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Influence of Photon Mass on Vacuum Birefringence Experiment *

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Influence of photon mass on vacuum birefringence experiment is analysed according to the nonlinearities of vacuum quantum electrodynamics for the light propagation through an intense electromagnetic field. It is shown that although the photon mass will cause a change of the refractive indices $n_\perp$ and $n_\parallel$ of vacuum birefringence, the difference $n_\parallel - n_\perp$ is unchanged, which means that the effect of photon mass cannot be observed in vacuum birefringence experiment.

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During the last few years the photon mass problem has attracted a great deal of experimental attention.[1,2] Although the introduction of a small photon mass may look unaesthetic, it can be shown that a theory of massive quantum electrodynamics (where $m_\gamma \neq 0$) is fully consistent theoretically, and its predictions go over smoothly to those for the massless case as $m_\gamma \to 0$. Hence the possibility of a nonzero value of photon mass can never be completely excluded, and a substantial experimental effort has been made to set an upper bound on photon mass. The most precise bounds on photon mass are based on the measurement of the planet magnetic field[3,4] and the ambient cosmic magnetic vector potential.[5,6] However, there is no direct analysis of the photon mass effect for the usual quantum electrodynamics (QED) experiments, such as the Lamb shift, measurements of the g-2 factor as well as the vacuum birefringence.

In this Letter, we only restrict ourselves to the analysis of the influence of photon mass on the vacuum birefringence experiment in QED. Our primal motivation is to find a possible limit for the photon mass by exploiting some phenomena in QED. QED predicts the existence of nonlinear effects between electromagnetic field in vacuum.[7−10] The creation of virtual electron-positron pairs results in the interaction of two photons and leads to additional terms in the Lagrangian, which was first obtained by Euler and Heisenberg,[8] and yielded to the vacuum birefringence in a transverse magnetic field.[11−14] Iacoppini and Zavattini first proposed to measure the vacuum birefringence in a 10 T magnetic field in the laboratory by a precision determination of the induced ellipticity on a laser beam down to 10^{-11}.[15] Subsequently there are several experiment groups (the PVLAS[16,17], the Fermilab P-877,[18] the Q&A[19]) devoting to the task in succession. Up to date, the vacuum birefringence has not been measured. Here we study the new situation of the vacuum birefringence considering the rest mass of photon. It is shown that although the photon mass will cause a change to the refractive indices $n_\perp$ and $n_\parallel$ of vacuum birefringence, the difference $\Delta n = n_\parallel - n_\perp$ is unchanged, which means that effect of photon mass cannot be observed in vacuum birefringence experiments.

Let us consider the simple example that a low-frequency wave traverses an intense and nearly constant applied electromagnetic field in the vacuum background. It is well known that the vacuum polarized by the external constant magnetic field acts like a birefringent medium.[7] This phenomenon is called "the vacuum birefringence". If the optical wave is a linearly polarized one, it will be changed into an elliptically polarized one after traversing the intense electromagnetic field. Here we assume that the photon rest mass is nonzero in the process. Let$F$ denote the total electromagnetic field $f_0 + f$, where f corresponds to the optical field and f_0 is the (nearly) constant applied field.[20] The total effective Lagrangian is

$$L = -\frac{1}{4}F^2 + \delta L, \tag{1}$$

with the additional Lagrangian density

$$\delta L = \frac{2\alpha^2}{45m^4}\Big[\frac{1}{4}(F^2)^2 + \frac{7}{16}(F \cdot F^*)^2\Big] + \frac{m_\gamma^2}{2}A_\alpha A^\alpha, \tag{2}$$

where α is the fine-structure constant, and m is the mass of the electron. $F^{*\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2$, $\varepsilon^{0123} = 1$, $F^2 = F_{\mu\nu}F^{\mu\nu}$ and $F \cdot F^* = F_{\mu\nu}F^{*\mu\nu}$; m_γ represents the photon mass and A_α is the four-dimensional vector potential of electromagnetic wave.

It is sufficient to keep in δL only those terms that are quadratic both in f_0 and f (since we are not interested in the direct effect of the incident wave on itself, or of the external field on itself). for the interaction

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this procedure gives[20]

$$\delta L = \frac{2\alpha^2}{45m^4}\Big[\frac{1}{2}f^2 f_0^2 + (f\cdot f_0)^2 + \frac{7}{8}(f\cdot f^*)(f_0\cdot f_0^*) + \frac{7}{4}(f\cdot f_0^*)^2\Big] + \frac{m_\gamma^2}{2}A_\alpha A^\alpha. \tag{3}$$

Here we should keep in mind that A_α represents the vector potential of the incident wave. Setting $\rho = 4\alpha^2/(45m^4)$, the modified Maxwell equation then reads

$$\begin{aligned}&(1-\rho f_0^2)\partial_\mu f^{\mu\nu} + 2\rho f_0^{\nu\mu}\partial_\mu f\cdot f_0 \\ &\quad + \frac{7}{2}\rho f_0^{*\nu\mu}\partial_\mu f\cdot f_0^* + m_\gamma^2 A^\nu = 0, \\ &\partial_\mu f^{*\mu\nu} = 0.\end{aligned} \tag{4}$$

We consider that a plane-wave mode with four-dimension wave vector k^μ. Using $\partial_\mu f^{*\mu\nu}=0$, we can set $f^{\mu\nu}(x) = (\varepsilon^\mu k^\nu - \varepsilon^\nu k^\mu)e^{ik\cdot x}$ and $A^\nu = i\varepsilon^\nu e^{ik\cdot x}$, where ε^μ describes the polarization of the wave. Then Eq. (4) turns to the form

$$\begin{aligned}&(1-\rho f_0^2)(k^2\varepsilon^\mu - k\cdot\varepsilon k^\mu) + 4\rho k_\nu f_0^{\nu\mu}(\varepsilon f_0 k) \\ &\quad + 7\rho k_\nu f_0^{*\nu\mu}(\varepsilon f_0^* k) - m_\gamma^2\varepsilon^\mu = 0,\end{aligned} \tag{5}$$

where $\varepsilon f_0 k$ represents $\varepsilon_\sigma f_0^{\sigma\tau}k_\tau$, and similarly for $\varepsilon f_0^* k$. In general, we can consider the form of ε^μ as[20]

$$\varepsilon^\mu = \xi_0 k^\mu + \xi_1 k_\nu f_0^{\nu\mu} + \xi_2 k_\nu f_0^{*\nu\mu} + \xi_3\lambda^\mu \tag{6}$$

in a field f_0. Compared with Ref. [20], we have added a new term $\xi_3\lambda^\mu$, which is relative to the non-zero photon mass. It should be noted that the four vectors $k^\mu, k_\nu f_0^{\nu\mu}, k_\nu f_0^{*\nu\mu}$ and λ^μ are chosen to be linear independent and can be considered as the complete base vectors. The expanded parameters ξ in Eq. (6) can be determined from Eq. (5). In order to simplify the expression for the solution of ε^μ, we assume λ^μ is perpendicular to the vectors $k^\mu, k_\nu f_0^{\nu\mu}$ and $k_\nu f_0^{*\nu\mu}$, i.e. $\lambda_\mu k^\mu = 0$, $\lambda_\mu k_\nu f_0^{\nu\mu} = 0$ and $\lambda_\mu k_\nu f_0^{*\nu\mu} = 0$, respectively. Inserting Eq. (6) into Eq. (5), we find with the help of some simple algebra that ξ_0, ξ_1, ξ_2 and ξ_3 are solutions to the homogeneous system

$$\begin{aligned}&\xi_1[(1-\rho f_0^2)k^2 + 4\rho k\cdot f_0\cdot f_0\cdot k - m_\gamma^2] \\ &\quad - \xi_2\rho k^2(f_0\cdot f_0^*) = 0, \\ &\quad -\xi_1\frac{7}{4}\rho k^2(f_0\cdot f_0^*) + \xi_2\Big[\Big(1+\frac{5}{2}\rho f_0^2\Big)k^2 \\ &\quad + 7\rho(kf_0f_0k) - m_\gamma^2\Big] = 0, \\ &\xi_3[(1-\rho f_0^2)k^2 - m_\gamma^2] = 0, \\ &\xi_0 m_\gamma^2 = 0,\end{aligned} \tag{7}$$

where obviously $k\cdot f_0\cdot f_0\cdot k$ means $k^\mu\cdot f_{0\mu\nu}\cdot f_{0\sigma}^{\nu}\cdot k^\sigma$.

For the nonzero photon mass m_γ^2, it is clearly that $\xi_0 = 0$ and λ^μ corresponds to the longitudinal polarization, since the freedom of gauge invariance is lost. To obtain the proper modes, we must set the determinant of the system (7) equal to zero. In the cases of practical interest (pure electric or pure magnetic field, for instance), $f_0\cdot f_0^* = -4\,\boldsymbol{E}\cdot\boldsymbol{B}$ will vanish and so does $k\cdot f_0\cdot f_0^*\cdot k$. We shall henceforth assume $f_0\cdot f_0^*=0$, and then we obtain the three proper modes corresponding to the nonvanishing of ξ_1, ξ_2 and ξ_3, respectively.

(i) Mode 1

$$\begin{aligned}&(1-\rho f_0^2)k^2 + 4\rho k\cdot f_0\cdot f_0\cdot k - m_\gamma^2 = 0, \\ &\varepsilon_1^\mu(k) = \xi_1 k_\nu f_0^{\nu\mu}.\end{aligned} \tag{8a}$$

(ii) Mode 2

$$\begin{aligned}&\Big(1+\frac{5}{2}\rho f_0^2\Big)k^2 + 7\rho k\cdot f_0\cdot f_0\cdot k - m_\gamma^2 = 0, \\ &\varepsilon_2^\mu(k) = \xi_2 k_\nu f_0^{*\nu\mu}.\end{aligned} \tag{8b}$$

(iii) Mode 3

$$(1-\rho f_0^2)k^2 - m_\gamma^2 = 0, \quad \varepsilon_3^\mu(k) = \xi_3\lambda^\mu. \tag{8c}$$

Let us consider the case of a pure magnetic field $\boldsymbol{B}$ and let θ be the angle between $\boldsymbol{B}$ and $\boldsymbol{k}$ by introducing the indices of refraction n with $k^\mu = (\omega,\boldsymbol{k}) = (\omega,\omega\boldsymbol{n})$ and $n = |\boldsymbol{n}|$. Then we solve Eqs. (8) for $\boldsymbol{n}_i$ and define the electric polarization $\boldsymbol{e}_i$ $(i=1,2,3)$ to first order in α^2 and m_γ^2 as follows:

(i) Mode 1 or transverse mode

$$n_1 = 1 + \frac{8\alpha^2}{45m^4}B^2\sin^2\theta - \frac{m_\gamma^2}{2\omega^2}, \quad \boldsymbol{e}_1 = \boldsymbol{n}_1\times\boldsymbol{B}. \tag{9a}$$

(ii) Mode 2 or parallel mode

$$\begin{aligned}&n_2 = 1 + \frac{14\alpha^2}{45m^4}B^2\sin^2\theta - \frac{m_\gamma^2}{2\omega^2}, \\ &\boldsymbol{e}_2 = \boldsymbol{B} - (\boldsymbol{B}\cdot\boldsymbol{n}_2)\boldsymbol{n}_2.\end{aligned} \tag{9b}$$

(iii) Mode 3 or longitudinal photon mode

$$n_3 = 1 - \frac{m_\gamma^2}{2\omega^2}, \quad \boldsymbol{e}_3 = B[\boldsymbol{n}_3 - (\boldsymbol{B}\cdot\boldsymbol{n}_3)\boldsymbol{B}/B^2]. \tag{9c}$$

For the first two modes we see that the terms transverse and parallel refer to the situation of the polarization with respect of the plane $(\boldsymbol{B},\boldsymbol{k})$. As $n_i - 1 + m_\gamma^2/2\omega^2$ is proportional to $\sin^2\theta$, it reflects the symmetry of the effect under the change of $\boldsymbol{B}$ into $-\boldsymbol{B}$. The birefringence effect vanishes at $\theta = 0$, and is maximal at $\theta = \pi/2$, perpendicular to the field. We can find that except for propagating along the external field direction ($\theta = 0$), the vacuum polarized by an external constant magnetic field acts like a birefringence medium.

A linearly polarized wave with frequency ω and wavelength $\lambda = 2\pi/\omega$ propagating along a distance L perpendicular to a constant magnetic field $\boldsymbol{B}$ would be transformed into an elliptically polarized wave according to $\boldsymbol{e}(t) = \alpha_1 \boldsymbol{e}_1 \cos(\omega t - n_1 \omega L) + \alpha_2 \boldsymbol{e}_2 \cos(\omega t - n_2 \omega L)$ if its initial polarization is $\boldsymbol{e}(t) = (\alpha_1 \boldsymbol{e}_1 + \alpha_2 \boldsymbol{e}_2) \cos \omega t$. The elliptical locus of the final polarization vector is

$$\left(\frac{x}{\alpha_1}\right)^2 + \left(\frac{y}{\alpha_2}\right)^2 - \frac{2xy}{\alpha_1 \alpha_2} \cos \Phi = \sin^2 \Phi \qquad (10)$$

with the phase shift ϕ given by

$$\Phi = 2\pi(n_2 - n_1)\frac{L}{\lambda} = \frac{\alpha}{15}\left(\frac{eB}{m^2}\right)^2 \frac{L}{\lambda}. \qquad (11)$$

We can also denote n_1 and n_2 by $n_\parallel$ and $n_\perp$, respectively. In the classical mode without photon mass, there are only two polarization modes: (i) mode 1 ($n_\perp$ mode) $n_\perp = 1 + 8\alpha^2 B^2 \sin^2 \theta / (45 m^4)$. For $\boldsymbol{e}_\perp = \boldsymbol{n}_\perp \times \boldsymbol{B}$. (ii) Mode 2($n_\parallel$ mode) $n_\parallel = 1 + 14\alpha^2 B^2 \sin^2 \theta / (45 m^4)$ for $\boldsymbol{e}_\parallel = \boldsymbol{B} - (\boldsymbol{B} \cdot \boldsymbol{n}_\parallel)\boldsymbol{n}_\parallel$. The difference $\Delta n = n_\parallel - n_\perp = 6\alpha^2 B^2 \sin^2 \theta / (45 m^4)$.

From the calculation and comparison above, we can see that the refraction indices of modes are dependent on the photon mass, while the polarization of the electromagnetic wave has the same form as that in the case of zero photon mass, except for a new longitudinal photon mode. Although the photon mass will cause a change to the refractive indices $n_\parallel$ and $n_\perp$ of vacuum birefringence, the difference $n_\parallel - n_\perp$ is unchanged. The emergent light's ellipticity of the polarized wave depends on $\Phi = 2\pi(n_2 - n_1)L/\lambda$, which is unchanged.

In conclusion, we have studied the influence of the photon rest mass on the vacuum birefringence experiment. Although some other methods have been used to detect the upper limits for the photon mass in classic electrodynamics,[1] it is unfeasible to detect the upper limits for the photon mass according to the experiments of vacuum birefringence.

References

[1] Tu L C, Luo J and Gillies G T 2005 *Rep. Prog. Phys.* **68** 77
[2] Tu L C, Ye H L and Luo J 2005 *Chin. Phys. Lett.* **22** 33057
[3] Fischbach E, Kloor H, Langel R A, Lui A T Y and Peredo M 1994 *Phys. Rev. Lett.* **73** 514
[4] Davis L, Goldhaber A S and Nieto M M 1975 *Phys. Rev. Lett.* **35** 1402
[5] Luo J et al 2003 *Phys. Rev. Lett.* **90** 081801
[6] Lakes R 1998 *Phys. Rev. Lett.* **80** 1826
[7] Euler E 1936 *Ann. Phys.* (Leipzig) **26** 398
[8] Heisenberg W and Euler E 1936 *Z. Phys.* **98** 714
[9] Kremer H F1965 *Phys. Rev.* **139** B254
[10] Schwinger J 1951 *Phys. Rev.* **82** 664
[11] Cantatore G, Valle F D and Milotti E 1991 *Phys. Lett.* B **265** 418
[12] Ni W T, Tsubono K and Mio N 1991 *Mod. Phys. Lett.* A **40** 3671
[13] Wu J S, Ni W T and Chen S J 2004 *Class Quantum Grav.* **21** S1259
[14] Adler S L 1971 *Ann. Phys.* (N.Y.) **67** 599
[15] Iacopini E and Zavattini E 1979 *Phys. Lett.* B **85** 151
[16] Pengo R et al 1998 *Frontier Tests of QED and Physics of the Vacuum* ed Zavattini E et al (Sofia: Heron Press) p 59
[17] Zavattini E and Zavattini G 2006 *Phys. Rev. Lett.* **96** 110406
[18] Nezrick F 1998 *Frontier Tests of QED and Physics of the Vacuum* ed Zavattini E et al (Sofia: Heron Press) p 71
[19] Ni W T 1998 *Frontier Tests of QED and Physics of the Vacuum* ed Zavattini E et al (Sofia: Heron Press) p 83
[20] Brezin E and Itzykson C 1971 *Phys. Rev.* D **3** 15