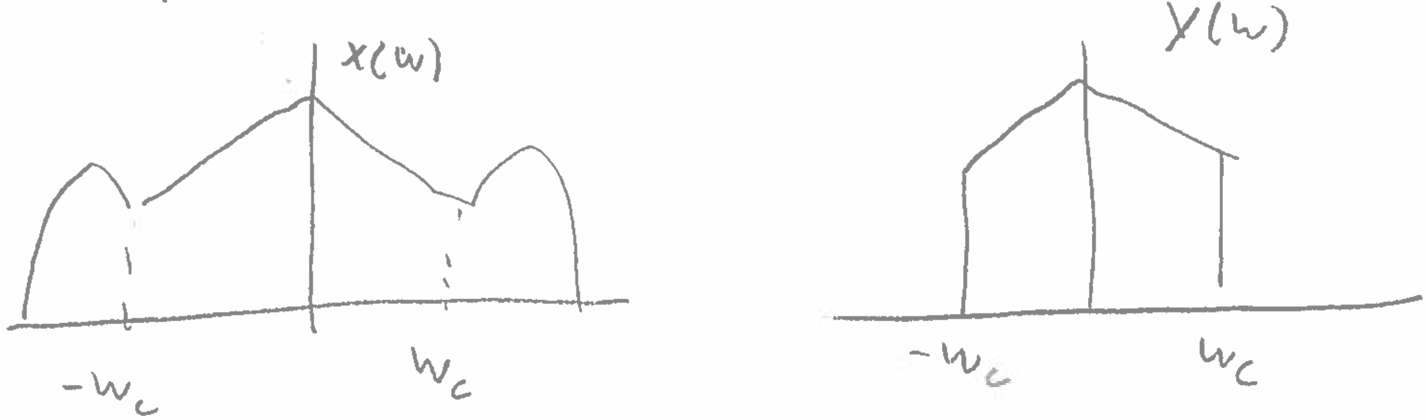
2.

a) $h(t) = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwt}\, dw =$

$h(t) = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwt} = \frac{1}{2\pi j t}\left[e^{jw_c t} - e^{-jw_c t}\right.$

$= \frac{1}{\pi t i} \sin(w_c t)$

b) convolution in the frequency domain is multiplication

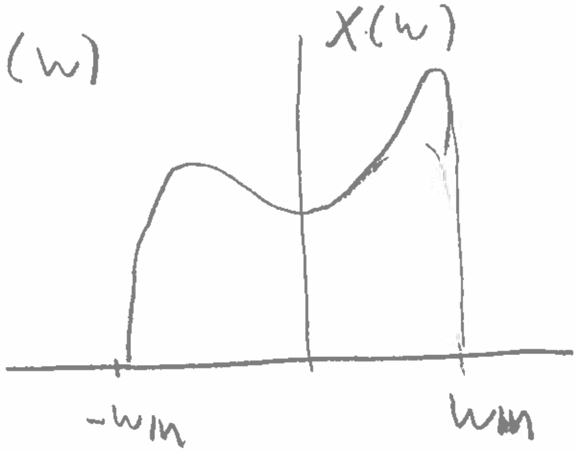


c) This LTI is an ideal low pass filter because it cuts off all of the frequencies above the cut off. There are no partial cut offs

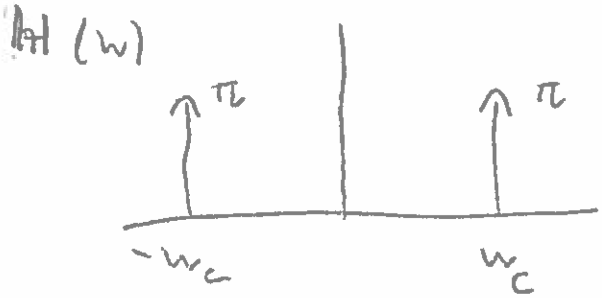
d) attached

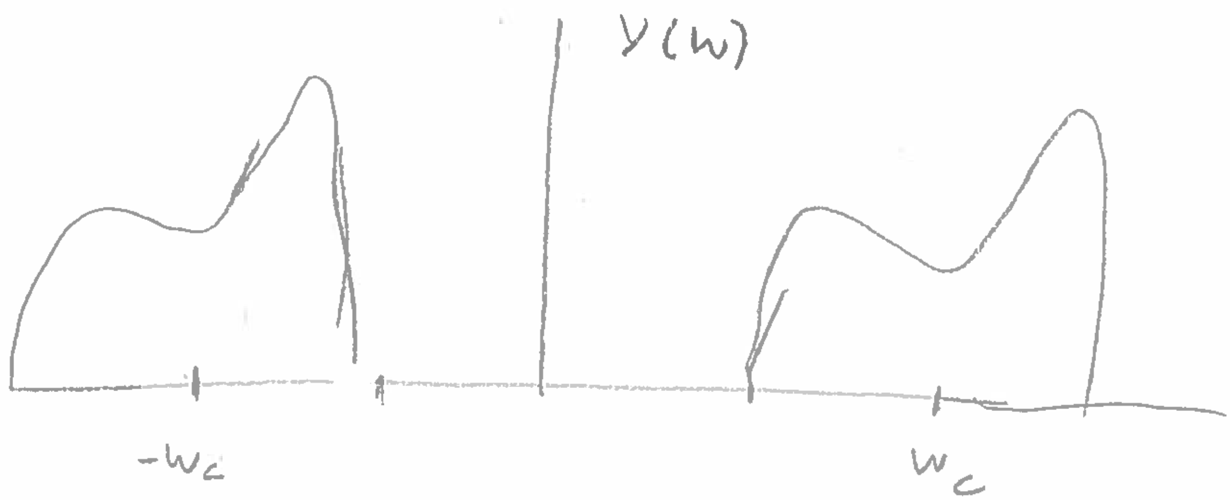
3 Let $y(t) = x(t)\cos(w_c t)$ where $w_c >> w_m$

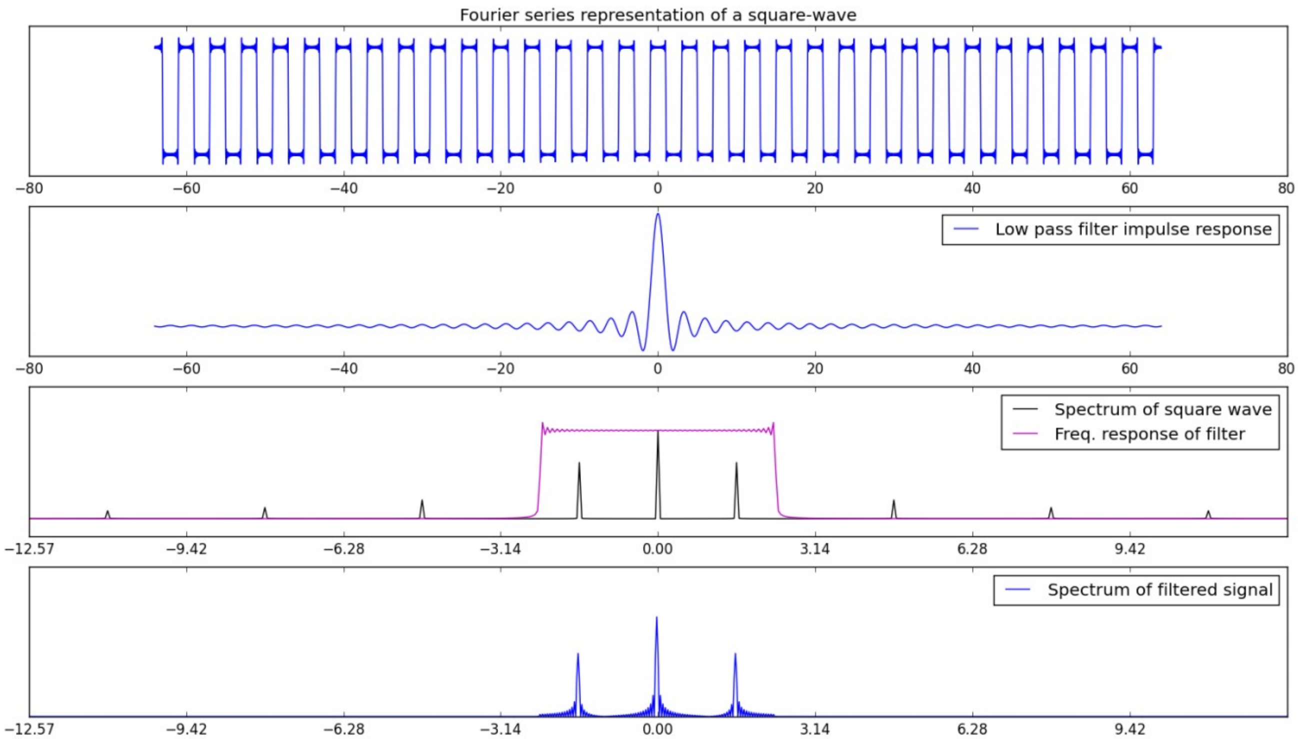
$Y(w)$

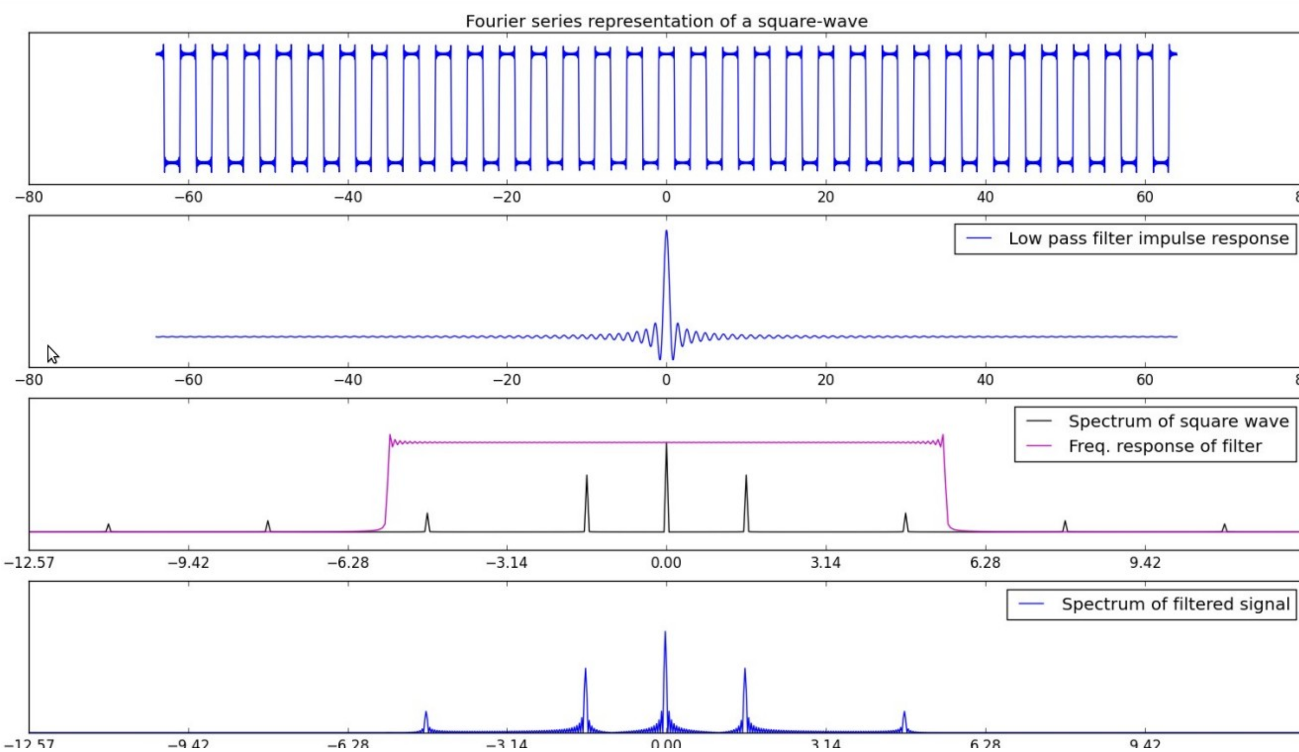


$$Y(w) = X \cdot h(w)$$
$$h(w) = w\,\delta(w_c t)$$
$$= \pi\,\delta(w - w_c) + \pi\,\delta(w + w_c)$$






Fourier series representation of a square-wave
−80 −60 −40 −20 0 20 40 60 80
Low pass filter impulse response
−80 −60 −40 −20 0 20 40 60 80
Spectrum of square wave
Freq. response of filter
−12.57 −9.42 −6.28 −3.14 0.00 3.14 6.28 9.42
Spectrum of filtered signal
−12.57 −9.42 −6.28 −3.14 0.00 3.14 6.28 9.42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