Computational Statistics

Exercise sheet 4

To be handed in for marking until tutorial of 17/05/2017

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Problem 1 - Generating Gaussian distributed random variables [7.5 points]

In this exercise, you will be applying the Box-Muller method for generating Gaussian distributed random variables.

- First, you will have to generate random uniform coordinates (x, y) from the unit circle using the rejection sampling procedure. Hence, your algorithm should reject any pairs (x, y) with $x^2 + y^2 > 1$.
- Then, for each accepted pair (x, y) evaluate the quantities $z_1 = x\Big(- 2\frac{\ln[x^2+y^2]}{x^2+y^2}\Big)^{1/2}$ and $z_2 = y\Big(- 2\frac{\ln[x^2+y^2]}{x^2+y^2}\Big)^{1/2}$
- The values z_1 and z_2 are each Gaussian distributed with zero mean and unit variance. Write an R script that implements this Box-Muller method and verify that the sampled values z_1 and z_2 are Gaussian distributed.

Problem 2 - Random Walk [7.5 points]

Write a R script to simulate a 1D random walk, in which a person starts at a point 0 and at each step randomly picks a direction (left or right) and moves 1 step in that direction. Take a positive integer n and terminate the simulation when the walk reaches n or $-n$. Decide the number of runs that you want to consider and, by averaging over this number, report an estimated average number of steps needed for the walk to terminate. Repeat this for different values of n, and plot the results to show qualitatively how rapidly the walk terminates, as a function of n.

Problem 3 - Lifetime of an element [5 points]

If τ is the lifetime of an element, which follows an exponential decay law with a characteristic decay time τ,

$$f(t) = A\text{exp}(-\text{t}/\tau), \tag{1}$$

1. find A such that $f(t)$ has the properties of a probability distribution functions;
2. what is the mean and standard deviation of this lifetime;
3. if $\tau = 5$ min, what is the probability that the lifetime is greater than or equal to 10 min;
4. estimate the probability of a lifetime of at least 10 min given that it has been observed for already 5 minutes: $P(T > 10|T > 5)$ (consider same τ as before);
5. generalize the previous question: estimate $P(T > (a + b)|T > a)$.