Introduction to Computational Physics SS2017

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Exercise 2 from April 26, 2017
Return by noon of May 5, 2017

1 Numerical Simulation of the 2-Body Problem

- Write a computer program that computes the relative motion of two point-like bodies under their mutual gravitational influence. Use a step-by-step Euler integration procedure. Set $G = 1$, $M_1 = M_2 = 1$.
- For which velocity v_0 can the two bodies rotate around each other in a circular fashion with a separation of 1?
- Now perform the numerical integration. Use, for the moment, a constant time step of $\Delta t = 0.01$. What happens with the numerical model if you choose $v_0/\sqrt{2}$ as the initial velocity? Make a plot of the orbits.
- Compute the eccentricity from the Runge-Lenz vector.
- What happens if you choose an initial velocity larger than $\sqrt{2}v_0$?
- Simulate another case, like e.g. the $v_0/3$ initial velocity case. Experiment with decreasing the time step (and simultaneously increasing the number of time steps) and see how the results change.

2 Error Analysis of Euler Scheme (HOMEWORK)

(a) Choose 3 different eccentricities and various different time steps (the latter spanning orders of magnitude!). Integrate the 2-body problem for 1 orbit. Plot, in a double-logarithmic fashion, the error in the energy at the end of this orbit as a function of Δt. Discuss the result, is it consistent with what one should expect? (10 points)

(b) Do the same as above, but now using the leapfrog integrator scheme. How does the result change? (10 points)

(c) The same as above, but now using the time-transformed leapfrog of the lecture notes. (voluntary supplement)

NOTE again, for this and all following homework exercises, which require writing a computer program, it is necessary to submit all parts of the program written by you, together with the results. It must be comprehensible for your tutor how your program generates your results.