informational closure

Information flow J_t from environment to a system at time t can be defined as

$$
\begin{aligned}
J_t(E \to S) \quad &:= I(Y_{t+1}; E_t|Y_t) \\
&= H(Y_{t+1}|Y_t) - H(Y_{t+1}|Y_t, E_t) \\
&= H(E_t|Y_t) - H(Y_t|Y_t, Y_{t+1}) \\
&= H(E_t|Y_t) - H(E_t|Y_t, Y_{t+1}) \\
&= I(Y_{t+1}; E_t) - (I(Y_{t+1}; Y_t) - I(Y_{t+1}; Y_t|E_t))
\end{aligned} \tag{1}
$$

Rewrite

$$I(Y_{t+1}; E_t|Y_t) = I(Y_{t+1}; E_t) - (I(Y_{t+1}; Y_t) - I(Y_{t+1}; Y_t|E_t)) \tag{2}$$

Trivial case

$$
\begin{aligned}
&I(Y_{t+1}; E_t) = 0 \\
&I(Y_{t+1}; Y_t) - I(Y_{t+1}; Y_t|E_t) = 0
\end{aligned} \tag{3}
$$

[[NTIC]]

In non-trivial cases,

$$I(Y_{t+1}; E_t) \neq 0, \tag{4}$$

This suggests that the process encodes information about the future state of the environment. Therefore, informational closure can be achieved by

$$I(Y_{t+1}; Y_t) - I(Y_{t+1}; Y_t|E_t) > 0 \tag{5}$$

And non-trivial information closure (NTIC) can be defined as

$$
\begin{aligned}
NTIC \quad &:= I(Y_{t+1}; Y_t) - I(Y_{t+1}; Y_t|E_t) \\
&= I(Y_{t+1}; E_t) - I(Y_{t+1}; E_t|Y_t)
\end{aligned} \tag{6}
$$

To maximize $NTIC$ is equivalent to

$$
\begin{aligned}
&\text{maximize} \quad I(Y_{t+1}; Y_t) \qquad and \\
&\text{minimize} \quad I(Y_{t+1}; Y_t|E_t)
\end{aligned} \tag{7}
$$

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$$
\begin{aligned}
J_t(E \to S) \quad &:= I(Y_{t+1}; E_t|Y_t) \\
&= H(Y_{t+1}|Y_t) - H(Y_{t+1}|Y_t, E_t) \\
&= H(E_t|Y_t) - H(Y_t|Y_t, Y_{t+1}) \qquad I(Y_{t+1}; E_t|Y_t) = I(Y_{t+1}; E_t) - (I(Y_{t+1}; Y_t) - I(Y_{t+1}; Y_t|E_t)) \\
&= H(E_t|Y_t) - H(E_t|Y_t, Y_{t+1}) \\
&= I(Y_{t+1}; E_t) - (I(Y_{t+1}; Y_t) - I(Y_{t+1}; Y_t|E_t))
\end{aligned} \tag{8}
$$