Cryptocurrency Portfolio Optimization with Modern-Portfolio-Theory-based Strategies and Wyckoff's Market Cycle of Bitcoin

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Abstract

This study analyzes the construction of optimal cryptocurrency portfolios using three Modern-Portfolio-Theory-based optimization strategies for investors holding cryptocurrency portfolios following the cryptocurrency crash in 2018. Using simulation and modeling of maximum Sharpe Ratio portfolio, minimum variance portfolio, and Markowitz portfolio, we found there is no big difference between the first two portfolios in terms of returns and variance, while Markowitz portfolio is not helpful for long-term investment. Furthermore, we applied Markowitz to each of the four phases of Wyckoff's market cycle for Bitcoin and the empirical result shows that the high-volatility feature of cryptocurrencies makes it more suitable for the four-phases approach of Markowitz and the portfolio return in each phase outperforms the classic Markowitz portfolio.

1. Introduction and Literature Review

In 1952, Harry Markowitz published his paper "Portfolio Selection" in the Journal of Finance, along with the cutting-edge breakthrough in his later book "Portfolio Selection: Efficient Diversification (1959), his work has provided a fundamental framework to the Modern Portfolio Theory. Ever since then, various portfolio optimization strategies have been derived from Modern Portfolio Theory (Markowitz, 1952). Motivated by the cryptocurrency crash in 2018, we want to investigate the optimal portfolio weights based on different optimization strategies to portfolios including cryptocurrencies. We will focus on the return-oriented Markowitz portfolio, the risk-return-oriented maximum Sharpe Ratio portfolio, and the risk-oriented minimum variance portfolio. In particular, in order to get more insights on short-term speculation between January 2018 and December 2019, we will also apply the four phases of Wyckoff's market cycle to bitcoin and analyze the Markowitz portfolio corresponding to each phase of the cycle.

Harry Markowitz described the "expected returns-variance of returns" rule for the first time in "Portfolio Selection" (Markowitz, 1952), which means investors prefer higher expected return and lower variance of return. This rule also implies a risk and return trade-off such that riskier assets require greater expected returns, and the efficient portfolio proposed by Markowitz' expected returns-variance of returns rule attains a risk-adjusted return depending on the individual investor's level of risk-aversion. He mentioned that expected return-variance of return rule provides a well-diversified efficient portfolio for a large and representative range of expected return and variance of returns. Therefore, Markowitz portfolio has been widely used to serve as a reference model in portfolio optimization. However, early studies also pointed out that Markowitz portfolios usually exhibit great weights for a small subset of the assets in the portfolio, and require highly precise parameter estimates since it is sensitive to small changes in mean and variance estimates (Jorion, 1985; Simmaan, 1997).

Among the set of efficient portfolios constructed along the efficient frontier of Markowitz, we can locate the tangent point that represents a portfolio with maximum risk-adjusted return, also known as maximum Sharpe Ratio portfolio. William Sharpe (1994) first brought up the idea of Sharpe Ratio to describe returns from per unit measurement of risk in the asset(s). A recent study has shown that Sharpe Ratios remain unaffected when adding cryptocurrencies to traditional portfolios, since the high risk in cryptocurrencies is compensated by high returns (Petukhina et al, 2020). At the same time, in Briere et al, (2015) and Akhtaruzzamen et al. (2019)'s studies, they found cryptocurrencies contribute to greater diversification and help to build more desirable risk-return performance. Therefore, maximizing Sharpe Ratio has become a standard modern strategy in cryptocurrency portfolio optimization.

In addition to the maximum Sharpe Ratio portfolio, the left-most point of the efficient frontier from Markowitz also provides a minimum variance portfolio (Markowitz, 1952). Instead of using risk as an adjustment tool for expected returns as in the maximum Sharpe Ratio portfolio, minimum variance portfolio allocates equal marginal risk contributions of each asset to the portfolio, without requiring specific information about the expected returns (R. Clarke et al., 2011). The findings of Liu (2018) show that minimum variance portfolio shows smallest volatility and also minimized maximum drawdown, while using cryptocurrencies for portfolio diversification.

In 1910, Richard D. Wyckoff proposed a four-phase market cycle, which consisted of accumulation, markup, distribution, and markdown (shown in Figure 1). In a recent study by Omar Gray and Eddy Breton in 2020, bitcoin exhibits the four phases of the Wyckoff market cycle. The Wyckoff trading method was shown to be successfully applied to the cryptocurrency market by using bitcoin as the trading asset. We will further discover differences in cryptocurrency portfolio optimization based on different phases of the Wyckoff market cycle in this study.

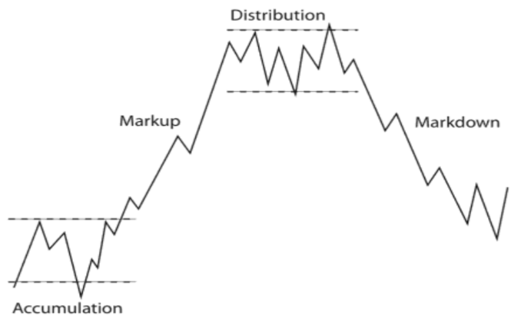


Figure 1. Wyckoff Cycle (Source: Adam Grimes, *The Art and Science of Technical Analysis: Market Structure, Price Action, and Trading Strategies*)

Most of the studies of cryptocurrency portfolio optimization either focus on all-crypto portfolios (Liu, 2019) or portfolios of one cryptocurrency (usually Bitcoin) with other traditional assets (Briere et al, 2015; Eisl et al. 2015; Burniske and White, 2016). In this study, we will use more than one cryptocurrency to provide diversification to a traditional portfolio consisting of stock indexes and commodities and find the optimal portfolio weights allocation of Markowitz portfolio, maximum Sharpe Ratio portfolio, and minimum variance portfolio. In particular, inspired by the previous study in Wyckoff market cycle theory and the application of the Wyckoff method in cryptocurrency, we will use Bitcoin's market cycle to find the Markowitz portfolio in each of the four phases of the Wycoff's market cycle for short term speculation.

2. Sample and Data

We examined daily returns for nine different assets from 2018-01-01 to 2019-12-31. These 9 assets can be further categorized into 3 different categories, which are market indices, commodities, and cryptocurrency. For market indices, we choose S&P 500, NASDAQ 100, and Russell 400 which will be a good measure of investors' sentiment of the stock market. For Commodities, what we choose is Crude Oil, Gold and Copper due to their high trading volumes and capacity. We use Bitcoin, Litecoin, and Ethereum as indicators of the cryptocurrency since they are more mature and have a longer time frame compared to the other cryptocurrencies. We are using web.DataReader to extract the data from yahoo finance.

1) **Variable Elimination**

   Each data set has a total of 7 variables. Here, we eliminated predictor variables “Open”, “High”, “Low”, and “Volume” from consideration because they are extraneous and less significant compared to “Close” in representing the price of the indicator. Since we also don’t need to consider stock split and “Adjusted closing” might not be steady with excessive “N.A” within the data, hence we will only use the variable “Close”.

2) **Handling Missing Values**

   Since certain days might not be a trading day for certain assets, there will be lots of “N/A” displayed in our extracted datasets. We performed missing data analysis by forward filling and backfilling all the “N/A " for any of the fields, leaving us with a total of 731 usable data points to analyze respectively.

Date	^GSPC	^RUT	NSDQ	LTC-USD	ETH-USD	BTC-USD	HG=F	CL=F	^GC=F
2018-01-01	2695.810059	1550.010010	76.739998	229.033005	772.640991	13657.200195	3.2560	60.369999	1313.699951
2018-01-02	2695.810059	1550.010010	76.739998	255.684006	884.443970	14982.099609	3.2560	60.369999	1313.699951
2018-01-03	2713.060059	1552.579956	77.660004	245.367996	962.719971	15201.000000	3.2370	61.630001	1316.199951
2018-01-04	2723.989990	1555.719971	78.690002	241.369995	980.921997	15599.200195	3.2425	62.009998	1319.400024
2018-01-05	2743.149902	1560.010010	79.209999	249.270996	997.719971	17429.500000	3.2070	61.439999	1320.300049

Figure1: This figure shows the final data frame we have after data-preprocessing, only displaying the first 5 rows.

Figure 2 shows the historical development of the prices of these 9 assets. We can see that the price of Bitcoin is much higher than the price of the other assets. However, since the price of the other assets are much lower and squeezed below, it is hard to compare the volatility of each asset.

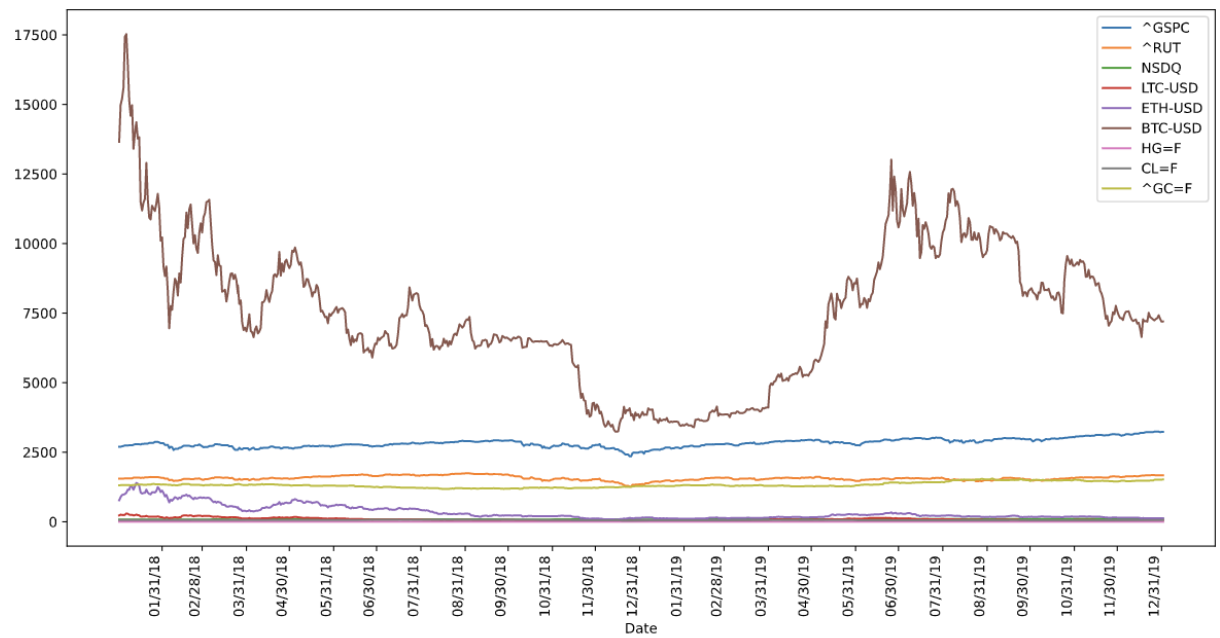


Figure 2: the historical development of the price of the 9 assets in the portfolio

To visualize the volatility, we can plot the daily returns. From Figure 3, we observe that the portfolio has a lot of distinctive positive spikes and a couple of negative ones. LTC-USD has a few quite high and obvious positive spikes and ETH-USD also has some spikes that stand out from the plot. From the above plot, we can roughly see that LTC-USD, ETH-USD look like quite risky assets compared to others.

Figure 3: the historical development of the daily returns of the 9 assets in the portfoli

3. Approach and Methodology

We hope that our model would provide useful insights to the asset's behaviors and risk profiles, so as to help future investors in their decision making through correct modeling. However, we do recognize certain limitations within our model since the data from 2018-2019 may not necessarily be extensible in the future. Especially given the high volatility of cryptocurrency. It may not accurately reflect current public sentiments towards the financial markets due to volatile market trends.

3. 1 Maximum Sharpe Ratio Portfolio & Minimum Variance Portfolio

We will find the returns, covariance and correlation matrix to understand the behavior of the assets and also for further calculation. We randomly generate 20000 portfolios, each with assets of randomly generated weightage using the np.random.random() function. Also the sum of the weightage of the total 9 assets will be 1. For each portfolio, we will calculate the return by multiplying the weight of the asset (which is randomly generated) by its return and summing the values of all the assets together.

- Expected return:

$$\mathrm{E}(R_p) = \sum_i w_i \, \mathrm{E}(R_i)$$

Portfolio Expected Return

We also calculate the variance, which is also the risk of that particular portfolio.

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij},$$

Portfolio Risk (Variance) | Rho — Correlation coefficient between asset i/j

There are two ways for us to choose the optimal weightage from the 20000 randomly generated portfolios.

3.1.1 Maximum Sharpe Ratio Portfolio

Here, we are maximizing the Sharpe ratio of the portfolio. The Sharpe ratio is an effective measure of the return over risk, with the formula of

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where:

R_p = return of portfolio

R_f = risk-free rate

σ_p = standard deviation of the portfolio's excess return

Generally, the higher the Sharpe ratio of the portfolio, the better in its risk adjusted performance and therefore more attractive is that portfolio. For the Sharpe ratio, we are here assuming a risk free rate of 0.01.

	^GSPC	^RUT	NSDQ	LTC-USD	ETH-USD	BTC-USD	HG=F	CL=F	^GC=F
Allocation	0.22	0.14	0.31	0.02	0.02	0.03	0.01	0.12	0.15
Annualised Return	0.0621								
Annualised Volatility	0.1057								

Figure 4: the final optimal portfolio with the highest Sharpe ratio

From Figure 4, we can see for the optimal portfolio, the annualised return is 6.21% with 10.57% of annualised volatility. S&P 500 has the highest weightage in the portfolio while the weightage of the 3 cryptocurrencies and copper are significantly lower.

3.1.2 Minimum Volatility Portfolio

The minimum volatility portfolio is a combination of assets with the lowest price movement. This portfolio is positioned for stable short-term growth. Therefore, this strategy is most preferable during the market downturns. In general, the optimal portfolio is chosen with the lowest variance.

	^GSPC	^RUT	NSDQ	LTC-USD	ETH-USD	BTC-USD	HG=F	CL=F	^GC=F
Allocation	0.19	0.01	0.24	0.02	0.02	0.03	0.24	0.03	0.24
Annualised Return	0.0406								
Annualised	0.0875								

Volatility	

Figure 5: the final optimal portfolio with the minimum volatility

From the earlier plot of daily return, we have seen that LTC-USD, BTC-USD and ETH-USD are having a higher volatility compared to the other assets. Therefore, in this minimum volatility portfolio, they are having a much smaller weightage. For the crude oil (CL=F), the weightage has also been greatly reduced for the min-variance portfolio compared to the sharpe ratio, we can deduce that crude oil is having a higher return associated with higher volatility.

If we plot out the graph of all the 20000 randomly generated portfolios. We obtained the following graph. We notice from the previous result that the annualised return for the highest Sharpe ratio portfolio is 6.21% while the annualised return for the minimum variance portfolio is 4.06%. The difference between these two portfolios is not much in terms of returns and variance.

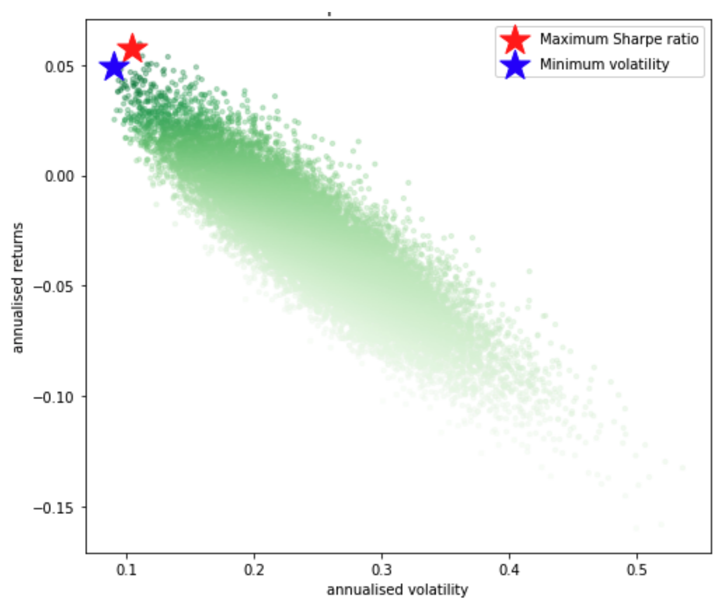


Figure 6: plotting of the simulated portfolios

3.2 Markowitz Portfolio

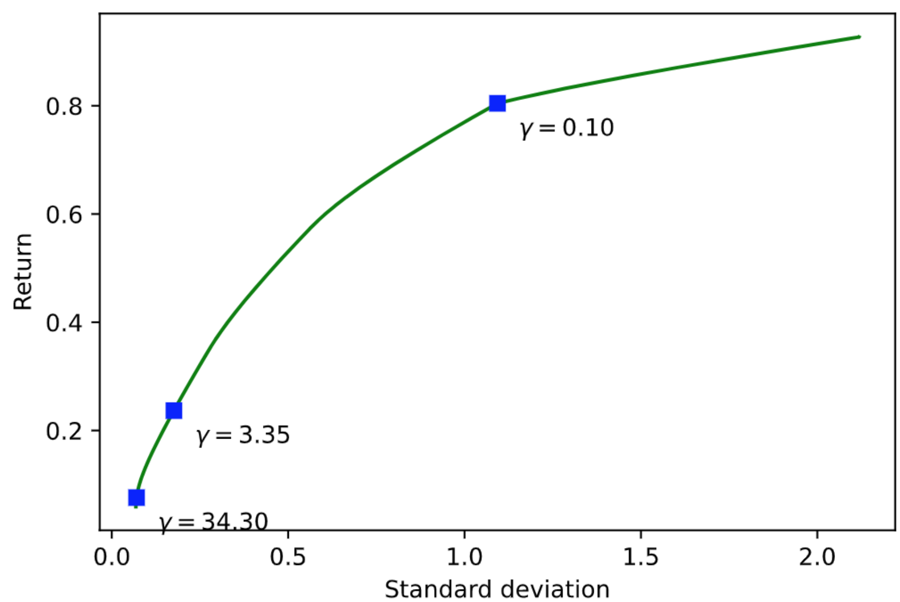
The main model we will be exploring and building is the Markowitz Portfolio Optimization Model. We believe that the Markowitz Model is a good way for us to find the maximum return portfolio while hedging the risk. In this section, we will talk about the main Markowitz Model which uses data from the whole time period, from 2018-01-01 to 2020-01-01. In later parts of the report, we will also explore more about the applications of Markowitz Model for different market phases, so that we can see in which phases cryptocurrency is a better way of investing. We will also find a maximum sharpe ratio portfolio, using the method of Monte Carlo simulation.

The Markowitz Portfolio Optimization Model is a convex optimization model that takes objective function and linear constraints as inputs. In this project, as suggested by the professor, we are using a Python optimization add-on called CVXPY to facilitate the creation of the optimization model. In this model, we decided to use the yearly statistics of the nine assets of our choice. That is because we believe, since we have a relatively long time span in the data, the yearly statistics are more accurate from the perspective of long-term investing. To get the statistics of annual asset returns and covariances, we multiplied the daily returns and covariance matrix by 250 to approximate the yearly data. That is because there are around 250 trading days a year.

After finishing the preprocessing of data, we proceed to build the model. The first thing to do is to create the decision variables "w", which represents the weights of assets in our portfolio. The portfolio return is then calculated by multiplying the yearly return matrix by the weights vector. Since the clients may have different risk aversions, we incorporated the risk aversion, called gamma, in our optimization model. Naturally, the portfolio will vary when we have different risk aversion values, so we will make plots to visualize the effect, as part of our model. The Sigma matrix is the covariance matrix of yearly returns, and it is later used to calculate the risk matrix, by multiplying with the weights vector.

The objective function of the model is then return minus gamma multiplied by risk. This makes sure we take the risk aversion into account when maximizing the objective function. The constraints we have on the asset weights are contained in the constraints list. Since we allow both long and short, the weights of the assets should be in the range of -1 to 1. Also, since the w variables represent the weights, the sum of them should be 1.

In the next part of the model, we compute the trade-off curve by running 100 iterations of the optimization model. For each iteration, we keep track of the weights, risk, and return, after solving the optimization problem we built previously. From the plot, we can see that the trade-off curve concaves down, which means that as the risk grows, the rate of return growth is less and less, although there will still be higher returns as the risk grows higher. The smaller the risk aversion, the large risk people are willing to take, and the more return they will gain. From the results, it is clear that the model will put more weight on the safer assets when the risk aversion is high, at the cost of having a lower return.



Another insight we gained from the model is that cryptocurrencies are not good assets for long term investment. The yearly return we get from the historical data is not positive, and this means that the overall return of cryptocurrencies is not very satisfying although they can give shockingly high returns in the short term. According to the results of our Markowitz Model, the cryptocurrencies are used to diversify the risk, rather than used to gain high returns. It is clear that long term investment in cryptocurrencies is not advised. On the other hand, this result also suggests the potential of using cryptocurrencies for short term speculation. The not-so-good long term return means that there must be periods that the cryptocurrencies are performing very well, and there are periods that they are crashing. Our next goal is to find these phases, and apply the Markowitz Model to each phase for better investment insights.

3.3 Four-phase Markowitz Portfolio with Wyckoff's Market Cycle

In the previous section, we found that Markowitz portfolio may be better used for short-term speculation. Therefore, we apply the same model structure of Markowitz portfolio optimization to each of the four phases of Wyckoff's market cycle of Bitcoin from January 2018 to December 2019. By comparing the historical development of closing price in Figure 2 with the Wyckoff cycle in Figure 1, we can approximate the time intervals for the four phases: the markdown phase is from January 1st 2018 to November 30th 2018, the accumulation phase is from December 1st 2018 to March 31st 2019, the markup phase is form April 1st 2019 to June 30th 2019, and the distribution phase is from July 1st 2019 to December 31st 2019. The efficient frontier of each phase is shown in Figure 3.

	^GSPC	^RUT	NSDQ	LTC-USD	ETH-USD	BTC-USD	HG=F	CL=F	^GC=F	Portfolio Return
Markowitz	1.000	-0.823	0.559	-0.044	0.168	0.180	-0.573	0.088	0.783	0.236
Markowitz-Markdown	0.721	-0.561	1.000	-0.144	-0.115	0.039	-0.341	-0.153	0.555	0.435
Markowitz-Accumulation	1.000	-0.567	-1.000	0.766	-0.050	-1.000	0.162	0.690	1.000	1.600
Markowitz-Markup	1.000	0.001	-0.049	-0.136	-0.038	0.864	-1.000	-0.643	1.000	2.356
Markowitz-Distribution	0.462	-0.058	0.388	-0.744	-0.139	0.565	-0.334	0.053	0.817	1.024

Table. 2 Asset Weight and Portfolio Return for Gamma=3.35

During the chosen time period, all cryptocurrencies had extremely large standard deviations, so we would like to explore the weights on each asset during this time for investors holding cryptocurrency portfolios. We assume that risk-averse investors would rather quit trading cryptocurrencies in the highly volatile period. Therefore, we choose and focus on comparing relatively risk-seeking Markowitz portfolios for the four phases of Bitcoin's market cycle. First of all, we found that during the markdown period, the highest weights on long positions are 100% for NASDAQ 100 and 72.1% for S&P 500. We also short sell Russell 400 for 56.1%, Copper for 34.1%, Crude Oil for 15.3%, Litecoin for 14.4%, and Ethereum for 11.5%. The portfolio return ends up at 43.5%. During the accumulation phase, we go with full long positions for S&P 500 and Gold, while take full short positions for NASDAQ 100 and Bitcoin.

Meanwhile, we have relatively high weights allocated in Litecoin and Crude Oil. As a result, the portfolio return achieved during the accumulation phase is 160%. Next, the Markowitz portfolio for the Markup phase has a full long position in both S&P 500 and Gold, and a full short position in Copper. Bitcoin has a high positive weight of 86.4%, and we short sell Crude for 64.3%. By the end of the markup phase, we attained a 235.6% portfolio return. Last but not least, we apply the Markowitz model to the distribution phase, and we have Gold with the highest weight of 81.7%, and Bitcoin with the second highest weight of 56.5%. In this case, we short sell Litecoin for 74.4%. At the end of the distribution phase, we can achieve a 102.4 % portfolio return.

Overall, portfolio return for each of the four phases is much higher than the return of the previous full-cycle Markowitz portfolio. In particular, the portfolio return for the markup phase is about 10 times the return of the full-cycle Markowitz portfolio, and even the markdown phase portfolio has twice as much return as the full-cycle portfolio.

Within the cycle, we noticed that Bitcoin has the highest weight in long position during the markup phase, and the highest weight in short position during the accumulation phase. This strategy takes advantage of the high volatility of Bitcoin, so we can hedge with Bitcoin when the market has a relatively low return period of accumulation and hold a long position of Bitcoin when the market is in an uptrend. Meanwhile, assets such as S&P 500 and Gold have low volatility and relatively high returns, and are allocated with high weights in long positions for all the phases, especially a full long position in accumulation and markup phases. This long position preference reflects that the optimal portfolio from the Markowitz model attains the lowest volatility and highest return for a given level of risk aversion. Among the three cryptocurrencies, Litecoin and Ethereum have relatively high weights in short position in the phase that Bitcoin has high weights in long position. Since both Litecoin and Ethereum have higher returns and volatilities than Bitcoin, it is used for hedging the position of Bitcoin when Bitcoin fluctuates.

By comparing the weights of each asset in the four phases with their weights in the full-cycle Markowitz model, we discovered that the weight allocation changes dramatically within a cycle. More importantly, by taking advantage of the high volatility of cryptocurrency, we can vary the relative weights and short/long position to attain a much higher portfolio return with the Markowitz model. In this case, we can identify the corresponding phase of the market cycle and construct an optimal portfolio that outperforms the classic full-cycle Markowitz portfolio.

Acknowledgement

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