Exercise No. 8

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1 Stability Analysis

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = N_i \left(a_i - N_i - \sum_j b_{i,j} P_j \right) \tag{1}$$

$$\frac{\mathrm{d}P_i}{\mathrm{d}t} = P_i \left(\sum_j c_{i,j} N_j - d_i \right) \tag{2}$$

1.1 fixed Points

to find the two fixpoints we can guess the trivial and test it

$$FP_1 : P = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, N = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

which gives us a solution for $\frac{\mathrm{d}N_i}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}P_i}{\mathrm{d}t} = 0$

the second fixed point we guess the following

$$FP_2 : P = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, N = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{4}$$

and with these values we also get $\frac{\mathrm{d}N_i}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}P_i}{\mathrm{d}t} = 0$

1.2 Jacobi Matrix A

$$A = \begin{pmatrix} a_1-2N_1-\sum_j b_{1,j}P_j & 0 & 0 & N_1b_{1,1} & N_1b_{1,2} & N_1b_{1,3} \\ 0 & a_2-2N_2-\sum_j b_{2,j}P_j & 0 & N_2b_{2,1} & N_2b_{2,2} & N_2b_{2,3} \\ 0 & 0 & a_3-2N_3-\sum_j b_{3,j}P_j & N_3b_{3,1} & N_3b_{3,2} & N_3b_{3,3} \\ P_1c_{1,1} & P_1c_{1,2} & P_1c_{1,3} & \sum_j c_{1,j}N_j-d_1 & 0 & 0 \\ P_2c_{2,1} & P_2c_{2,2} & P_2c_{2,3} & 0 & \sum_j c_{2,j}N_j-d_2 & 0 \\ P_3c_{3,1} & P_3c_{3,2} & P_3c_{3,3} & 0 & 0 & \sum_j c_{3,j}N_j-d_3 \end{pmatrix} \tag{5}$$

$$\overset{FP_2}{=} \begin{pmatrix} -1 & 0 & 0 & 20 & 30 & 5 \\ 0 & -1 & 0 & 1 & 3 & 7 \\ 0 & 0 & -1 & 4 & 10 & 20 \\ 20 & 30 & 35 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 7 & 8 & 20 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$

1.3 EVals and EVecs

i	λ_i
1	-33.06613885
2	32.06613885
3	-9.99024416
4	8.99024416
5	-0.86313363
6	-0.13686637

and

$$v_i = \begin{pmatrix} 0.5562403 & -0.5463627 & 0.82241683 & -0.80651197 & 0.48435472 & 0.09294963 \\ 0.10307497 & -0.10124458 & -0.1459207 & 0.14309872 & -0.29627361 & -0.0568561 \\ 0.32248203 & -0.31675546 & -0.36432112 & 0.35727545 & -0.0551648 & -0.01058635 \\ -0.67377404 & -0.68244818 & -0.29655807 & -0.3231716 & 0.67224106 & 0.81356161 \\ -0.08907577 & -0.09022253 & -0.09374396 & -0.10215667 & -0.46197821 & -0.55909667 \\ -0.33774498 & -0.3420931 & 0.26995038 & 0.2941761 & 0.09616338 & 0.11637914 \end{pmatrix} \tag{7}$$

where every column is the corresponding Eigenvector