Solar Car Optimization For the World Solar Challenge

Keywords: Solar car, World Solar Challenge, optimization, dynamic model, battery management, solar energy.

Abstract. This paper presents the details of an optimization method for the solar car's speed and battery management. The optimization method is demonstrated for the World Solar Challenge along a 3000km from Darwin to Adelaide, Australia. The optimization can be performed at any time during the race based on the current location, time, and battery status. The method predicts the required car speeds between the road land marks, taking into consideration the mandatory stops. The optimization includes the terrain inclination, the car rolling resistance and aerodynamic resistance, and predicts available solar energy as function of position and time.

I. Introduction

The World Solar Challenge (WSC) is a solar-powered car race which covers 3,021 km from Darwin to Adelaide [1], Australia, as shown in Fig. 1. The purpose of the WSC is to stimulate research into, and development of sustainable transportation. The race calls for the design and construction of a Solar Electric Vehicle (EV) within given design parameters [2], and driving the Solar EV across the continent of Australia. WSC attracts teams from around the world, most of which are teams from universities or corporations.

The first race event took place in 1987, and repeated every two years since then. The last solar car race took place in October, 2015, starting from Darwin in the north and ending near Adelaide in the south [1]. In addition to the certain mandatory design



Fig. 1. 3,000km route of World Solar Challenge

constraints, the race is subject to specific regulations and instructions. Driving time is specified to be between 8:00 am and 5:00 pm. In order to select a suitable place for the overnight stop (alongside the highway) it is possible to extend the driving period for a maximum of 10 minutes. This extra driving time should be compensated by a starting time delay the next day.

At various points along the route there are checkpoints where cars can stop during a mandatory 30 minutes pause period. Only limited maintenance tasks (no repairs) are allowed during this compulsory stop.

For optimization purpose, 56 land marks (towns, cities, features, etc.) were identified along the road, [3]. These land marks are listed in Appendix A together with their distance from start, latitude, longitude, and elevations. Fig. 2 is produced by plotting their elevations versus distance.

Observing the road elevations in Fig. 2, it is clear that the most difficult day is the second day, while the least demanding day is the third day (assuming about 750-850 km /day). The daily consumption of the battery charge should be planned to provide the excess energy to overcome the expected road difficulties.

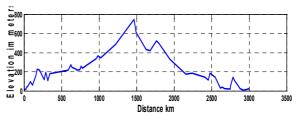


Figure 2. Road elevation

The capacity of the batteries is limited to approximately 5 kWh maximum. At the start of the race, the batteries may be fully charged. More details about limitations and regulations can be obtained from the official race website [1]. Based on the road difficulties, the proposed battery consumption is 24% first day 40% second day, 10% third, and 21 % last day. Leaving 5% in the battery near the end of the race. Accordingly, the recommended battery status at 5:00 pm during the race days will be (0.76, 0.36, 0.26, 0.05) of the full capacity. The additional charging of the battery during the stop periods should

also be consumed during the day. The following are the assumed battery characteristic [3]:

 B_{max} =5 kWh; Battery charging and discharging efficiency η_{bt} = η_{bd} =0.96; the state equation of the battery is given by

$$\dot{B}_{t}(t) = -\eta_{bd}(P_{c}(t) - P_{s}(t)), \text{ during discharging} \dot{B}_{t}(t) = \eta_{bt}P_{s}(t), \text{ during charging,}$$
(1)

where P_c is the power consumed by the car, and P_s is the power delivered by the solar panels.

During the stop periods, the solar energy from the solar panels can be used to charge the batteries. The stop periods include a mandatory 30 minutes period from 8:00 am to 5:00 pm, from 5:00 pm to sunset, and from sunrise to 8:00 am.

This paper presents an optimization approach for the available solar energy and management of battery power for solar cars in a given mission.

In the next section we introduce the fundamental equations for estimation of the available solar energy during driving and during the stop periods. Then in Section III we present the dynamic model of the solar car. Section IV provides the optimization algorithm. The simulation results and discussion are presented in Section IV.

II Solar Energy Equations

Calculation of the expected solar radiation at any place on earth and at any time [4], can be used to estimate the solar energy available to the solar car along the race. The main input parameters to the solar insolation calculations are the following:

- Date (Year Yr, month Mn, day D).
- Current time (Hours Hr, Minutes Mn).
- Time zone (T_z in hours, East/West of Greenwich).
- Latitude (Φ, L_{at}) : the angular location north or south of the equator; $-90^{\circ} \le \Phi \le 90^{\circ}$
- Longitude (L_{ng} degrees, East or West).
- Solar panels surface tilt β , the angle between the solar panel surface and the horizontal.
- Solar panel surface azimuth angle γ, the deviation of the projection of the normal to the surface on the horizontal plan from local meridian, with zero south, east negative.

The calculated solar energy assumed perfect sunny and clear condition, and does not take into consideration the actual weather condition.

The basic algorithm proceeds as follows:

 First, calculate the day of the year B
 Define B=(N-1)360/365; where N is the nth day of the year

2) Find the standard meridian

$$L_{\rm st} = \begin{cases} \left(\text{Time difference in hours } T_{\rm z} \right) * 15 & \text{if East} \\ L_{\rm st} = 360 - L_{\rm st} & \text{else} \end{cases}$$

3) Calculate the solar local time (T_s)

4) Calculate ω : Hour angle (solar time in angles w.r.t. the local meridian (morning negative); Calculated as 15 degrees per hour, measured from the noon time. For example 10:30 am solar time becomes -22.5 degrees.

$$\omega = \begin{cases} \left(12:00 - T_s \right) * 15 \text{ per hour} & \text{if AM} \\ T_s * 15 \text{per hour} & \text{else} \end{cases}$$
 (4)

5) Calculate sun declination angle δ : Declination with respect to the plane of equator (depends on the day of the year).

$$\delta = 23.45\sin(360\frac{284 + N}{365}). \tag{5}$$

The solar flux is given by $G_{\rm sc}$ = 1366 watt/m₂ is the solar constant. The solar constant is the average amount of energy striking one square meter (perpendicular to the suns' rays) each second at the top of the earths' atmosphere.

If we include extra-terrestrial radiation, then the total radiation energy reaching earth

$$G_{\rm s1} = G_{\rm sc} \left(1 + 0.033 * \cos(360N/365) \right)$$
 (6)

Of this energy reaching the top of the atmosphere, a good portion can be absorbed and reflected by the atmosphere. Solar insolation is the amount of the solar energy received at the earths' surface. On a clear day ~1000 W/m2 reaches a surface perpendicular to the incoming radiation. This energy varies due to the angle of the incoming radiation and cloud cover. For practical purpose we will use

$$\hat{G}_S = 0.7 * G_{S1}$$

The beam radiations \hat{G}_S is the solar radiations that travels from the sun to the surface of the earth without being scattered by the atmosphere

The solar insolation at any place and at any time is given by

$$I_{s} = G_{s1}(\cos(\varphi)\cos(\delta)\sin(\omega) + \sin(\varphi)\sin(\delta))$$
Watt / m2
(7)

where ω is hour angle.

The Average daily solar energy can be obtained by integrating Eq. (7) from sunrise to sunset, and it is given by

$$H_{d} = \frac{24*3600}{\pi} G_{s1}(\cos(\varphi)\cos(\delta)\sin(\omega_{s}) + \frac{\pi\omega_{s}}{180}\sin(\varphi)\sin(\delta))^{2})$$
(8)

where ω_s is the sunset hour angle in degrees

$$\cos(\omega_{\rm s}) = -\tan(\varphi)\tan(\delta). \tag{9}$$

The sunrise hour angle is the negative of the sunset hour angle. The daylight hours is given by

$$N_{hrs} = (2/15)\cos^{-1}(-\tan(\varphi)\tan(\delta)).$$

Example 1: at Darwin (Latitude 12.4667 south, longitude 130.833 east), the expected daily solar insolation on October 1, 2015 is 11.04 kWh/m² (39.75 MJ/m²).

The next equation calculates the expected solar insolation between two angle hours ω_1 and ω_2 ;

$$E_s(t_2, t_1) = \frac{12*3600}{\pi} G_{s1}(\cos(\varphi)\cos(\delta)(\sin(\omega_2))$$

$$-\sin(\omega_1) + \frac{\pi(\omega_2 - \omega_1)}{180}\sin(\varphi)\sin(\delta)). \tag{10}$$

Equation (10) gives the Solar energy in the given time period and at the given location per m2.

Example 2: on October 1, 2015, at Darwin, the available solar energy for battery charging between 12:30 and 1:00 $E_s(t_2, t_1) = 0.705 \text{ kWh/m}^2$.

The net collected energy from the solar panels of the car is given by

$$E_n(t_2, t_1) = \eta_S A_S I_h. \tag{11}$$

where: $\eta_S = 0.227$: Efficiency of the solar panels and $A_s = 6.0 \text{ m}2$: Area of the solar panels.

Accordingly, using eq. (11), the actual collected solar energy for battery charging=0.96kWh.

The optimization algorithm relies on Eq. (10) to calculate the available solar energy between land marks, and to calculate the available solar energy to charge the battery during the 30 minutes stop period. The above equations assumes the solar cells are horizontal. Additional solar energy can be captured if the solar cells are allowed to tilt to face the sun.

Let θ be the incidence angle between beam of solar radiation and normal to the solar panel surface. This angle can be found by the following equation [4]

 $\cos \theta = \sin \varphi \sin \delta \cos \beta - \cos \varphi \sin \delta \sin \beta \cos \gamma$

 $+\cos\varphi\cos\delta\cos\beta\cos\omega$

$$+\sin\varphi\cos\delta\sin\beta\cos\gamma\cos\omega$$
 (12)

 $+\cos\delta\sin\beta\sin\gamma\sin\omega$

Beam radiations on the tilted surface is calculated by multiplying the beam radiations on flat surface by the shape factor of the inclined surface R_b which is a function of zenith and beam ray incident angles follows

$$R_b = \cos(\theta) / \cos(\theta_z) \tag{13}$$

The beam radiation on tilted surface I_{bT} is then given by the following equation

$$I_{bT} = R_b \hat{G}_S \tag{14}$$

A more accurate equation for R_b for calculation of the solar irradiation on a surface of any tilt and azimuth angle is given by Klien [5].

For practical purpose, the available solar energy per m²can be approximated by taking the average of the sun azimuth angles at the start and at the end period, as shown in Eq. (13).

$$E_{St}(t_2, t_1) \cong 2E_S(t_2, t_1) / (\cos(\theta_{z1}) + \cos(\theta_{z2}))$$

$$\cos(\theta_{\tau i}) = \cos(\varphi)\cos(\delta)\cos(\omega_i) + \sin(\varphi)\sin(\delta)$$
 (13)

In Example 2: if the panels were tilted perpendicular to the sun, the solar insolation would only increase from 0.705 to 0.707 kWh/m2 because during this noon period the sun is almost vertical.

On the other hand, the battery can also be charged from sunrise to 8:00 am and from 5:00 pm to sun set. Again at Darwin on October 1, the sunrise and sunset are found to be 6:30 and 18:42 respectively. The estimated solar energy on a horizontal surface from sunrise to 8:00 am came to 0.44 kWh/m2, while the estimated solar energy from 5:00 pm to sunset came to 0.556 kWh. The following Table provides the net contribution of each period to battery charging by combining equations (10), (11), and (1).

Table 1. Estimated battery charging (kWh) during the stop periods.

the stop perior	as.			
Energy in	Sunrise-	12:00-	5:00	Total
kWh	8:00 am	12:30	pm to	
/Period			sunset	
E_{s} ,	0.44	0.705	0.556	1.701
Eq(10)				kWh
$E_{\rm a}$,	0.599	0.960	0.757	2.137
Eq.(11)				
ΔB in kWh	0.575	0.922	0.727	2.224
Eq. (1)				
$\Delta B\%$	11.51	18.44	14.54	44.48%
contribution				

The above table is given as an example. During the race the car position moves away from the equator (latitude increases from 12.4 to 34.8 south), affecting the available solar energy. In general, the 30 minutes mandatory stop in the middle of the day would add 15 to 20 % to the battery, while the charging of the battery after the end of the daily race time (from 5:00 to sunset) and before the start of the race (sun rise to 8:00 am) would contribute additional 20-28%. In all, about 35-45% additional charge is assumed to be available for the battery. However, for the first day, only charging in the midday stop is added with an average of about 18%. Although battery could be charged during travel, the optimization algorithm tends to utilize all the available solar energy to maximize speed. If the solar panels are properly titled before and after the race period, the available solar energy can be doubled, making the battery fully charged again at the start of the next race period.

III. Dynamic Model of the Solar Car

The dynamic model of the car can be described by the equation

$$m\ddot{x} + \lambda_a \dot{x}^2 + C_r mg \cos(\theta(x)) + mg \sin(\theta(x)) = F(t).$$
(15)

where m: mass of the car +driver = 240 kg; λ_a : aerodynamic resistance coefficient =0.045; C_r : roll constant =0.003, and θ : road inclination. Since most of the time the car will go at steady velocity, then equation (10) can be simplified to

 $\lambda_a \dot{x}^2 + C_r mg \cos(\theta(x)) + mg \sin(\theta(x)) = F(t)$. (16) Multiplying Eq. (16) by the velocity

$$\lambda_a \dot{x}^3 + C_r mg\dot{x}\cos(\theta(x)) + \dot{x}mg\sin(\theta(x)) = P_c(t)$$

$$= \eta_m (P_s(t) + P_b(t)) \tag{17}$$

 $P_{\rm s}$ is the solar power and can be calculated as outlined before, $\eta_{\rm m}$ is the motor/gear efficiency, and $P_{\rm b}$ is the power delivered by the battery. The calculation of $P_{\rm s}$, as outlined in Section II, requires year, month, day, hour, minutes, time zone, latitude, and longitude.

Recall that the road is divided by N land marks. Each land mark is identified by its GPS position, distance from beginning, and its elevation. The velocity is assumed to be constant between these land marks. Let v(i) be the velocity between land mark Ms(i) and Ms(i-1), Ms(0) is the the starting point. Let t_i be the time of travel from Ms(i-1) to Ms(i) with speed v(i).

$$t_i = \frac{Ms(i) - Ms(i-1)}{v(i)} = \frac{d_i}{v_i}$$
; where d_i is the

distance between Ms(i) and Ms(i-1).

Integrating Equation (17) over the period t_i

$$t_i(\lambda_a v_i^3 + C_r mg \cos(\theta_i) v_i + mg \sin(\theta_i) v_i) = E_c(i)$$

= $E_s(i) + E_b(i)$ and $f_i(v_i) = E_c(i)$ (18)

$$\tan(\theta_i) = \frac{(h_i - h_{i-1})}{d_i}$$
, where h_i is the elevation of the

ith land mark, and $E_s(i)$ and $E_b(i)$ are the available solar energy from the solar panels, and the energy drained from the battery during travel from Ms(i-1) to Ms(i).

IV. Optimization of the Velocity Profile

In the following we formulate an optimization problem to select the speeds between the land marks while managing the available energy budget from the solar energy and the battery consumption. The optimal speeds are selected to collectively maximize the travelled distance from the current position until 5:00 pm, while fully utilizing the available solar energy and the allocated battery budget. The optimization can be performed dynamically at any time and any point along the road for continuous adjustment based on the current circumstances and battery status. The algorithm can work any time provided that it is given the date, current time, current

position, and current battery status, and it tries to optimize the velocities until 5:00 pm, or until the end of race in the final day.

The optimization is based on the following assumptions:

- Velocities are maintained constant between land marks
- Sunny and clear weather condition
- Daily End of travel is 5:00 pm
- Effect of wind speed is ignored.
- Land marks are those given in Appendix A. More details can be included if available.
- Battery could be charged during driving if solar energy exceeds the required motor power.
- Car stops on or after 12:30 pm for 30 minutes at the nearest land mark. During this time Battery is charged.
- Battery should not be drained to less than 5% of its full capacity (5 kWh).

The approach to solve this optimization problem relies on defining a cost function which can be evaluated for any given velocities between the land marks until the end of the race day. Then, we use a suitable search-based optimization techniques to find the optimal velocities which minimize the cost function.

Let $V = [v_1, v_2, ..., v_{Nms}]$ be a given velocity profile from the current position to the end of the day, where *Nms* can be up to 23 mile stones (or until the end of race), and let i_f to be the index of the last land mark to be seen before stopping at 5:00 pm.

The distance travelled until T_i =5:00 pm is the given by

$$t_{i} = \frac{d_{i}}{v_{i}}; \quad T_{i_{f}} = \sum_{j=1}^{i_{f}} t_{j}; \quad T_{i_{f}} < T_{f}$$

$$D(V) = \sum_{i=1}^{i_{f}} d_{i} + (T_{f} - T_{i_{f}}) v_{i_{f}} + 1.$$
(19)

where t_0 is starting time (or the current time). The second term in the expression of the distance D(V), is the distance to be travelled from the current position until stopping at 5:00 pm. The cost function J(V) is defined as follows

$$J(V) = W_1 abs(B_t(T_f) - B_t(dy)) + W_2 / D(V) + J_2.$$
 (20)

where W_1 and W_2 are suitable weights $,B_t(dy)$ is the target battery status at the end of the racing day number "dy", B=[0.75, 0.40, 0.30, 0.1] J_2 includes additional penalties if battery status gets too low, or the solar energy is not fully utilized; that is if $P_c < P_s$; or if $B_t(t) < 0.05$ B_{full} . The algorithm tests up to 23 milestones per day.

 $B_t(T_f)$ is the battery status at the end of the day at $T_{\rm f}$, and is calculated by subtracting the battery consumption during travelling, using Eq (18), and adding the recharging, E_r , during the 30 minute break.

$$B_t(T_f) = -\sum_{i=1}^{i} E_b(i) / \eta_b + E_r$$
 (21)

Although there is additional battery charging during other stop periods; between 5:00 pm and sunset, and between sunrise and 8:00 am, these changes in the battery status are automatically included as the initial battery status during the next day optimizations.

Next, the cost function defined in Eq. (20) is minimized using one of the known search-based optimization techniques, e.g. particle swam, genetic algorithm, or the traditional simplex method or the Interior Point Algorithm. We used MATLAB's "fmincon.m", constrained optimization function with the 'interior-point', and setting the search limits of velocity between 10 to 40 m/sec.

V. Simulation results and discussions

We consider a first day starting at 8:15 am and ending at 5:00 pm, expected to stop 30 minutes after 12:30 at the nearest land mark. Battery status at the start is full =1

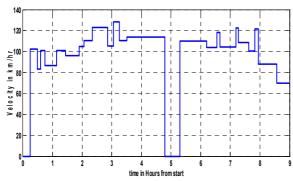


Fig.3. Optimized velocity profile. Time starts at 8:00 am.

The algorithm provided the recommended speeds as shown in Fig. 3, where we assumed the car starts to move at 8.15 am. Fig. 4 shows the travelled distance against time, where t=0 is 8.00 am. The travelled distance during the first day came to 852 km.

The variations of the road elevation as function of the travelling time is given in Figure 5.

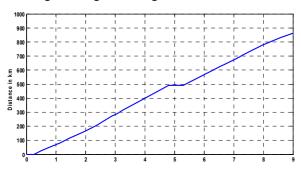


Figure 4. Travelled distance versus time.

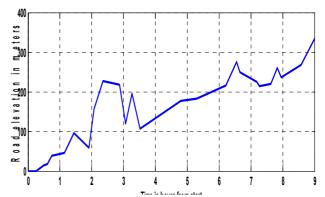


Fig.5. Road elevation versus travel time.

The battery status versus travelling time is presented in Figure 6. The figure shows that the battery status at the end of the day came to about 0.76 as the planned battery daily budget. Notice also the road gets more steep at the end of the day when the available solar energy is substantially reduced. To compensate for the deficiency in the solar energy, the program draws more power from the battery. This situation is clearly illustrated in Fig. 6, where the car is drained quickly at the end of the day. The algorithm shows also good battery management, as it tries to maintain the battery when there is enough solar energy and use the savings during the difficult time at the end of the day.

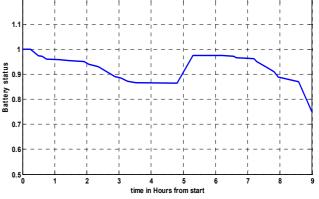


Fig.6. Battery status.

Conclusion

The paper presented an optimization approach for the available solar energy and management of battery power for solar cars in a given mission. We demonstrated the optimization approach for the 3000 km World Solar Challenge. The algorithm tries to maximize the speeds between land marks, by estimating the available solar energy and battery power. Every time the algorithm runs during the race it updates the remaining velocities taking into consideration the current status to accommodate the uncertainty in the road condition and in the available solar energy. We have used about 56 road land marks. Better results can be obtained if more information on land marks is available. We assumed

the midday mandatory stop to be at the nearest land mark after 12:30, although stops could be made at any other time with a slight change in the calculations. The accuracy of calculations depends very much on accurate knowledge of the roll constants and the aerodynamic drag of the car, and use of more accurate solar energy estimation equations. Accurate values of the battery efficiency during charging and discharging are important as well for accurate management of the battery budget during the race. These values can be re-estimated online if measurement of voltages and currents are monitored by a suitable data acquisition system. The presented optimization algorithm can lead to better planning and efficient management of the solar energy during the WSC race.

Acknowledgement

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Appendix A: LAND MARKS

Name	Lat S	Lng	Elev.	Distance	Name	Lat S	Lng	Elev.	Distance
Darwin	12.4	130.92	13.41	0.00	Barrow Creek	21.62	133.85	503.22	1237.52
Palmerston	12.47	130.97	17.07	24.96	Harry Greek	23.22	133.75	713.23	1435.39
HW36	12.55	131.07	37.49	34.52	X12	23.47	133.83	743.71	1466.55
HW34	12.67	131.07	46.02	48.37	Alice Springs	23.68	133.87	611.73	1492.58
Crocodile									
farm	12.97	131.10	94.49	83.01	X13	24.73	133.17	434.04	1626.02
Adelaide									
River	13.23	131.10	58.52	113.02	Eridunda	25.17	133.17	420.62	1679.95
Douglas hot									
springs	13.53	131.42	156.36	157.82	Kulgere	25.85	133.28	521.51	1763.51
Emerald									
springs	13.62	131.62	227.38	174.77	X7	26.08	133.18	500.18	1792.21
pine Creek	13.80	131.83	216.71	205.94	Maria	27.32	133.65	330.40	1946.39
X1	14.18	132.03	118.57	269.87	Coober pedy	28.97	134.75	173.74	2158.40
X2	14.30	132.08	195.38	290.00	X8	29.62	135.15	181.97	2241.61
Katherine	14.45	132.25	106.98	316.01	Glengambo	30.97	135.73	143.87	2410.93
Venn	14.57	132.50	176.78	343.99	X9	31.23	136.40	112.47	2456.52
Daly water	16.25	133.38	215.49	590.03	Nurrungu	31.25	136.80	185.93	2478.46
X3	16.55	133.35	275.23	625.68	Mount Gunson	31.50	137.98	142.95	2548.29
Dunmassa	16.65	133.40	249.02	637.41	Baroota	32.17	137.97	26.52	2618.84
Henry Walker	17.13	133.45	225.86	693.18	Hesso	32.20	137.58	41.45	2640.94
Causeway									
dam	17.22	133.45	213.36	703.64	Port Augusta	32.48	137.75	21.34	2671.93
Elliot	17.55	133.53	219.46	742.06	B89	33.17	138.07	16.76	2746.84
X4	17.72	133.63	259.69	762.12	WarnerTown	33.22	138.12	56.08	2752.84
X5	17.85	133.68	236.22	777.32	X10	33.30	138.17	143.26	2784.77
HW66	19.43	134.20	333.45	961.82	SnowTown	33.52	138.20	109.42	2800.83
Tennant									
Creek	19.63	134.18	368.81	987.67	Lochiel	33.92	138.15	87.78	2826.98
Warumungu	19.90	134.20	347.78	1021.06	Beufort	34.07	138.98	25.30	2878.33
X6	20.20	134.22	370.03	1058.16	Port Wakefield	34.13	138.22	6.40	2923.64
Ankweley	21.57	133.72	488.59	1228.96	Dublin	34.45	138.35	14.63	2958.20
Barrow Creek	21.62	133.85	503.22	1237.52	End	34.83	138.58	22.86	3000.51