Assignment about stellar evolution models

based on that proposed by Sergio Simón Díaz

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3 December 2024

In order to do the following assignment, you will have to download the evolutionary models in the "Selection of Geneva evolutionary models" (Ekström et al. 2012) file from the "Aula Virtual".

Within the downloaded compressed file you will find files with names such as M1p25Z14V0.dat, where M1p25 means that the model refers to a $M = 1.25\,\mathrm{M}_\odot$ star, Z14 implies a solar metallicity ($Z = 0.014$), and V0 refers to no rotation. Some peculiarities of the models:

- For low-mass stars the integration ends at the helium flash.
- For intermediate-mass stars the integration ends at some point druing the early AGB phase.
- For massive stars the integration stops when carbon ignites.

Each file includes a table that describes several properties of stars as a function of time, which appears in column number 2. For example, if you would like to study the evolution of the effective temperature of a star, you would have to plot column 5 against column 2.

You will first need to create a code that allows you to read the information in each of the tables. In `python` the instruction `read` within the `astropy` package might be a good way to go.

```
from astropy.io.ascii import read
table_M5 = read("M005Z14V0.dat",data_start=2) #Reading the table for
                                               #a M=5M_sun star
#Above, we start reading the data at line=2
#(0 contains the labels, and 1 contains the units)
print(table_M5.columns) #Examining the contents of the table
```

The assignment will be evaluated based on its structure, the figures, and the relations established between them and the concepts taught in the theory lectures.

The following homology relations will be required:

- From Chapter 7 for stars dominated by an electrons scattering opacity

$$L \propto M^3. \tag{1}$$

- From Chapter 7 for stars dominated by a Kramers' opacity

$$L \propto M^{(10n+31)/(2n+5)}. \tag{2}$$

- From Chapter 7

$$\rho_* \propto \frac{M}{R_*^3}. \tag{3}$$

- From Chapter 7

$$T_* \propto \frac{M}{R_*}. \tag{4}$$

- From Chapter 7 for stars by an electron scattering opacity

$$R_* \propto M^{(n-1)/(n+3)}. \tag{5}$$

- From Problem 6 in Chapter 7 for stars dominated by a Kramers' opacity

$$R_* \propto M^{(2n-7)/(2n+5)}. \tag{6}$$

Do the following:

1. Plot the time evolution of the central hydrogen and helium abundances for stars with $M = 5\,\mathrm{M}_\odot$ and $M = 9\,\mathrm{M}_\odot$. How long does the main sequence last for the two stars? Is this the result that you expected? Why?

2. For the models with $M = 0.8, 1, 1.25, 1.5, 2, 3, 4, 5, 7, 9, 15, 25, 40\,\mathrm{M}_\odot$:

   (a) Plot the evolutionary tracks in an HR diagram.

   (b) Plot the ZAMS and the TAMS. What is the cause of the different main sequence behaviours for stars in different mass ranges?

   (c) What is the age of the stars when they reach the ZAMS?

   (d) What is the age of the stars as they leave the main sequence?

   (e) Plot the $L - M$ relation for ZAMS stars. Compare the slopes to those expected from the homology relations.

   (f) Plot the $\rho_\mathrm{c} - M$ relation for ZAMS stars. Compare the slopes to those expected from the homology relations.

   (g) Plot the $T_\mathrm{c} - M$ relation for ZAMS stars. Compare the slopes to those expected from the homology relations

3. For the models with $M = 1, 3, 9, 40\,\mathrm{M}_\odot$:

   (a) Plot the tracks of the centres of the stars in the $(\log T, \log \rho)$ plane.

   (b) Mark the location of the centre of the stars when at the ZAMS.

   (c) Mark the location of the centre of the stars when at the TAMS.

(d) Plot the areas dominated by a classic ideal gas, a degenerate electron gas, a degenerate relativistic electron gas, and a radiation gas. Assume $\mu_e = 2$ when dealing with the limit between the classic ideal and the degenerate gases and $\mu = 0.61$ when looking for the separation between the classic ideal gas and the radiation-dominated region.

4. For the model with $M = 4\,\mathrm{M}_\odot$:

(a) Plot the central abundances of hydrogen, helium, carbon, and oxygen as a function of time. Indicate when the star is in the main sequence (MS), the Hertszprung gap + RGB phase (RGB), the blue loop (BL), and the AGB phase (AGB). You can do that with vertical lines separating the evolutionary stages and using labels. Since the MS phase is much longer than the others, you can plot a fraction of it only.

(b) Plot the evolutionary track in the HR diagram. Indicate with different colours the main sequence (MS), the Hertszprung gap + RGB phase (RGB), the blue loop (BL), and the AGB phase (AGB).

(c) Plot the evolution of the centre of the star in the $(\log T, \log \rho)$ plane. Indicate with different colours the main sequence (MS), the Hertszprung gap + RGB phase (RGB), the blue loop (BL), and the AGB phase (AGB).

(d) Show the evolution of the radius of the star as a function of time. You can use the same vertical lines and labels as in the plot for the chemical abundances to separate the different evolutionary stages.

(e) Explain why there are two moments in which the radius of the star evolves fast.

(f) What happens once the star ends its evolution?