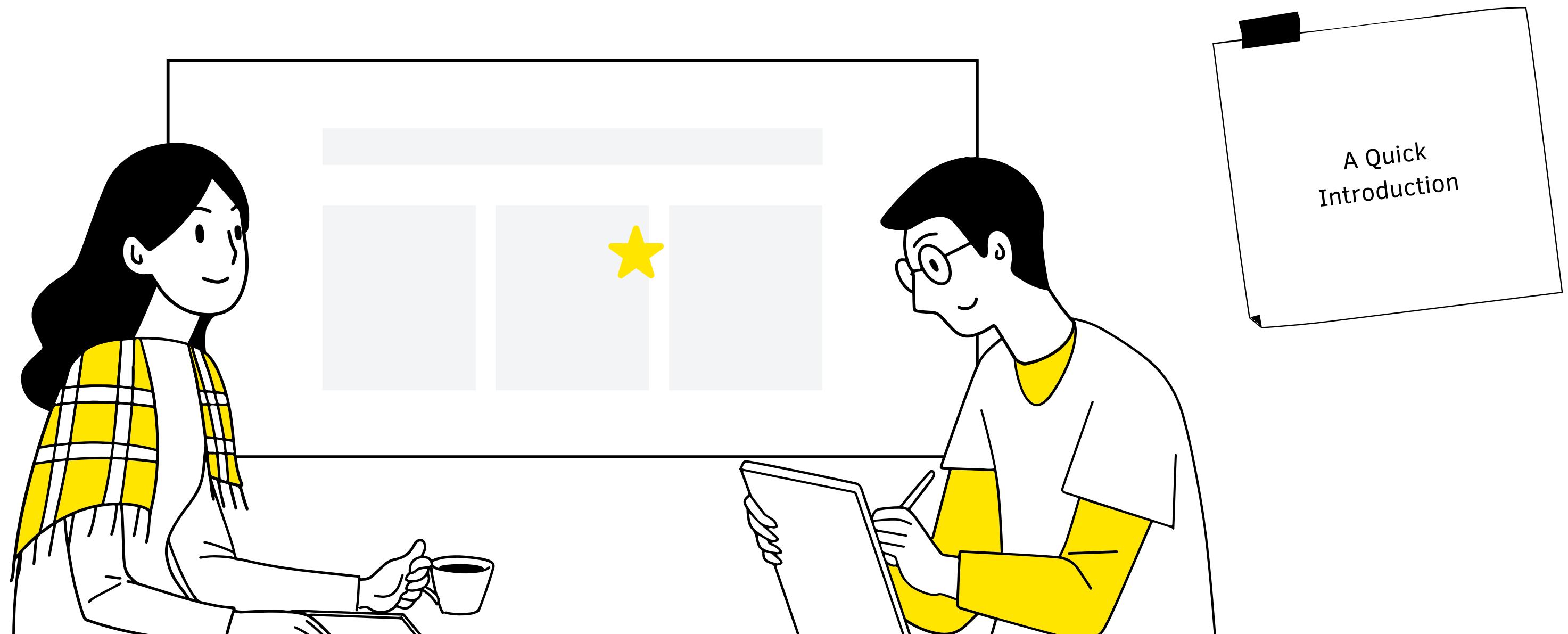


# Game Theory

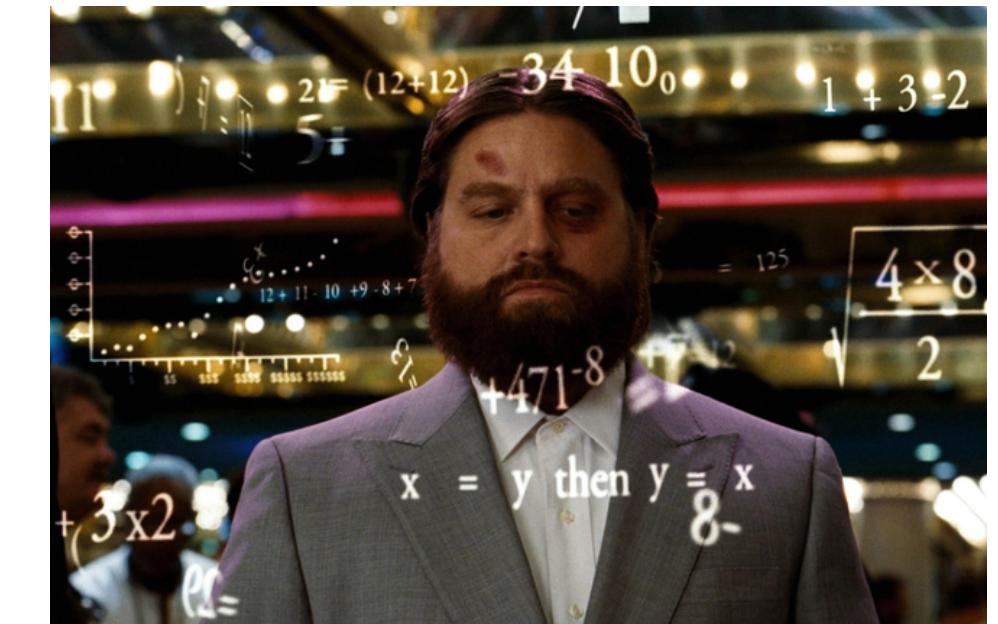




# What my mom thinks I do



# What my friends think I do



# What society thinks I do



# What my teachers think I do



# What I think I do



# What I actually do

# We'll discuss

1

What is game theory?

3

Zero-sum games

2

Combinatorial games

4

General-sum games

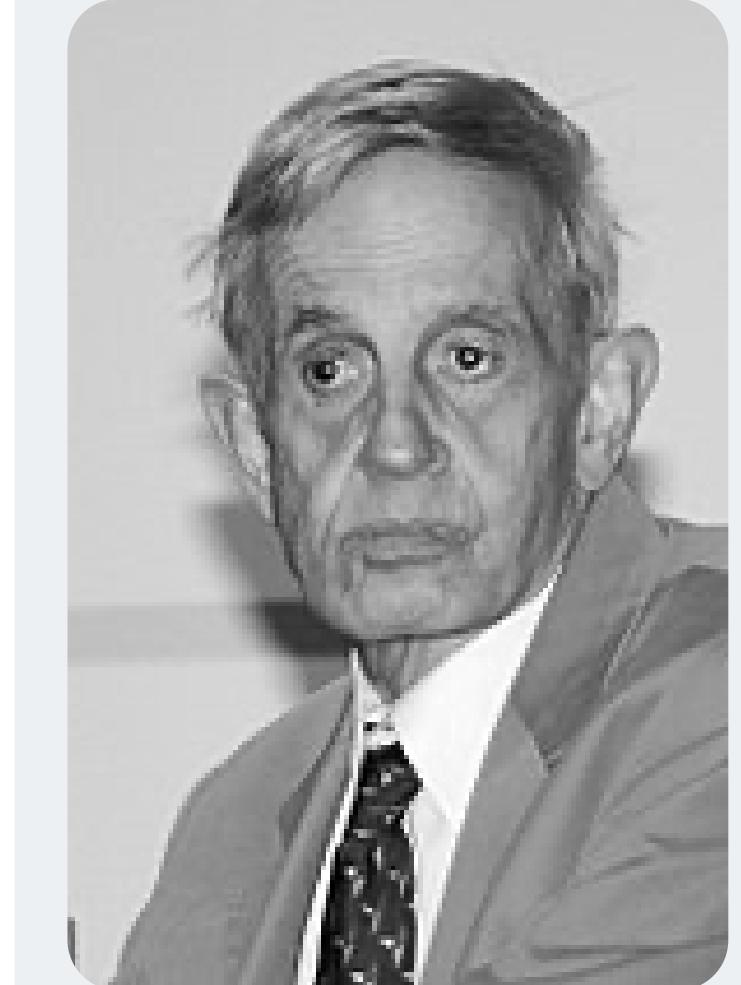
# Game Theory

Study of mathematical models of strategic interactions among **rational** decision-makers

von Neumann



John Nash



# Fields

Economics

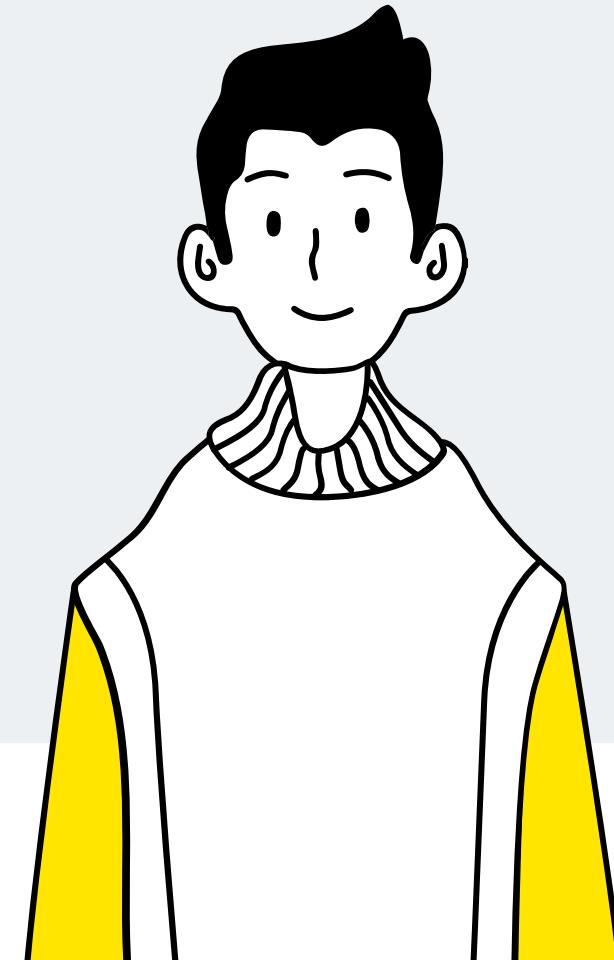
Biology

Political  
Science

Sociology

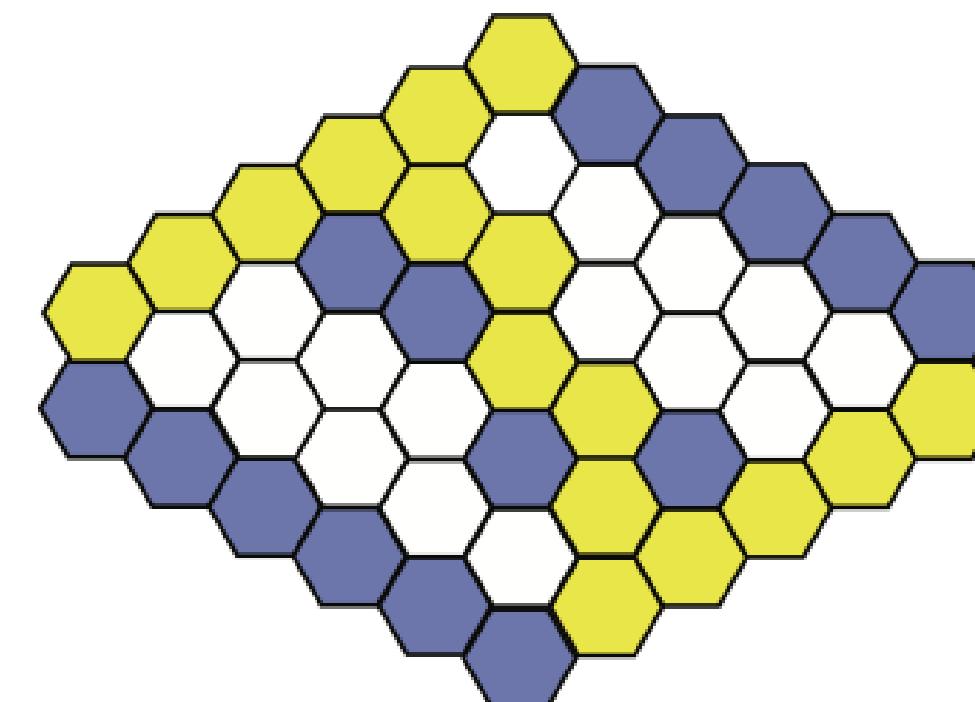
# Combinatorial games

- Two players
- Set of positions
- Set of legal moves between positions
- Some of positions are terminal
- Each terminal position is labelled as winning for either player I or II



# The game of Hex

- Each player is assigned a color, and two opposing sides of the board
- The players take turns coloring in empty hexagons
- The goal for each player is to link two sides of the board with a chain of hexagons in his color



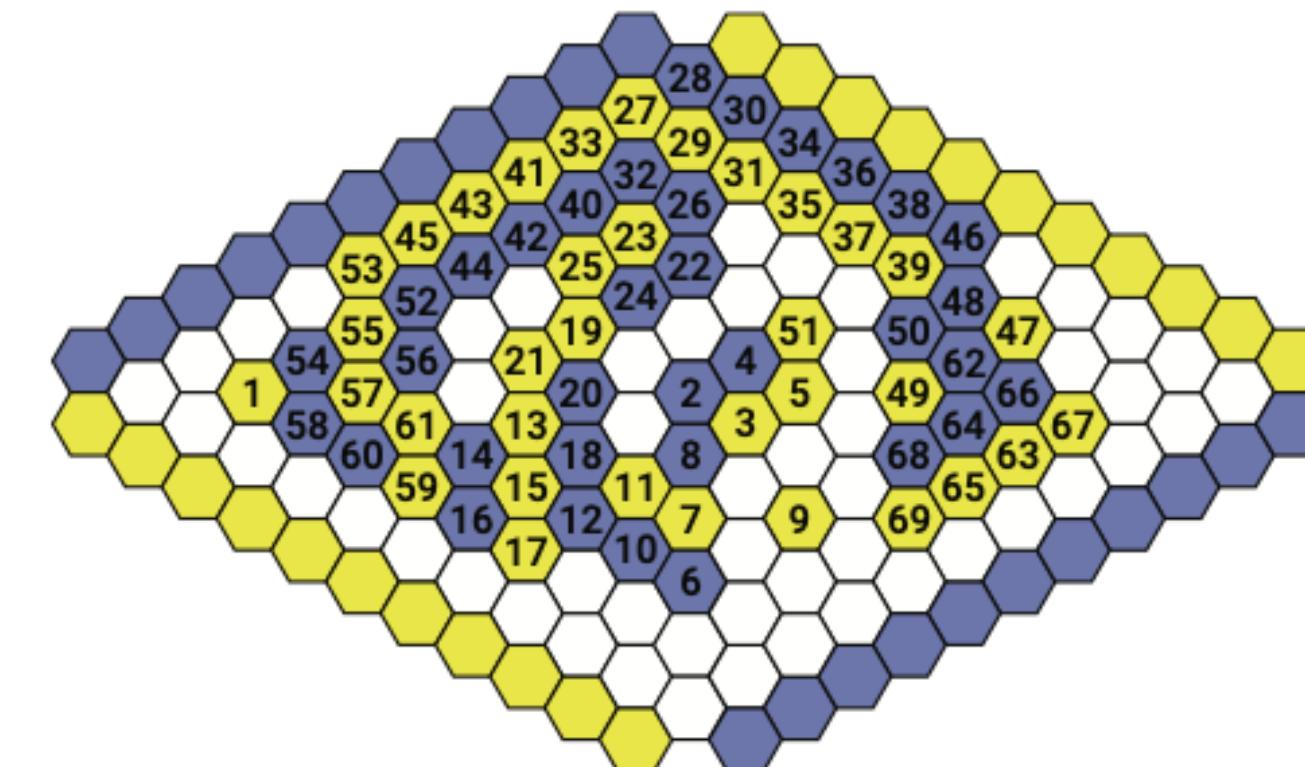
# The game of Hex

## Statement

Blue moves next, but Yellow has a winning strategy

## Theorem

The player who moves first has a winning strategy



# Zero-sum games

- Two players
- One player's loss is the other player's gain.
- Can be represented by an  $m \times n$  payoff matrix A, whose rows are indexed by the m possible actions of player I and whose columns are indexed by the n possible actions of player II.

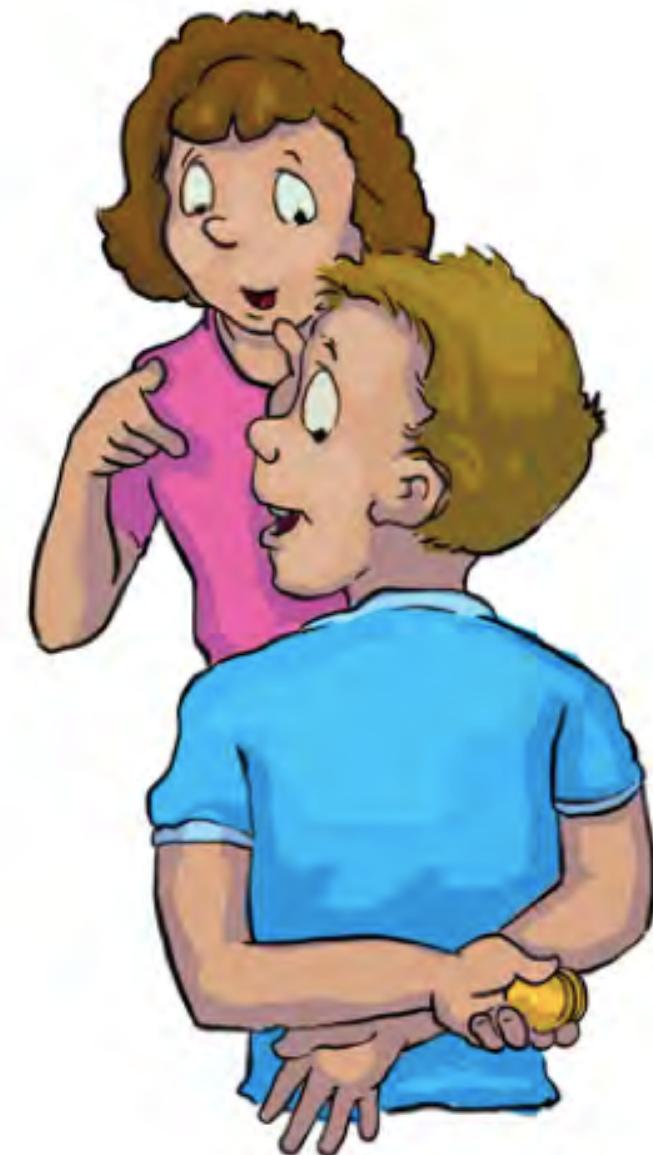


# Pick a hand

- There are two players, Chooser and Hider
- Hider has two gold coins in his back pocket
- Either takes out one coin and holds it in his left hand or takes out both and holds them in his right hand
- Chooser picks a hand and wins any coins the hider has hidden there



**How much should Chooser be willing  
to pay in order to play this game?**



		Hider	
		L1	R2
Chooser	L	1	0
	R	0	2

No matter how Chooser plays, Hider can guarantee himself an expected loss of at most  $2/3$

# Zero-sum games

## Theorem

Even if each player's strategy is known to the other, there is an amount that one player can guarantee as her expected gain, and the other, as his maximum expected loss.



# Thank you for your attention

