PS06

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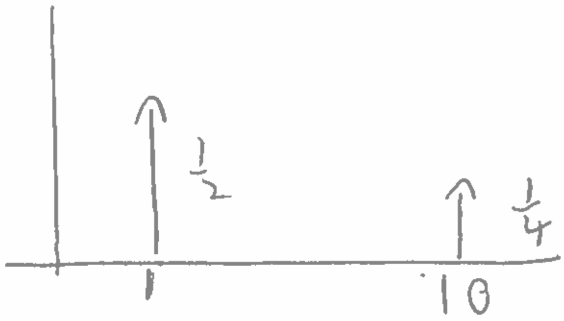
1. Impulse response refers to the reaction of any dynamic response to some external change. The external change basically refers to impulse. ~~The~~ With audio signal of the gun shot (impulse) and the recording of the gunshot in the range (impulse response) we can develop a "transfer" function that describes the system. We can convolve this transfer function with ~~the~~ any audio signal, in this case a violin recording and produce an impulse response, which is the sound of violin played in the gun range

2. $y(t) = \frac{1}{2}x(t-1) + \frac{1}{4}x(t-10)$

In terms of the equation, it definitly makes sense as an echo channel $\frac{1}{2}x(t-1)$ shows that input $x(t)$ is delayed by a second with half of the amplitude. $\frac{1}{4}x(t-10)$ shows that the input $x(t)$ is delayed by 10 seconds ~~and~~ with $\frac{1}{4}$ of the amplitude.

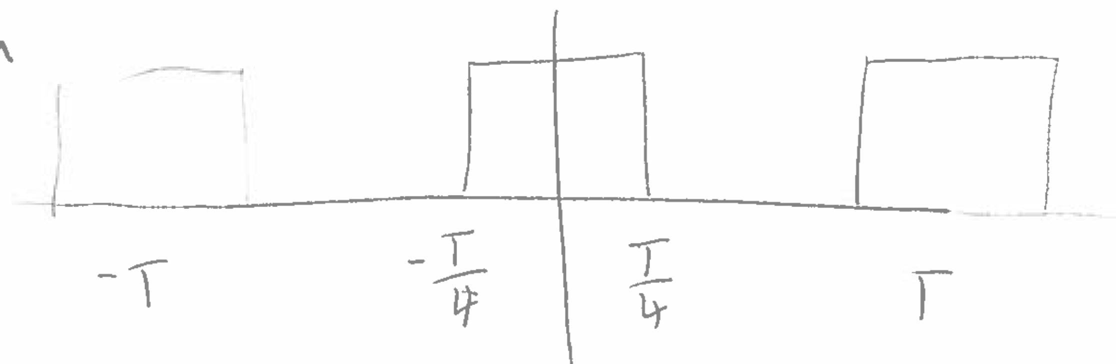


$$\overset{t}{y(t) = x * h(t) = h * x(t)}$$

$$y(t) = \frac{1}{2}x(t-1) + \frac{1}{4}x(t-10)$$

impulse $= \frac{1}{2}\delta(t-1) + \frac{1}{4}\delta(t-10)$

3a



integral of 0 is 0

$$c_k = \frac{1}{T}\int_{-T/4}^{T/4} e^{-j\frac{2\pi}{T}kt}\,dt$$

$$= \frac{1}{T}\,\frac{1}{-j\frac{2\pi}{T}k}\,e^{-j\frac{2\pi}{T}kt}\Big|_{-T/4}^{T/4}$$

$$\frac{1}{T}\,\frac{T}{-j2\pi k}\left(e^{-j\frac{2\pi}{4}} - e^{j\frac{2\pi}{4}k}\right) = \frac{1}{j2\pi k}\left(e^{j\frac{\pi}{2}k} - e^{-j\frac{\pi}{2}k}\right)$$

$$\frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) = \sin\theta$$

$$\frac{1}{\pi k}\left(\frac{1}{2j}e^{j\frac{\pi}{2}k} - \frac{1}{2j}e^{-j\frac{\pi}{2}k}\right) = \frac{\sin(\frac{\pi}{2}k)}{\pi k}$$

$K = \text{odd} \Rightarrow \frac{1}{\pi k}$ or $-\frac{1}{\pi k}$

$k = \text{even} = 0$

$$C_k = \frac{\sin(\frac{\pi}{2} k)}{\pi k}$$

see graph

$$\tilde{x}(t) = \sum_{k=-K}^{K} C_k e^{-j\frac{2\pi}{T}kt}$$

eval at when

$K = 5, 17, 257$

c. The edge has a spike that we can't seem to get rid of no matter how many terms we add. As $k \to \infty$, the edges of the discontinuity decreases but it never seems to go away

4.a $y(t) = x(t - T_1)$ $|T_1| < T$

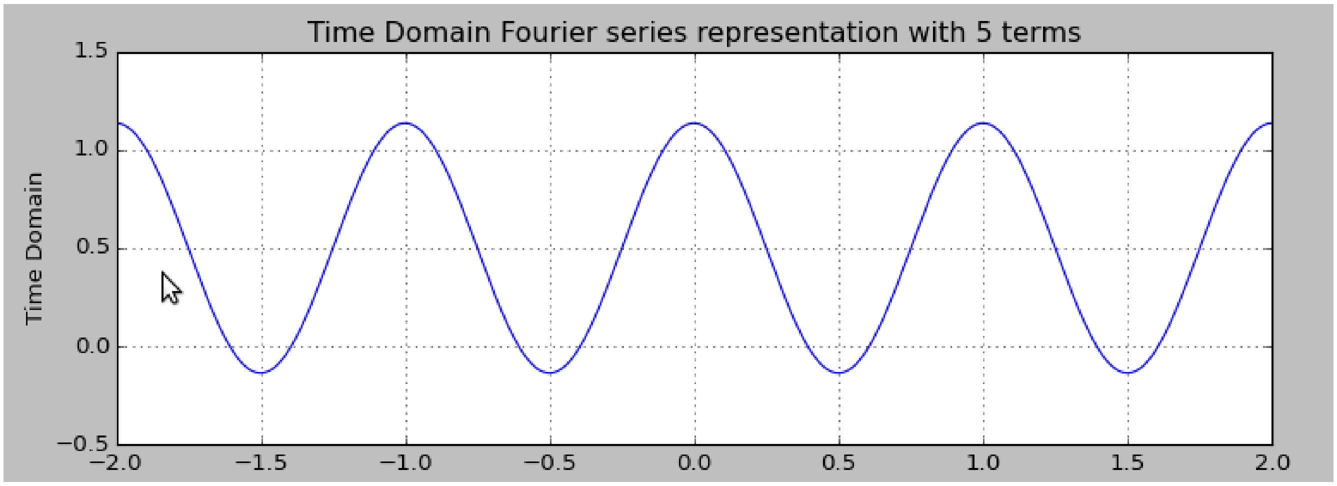
$u = t - T_1 \rightarrow du = dt$

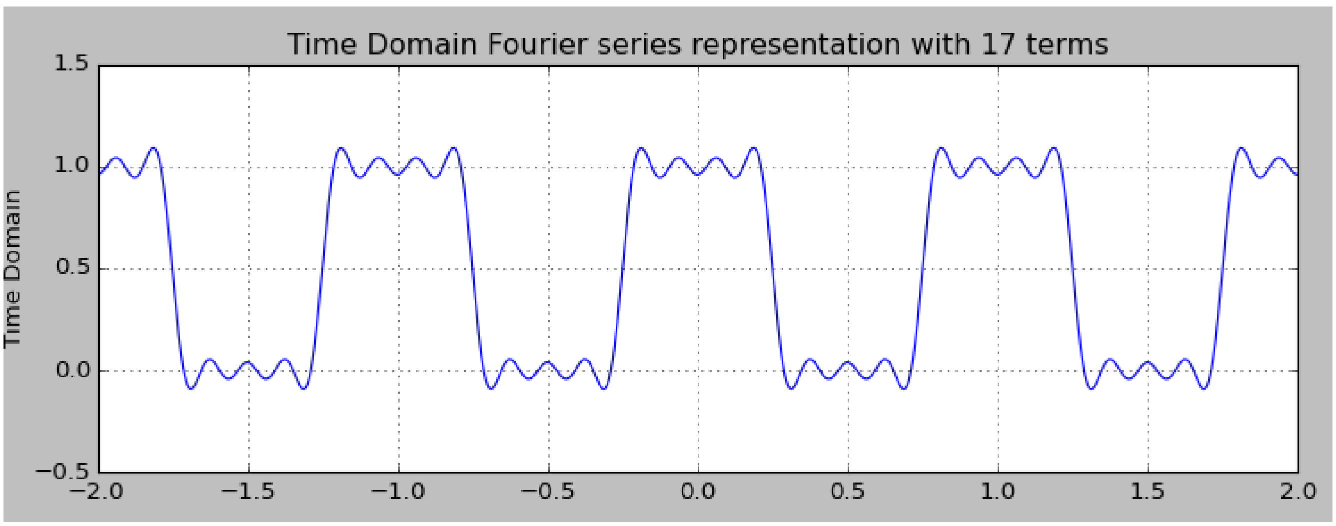
$$\frac{1}{T}\int_{-T/2}^{T/2} x(t-T_1)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

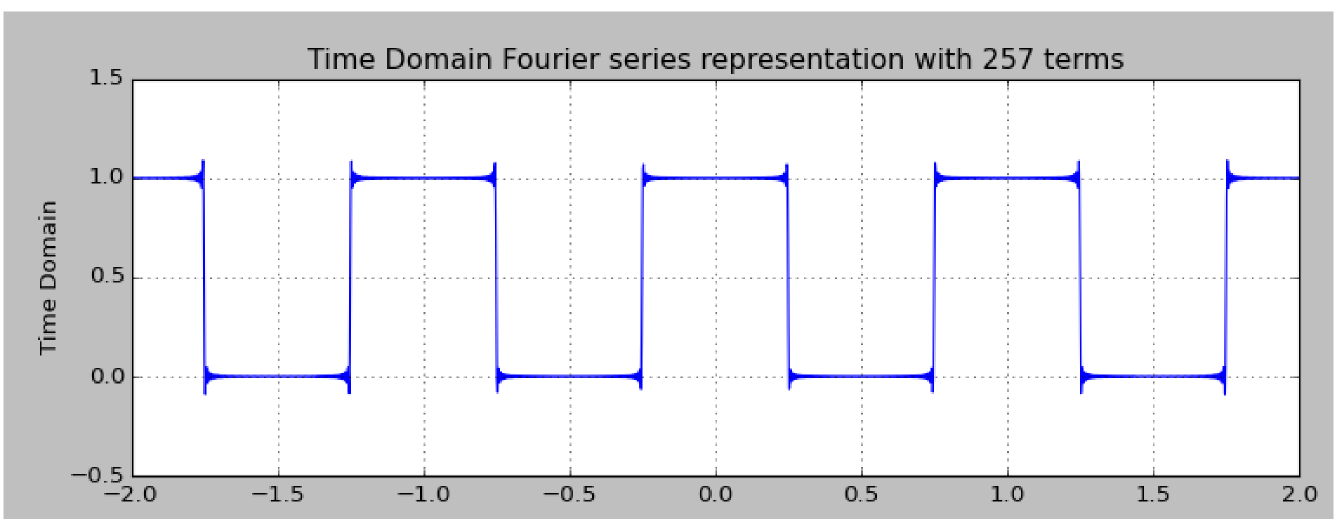
$$= \frac{1}{T}\int_{-T_2 - T_1}^{T/2 - T_1} x(u)\, e^{-j\frac{2\pi}{T}k(u+T_1)}\, du$$

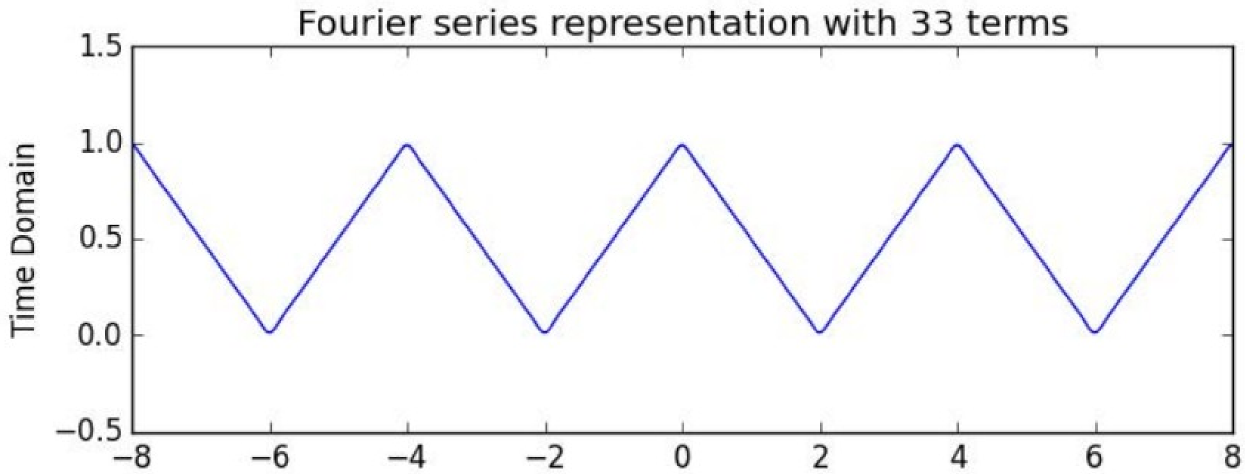
$$= e^{-j\frac{2\pi}{T}kT_1}\Big[\underbrace{\frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} x(u)\, e^{-j\frac{2\pi}{T}ku}\, du}_{C_k}$$

$$= \boxed{e^{-j\frac{2\pi}{T}kT_1}\, C_k}$$


Time Domain Fourier series representation with 5 terms
Time Domain
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0


Time Domain Fourier series representation with 17 terms
Time Domain
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0


Time Domain Fourier series representation with 257 terms
Time Domain
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0



I added x = x + np.exp(1j*2*np.pi/T*k)*Coeff*np.exp(1j*2*np.pi/T*k*ts) to FourierSeries1.ipynb