The Newtonian Limit in CDT

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Does CDT have a Newtonian Limit?

- CDT looks like GR at cosmological scales, does it have a Newtonian limit?

$$F = \frac{G m_1 m_2}{r^2}$$

- At first glance, this is hard:
  - CDT is not well suited for approximating smooth classical space-times
  - We don't have the time or resolution to watch objects fall

A trick from GR

- The static axisymmetric Weyl metric

$$ds^2 = e^{2\lambda}dt^2 - e^{2(\nu-\lambda)}\left(dr^2 + dz^2\right) - r^2 e^{-2\lambda} d\phi^2$$

- With two-body solutions

$$\lambda(r,z) = -\frac{\mu_1}{R_1} - \frac{\mu_2}{R_2}, \quad R_i = \sqrt{r^2 + (z-z_i)^2}$$

$$\nu(r,z) = -\frac{\mu_1^2 r^2}{R_1^4} - \frac{\mu_2^2 r^2}{R_2^4} + \frac{4\mu_1\mu_2}{(z_1-z_2)^2}\left[\frac{r^2 + (z-z_1)(z-z_2)}{R_1 R_2} - 1\right]$$

- Leads to a “strut”

$$\nu(0,z) = \frac{4\mu_1\mu_2}{(z_1-z_2)^2}$$

- With a stress

$$T_{zz} = \frac{1}{8\pi G}\left(1 - e^{-\nu(r,z)}\right) 2\pi\delta(r)$$

- That can be integrated to get the Newtonian force

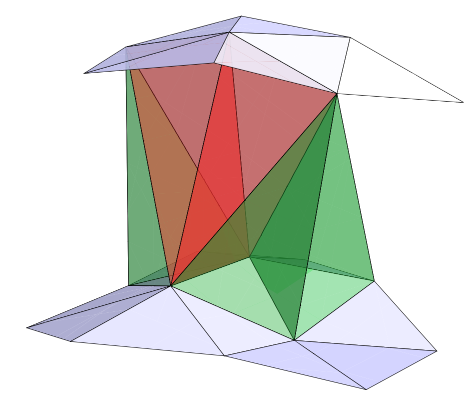
$$F = \int T_{zz} dA = \frac{1}{4G}\left(1 - e^{-\nu(r,z)}\right) = \frac{G m_1 m_2}{(z_1-z_2)^2} \quad \text{for} \quad \mu_i = G m_i$$

Measuring Mass in CDT

Mass ➔ Epp quasilocal energy

$$E_E \equiv -\frac{1}{8\pi G}\int_\Omega d^2x\sqrt{|\sigma|}\left(\sqrt{k^2-l^2}-\sqrt{\bar{k}^2-\bar{l}^2}\right)$$

$$l \equiv \sigma^{\mu\nu} l_{\mu\nu} \quad k \equiv \sigma^{\mu\nu} k_{\mu\nu}$$



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- In 3+1 CDT, extrinsic curvature at a face is proportional to the number of connected pentachorons (4-simplices)

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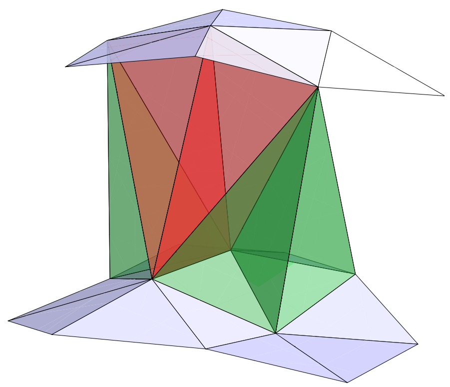

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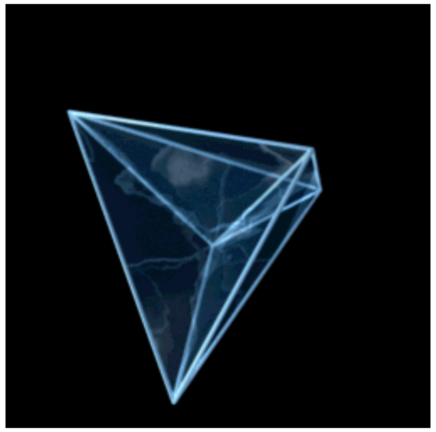

Some Computational Methods

- Distance
  - Calculate single-source shortest path between the two masses using Bellman-Ford algorithm
  - Modify allowed moves in a sweep to not permit successive moves which increase or decrease distance
- Stress
  - Hmmm ...

On-Going work

- Based on Rajesh Kommu's CDT implementation
  - arxiv.org/abs/1110.6875
- Code is on GitHub (warning: messy!)
  - https://github.com/ucdavis/CDT
- Work in early stages, results "soon"
- Thanks to Markus Afshar, Professor Steve Carlip, Joshua Cooperman, Colin Cunliff, Henrique Gomez, Rajesh Kommu, Jonah Miller, and Charles Pierce for many helpful discussions