Time Series Forecasting with Autoregressive Processes

A hands-on tutorial on AR(p) process for time series analysis in Python

Marco Peixeiro Follow
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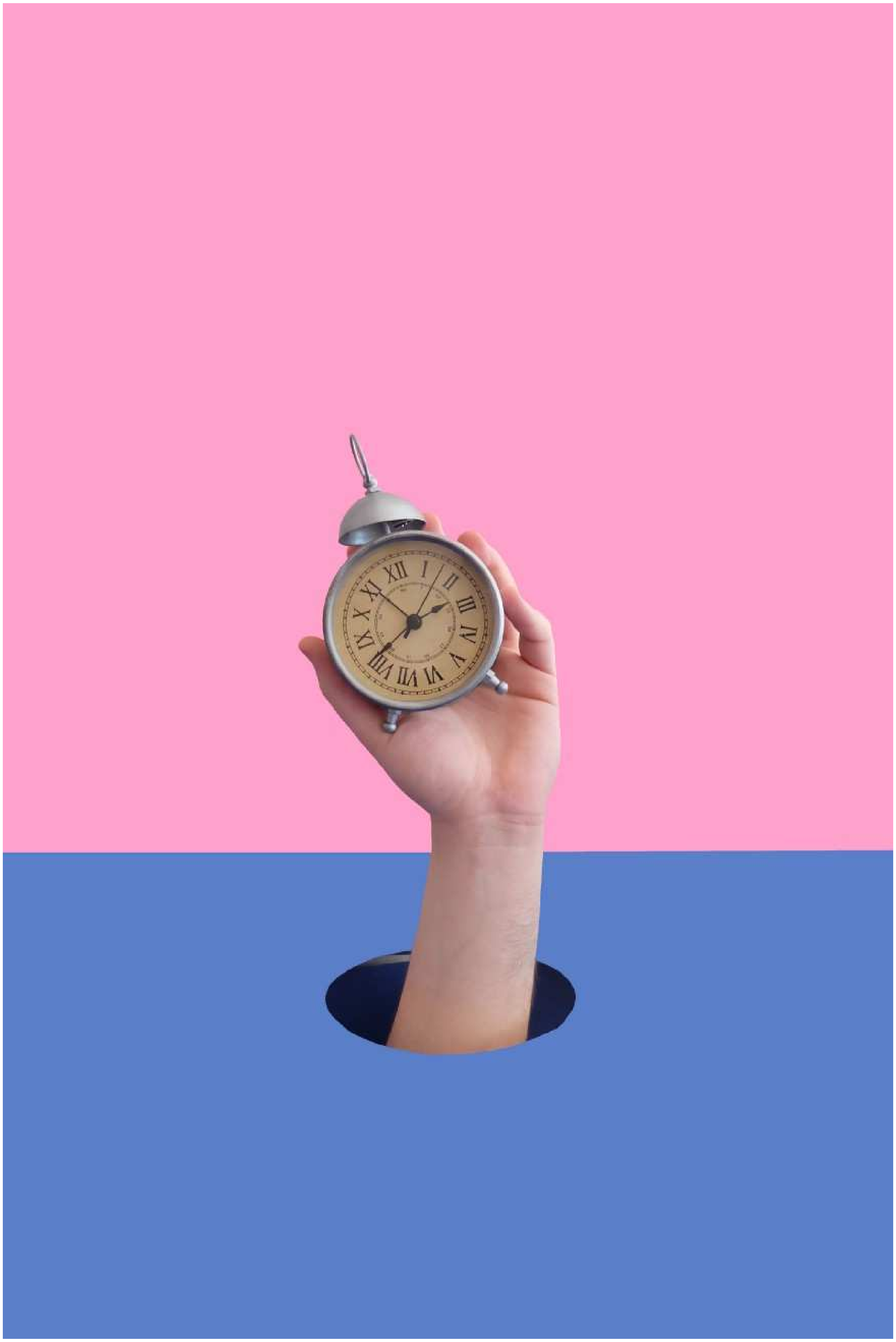
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Introduction

In this hands-on tutorial, we will cover the topic of time series modelling with autoregressive processes.

This article will cover the following key elements in time series analysis:

- autoregressive process
- Yule-Walker equation
- stationarity
- Augmented Dicker-Fuller test

Make sure to have a Jupyter notebook ready to follow along. The code and the dataset is available here.

Let’s get started!

> For hands-on video tutorials on machine learning, deep learning, and artificial intelligence, checkout my YouTube channel.

Autoregressive Process

An **autoregressive model** uses a linear combination of past values of the target to make forecasts. Of course, the regression is made against the target itself. Mathematically, an AR(p) model is expressed as:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t$$

Autoregressive model

Where:

- p: is the order
- c: is a constant
- epsilon: noise

AR(p) model is incredibly flexible and it can model a many different types of time series patterns. This is easily visualized when we simulate autoregressive processes.

Usually, autoregressive models are applied to stationary time series only. This constrains the range of the parameters *phi*.

For example, an AR(1) model will constrain *phi* between -1 and 1. Those constraints become more complex as the order of the model increases, but they are automatically considered when modelling in Python.

Simulation of an AR(2) process

Let's simulate an AR(2) process in Python.

We start off by importing some libraries. Not all will be used for the simulation, but they will be required for the rest of this tutorial.

```
from statsmodels.graphics.tsaplots import plot_pacf
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.stattools import pacf
from statsmodels.regression.linear_model import yule_walker
from statsmodels.tsa.stattools import adfuller
import matplotlib.pyplot as plt
import numpy as np

%matplotlib inline
```

We will use the *ArmaProcess* library to simulate the time series. It requires us to define our parameters.

We will simulate the following process:

$$y_t = 1 + 0.33\, y_{t-1} + 0.5\, y_{t-2}$$

AR(2) process

Since we are dealing with an autoregressive model of order 2, we need to define the coefficient at lag 0, 1 and 2.

Also, we will cancel the effect of a moving average process.

Finally, we will generate 10 000 data points.

In code:

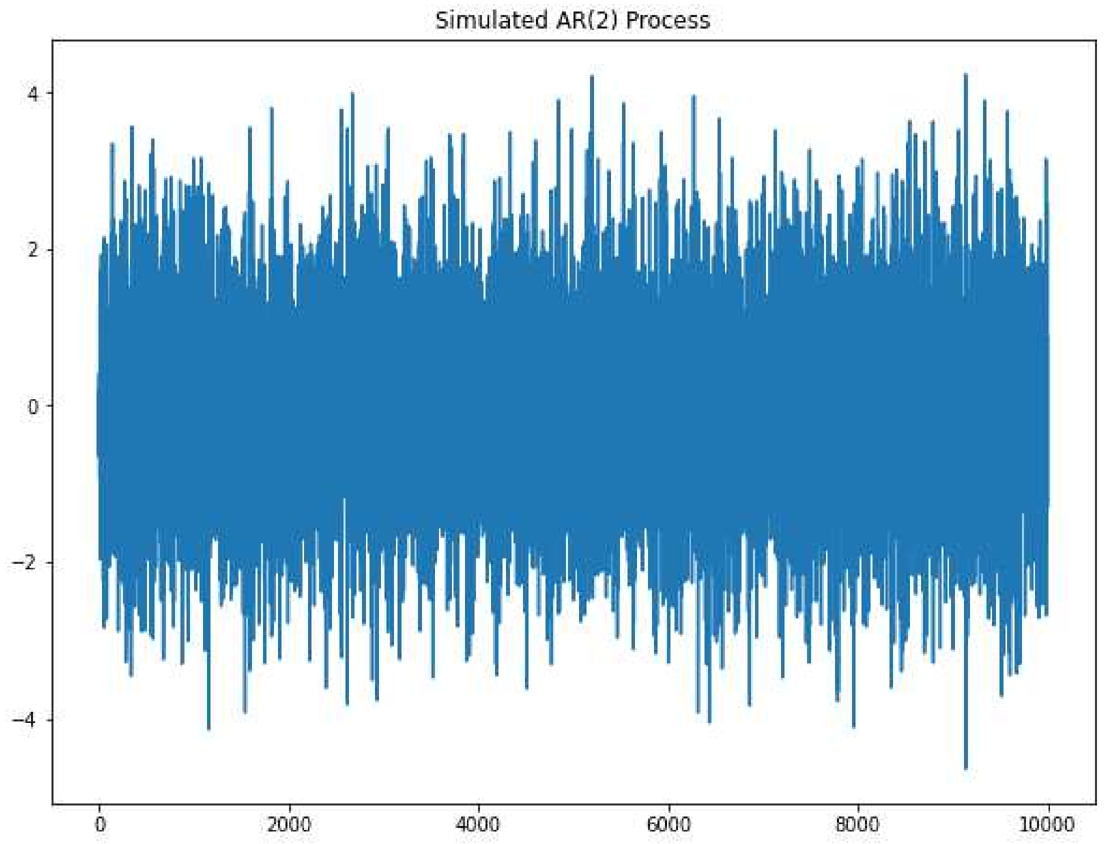
```
ar2 = np.array([1, 0.33, 0.5])
ma = np.array([1])

simulated_AR2_data = ArmaProcess(ar2,
ma).generate_sample(nsample=10000)
```

We can plot the time series:

```
plt.figure(figsize=[10, 7.5]); # Set dimensions for figure
plt.plot(simulated_AR2_data)
plt.title("Simulated AR(2) Process")
plt.show()
```

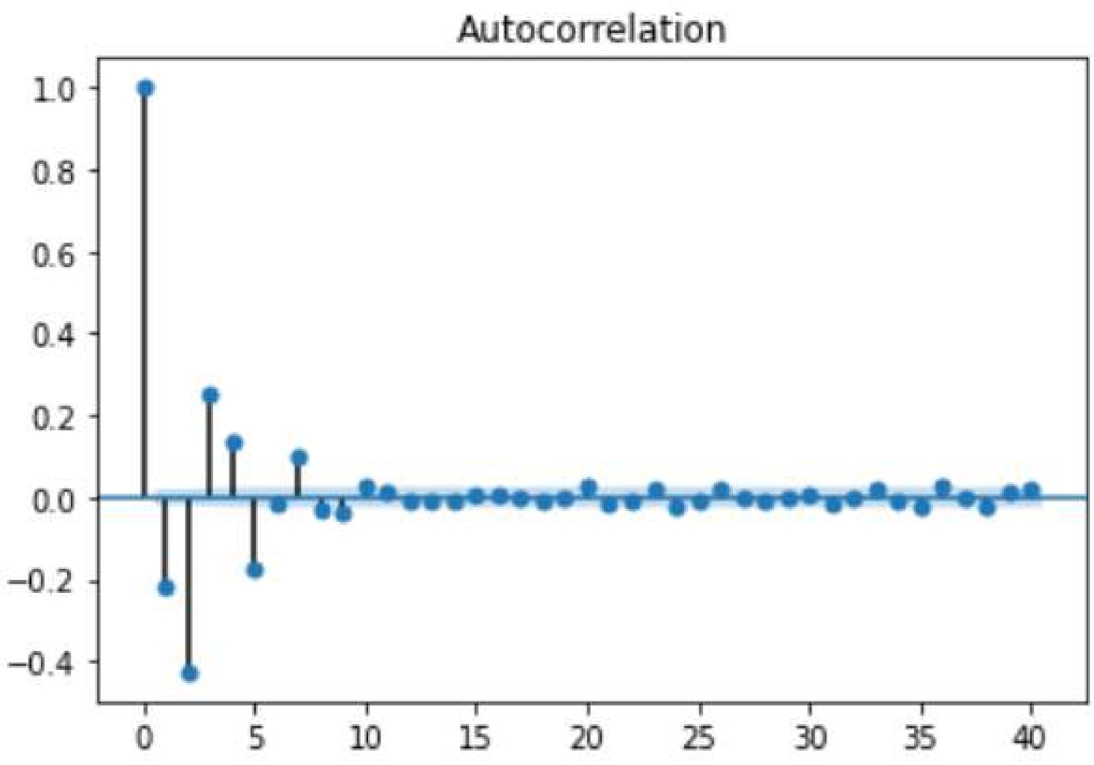
And you should get something similar to this:



Plot of the simulated AR(2) process

Now, let’s take a look at the autocorrelation plot (correlogram):

```
plot_acf(simulated_AR2_data);
```

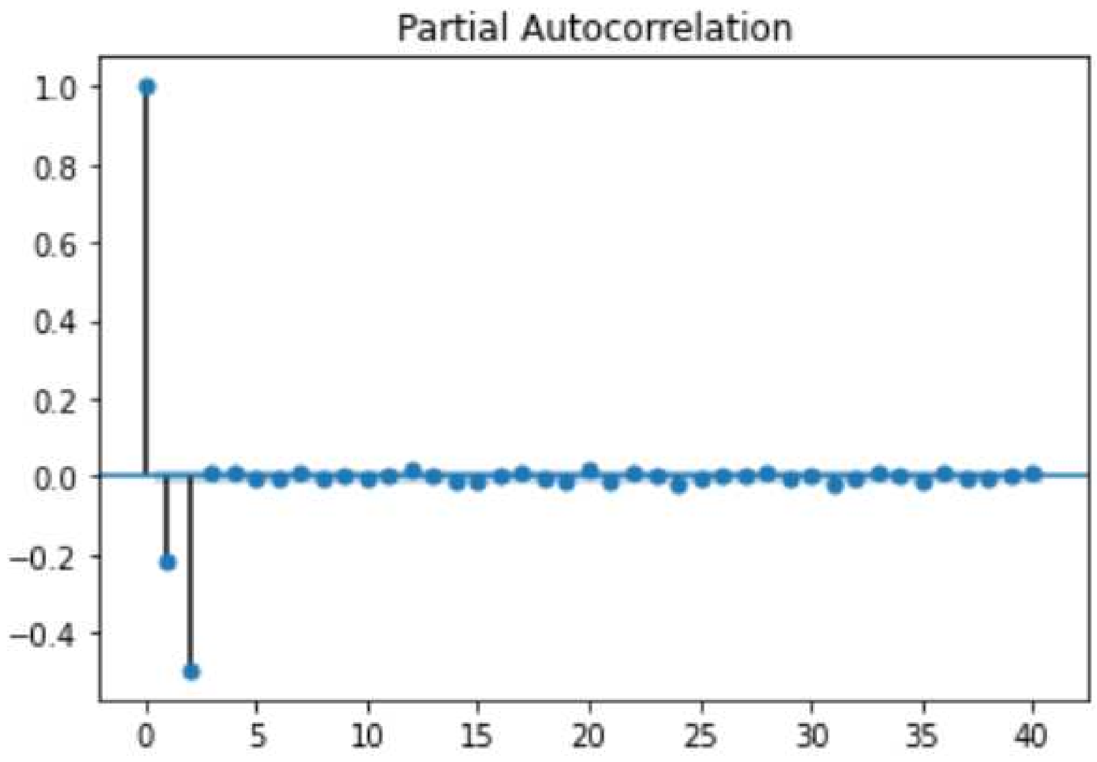


Correlogram of an AR(2) process

You can see that the coefficient is slowly decaying. This means that it is unlikely a moving average process and it suggests that the time series can probably be modelled with an autoregressive process (which makes sense since that what we are simulating).

To make sure that this is right, let’s plot the partial autocorrelation plot:

```
plot_pacf(simulated_AR2_data);
```



Partial autocorrelation plot for an AR(2) process

As you can see the coefficients are not significant after lag 2. Therefore, the partial autocorrelation plot is useful to determine the order of an AR(p) process.

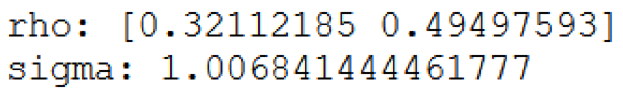
You can also check the values of each coefficients by running:

```
pacf_coef_AR2 = pacf(simulated_AR2_data)
print(pacf_coef_AR2)
```

Now, in a real project setting, it can be easy to find the order of an AR(p) process, but we need to find a way to estimate the coefficients *phi*.

To do so, we use the Yule-Walker equation. This equations allows us to estimate the coefficients of an AR(p) model, given that we know the order.

```
rho, sigma = yule_walker(simulated_AR2_data, 2, method='mle')
print(f'rho: {-rho}')
print(f'sigma: {sigma}')
```



```
rho: [0.32112185 0.49497593]
sigma: 1.006841444461777
```

Yule-Walker coefficient estimates

As you can see, the Yule-Walker equation did a decent job at estimating our coefficients and got very close to 0.33 and 0.5.

Simulation of an AR(3) process

Now, let's simulate an AR(3) process. Specifically, we will simulate:

$$y_t = 1 + 0.33\, y_{t-1} + 0.5\, y_{t-2} + 0.07\, y_{t-2}$$

AR(3) process for simulation

Similarly to what was previously done, let's define our coefficients and generate 10 000 data points:

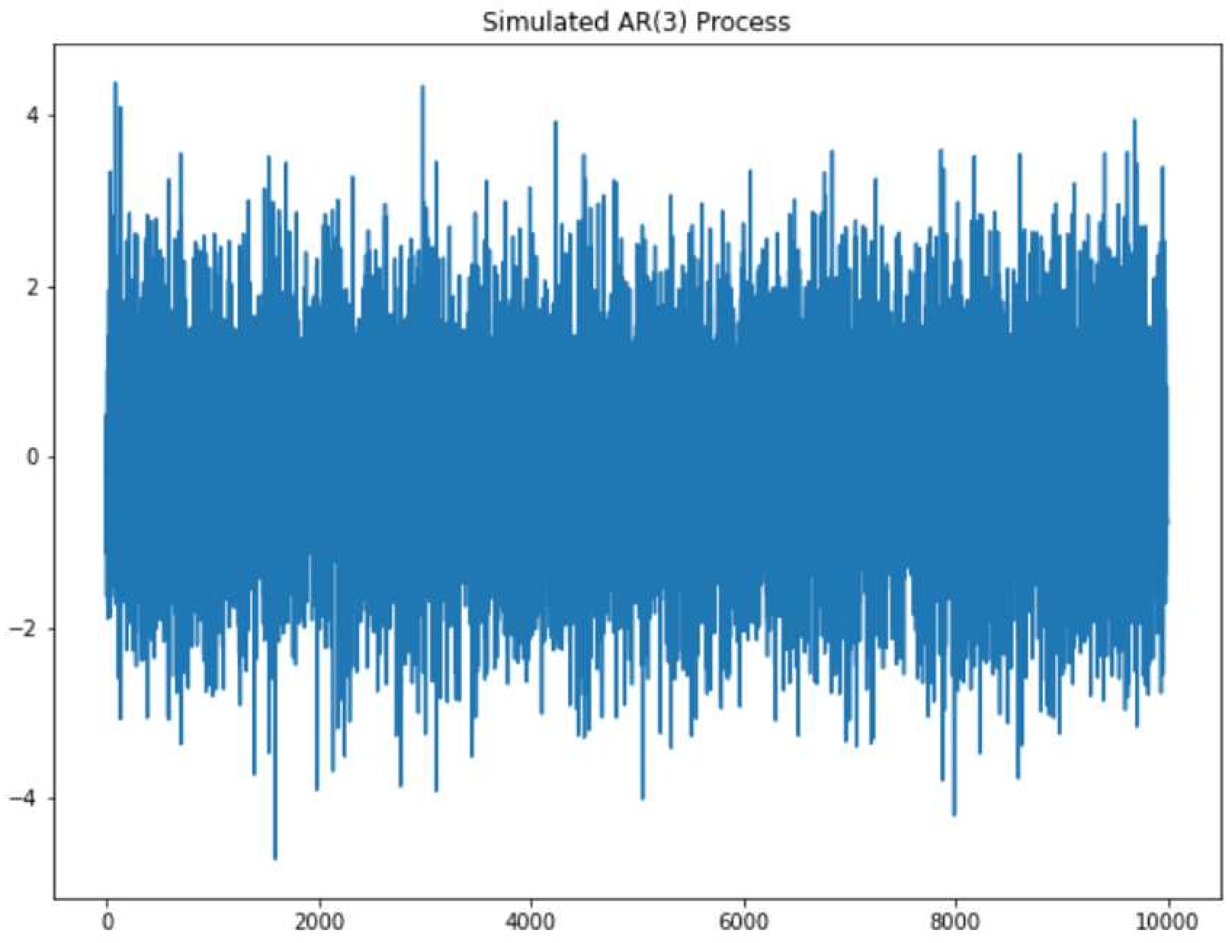
```
ar3 = np.array([1, 0.33, 0.5, 0.07])
ma = np.array([1])

simulated_AR3_data =
ArmaProcess(ar3,ma).generate_sample(nsample=10000)
```

Then, we can visualize the time series:

```
plt.figure(figsize=[10, 7.5]); # Set dimensions for figure
plt.plot(simulated_AR3_data)
plt.title("Simulated AR(3) Process")
plt.show()
```

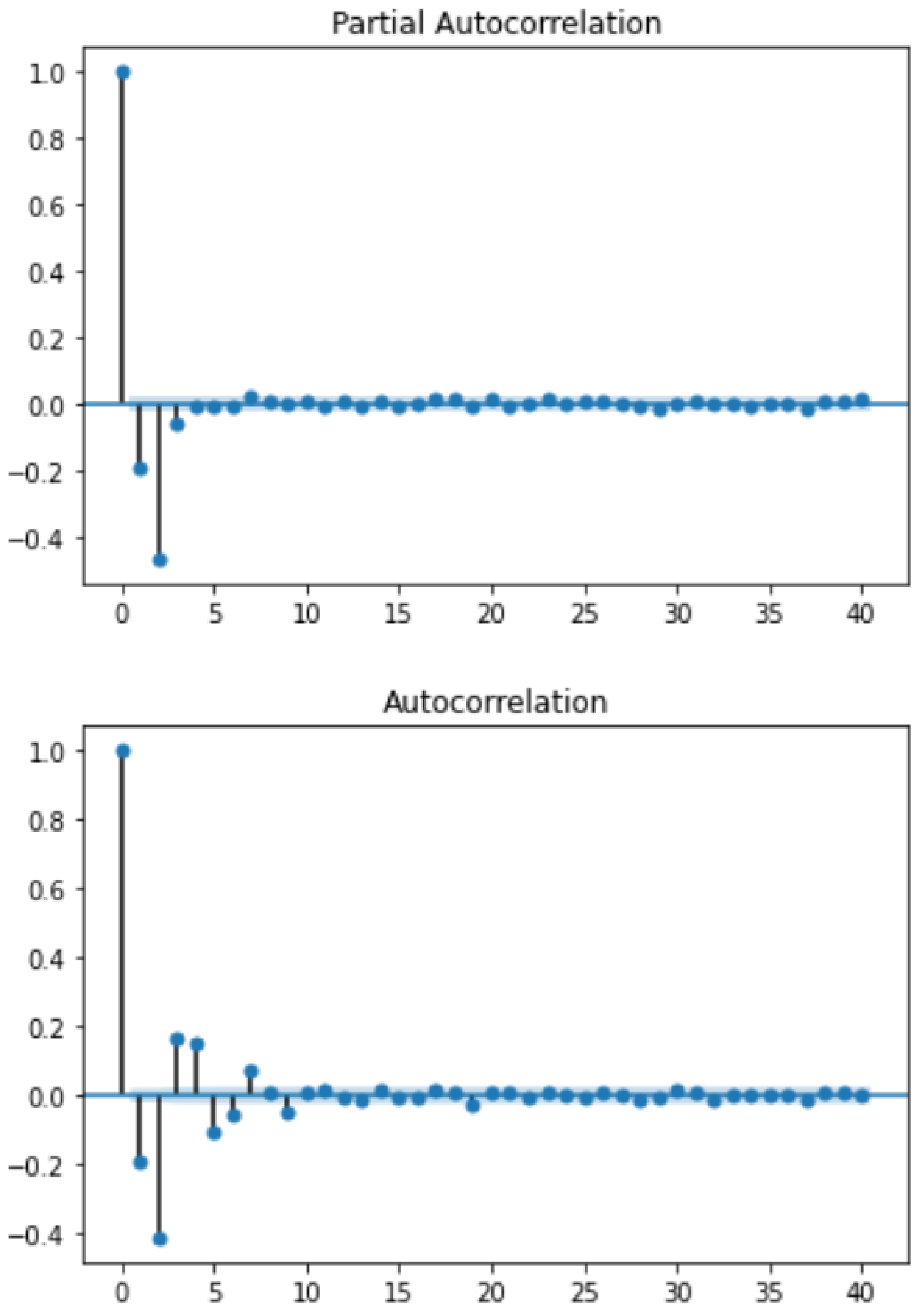
And you should see something similar to:



Simulated AR(3) process

Now, looking at the PACF and ACF:

```
plot_pacf(simulated_AR3_data);
plot_acf(simulated_AR3_data);
```

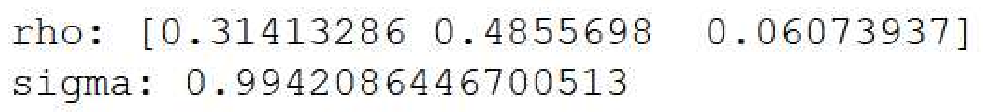


PACF and ACF for an AR(3) process

You see that the coefficients are not significant after lag 3 for the PACF function as expected.

Finally, let's use the Yule-Walker equation to estimate the coefficients:

```
rho, sigma = yule_walker(simulated_AR3_data, 3, method='mle')
print(f'rho: {-rho}')
print(f'sigma: {sigma}')
```



```
rho: [0.31413286 0.4855698  0.06073937]
sigma: 0.9942086446700513
```

Yule-Walker coefficient estimates

Again, the estimations are fairly close to the actual values.

Project — Forecasting the quarterly EPS for Johnson&Johnson

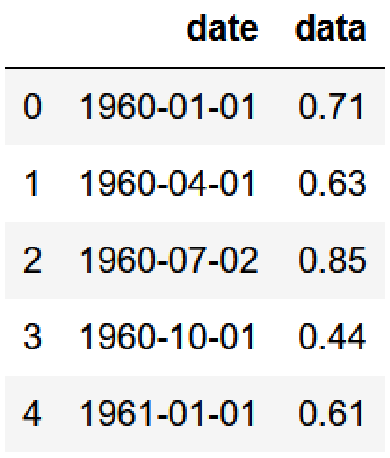
Now, let's apply our knowledge of autoregressive processes in a project setting.

The objective is to model the quarterly earnings per share (EPS) of the company Johnson&Johnson between 1960 and 1980.

First, let's read the dataset:

```
import pandas as pd


data = pd.read_csv('jj.csv')
data.head()
```

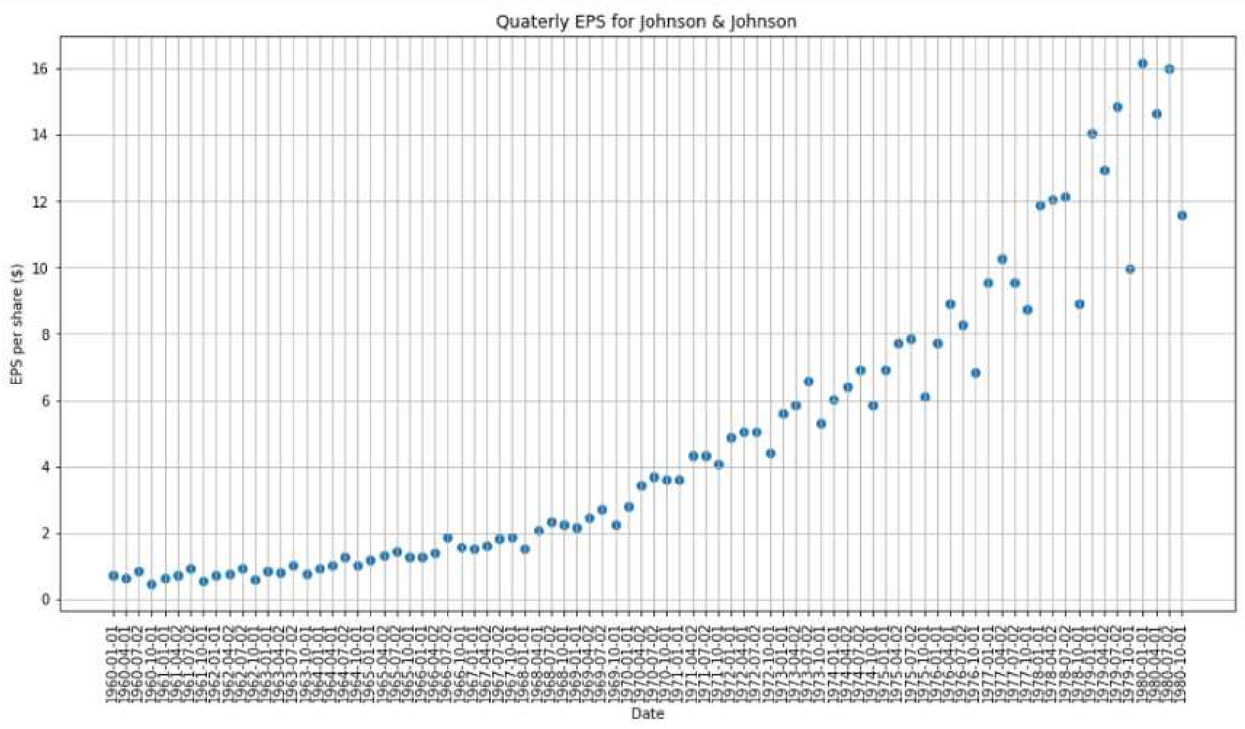


	date	data
0	1960-01-01	0.71
1	1960-04-01	0.63
2	1960-07-02	0.85
3	1960-10-01	0.44
4	1961-01-01	0.61

First five rows of the dataset

Now, the first five rows are not very useful for us. Let’s plot the entire dataset to get a better visual representation.

```
plt.figure(figsize=[15, 7.5]); # Set dimensions for figure
plt.scatter(data['date'], data['data'])
plt.title('Quaterly EPS for Johnson & Johnson')
plt.ylabel('EPS per share ($)')
plt.xlabel('Date')
plt.xticks(rotation=90)
plt.grid(True)
plt.show()
```



Quaterly EPS for Johnson&Johnson between 1960 and 1980

Awesome! Now we can the there is clear upwards trend in the data. While this may be a good sign for the company, it is not good in terms of time series modelling, since it means that the time series is not stationary.

As aforementioned, the AR(p) process works only for stationary series.

Therefore, we must apply some transformations to our data to make it stationary.

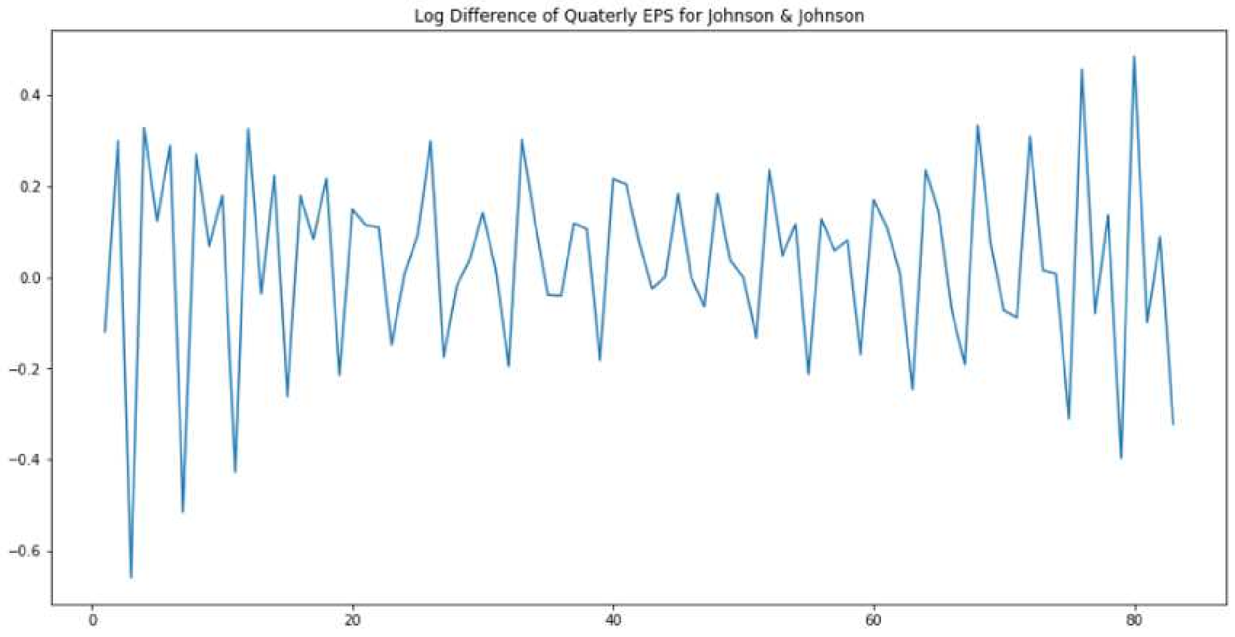
In this case, will take the log difference. This is equivalent to taking the log of each value, and subtracting the previous value.

```
# Take the log difference to make data stationary


data['data'] = np.log(data['data'])
data['data'] = data['data'].diff()
data = data.drop(data.index[0])
data.head()
```

Plotting the transformed time series:

```
plt.figure(figsize=[15, 7.5]); # Set dimensions for figure
plt.plot(data['data'])
plt.title("Log Difference of Quaterly EPS for Johnson & Johnson")
plt.show()
```

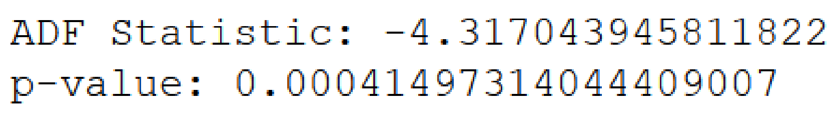


Log difference of quarterly EPS for Johnson&Johnson

Now, it seems that we removed the trend. However, we have to be sure that our series is stationary before modelling with an AR(p) process.

We will thus use the augmented Dicker-Fuller test. This will give us the statistical confidence that our time series is indeed stationary.

```
ad_fuller_result = adfuller(data['data'])
print(f'ADF Statistic: {ad_fuller_result[0]}')
print(f'p-value: {ad_fuller_result[1]}')
```



```
ADF Statistic: -4.317043945811822
p-value: 0.00041497314044409007
```

Results of the ADF test

Since we get a large negative ADF statistic and *p-value* smaller than 0.05, we can reject the null hypothesis and say that our time series is stationary.

Now, let's find the order of the process by plotting the PACF:

```
plot_pacf(data['data']);
plot_acf(data['data']);
```

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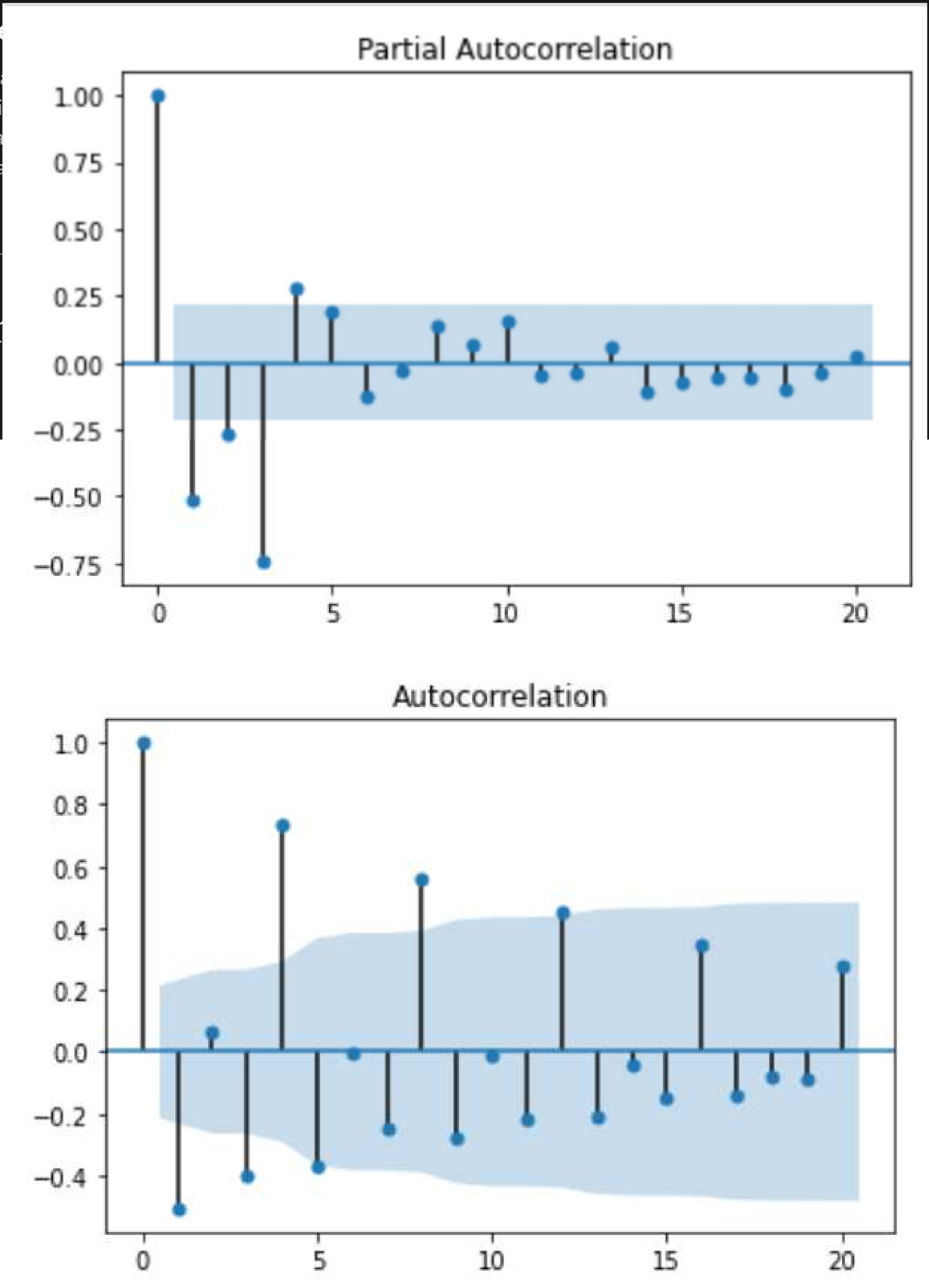
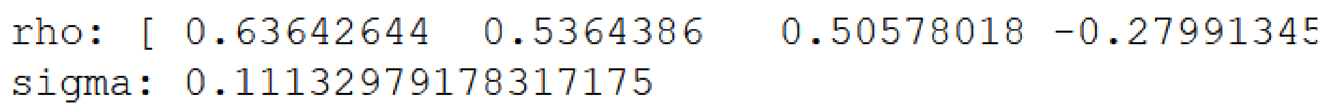


PACF and ACF

As you can see, after lag 4, the PACF coefficients are not significant anymore. There
we will assume an autoregressive process of order 4.

Now, we will use this information to estimate the coefficients using the Yule-Walke
equation:

```
# Try a AR(4) model
rho, sigma = yule_walker(data['data'], 4)
print(f'rho: {-rho}')
print(f'sigma: {sigma}')
```



```
rho: [ 0.63642644  0.5364386   0.50578018 -0.2799134
sigma: 0.11132979178317175
```

Yule-Walker coefficient estimates

Therefore, the function is approximated as:

$$y_t \approx 0.01 + 0.64\, y_{t-1} + 0.54\, y_{t-2} + 0.51\, y_{t-3} - 0.28$$

Note that this equation models the transformed series.

Conclusion

Congratulations! You now understand what an autoregressive model is, how to rec
an autoregressive process, how to determine its order, and how to use it to model a
life time series.

Learn more about time series with the following resources (I earn a commission if
opt in one of the courses below):

- Practical Time Series Analysis
- Sequences, Time Series and Prediction

Cheers 🍺