Computational Statistics
Exercise sheet 7

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Problem 1 - Fisher vs. Full Likelihood

Sn Likelihood

The dimensionless Hubble function is:

$$E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_M(1+z)^3 + \Omega_L\,, \tag{1}$$

where $\Omega_L = 1-\Omega_M$, that is we assume here flatness. The luminosity distance and distance modulus are then:

$$d_L(z,\Omega_M) = (1+z)\int_0^z \frac{dz'}{E(z)} \quad \text{and} \quad m_t(z,\Omega_M) = 5\log_{10} d_L(z,\Omega_M)\,. \tag{2}$$

The helper function S_n is:

$$S_n(\Omega_M) = \sum_i \frac{[m_t(z_i) - m_i]^n}{\sigma_i^2}\,, \tag{3}$$

where m_i and σ_i are the distance modulus and error of the i-th supernova at redshift z_i. We can then build the likelihood, after marginalization over a constant:

$$\ln L(\Omega_M) = -\frac{1}{2}\chi^2 \quad \text{where} \quad \chi^2 = S_2 - \frac{S_1^2}{S_0}\,. \tag{4}$$

Fisher approximation

The likelihood L can be approximated with the following likelihood

$$L \approx L_f \equiv \exp\left[-\frac{1}{2}(\theta_\alpha - \hat{\theta}_\alpha)F_{\alpha\beta}(\theta_\beta - \hat{\theta}_\beta)\right]\,, \tag{5}$$

where $\boldsymbol{\theta}$ is the vector of theoretical parameters and the Fisher matrix is defined as

$$F_{\alpha\beta} \equiv -\left.\frac{\partial^2 \ln L(\boldsymbol{\theta})}{\partial\theta_\alpha\partial\theta_\beta}\right|_{\mathrm{ML}}\,. \tag{6}$$

In our case $\theta = \Omega_M$, and the previous expressions simplify to:

$$\ln L_f \approx -\frac{1}{2}(\Omega_M - \Omega_M^{\text{b.f.}})^2 F\,, \tag{7}$$

and

$$F = -\left.\frac{\partial^2 \ln L(\Omega_M)}{\partial{\Omega_M}^2}\right|_{\Omega_M^{\text{b.f.}}}\,. \tag{8}$$

Assignment

1. Find F and plot/compare L versus L_f. Calculate the needed derivatives analytically, or numerically if you find it too complicated. Use 1 entry every 15 of the dataset "supernovae.csv" available in the dropbox.

2. Compute the 95% c.l. interval for Ω_M using L_f and L. Compare the two constraints. What is the origin of the difference?

Problem 2 - Complex Gaussian distribution

Suppose we have a complex-valued random variable z with statistically independent real and imaginary parts x and y, each following a Gaussian distribution.

- Suppose z has expectation value $\mu = 0$. Define a covariance matrix $C = \langle z z^\dagger \rangle$, where the superscript $\dagger$ denotes Hermitian conjugation. Show that C is diagonal if you express it in terms of x and y;

- Is the determinant of C always positive? Give an explicit expression for the inverse of C.