1 Non-uniform distributions

using the following code we plotted $p(\theta, g)$ und $p(\eta, g)$, as well as performed sampling with g=0.5

```
ntests = 1000

# part 1
scatter_prb <- function(theta, g){
    1/(4*pi)*(1-g^2)/(1+g^2-2*g*cos(theta))^(3/2)
}

png('6-2_scatter_prob.png')
xdata = seq(0, pi, length.out=100)
#plot(0,type='n', xlim=c(0,pi), xlab="theta / pi", ylab="p(theta)", xaxt='n')

data = sample(xdata,ntests,rep=TRUE,prob=2*pi*sin(xdata)*scatter_prb(xdata, .5))
h = hist(data, breaks=seq(0,pi, length.out=20))
plot(h$mids, h$density, xaxt='n', xlab="theta / pi", ylab="p(theta)", pch=20)

for (g in seq(.1,.5,.2)) {
    lines(xdata, 2*pi*sin(xdata)*scatter_prb(xdata, g), col=1+g*10)
}
axis(1, at=seq(0,pi,pi/4),labels=seq(0, 1, 1/4), las=2)

#mean and variance
paste('theta_mean = ', mean(data))
paste('theta_var   = ', var(data))

# part 2
scatter_eta <- function(eta, g){
    scatter_prb(acos(eta),g)
}
xdata = seq(-1,1,.01)

png('6-2_scatter_eta.png')
#plot(0,type='n', ylim=c(0,.8*ntests), xlim=c(-1,1), xlab="eta", ylab="p(eta)")
data = sample(xdata,ntests,rep=TRUE,prob=scatter_eta(xdata,.5))
h = hist(data, breaks=seq(-1,1, length.out=20))
plot(h$mids, h$density/4, xlim=c(-1,1), xlab="eta", ylab="p(eta)", pch=20)

lines(xdata, scatter_eta(xdata, .5), col=2)
```

2 Higher order moments of multivariate Gaussian

$$E[x_1 \cdot x_2 \cdot \ldots x_{2n}] = \sum_{1}^{\binom{2n}{2}} \left(\prod_{1}^{n} E[x_i x_j] \right) \tag{1}$$

2.1 a)

$$\begin{aligned} E[x_1^4 x_2^2] =& \binom{3}{1}_{perm} 2^0_{rot} E[x_1^2]^2 E[x_2^2] && (2)\\ &+ \binom{3}{1}_{perm} 2^2_{rot} E[x_1^2] E[x_1 x_2]^2 && (3)\\ =& 3\sigma_1^4\sigma_2^2 + 12(\varrho_{12}\sigma_1\sigma_2)^2\sigma_1^2 && (4)\\ & && (5) \end{aligned}$$

2.2 b)

$$\begin{aligned} E[x_1^3 x_2] =& \binom{4}{2} E[x_1^2] E[x_1 x_2] && (6)\\ =& 6\sigma_1^2(\varrho_{12}\sigma_1\sigma_2) && (7) \end{aligned}$$