Problem Set 10

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1. $\dot{y} + y = x = Ys + Y = X$

$H(x) = \frac{Y}{X} = \frac{1}{1+s} \rightarrow \int_0^\infty \frac{1}{1+s} e^{-st} dt$

$= \frac{1}{1+s} \quad \frac{1}{-s} e^{-st}\Big|_0^\infty = \frac{1}{s(1+s)} (0 - 1) = \frac{1}{s(1+s)}$

$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} \rightarrow A(s+1) + Bs = 1$

$A + B = 0$

$A = 1$

$\boxed{\begin{matrix} A = 1 \\ B = -1 \end{matrix}}$

$\frac{1}{s} - \frac{1}{s+1} \rightarrow \frac{1}{s} = u(t) - e^{-1t} u(t)$

$= \boxed{(1 - e^{-t}) u(t)}$

2. A. $k(Y_{sp} - HX) = X$

$\frac{X}{Y_{sp}} = \frac{k}{1+kH} \qquad Y = HX \qquad \frac{Y}{Y_{sp}} = \boxed{\frac{kH}{1+kH}}$

B. $H(s) = \frac{1/\tau}{s - 1/\tau}$ $\qquad \frac{Y(s)}{Y_{sp}(s)} = ?$ $\qquad K >> 1/\tau$

$$\frac{K\frac{1/\tau}{s+1/\tau}}{1+K\frac{1/\tau}{s+1/\tau}} = \frac{K/\tau}{s+(K+1)/\tau} = \frac{K}{s\tau+K+1}$$

$K = K_I/s$ for integral

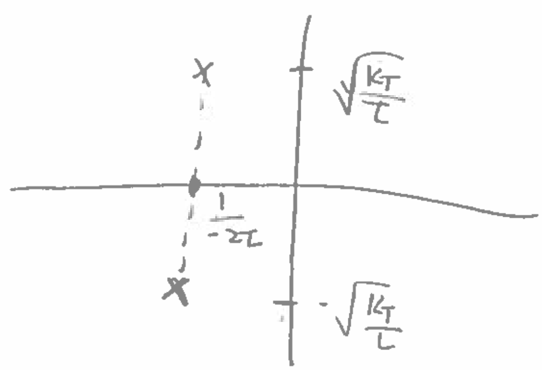
$$\frac{K_I/s}{s\tau + K_I/s + 1} = \frac{K_I}{s^2\tau + K_I + s}$$

when this equation goes to infinite

when $s^2\tau + K_I + s = 0$ quadratic

$$\frac{-1 \pm \sqrt{1-4\tau K_I}}{2\tau} = -\frac{1}{\tau} \pm \sqrt{\frac{1}{\tau^2} - \frac{4K_I}{\tau}} \rightarrow -\frac{1}{2\tau} \pm \sqrt{\frac{-4K}{\tau}}$$

$$-\frac{1}{2\tau} \pm i\sqrt{\frac{K_I}{\tau}}$$



4 $\qquad H(s) = \frac{1}{s^2 - .01s + 1}$ $\qquad$ coeff → [1, -.01, 1+Kp

B. proportional control $\quad K = K_p$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{KH}{1+KH} = \frac{K_p H}{1+K_p H} = \frac{\frac{K_p}{s^2-.01s+1}}{1+\frac{K_p}{s^2-.01s+1}} = \boxed{\frac{K_p}{s^2 - .01s + 1 + K_p}}$$

4c. Integral $K = K_I/s$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{KH}{1+KH} = \frac{K_I/s\, H}{1+K_I/s\, H} = \frac{\frac{K_I}{s(s^2-.01s+1)}}{1+\frac{K_I}{s(s^2-.01s+1)}}$$

$$= \boxed{\frac{K_I}{s^3-.01s^2+s+K_I}} \rightarrow \text{coeff} \rightarrow [1, -.01, 1, K_I]$$

4b derivative $K = sK_d$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{KH}{1+KH} = \frac{sK_dH}{1+sK_dH} = \frac{\frac{sK_d}{s^2-.01s+1}}{1+\frac{sK_d}{s^2-.01s+1}}$$

$$= \frac{K_d}{s^2-.01s+1+sK_d} = \boxed{\frac{K_d}{s^2+(K_d-.01)s+1}}$$

coeff $\rightarrow [1, K_d-.01, 1]$

Integral works. It stabilizes the step function

In [1]:

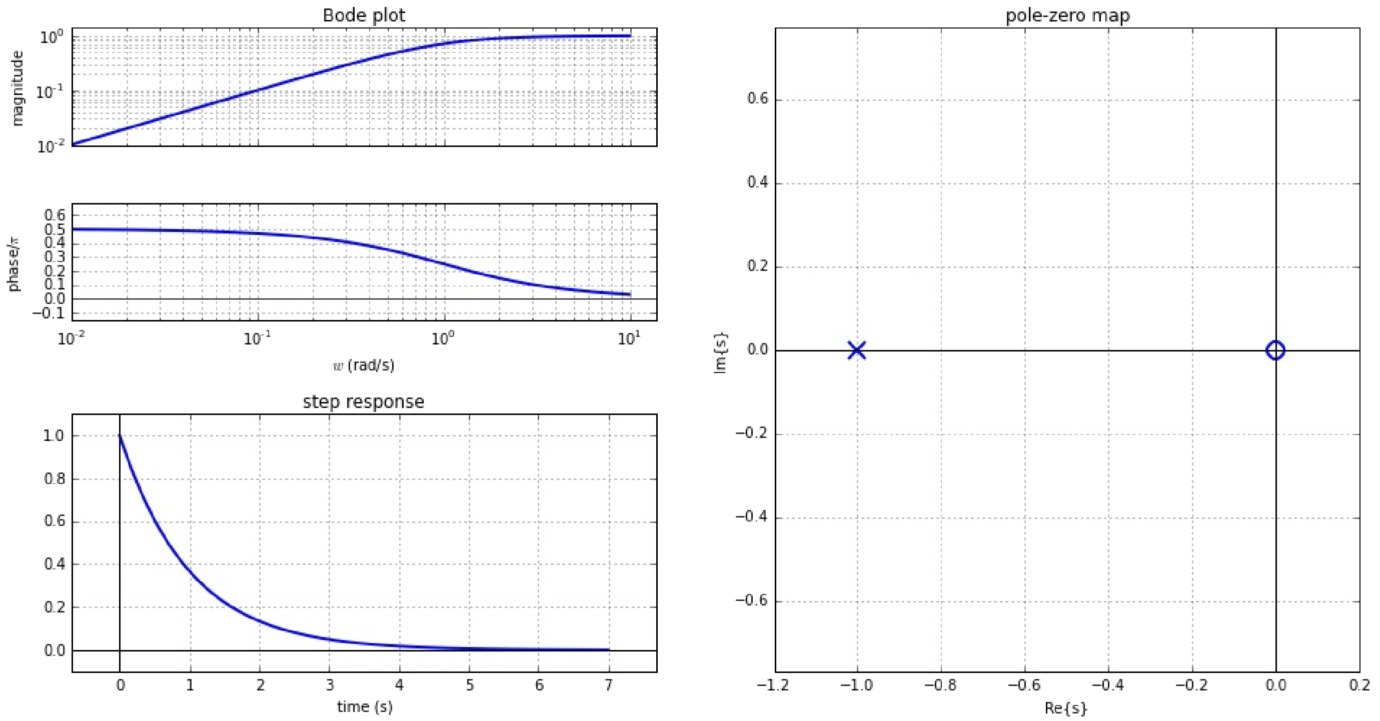
```
%matplotlib inline
%run convenience.ipynb
from __future__ import print_function
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal

np.set_printoptions(precision=2,suppress=True) # numpy output options

pi=np.pi
j=1j
```

System characterization with Bode plot, step response and pole-zero map

In [2]:

```
# 3A
combinedplot(signal.lti([1,0],[1,1]))
```

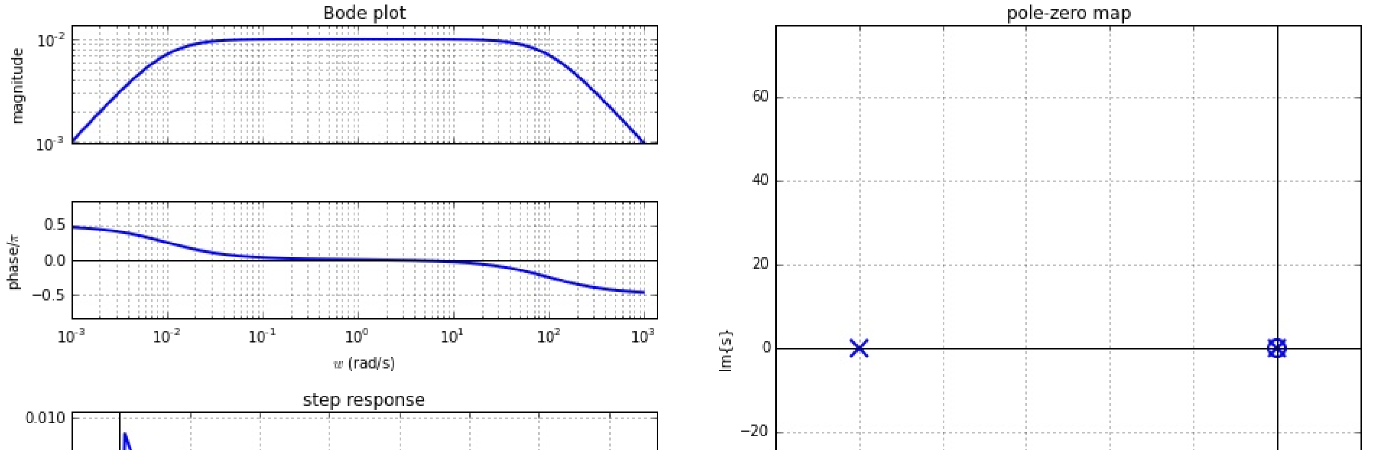
```
/home/james/anaconda/lib/python2.7/site-packages/matplotlib/axes/_axes.py:475: UserWarning: No labelled objects found. Use label='...' kwarg on individual plots.
  warnings.warn("No labelled objects found. "
```

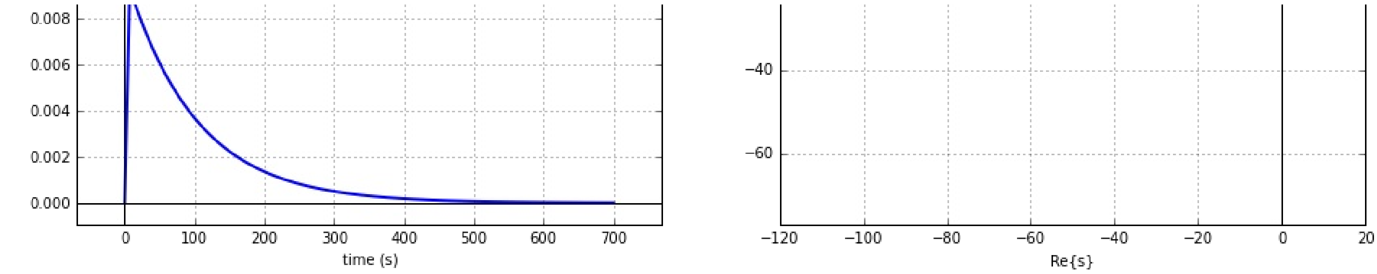


It is a linear system that act as a high pass filter. There are one poles and one zero both on the real axis.

In [3]:

```
# 3B
combinedplot(signal.lti([1,0],[1,100, 1]))
```

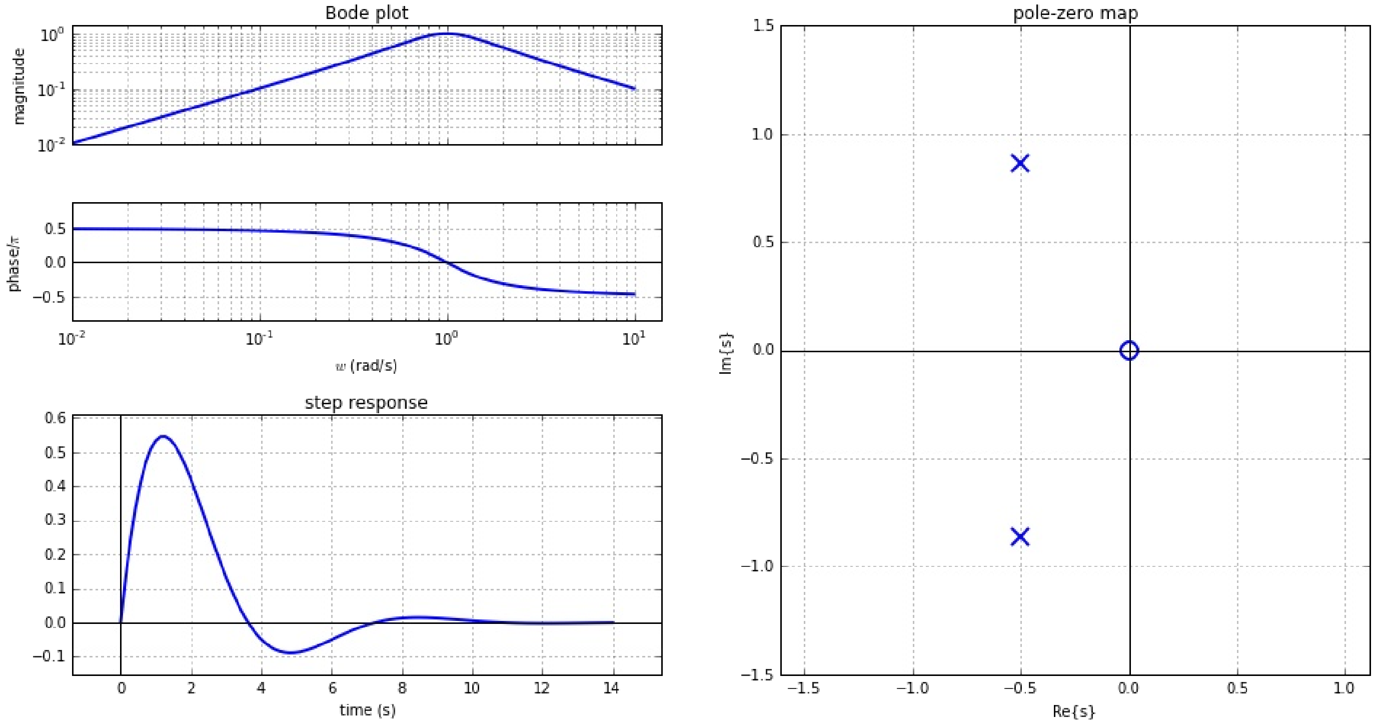




This is a second order system that act as a band pass filter. The pole and zero are both on the real axis. The step response slowly die off.

In [4]:

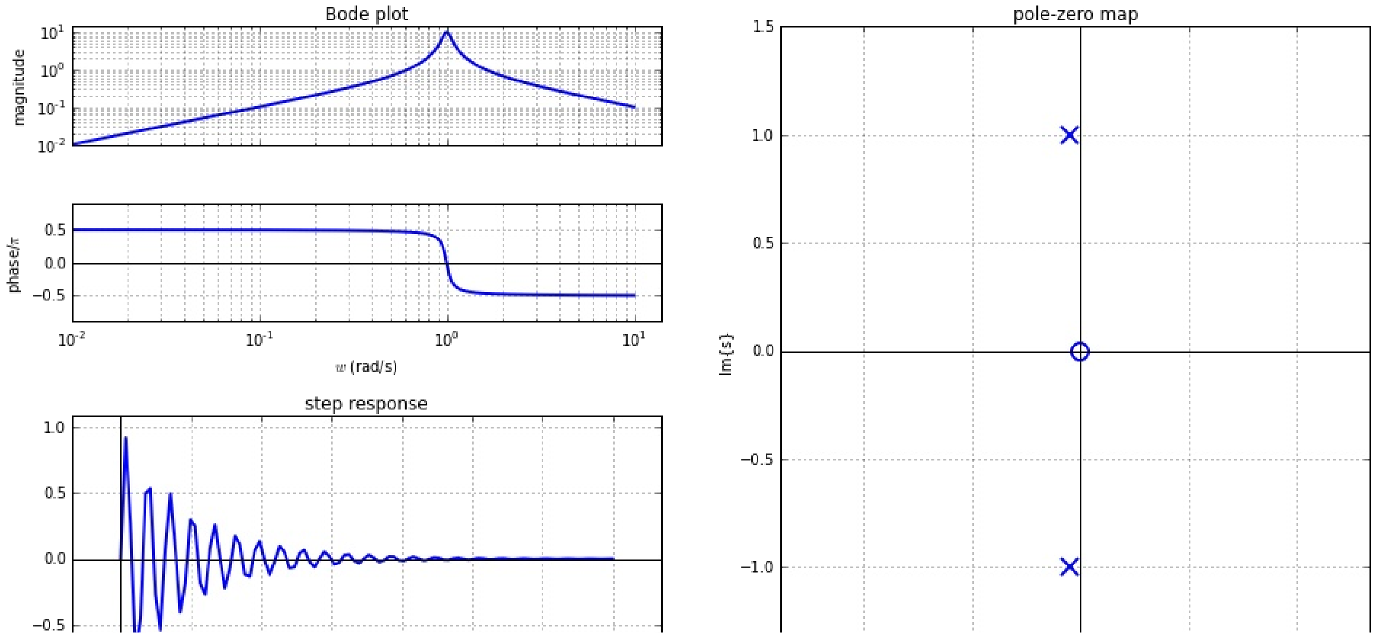
```
# 3C
combinedplot(signal.lti([1,0],[1,1, 1]))
```

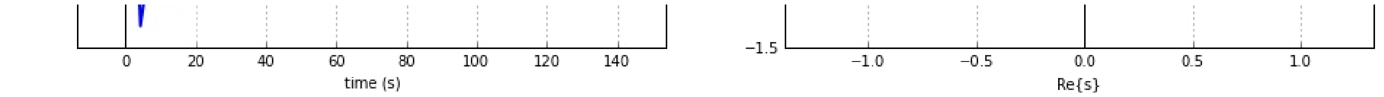


Another second order system. The step response shows a damped oscillation. Two imaginary poles and one real zero. It seems like the imaginary part causes the oscillation.

In [5]:

```
# 3D
combinedplot(signal.lti([1,0],[1,.1, 1]))
```

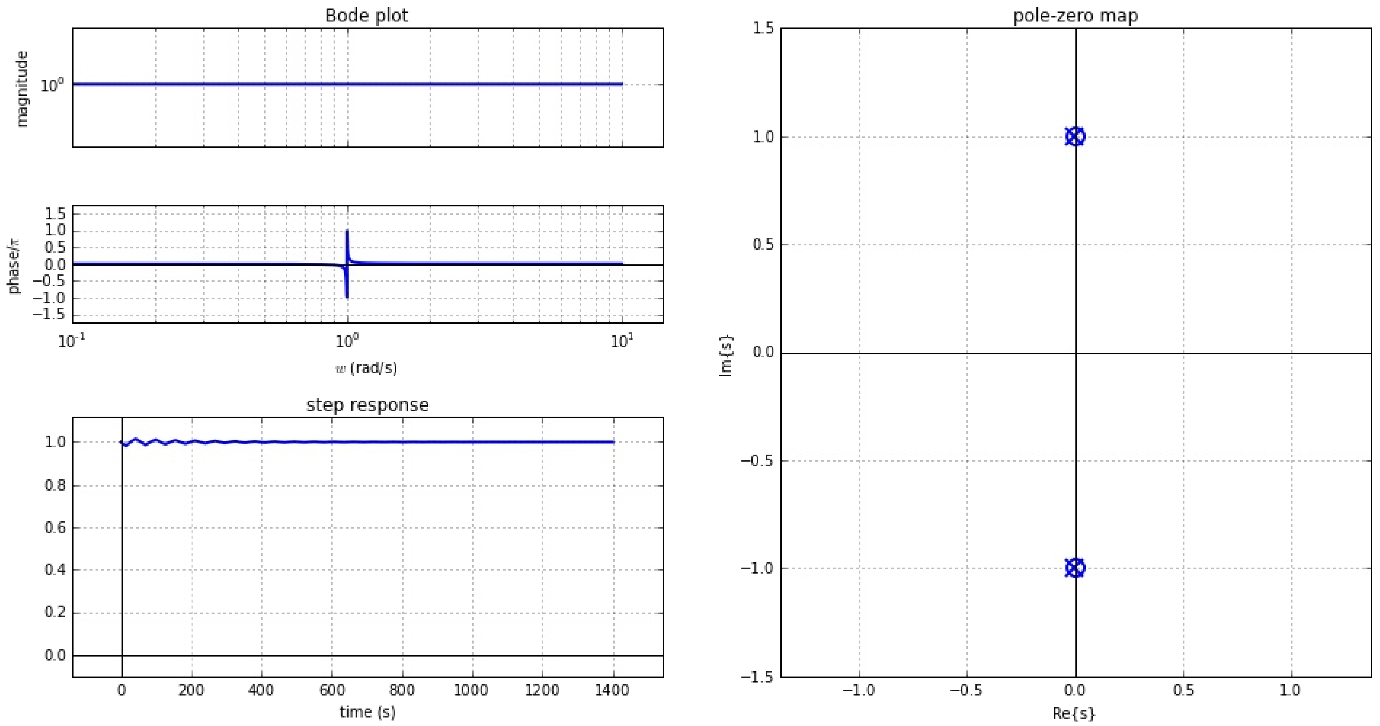




A second order systme that acts like 3C with an oscillation that slowly dies out. The poles are closer to the zero on the imaginary axis which seems to cause the stronger oscillation .

In [6]:

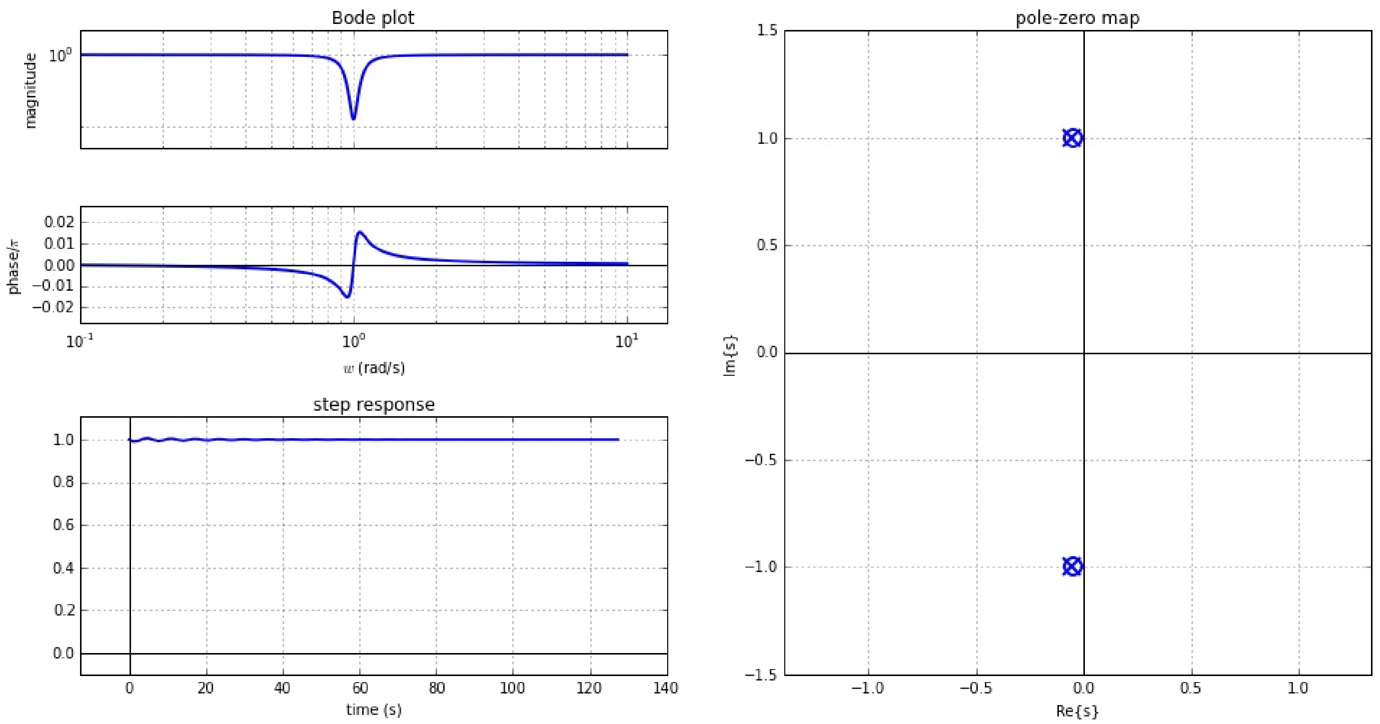
```
# 3E
combinedplot(signal.lti([1,-0.01, 1],[1,.01, 1]))
```



A second order systme that doesnt kill signals and shifts the phase. There seem to be a slight oscillation that dies out pretty quickly.

In [7]:

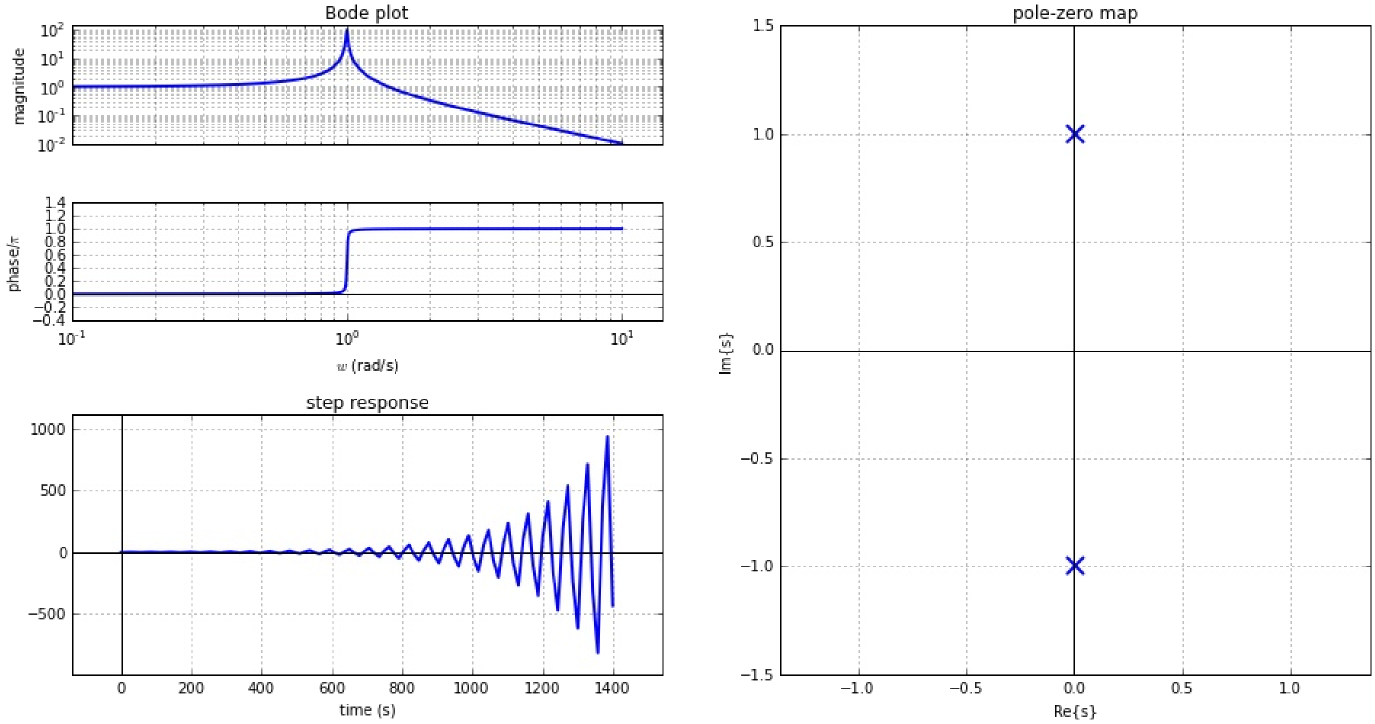
```
# 3F
combinedplot(signal.lti([1,0.1, 1],[1,.11, 1]))
```



A second order systme that kills a specific frequency. There is a pair of imaginary pole and a pair of imaginary zeros.

In [8]:

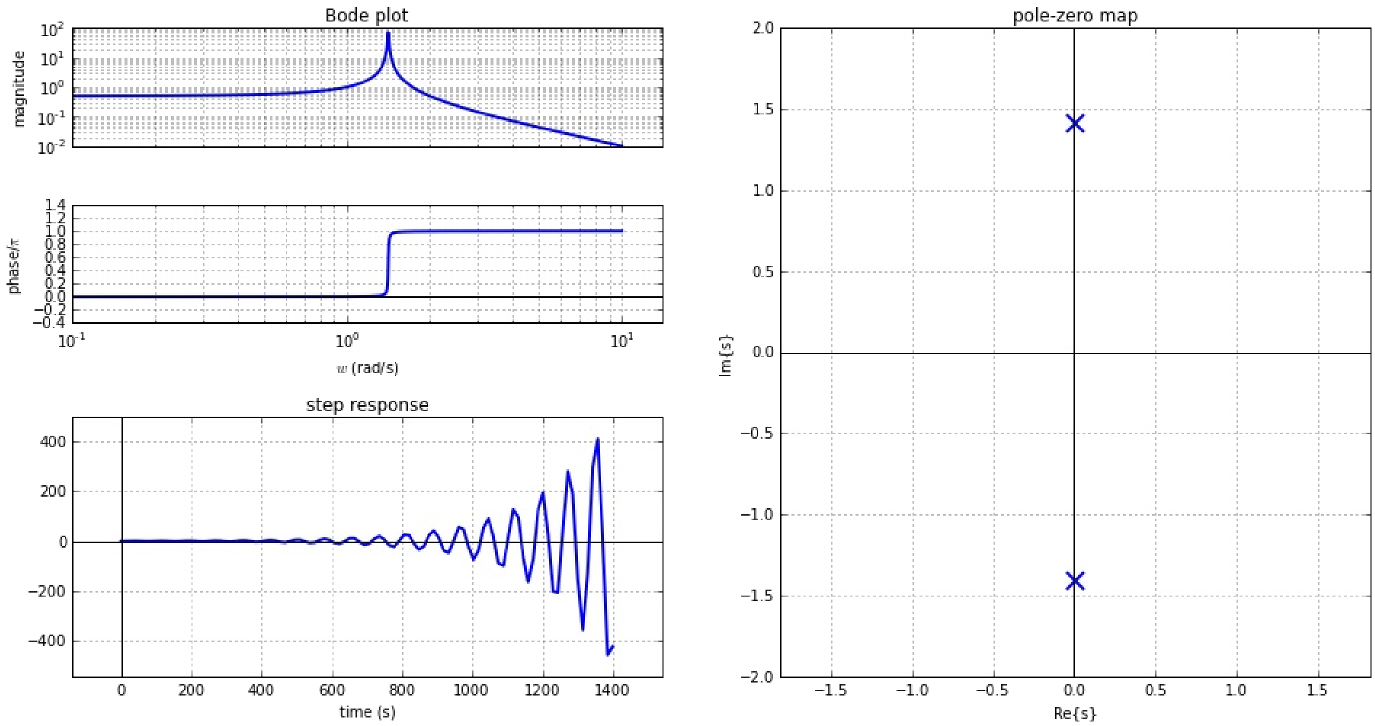
```
# 4A
combinedplot(signal.lti([1],[1,-.01, 1]))
```



It has two imaginary poles. The step response shows an unstable oscillation that increase in amplitude.

In [9]:

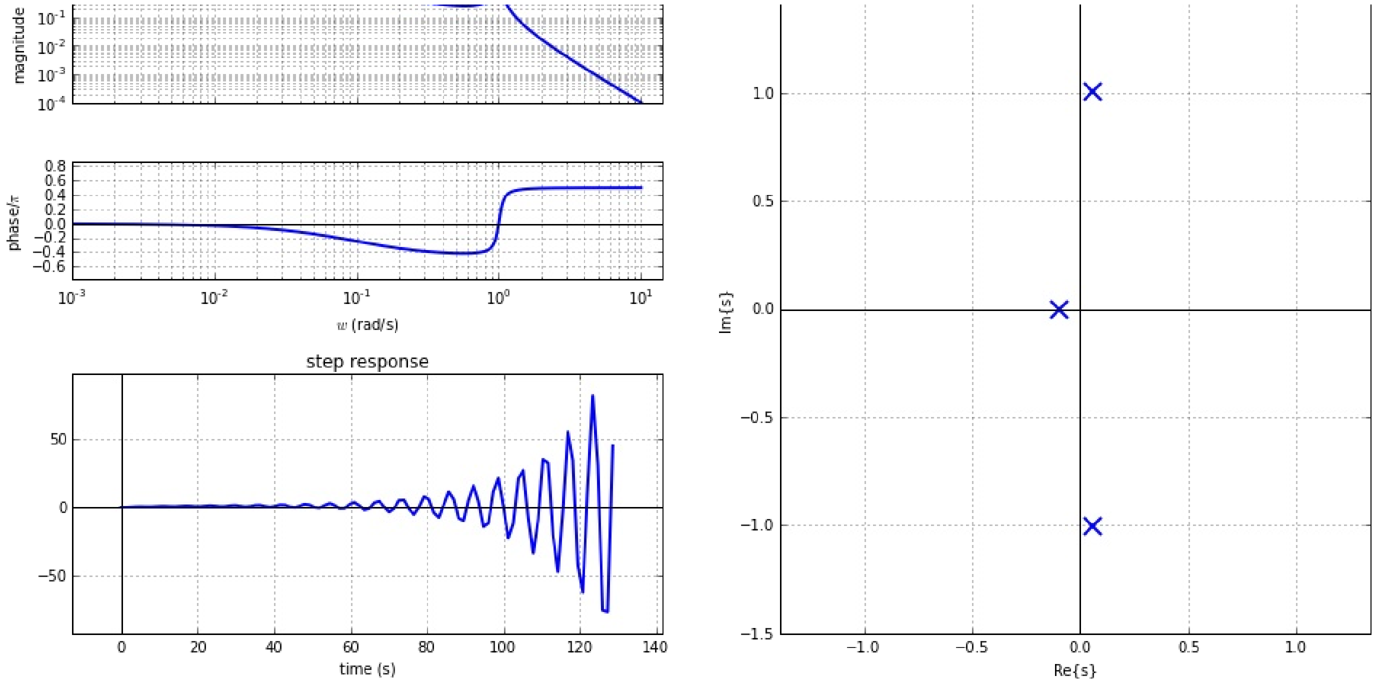
```
# 4B
Kp = 1
combinedplot(signal.lti([Kp],[1,-.01, 1+Kp]))
```



The proportional control is not able to stabilize the system.

In [10]:

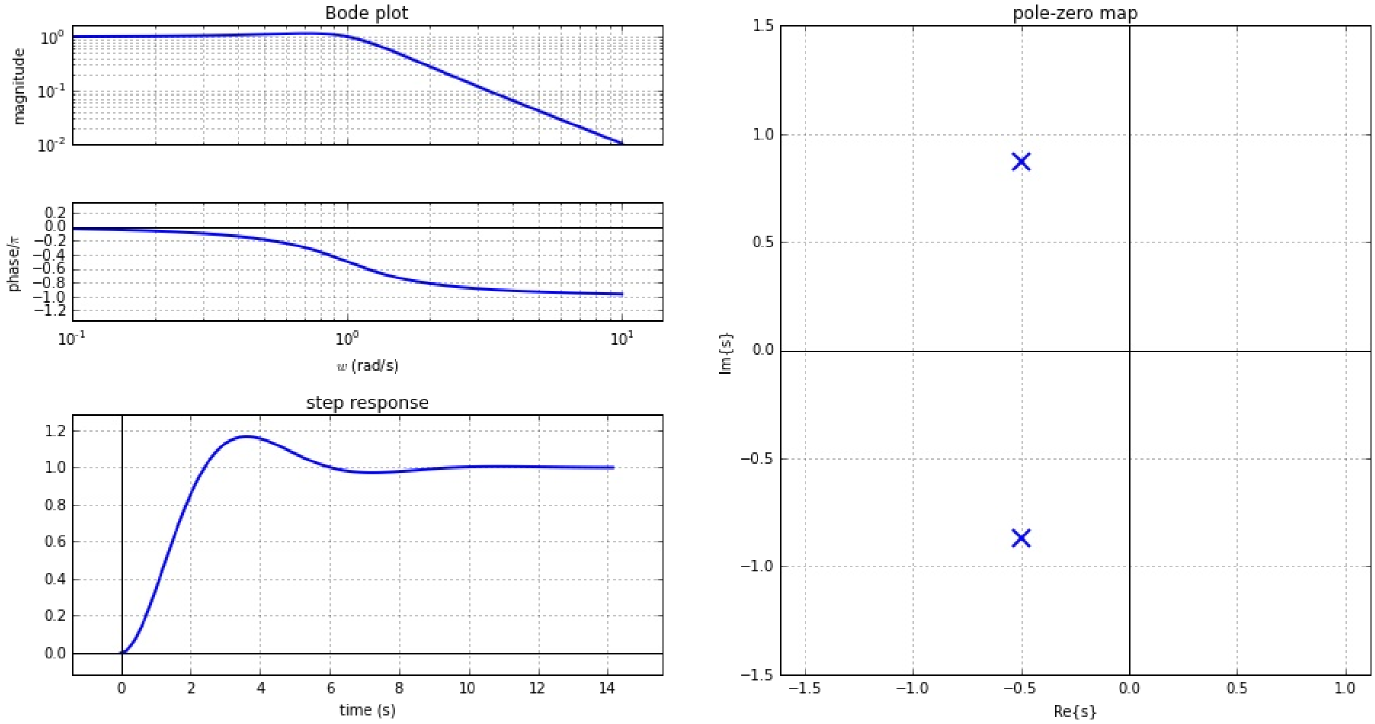
```
# 4C
Ki = .1
combinedplot(signal.lti([Ki],[1,-.01, 1, Ki]))
# second order two imaginary poles the system oscillates and gets bigger
```



This is a second order that act as a band pass filter. The pole and zero are both on the real axis. The step response slowly die off.

In [11]:

```
# 4D
Kd = 1
combinedplot(signal.lti([Kd],[1,-.01 + Kd, 1]))
```



The derivative control is able to stablize the system. The poles are moved from the right of the imaginary axis to left and this seems to stablize the system.