Michael W. Daniels and P.R. Kumar

Racing With the Sun: The Optimal Use of the Solar Powered Automobile

June 1997 marked the fourth running of Sunrayce, a biennial ten-day cross-country race of solar powered vehicles. In 1997, thirty-six solar cars from different colleges and universities raced from Indianapolis to Colorado Springs. These cars were designed to meet very specific electrical and mechanical regulations outlined in the Sunrayce rules (see [1]). Each car represented a different university's effort to build the most efficient solar powered vehicle in the country. The University of Illinois' entry in Sunrayce was the "Photon Torpedo," see the illustration on this page and Fig. 1. Due mostly to reliability problems, the Photon Torpedo finished a disappointing fourteenth.

Sunrayce resembles the Tour de France in that each team races against the clock rather than directly racing against the other teams. Each team is started at a dif-

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Daniels was formerly with the Department of Mechanical Engineering and Coordinated Science Laboratory, University of Illinois. He is now with Aircraft Division, Moog, Inc., Buffalo, NY. Kumar is with the Department of Electrical and Computer Engineering and the Coordinated Science Laboratory, University of Illinois (prkumar@decision.csl.uiuc.edu).

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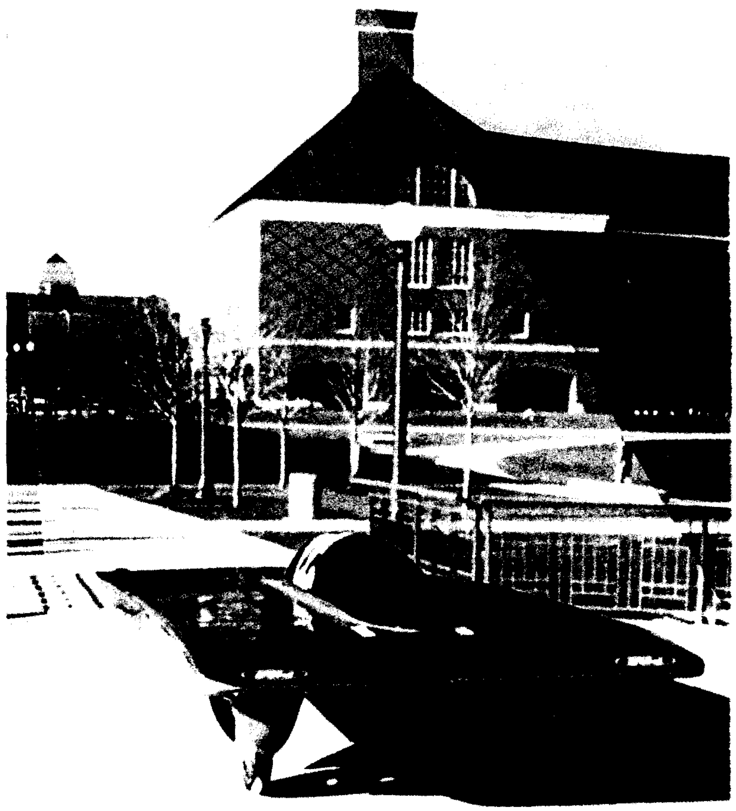

ferent time and timed separately as the race progresses. The race consists of nine racing days (and one rest day) with each day's route running about 150 miles. At a certain location near the middle of each day's leg, a fifteen minute rest period is required and each team is required to finish each leg within eight hours of its start time.

Scoring for each day's leg is the time taken to complete that day's course, plus time for any penalties that are assessed. Included in these is a penalty for not crossing the finish line within eight hours. This penalty is scored as eight hours plus three minutes for each mile not covered on that day's race. The team to complete the race in the least total time is declared the winner.

The primary power source for a solar car is, of course, solar power from photovoltaic cells. Part of the time, the current from the array is completely devoted to powering a DC motor which, in turn, powers the car. However, each car is also allowed to have several lead-acid batteries. At the beginning of the race, these batteries are fully charged. However, once the race has begun, only solar power may be used to charge them.

Battery power is essential to success. It provides the excess current needed when the car's power requirements exceed the power provided by the solar cells. This will occur while accelerating, climbing hills, or just traveling at a high speed. In addition, batteries are heavily used during cloudy days.

The batteries are charged during racing if the solar power exceeds the car's power requirements, but most battery charging occurs while static charging. This entails removing the solar array from the car and pointing it directly at the sun to charge the batteries. Before and after each day's leg, teams are allowed to static charge. This allows teams to re-charge their batteries before the next day's race. Of course, this assumes that the team finishes that day's leg before sundown. Teams may also static charge during the midday rest period and during the rest day.

Once a solar car has been built and tested, the question arises as to how to race with it. Since most teams are extremely energy limited for the race, strategy becomes critical if one is to win. In Sunrayce '97, most of the terrain, as well as the data on stop signs and stop lights, was known by the Illinois team many months before the race. We needed to determine at what speed we should be traveling during all parts of the race in order to use energy most efficiently and finish the race in the least amount of time. In what way should one optimally accelerate from a stop? Should we cruise at a constant speed, at a constant power, or in some other way? How should one climb and descend hills? Also, given a weather prediction for tomorrow, how much energy should one aim to consume today?

These are some of the questions we answer through our analysis. In the next section, some of the prior work is described, followed by derivations of the equations modeling the car. The following section formulates the optimal control problem when the weather conditions are known, followed by solutions for various scenarios of interest. The next two sections first compare the results with different rule of thumb strategies and then obtain the optimal strategy given probabilistic weather predictions using stochastic dynamic programming. A post-race analysis is then provided.

Prior Work

The Photon Torpedo was the University of Illinois' second entry in Sunrayce. The university first entered the race in 1995 with a car dubbed "Sunchief" (which finished 25th in a field of 40 cars). A spreadsheet was developed by Wright [2] to help deal with the daunting task of predicting Sunchief's power consumption throughout the race, and it became affectionately known as "Sunguesser." The main input to Sunguesser, along with the car's physical parameters, was the "cruise" velocity. Sunguesser also computed the energy used in starts and stops along the route and the energy consumed on the average road grade for that day. In addition, using the predicted time on the route and the position of the sun in the sky relative to the solar array, the spreadsheet attempted to determine the energy input from the sun while racing and while static charging. Sunguesser used a built-in optimizer in the spreadsheet to determine the best cruise speed to be used on each day. The optimizer attempted to minimize the total driving time for the entire race with the given energy constraints. For example, if, at the end of any day, the energy left in the battery was less than a specified minimum, a heavy penalty was added to that day's time. In this way, the optimizer would move away from disallowed solutions towards a feasible and optimal solution. Sunguesser was used daily during the 1995 race to determine an updated best cruise speed for the next segment of the race.

A continuation of this Sunguesser work was done in parallel with the work described in this paper. Before the 1997 race, it was decided that a more complex, but easier to use, computer program should be written to replace Sunguesser. The team was interested in developing a more accurate driving model which would take advantage of very specific route information. A program called NOMAD (Numeric Optimizing and Motion Analyzing Discretizer) was developed by M. Daniels, R. Singhal and S. Fisher. NOMAD uses the same theoretical equations as Sunguesser, but, rather than just using the number of stops and average road grade to compute energy consumption, it integrates discrete instantaneous powers over the race route using actual incremental road data. This provides a more accurate computation of the power used while driving. In addition, NOMAD uses a program, written by D.F. Brown, to calculate the sun intensity, given the weather conditions, over various parts of the day [3]. This information is also integrated to determine the amount of energy that will be obtained from the sun.

An option was built into NOMAD to use a Genetic Algorithm (GA) to optimize the cruise speeds for the least total racing time. However, the genetic algorithm proved troublesome in that it converged to non-optimal local minima. For this reason, the GA was not used extensively, but NOMAD was instead used for its simulation and predictive capabilities.

While both these programs were helpful to the teams in their respective races, neither considers the problem from a "rule of thumb" point of view. Optimal control of solar powered cars was investigated for the 1987 World Solar Challenge, a race across Australia [4]. Cassidy and Gurley [5] used calculus of variations to derive a Lagrange multiplier which represented the "utility of battery charge in terms of distance covered." In this way they

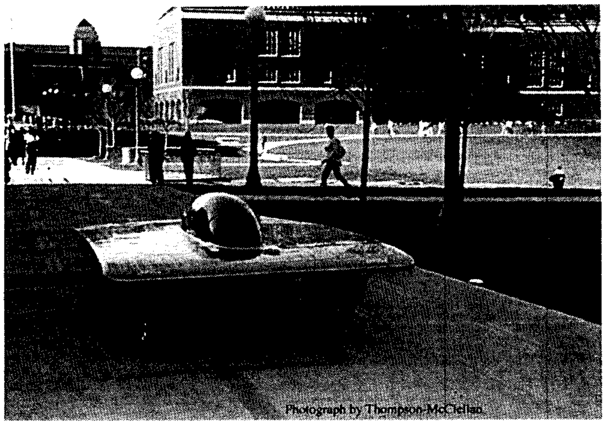

Fig. 1. The Photon Torpedo.

were able to determine the necessary velocities so that the GM Sunraycer could maximize its distance in a given time.

The goal of the work described in this paper was to build on past work and to give the University of Illinois a strategic advantage over other teams in Sunrayce '97.

Dynamics of the Solar Car

In this section we derive the equations modeling the car and the battery.

Power Lost and Supplied

The need for a racing strategy quickly becomes apparent when one looks at the power losses the car experiences during racing. First, rolling resistance representing the power lost in the system due to friction between the tires and the road, and friction between the shaft and the bearing, draws power linearly as the car moves at higher velocities: It is the force required to keep the car moving at a constant velocity on flat ground in a vacuum. Given as a power,

$$P_{roll} = mgVC_r,$$

where m is the mass of the car, g is the acceleration due to gravity, V is the car's velocity, and constant C_r is the coefficient of rolling resistance generally given in (lbs of resistive force)/(1000 lbs of car weight).

Another power loss which increases linearly with velocity is the power lost due to a road grade:

$$P_{grade} = mgzV,$$

where z is the road grade (i.e., the slope of the road).

Finally, and ultimately most importantly, the car also experiences aerodynamic losses. However, unlike the previous power losses, the aerodynamic loss is not linear with velocity. Rather it is *cubic* in velocity. This means a slight increase in velocity can lead to a dramatic increase in power losses due to aerodynamic drag. This power loss is given as

$$P_{aero} = 0.5\rho_{air}C_dA_fV^3, \qquad (1)$$

where ρ_{air} is the density of air, C_d is the coefficient of aerodynamic drag, and A_f is the frontal area of the car.

Because of this nonlinearity with respect to velocity, racing strategy is critical. For example, if today is sunny and tomorrow is cloudy, a team could travel at a high velocity today, but be forced to a very slow velocity tomorrow due to energy limitations. However, if that team were to travel more slowly today, they could save their energy and travel at a higher velocity tomorrow achieving a *lower total time.*

From these power formulas, it can be seen that the total power needed to propel a solar car at a *constant* velocity is

$$P(V) = P_{aero} + P_{grade} + P_{roll}. \qquad (2)$$

Next, consider the power required to *accelerate* the solar car: It is

$$P_{accel} = mAV,$$

where A is the instantaneous acceleration.

Consequently, the power that is used at the output of the car's wheels is

$$P_{out} = mAV + P(V). \qquad (3)$$

Power Supplied

As discussed previously, power is supplied from two sources: the solar array and the batteries (the battery is discussed further below). Here, solar power $P_s(t)$ and battery current $I_B(t)$ will be modeled as functions of time. The battery voltage $E_B[I_B(t)]$, in turn, can be modeled as a function of the battery current. This is due to internal resistance within the batteries. Consequently, the power supplied to the solar car is

$$P_{supplied} = P_s(t) + I_B(t)E_B(I_B(t)).$$

It should be noted that solar power could also be modeled as a function of position, $P_s(x)$. This will be discussed below (see the subsection "Non-constant Solar Power").

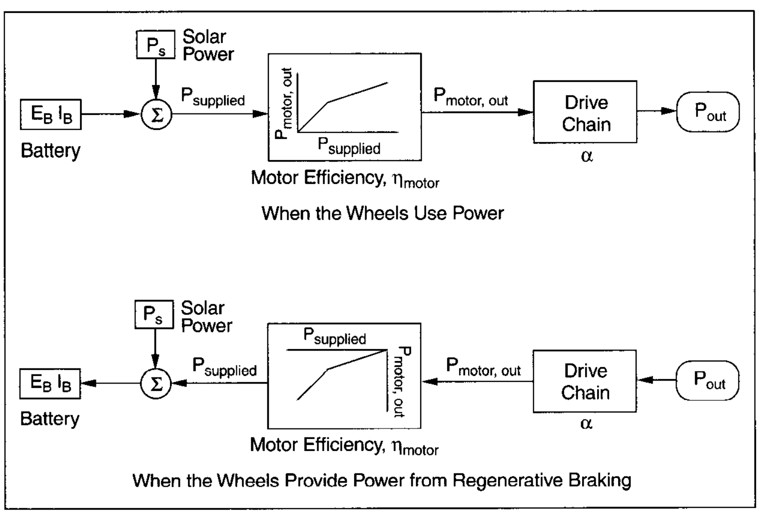

Fig. 2. Illustration of the power flow through the system.

Motor and Drive Chain Power Losses

The motor and chain drive each have associated efficiencies which, when multiplied together, determine the relationship between $P_{supplied}$ and P_{out}. For simplicity, the drive chain efficiency is assumed to be a constant, α. $P_{motor_{out}}$ is defined as the power at the output of the DC motor before the drive chain. $P_{motor_{out}}$ can be negative when regenerative braking is applied. The relationship between P_{out} and $P_{motor_{out}}$ is

$$P_{out} = \begin{cases} \alpha[P_{motor_{out}}], \text{ for } P_{motor_{out}} > 0 \\ \frac{1}{\alpha}[P_{motor_{out}}], \text{ for } P_{motor_{out}} < 0. \end{cases}$$

This can be seen more easily in Fig. 2, which illustrates the power flow through the system both when the car requires power from the sun and batteries, and when regenerative braking is applied.

The DC motor efficiency is a function of the motor's rpm and $P_{supplied}$. By studying the efficiency curves for a variety of motor rpm's, we determined that, for positive powers, the relationship between $P_{supplied}$ and $P_{motor_{out}}$ was well modeled by two intersecting straight lines. However, these two lines would be different at every value of the rpm. Therefore, linear interpolation was used to construct efficiency lines between the known data at certain rpm values.

Besides positive power consumption, negative power consumption needs to be considered as well. Most solar cars are equipped with regenerative braking systems which apply negative torque to the motor, acting as a generator to slow the car. This power is used to charge the batteries. However, since this power effectively goes "through" the chain drive and motor on its way to the battery, it is also subject to the same efficiency losses. Therefore, the two lines that model the positive power efficiency are mirrored across the origin so that four lines are used as follows:

$$P_{out} = \begin{cases} \alpha[a_2 P_{supplied} + b_2], \text{ for } P_{supplied} \in \left[\frac{b_2 - b_1}{a_1 - a_2}, +\infty\right) \\ \alpha[a_1 P_{supplied} + b_1], \text{ for } P_{supplied} \in \left[\frac{b_1 - \alpha^2 a_1 b_1}{\alpha^2 a_1^2 - 1}, \frac{b_2 - b_1}{a_1 - a_2}\right] \\ \frac{1}{\alpha}\left[\frac{1}{a_1} P_{supplied} + \frac{b_1}{a_1}\right], \text{ for } P_{supplied} \in \left[\frac{a_1 b_2 - a_2 b_1}{a_2 - a_1}, \frac{b_1 - \alpha^2 a_1 b_1}{\alpha^2 a_1^2 - 1}\right] \\ \frac{1}{\alpha}\left[\frac{1}{a_2} P_{supplied} + \frac{b_2}{a_2}\right], \text{ for } P_{supplied} \in \left(-\infty, \frac{a_1 b_2 - a_2 b_1}{a_2 - a_1}\right]. \end{cases} \tag{4}$$

Here, a_1 and b_1 are, respectively, the slope and intercept of one of the two positive lines, and, likewise, a_2 and b_2 represent these parameters for the other line. These lines, plotted for 800 rpm, can be seen in Fig. 3. Note that the chain drive efficiency is included in these equations. One should also note that lines two and three *do not* intersect at the origin. Instead this intersection falls in the fourth quadrant in which a negative P_{out} requires a positive $P_{supplied}$. This is not technically correct, but only a small portion of the curves fall into this region and we neglected it. Note that we can define the motor efficiency as

$$\eta_{motor} := \frac{P_{motor_{out}}}{P_{supplied}}.$$

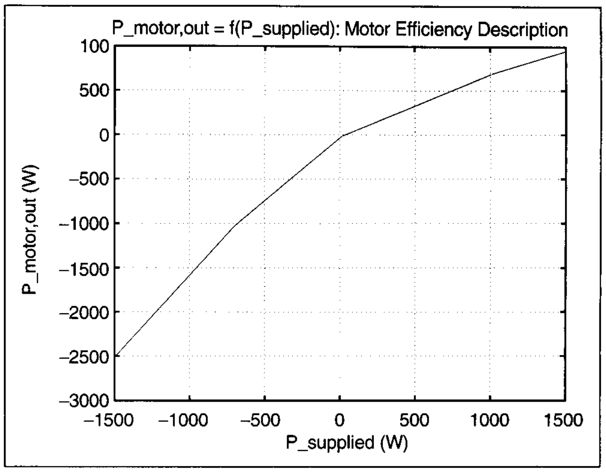

Fig. 3. Plot of $P_{motor_{out}} = f(P_{supplied})$ for 800 rpm.

It is not a constant, but is a function of either $P_{motor_{out}}$ or $P_{supplied}$, and rpm. Again, for a more complete picture of the system's power flow, see Fig. 2.

Battery Model

Batteries are not ideal energy receptacles. The amp-hours which are put into the batteries *are* conserved within the batteries themselves, but there is an energy "penalty" associated with drawing energy from the batteries at anything higher than an infinitesimal rate. In other words, the *capacity* of the batteries decreases as the current draw increases. After extensive testing of the valve-regulated lead acid batteries used by the Photon Torpedo, it was estimated that a current of i amps should actually be inflated to $i(1 + 0.01i)$ amps in calculating the capacity used. For example, if the battery capacity of the solar car was 75 amp-hours with current drawn at an infinitesimal rate, it would drop to 50 amp-hours when drawing at 50 amps. (A current draw of 50 amps would be considered quite high. A more typical cruising current draw is 10 amps). Battery efficiency will not be included in the model of the car, but will instead be included in the cost function, discussed in the next section.

In addition to this battery efficiency, one must also account for the fact that as higher currents are drawn from the battery, the voltage across the battery decreases. A linear voltage drop of the form

$$E_B(I_B) = E_0 - RI_B \tag{5}$$

was adopted. Here E_0 is the nominal battery voltage (internal emf) and R is an internal battery resistance. These values were determined from empirical data for the Photon Torpedo's batteries.

The System Equations

From the above equations for power supplied, power losses, and power required for acceleration, the system's power balance equation can be written as

$$\alpha\eta_{motor}(P_s(t) + I_B(t)E_B(I_B(t)), V(t))[P_s(t) + I_B(t)E_B(I_B(t))]$$
$$= P(V(t)) + mA(t)V(t),$$

where the motor efficiency η_{motor} is, as defined earlier in this section, a function of the power supplied $P_s(t)+I_B(t)E_B(I_B(t))$, as well as the velocity $V(t)$ (through the rpm). Above and throughout, functional dependence is specified by $(\cdot)$, while square brackets $[\cdot]$ are reserved for parsing algebraic expressions. Solving for the acceleration A we get

$$A(t) = \frac{\alpha\eta_{motor}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))[P_s(t)+I_B(t)E_B(I_B(t))]-P(V(t))}{mV(t)}$$

Thus,

$$\begin{aligned}&P_{out}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))\\&=\alpha\eta_{motor}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))[P_s(t)+I_B(t)E_B(I_B(t))]\\&=f(P_s(t)+I_B(t)E_B(I_B(t)),V(t)).\end{aligned}$$

Hence, noting that P_{out} is a function of $P_s(t)+I_B(t)E_B(I_B(t))$ and $V(t)$, one can interpolate from the motor efficiency curves and write the acceleration equation as

$$A(t)=\frac{P_{out}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))-P(V(t))}{mV(t)}.$$

Realizing that the acceleration is just the time derivative of the velocity and, in turn, the velocity is the time derivative of the position, the following state equations are derived:

$$\frac{dV(t)}{dt}=\frac{P_{out}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))-P(V(t))}{mV(t)},$$

$$\frac{dx(t)}{dt}=V(t),$$

where $x(t)$ is the position of the solar car at time t.

Formulation of the Deterministic Optimization Problem

The input to the system is the battery current $I_B(t)$, and the output is the car's velocity $V(t)$. The ending state of the car is essentially described by four variables: the final position x of the car, its final velocity V, the final charge Q remaining in its batteries, and the final time T.

One may optimize with respect to any of the above four quantities, subject to holding the other three fixed. Indeed any optimal solution, in the racing sense of the term, necessarily has to be optimal with respect to any of the above four choices; thus the optimal solution to the problem can be found by fixing three of the objectives and solving for the fourth. For example, if one knows the ending time, the finishing position, and the terminal velocity, then one can solve for the optimal velocity profile which will minimize the amount of charge used. On the other hand, if one wishes to determine the distance that can be traveled in a certain amount of time while terminating with a certain amount of charge and at a particular velocity, then one could solve for the optimal velocity profile which will transport the car as far as possible under those constraints. In other words, the optimal solution is optimal with respect to any of the three objectives.

Our approach to solving these problems was via the Pontryagin Maximum Principle [6]. We will consider the variant where one fixes the final position $x(T)$, final velocity $V(T)$, and final time T. So the problem considered is to minimize the charge needed to satisfy these terminal state requirements. In this case, because of the penalty for drawing high current from the batteries, the cost function will be

$$\int_0^T [1+0.01\,I_B(t)]\,I_B(t)dt \quad (Ampere-seconds), \tag{6}$$

where T is the final time.

To summarize, the problem considered is to minimize (6) for the system

$$\frac{dx(t)}{dt}=V(t) \tag{7}$$

$$\frac{dV(t)}{dt}=\frac{P_{out}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))-P(V(t))}{mV(t)}, \tag{8}$$

with initial and terminal states

$$x(0)=x_o \tag{9}$$

$$x(T)=x_f \tag{10}$$

$$V(0)=V_o \tag{11}$$

$$V(T)=V_f, \tag{12}$$

by choosing the control variable $I_B(t)$ on $[0,T]$.

The Hamiltonian is

$$H(I_B,V,x,t)= \lambda_1\frac{P_{out}(P_s(t)+I_BE_B(I_B),V)-P(V)}{mV}+\lambda_2V+I_B[1+0.01I_B] \tag{13}$$

where λ_1 and λ_2 are the Lagrange multipliers, and

$$P(V)=0.5\rho_{air}C_dA_fV^3+mgzV+mgVC_r. \tag{14}$$

If the road slope z depends on x, then we should write the above more properly as $P(V,x)$.

From the Pontryagin Maximum Principle for a system with prescribed initial and terminal states and terminal time, and an integral cost (Theorem 5.10, pp. 299 of Athans and Falb [7]), we obtain the following differential equations for the Lagrange multipliers:

$$\frac{d\lambda_1}{dt}=\lambda_1\frac{P_{out}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))-P(V,x)}{mV^2} -\lambda_1\frac{\frac{\partial P_{out}}{\partial V}(P_s(t)+I_B(t)E_B(I_B(t)),V(t))-\frac{\partial P(V,x)}{\partial V}}{mV}-\lambda_2, \tag{15}$$

$$\frac{d\lambda_2}{dt} = \lambda_1 P'(x) = \lambda_1 g z'(x). \quad (16)$$

Including the two state equations (7), (8), we have four differential equations which describe the optimized system. We thus need four boundary conditions, which are given by the initial as well as terminal conditions (9)-(12) on the two state equations. That is, we have a Two-Point Boundary Value Problem (TPBVP).

To solve the TPBVP, a shooting method has been employed. We search for appropriate *initial conditions* on the remaining two variables λ_1 and λ_2 until we obtain satisfaction of the desired terminal constraints on $V(T)$ and $x(T)$. From the maximum principle, $I_B(t)$ is chosen to minimize the Hamiltonian at each time t.

The solution of the differential equations is extremely sensitive to the initial value of λ_1. This creates a problem in that, to satisfy the terminal conditions, convergence problems precluded in most cases a time horizon longer than fifteen minutes of drive time. Indeed, even with a tolerance of 10^{-15} on λ, some solutions would not converge because even greater precision is needed on the initial condition.

Solutions for Scenarios of Interest

The following are solutions for various scenarios which are of particular interest to a solar car racing team. Boundary conditions were chosen arbitrarily for each case, but in such a way as to provide sufficient time and distance for the car to complete its desired task. The goal of these solutions is not to observe a specific quantitative answer, but rather to examine the shapes of curves. All solutions provided are typical of each scenario.

Constant Conditions

This is the problem of driving under constant conditions over flat ground. It is desired that the car starts initially at 17 m/s and terminates at the same velocity in 1200 seconds after traversing 20400 m.

The problem consists of solving the differential equations (7), (8), (15), (16), where I_B is chosen to minimize (13) at each instant t, and the initial conditions are $x(0) = 0$, $V(0) = 17$. The initial conditions $\lambda_1(0)$ and $\lambda_2(0)$ are then determined so that the terminal conditions $x(1200) = 20400$, $V(1200) = 17$ are met.

The question is whether it is preferable to maintain a constant speed throughout the run or to vary it in some way. Since it is suspected that variations in speed are inherently inefficient, we expect a constant speed solution. In fact, this is the result that the maximum principle delivers, a constant speed of 17 m/s and a constant current of 9.166 A.

Acceleration from a Stop

Consider the problem of accelerating from a stop to a certain cruising velocity. This is to occur in 250 seconds and in 3817 m. The optimal solution can be seen in Fig. 4.

The optimal velocity profile shows a fairly constant acceleration of about 0.3 m/s for 50 seconds, and then the profile smoothly rounds off the curve and proceeds at a constant 17 m/s for the rest of the allotted time. To achieve this, the current profile spikes up to close to 20 A in the first 10 seconds but then settles down to the steady state value of 9.26 A.

Deceleration to a Stop: Regenerative Braking

Here, two different scenarios will be studied. First, consider a scenario in which the driver has ample warning that he must stop. The driver has 150 seconds and 855 m to stop from a velocity of 17 m/s. Here the optimal solution is to use regenerative braking for about the first 40 seconds, until the velocity is reached where the driver can apply virtually zero current and the car will come to a rest at the desired position and time (see Figs. 5 and 6).

For the other stopping situation of interest, the driver is given only 30 seconds and 300 m to stop from 17 m/s. As one would suspect, the optimal solution applies regenerative braking for the entire 30 seconds to stop in time.

Minor Decelerations

In some situations, e.g., entering a town, the driver must slow the car to comply with the local speed limit. For this situation, the car is constrained to slow from 17 to 12 m/s in 250 seconds while traversing 3278 m. The optimal solution to the problem can be found in Fig. 7. The velocity decreases smoothly to 12 m/s.

Climbing Up Hills

One of the main questions in dealing with energy management is how to climb and descend hills. To answer the question of climbing a hill, a road profile was studied that starts out flat for 1500 m, increases to a 2% grade from 1500 to 2000 m, and stays at this grade until 2500 m. The profile then decreases its grade until it is 0% at 3000 m and remains that way until the end. With an initial and final velocity of 17 m/s, a time horizon of 300 seconds, and a distance of 4995 m, the optimal result is shown in Figs. 8 and 9. To understand the figures, it should be noted that the car is at 0 m at 0 sec, 1500 m at 85 sec, 2000 m at 112 sec, and at 3000 m at 178 sec. Noting the slopes at these points, this solution suggests increasing velocity in anticipation of the hill and allowing gravity to slow the car as it ascends by decreasing the current. The current is decreased so that it will reach the steady state value for traveling at 17 m/s on flat ground and, once on top of the hill, the car will accelerate back to 17 m/s.

Descending Down Hills

The profile used to study descending hills begins with a flat portion for 1500 m, decreases to a -3% grade from 1500 to 2000 m, and remains at this down grade until 2500 m. Then it increases its grade back to 0% at 3000 m and stays this way until the end. The time constraint was 250 seconds, the distance to traverse was 4431 m, and the initial and final velocities were 17 m/s. The optimal solution can be seen in Figs. 10 and 11. Again, to understand the figures, a time needs to be associated with each change in the hill's slope. The car is at 0 m at 0 sec, 1500 m at 92 sec, 2000 m at 125 sec, and 3000 m at 175 sec. The optimal velocity profile generated is very much opposite of the one generated for ascending a hill. It advises first slowing down in anticipation of the hill and then coasting, allowing gravity to accelerate the car as it proceeds down the hill. It then increases its current back to the flat ground steady state level associated with a speed of 17 m/s. Hence, one does not use regenerative braking unless it is necessary to keep the car from exceeding the speed limit.

Non-Constant Solar Power

A very important strategic question is how to most efficiently use the solar power input. We treat the solar power as a function of position rather than time. For example, if the car is currently getting 300 W of incident radiation from the sun, but it is known

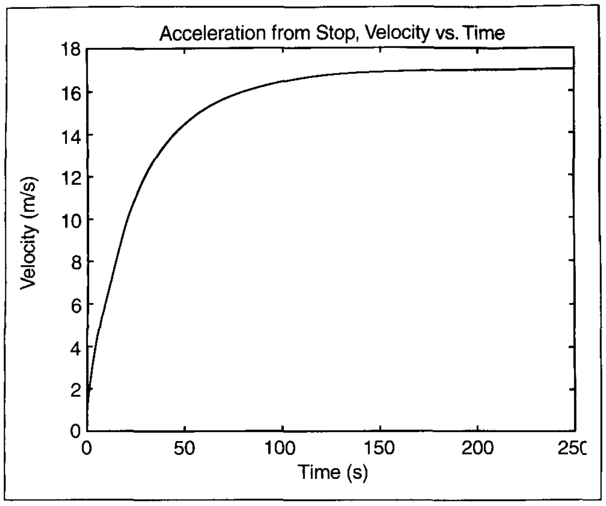

Fig. 4. Acceleration from stop (plot of velocity).

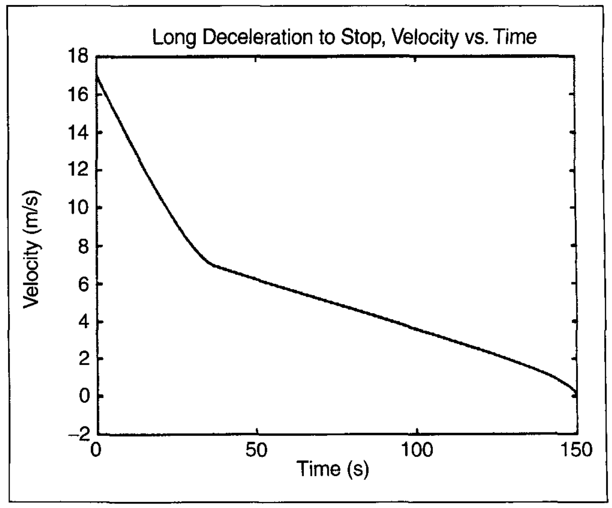

Fig. 5. Long deceleration to stop (plot of velocity).

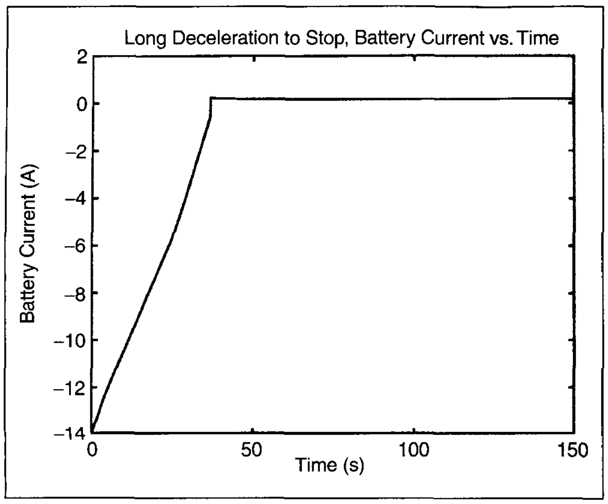

Fig. 6. Long deceleration to stop (plot of current).

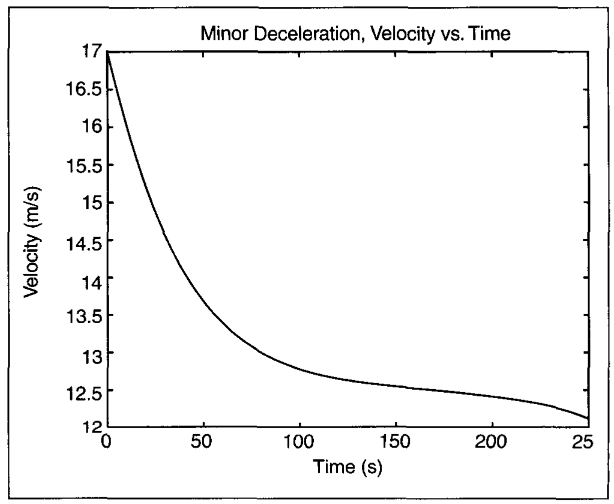

Fig. 7. Minor deceleration (plot of velocity).

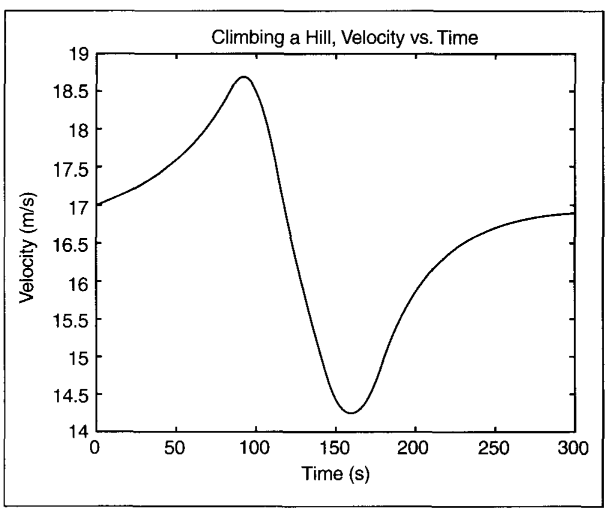

Fig. 8. Ascending hill (plot of velocity).

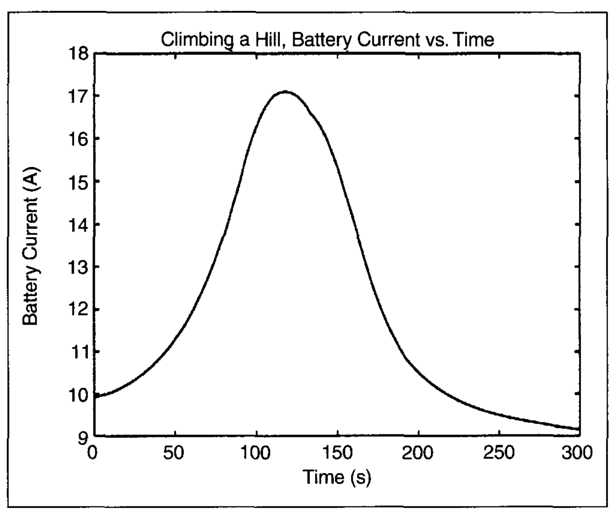

Fig. 9. Ascending hill (plot of current).

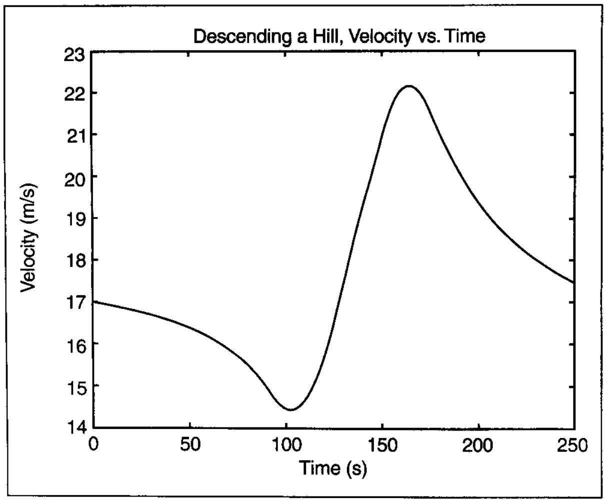

Fig. 10. Descending hill (plot of velocity).

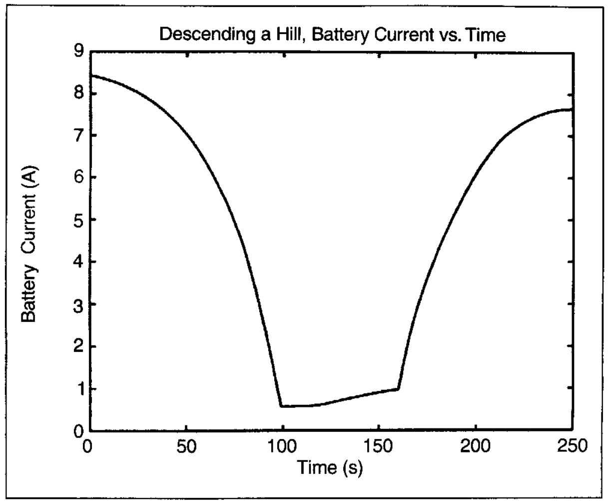

Fig. 11. Descending hill (plot of current).

that in 4000 m the sun's radiation will increase to 800 W, should the driver increase speed to get out quickly from under the cloud and charge on the other side, or decrease speed now with the expectation that he can go faster once he passes 4000 m? To answer this question, this sun intensity profile is used with a flat ground profile in which the initial and terminal velocities are each 17 m/s. The time is 500 seconds for a distance of 8125 m. The velocity plot from the maximum principle solution is shown in Fig. 12.

The solution suggests that the driver should slow down while under the cloud and increase speed again once he is clear of it. So in the case of reasonable velocities, it is more beneficial to use the solar power while one has it to increase speed slightly than to store it in the batteries. However, as speeds increase, the energy penalty for the increased speed is much greater, so a limit on the velocity may exist above which it is better to store the sun's energy for later use.

Passing Other Cars

Consider now a situation that arises when passing another car. The desired velocity profile is such that the average velocity is higher than either the initial or terminal velocities. The initial velocity is set to 17 m/s and the terminal velocity is 17.34 m/s. The run is 200 seconds with an ending position of 3522 m. So the average velocity is 17.61 m/s. This can be seen in Fig. 13. The solution shows a smooth acceleration and deceleration and spends very little time at its maximum velocity.

Comparisons of Rule of Thumb Strategies with Optimal Solutions

We now compare the optimal solutions with rule of thumb strategies, such as "maintain constant velocity," "maintain constant acceleration," or "maintain constant current."

Constant Conditions

We look at the effects of varying the velocity while driving under constant conditions. The maximum principle solution for a flat ground problem in which the initial velocity equals the final velocity, is a constant velocity profile. For a 17 m/s initial and final velocity, with an ending position of 8517 m, and time of 500 seconds, the cost for the maximum principle solution is 5003 A-s. To determine the effects of varying this velocity, we compare this with a velocity profile which varies above and below this constant velocity. This roughly models the effect of using the driver's foot to control speed rather than a cruise control. We will therefore use a velocity profile that starts at 17 m/s for 100 seconds, accelerates to 20 m/s in 50 seconds, and spends 50 seconds at this velocity. The profile next decelerates to 14 m/s in 100 seconds, spends 50 seconds at this velocity, and accelerates back to 17 m/s. Since the profile has an average speed of 17 m/s, it allows a direct comparison to the maximum principle solution. The cost of this profile is 5150 A-s, so the Maximum Principle solution of constant velocity represents a 2.9% cost savings.

Acceleration from a Stop

We will look at the effects of varying the optimal acceleration solution. The optimal solution for accelerating from rest (or close to it) is a smooth curve (Fig. 4) which can be closely approximated by two straight lines, one of constant finite acceleration and one of constant velocity. We will compare solutions for accelerating from 1 to 17 m/s in 250 seconds and in 3817 m. The

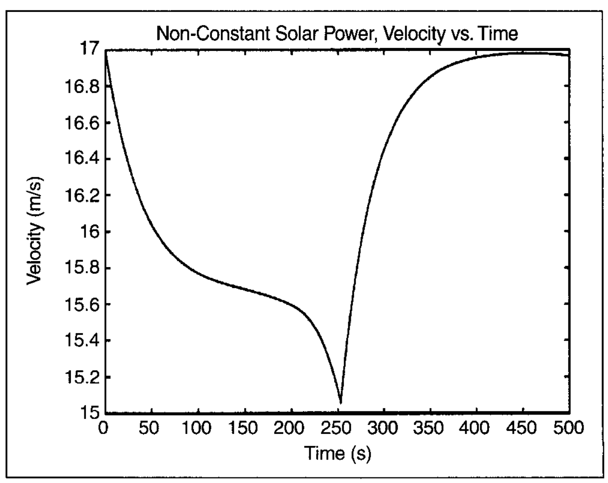

Fig. 12. Non-constant solar power (plot of velocity).

maximum principle solution requires a cost of 3036 A-s. The equivalent straight line solution, with the first line accelerating at 0.2953 m/s^2 for 54.175 seconds and the second having zero acceleration for the remaining 195.825 seconds, requires a cost of 3134 A-s.

Another obvious possibility is to accelerate more quickly at first, say at 0.35 m/s^2 for 43.82 seconds, and then use a small positive acceleration (0.0032 m/s^2) for the remaining time. The cost of this is 3115 A-s, which is 2.6% higher than the optimal solution. Alternatively, one may accelerate more slowly at the beginning, 0.2 m/s^2 for 92.15 seconds, and decelerate at -0.015 m/s^2 for the remaining 157.85 seconds to meet the problem requirements. The cost of this scenario is 3251 A-s. In this case, one can achieve about a 7% energy savings by using the maximum principle solution.

Hill Climbing

When climbing a hill, the optimal solution accelerates in anticipation of the hill, decelerates while climbing the hill, and speeds back up once the summit is reached (Fig. 8). Another strategy that one might consider is a constant current strategy. When one applies a constant current of 12.72 A, the same position, 4995 m, is reached in 300 seconds. The cost of the optimal solution is 4061 A-s. On the other hand, the cost associated with the constant current solution is 4303 A-s. Therefore, the maximum principle solution gives a 6% energy savings over the constant current strategy.

Stochastic Dynamic Programming for Optimizing Weather Strategy

In order to create long term driving strategies in which weather predictions are utilized, a stochastic dynamic programming approach [8] was used. Weather influences are modeled through probabilities describing the uncertainty.

The goal is to minimize the total time needed to complete the race course. The question is one of how best to use a limited amount of energy to accomplish this goal. A good illustration of the problem is presented in [2]. Consider the curve in Fig. 14 that details the driving time for the day as a function of the energy consumed that day. Since the power loss due to aerodynamic drag rises as a cubic function of velocity, as the time to complete the day decreases, the power needed to meet this objective increases dramatically.

Consider a two day race and suppose that through weather forecasting, it is determined that the car has a 6000 W-hr energy budget today and will have a 5000 W-hr energy budget tomorrow. If the team were to use their entire 6000 W-hr energy budget on day one, finishing that day in 4.5 hours, then 5000 W-hr would be left for day two, which provides a finishing time of 6 hours. So the two day time is 10.5 hours. However, suppose the team was to save 500 W-hr on day one to use on the second day. Then, on each day the team would consume 5500 W-hr of energy and finish in 5 hours. Consequently the total race time would be 10 hours, saving 0.5 hour over the race. This strategic problem is the motivation for this analysis.

The problem requires several inputs to incorporate the probabilistic weather predictions. The first input is a matrix of the probabilities of several weather scenarios for each day.

(It should be noted that the atmospheric science experts assisting the team were willing to provide, every day, a probability matrix $[p(s,n)]$, where s ranges over a small set of scenarios and n ranges over a short horizon. Each entry $p(s,n)$ is the probability for a scenario s occurring n days hence. However, conditional probabilities of the type $p(s,n|s',n')$ were outside of what they could entertain.)

In addition, a matrix is needed for each day that contains the driving times for that day given the weather scenario and the total battery energy consumed. Also, one must assign the beginning battery charge, the current day, the weather for the current day, and the ending race day for the analysis. For example, one may wish to end the analysis at the end of the fourth day with the assumption that the batteries can be completely recharged on the rest day. The output to the system is the target ending energy for today and the corresponding average speed for the day given the probabilistic weather analysis.

The detailed NOMAD model was used to determine the driving time for each leg given a particular energy budget for that day and various weather scenarios. The following dynamic programming type equation was used to determine the array of possible times at each stage for discrete energy uses in that stage,

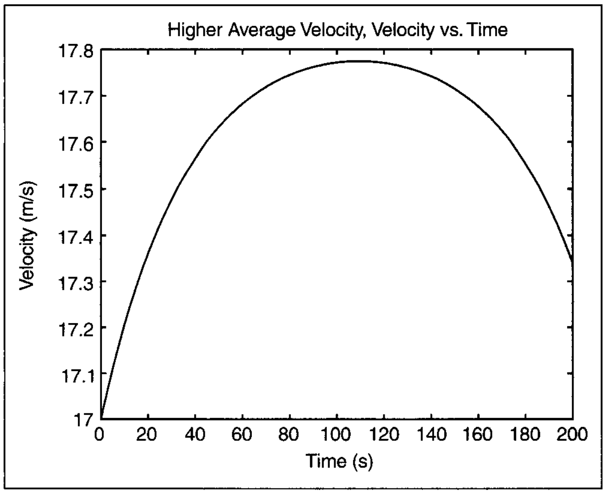

Fig. 13. Passing other cars (plot of velocity).

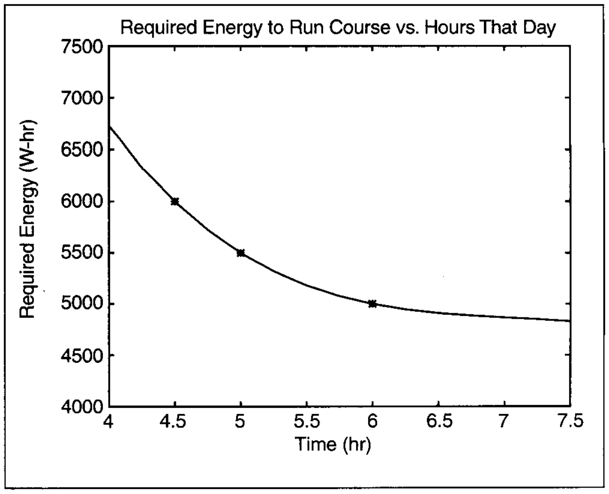

Fig. 14. Example plot of the required energy to finish day in the given time.

$$T_n(C_n) = \sum_W [P_n(W) \times \mathrm{Min}_{C_{n+1}} [D_n(C_n - C_{n+1}, W) + T_{n+1}(C_{n+1})]],$$

where $T_n(C_n)$ is the expected remaining race time starting from day n when C_n is the charge state on that day. The quantity $D_n(C_n - C_{n+1}, W)$ is the time taken on day n with weather W, an initial charge C_n, and the next day's starting charge, C_{n+1}. In addition, $P_n(W)$ is the probability that weather scenario W will occur on day n. Once the user knows the current energy charge and has a nearly 100 percent sure weather prediction for that day, he can use tomorrow's T array to determine how the car should drive today. This is done by adding the possibilities for today's times to the corresponding times in the T array (linked by charge used) to find the option which will take the least total race time.

Therefore, all one had to do to make daily strategic decisions was to compile information from NOMAD, get the weather probabilities, and complete as above. This would output the optimal average speed for that day given the weather probabilities for the following days.

Post-Race Analysis

In order to make good strategic decisions, it is imperative that the team have a good model of the vehicle. This became a major problem for the Photon Torpedo team because of insufficient testing time. Therefore, at the time of the race, the team thought that the car was more competitive than it actually was. Of particular importance was the determination of the aerodynamic drag and frontal area, commonly known as the $C_d A_f$ (see (1)). This is crucial to the calculations because this number is multiplied by the cube of the velocity when determining the aerodynamic power loss.

In limited testing done before the race, the drag was estimated to be about 0.15, and this was the value used throughout the race in making driving decisions. However, in reviewing the telemetry from the race, it is now estimated that the drag was actually about 0.18. Apparently, ground effects were insufficiently considered.

Even after the race, it is difficult to determine the exact weather and driving conditions seen during the race. However, in an attempt to quantify the team's strategic successes and failures, and to determine the team's best possible placement given the physical limitations of the Photon Torpedo, we compare the "optimal" strategic solution to the actual race results.

Following is a day-by-day analysis of how the University of Illinois *should* have run the race given the actual weather and no breakdowns. (We use kilowatt-hours (kW-hr) rather than amp-hours to measure the cost in this analysis, i.e., we are effectively assuming a constant battery voltage, which is a reasonable assumption for the battery over most of the discharge range. Therefore, either quantity will effectively represent the cost.) Note that the energy "used" on each day includes the energy input from the sun obtained while driving and while static charging at the end of the day and the next morning. It is the *net* energy consumed before the driving begins the next day.

Day 1

On day 1, we cruised at 55 mph but made a mistake on our hill strategy. We slowed down on the ascent and sped up on the descent. However, since day 1 was so short and the weather was relatively sunny, the hill climbing strategy described in this paper should have been ignored. Instead, the hills should have been driven more aggressively (i.e., at the speed limit) because there was plenty of energy available. We finished the day in 1.75 hours.

Day 2

On day 2, the dynamic programming strategy suggests that we should have used about 2 kW-hr of the battery and driven the 169 mile course in 5.08 hours. This implies an average speed of 35 mph and a cruising speed of 38 mph. This is fairly slow because of a strong headwind this day. Due to our incorrect model (and foolish optimism), we drove much too fast on this day, draining the battery pack. We completed the day in 4.87 hours which corresponds to an average speed of about 35 mph. However, this is misleading because we were forced to stop several times due to mechanical failure.

Day 3

Day 3 was mostly cloudy during the day. Therefore the recommended driving speed is slow, a 33 mph cruise speed. This will consume about 1 kW-hr and take 5.06 hours to complete the 165 mile course. Because of our poor judgment on Day 2 and our inability to recover this lost energy, we had very limited power on Day 3 and completed the course in 6.58 hours.

Day 4

Day 4 was mostly cloudy during the morning and then clear after midday. The strategy program suggests driving the 156 mile leg in 3.78 hours. This means driving at a cruise speed of 44 mph and consuming 3 kW-hr of energy. Of course, following the strategy of changing solar power suggests driving more slowly in the morning and faster in the afternoon. In the actual race, the car suffered a major array failure and was only barely able to complete this day's course. The actual time to complete the leg was 6.9 hours.

Day 5

This was the rest day. It was sunny, so full recharging was easily accomplished.

Because of breakdowns and strategic mistakes, the University of Illinois finished 16th (out of 36) in the first half of the race with a time of 20.1 hours. However, this analysis says that if the proper strategy model was followed and no breakdowns had occurred then the Photon Torpedo would have finished a more respectable 10th in a time of 15.3 hours. This significant improvement is caused mostly by the elimination of breakdowns, but the improved strategy would have helped as well, especially given the improper use of energy on Day 2.

Now the second half of the race will be analyzed. Unfortunately, we do not have road data for Days 9 and 10 of the race available for analysis. Data for all other days was gathered by the team before the race. However, legs 9 and 10 were not driven beforehand to collect elevation and stops data. We will therefore study Days 6, 7, and 8 and constrain Day 8 to end with the battery charge with which we actually started Day 9. This way we can make a second half race comparison even though some data is missing.

Day 6

Starting Day 6 with a full charge of 6 kW-hr, the dynamic programming model instructs the team to drive at a cruise speed of 33 mph while consuming about 2 kW-hr of the battery's energy on the 148 mile leg. This would have given us a finishing time of

5.02 hours for the day. In the actual race we used about 1.5 kW-hr of the energy and completed in a time of 5.17 hours. This day was fairly cloudy so speed was "expensive" in terms of energy.

Day 7

The model suggests that we consume a net of 0 kW-hr of energy on the 151 mile long Day 7. That is, we should drive in such a way as to recover all energy expended before the beginning of Day 8. This would have allowed us to finish with a time of 3.68 hours using a cruise speed of 43 mph. The weather for this day was very good (including a tailwind) so the times are much shorter than those for Day 6. In the actual race we *recovered* about 0.5 kW-hr of energy on this day (ending with 5.0 kW-hr in the pack) and finished with a time of 4.05 hours. This time included 15 minutes spent changing a tire.

Day 8

On Day 8 the team consumed about 1 kW-hr of energy to finish the 165 mile long day in 3.87 hours. We were very competitive this day because of excellent weather. The model suggests that we should have again used a net total of 0 kW-hr this day and ended with a time of 4.09 hours. So in both the dynamic programming model and in the actual race, we end Day 8 with 4 kW-hr of energy in the battery. As stated above, this was constrained so that we can make a comparison with the actual race results.

Day 9

We started Day 9 with approximately 4 kW-hr of energy in the battery and ended with 5.5 kW-hr. We gained energy on this day because of sunny conditions while static charging. This led to a race time of 3.32 hours for this 140 mile leg. We drove this day much too conservatively and left ourselves too much energy for Day 10.

Day 10

On Day 10, we cruised at 55 mph using approximately 4 kW-hr of the final 5.5 kW-hr left in the battery. We finished the 77 mile leg with a time of 2.13 hours. However, this time is inflated by about 15 minutes over the ideal race time for this charge consumption due to a motor fan problem that occurred as we entered Colorado Springs.

In the actual race, we finished 8th in the second half with a total time of 18.54 hours, for that half. The dynamic programming model says that we should have ideally been able to finish this half in 18.24 hours which would have also put us in 8th place (assuming we use the actual race times for Days 9 and 10). So we actually did very well on Days 6, 7, and 8 according to the strategy model. Of course, the model may have produced different results if it was not constrained by our actual performance on Days 9 and 10.

Overall, with the assumption for Days 9 and 10, the model estimates that Illinois should have ideally been able to finish the race in 33.64 hours, which would have put us in 9th place overall.

Concluding Remarks

The University of Illinois once again has high hopes for Sunrayce '99. The team members still involved in the project feel that the experience gained in 1997's race will provide a strong foundation for the future of solar car racing at this university. Hopefully they can continue the work discussed here and use the wisdom acquired by the 1997 team to compete well in the future.

Acknowledgments

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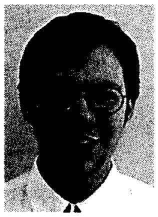

Michael W. Daniels obtained the B.S. degree from the University of Illinois in 1996, and the M.S. degree in Mechanical Engineering, also from the University of Illinois, in 1998. He is currently employed at Moog, Inc., in East Aurora, New York.

P. R. Kumar is a Professor of ECE and a Research Professor in CSL at the University of Illinois. His current research interests are in wireless networks, manufacturing systems, stochastic systems, learning, adaptive systems, and financial economics. He serves on several editorial boards and is a past Associate Editor at Large for *IEEE Transactions on Automatic Control*, and Associate Editor of *SIAM Journal on Control and Optimization*, *Systems and Control Letters*, and *IEEE Transactions on Automatic Control*. He has presented plenary lectures at the IEEE CDC, SIAM Conference on Optimization, and SIAM Annual Meeting. He was a recipient of the Eckman Award of the AACC.