Background Independent Quantum Gravity

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Why Quantum Gravity?

General Relativity and Quantum Mechanics are Fundamentally Incompatible

- GR violates the Uncertainty Principle
- QM violates Causality

Eppley and Hannah, 1977: A classical gravitational measurement collapses the quantum wave function. Then momentum is not conserved. Measure its position by scattering very short wavelength gravity wave, causing state to change with small uncert in x and large uncert in p. Classical gravity allows arbitrarily small momentum.

A classical gravitational measurement does not collapse the quantum wave function: then signals can be sent faster than light. Place a proton in a box in state equally likely left or right. Split the box in half and carry one half to a remote location. Monitor your half continuously with gravitational measurements, while a colleague performs a nongravitational measurement of the other half. Your colleague's measurement will collapse the wavefunction, causing an instantaneous and detectable change in the half of the box you are measuring.

Wavefunction does not collapse (Everett many-worlds). Then gravity will not have localized source. Consider a gravitating mass in a superposition of two widely separated eigenstates. If its classical gravitational field depends on its quantum wave function, its gravitational attraction should point to some intermediate average location.

Why Quantum Gravity?

General Relativity and Quantum Field Theory give different answers

- Black Hole Information Paradox
- Wormholes, Time Machines, and Warp Drives
- Origins and structure of the universe

QFT – Information never destroyed
GR – Information destroyed by singularity

GR – 1935 wormhole solutions discovered in GR (Einsten-Rosen bridges) A wormhole can be created from a black hole by placing a shell of negative stress-energy at the event horizon. But ANEC cannot be violated.
QFT – ANEC is violated by the Casimir effect and the Inflaton
Time machines can be made from wormholes by relativistically accelerating one mouth with respect to another and bringing them close together. Warp drives can be created by a metric with a region of ANEC-violating stress-energy

GR – Spinning black holes produce ring singularities, allowing gateway to another part of spacetime. Passing through the ring you witness the entire future history of the universe in finite time (probably fried by blue-shifted radiation)
Wave function of the universe: Hartle-Hawking state, no time before BB, no-boundary proposal

LQG – Recent paper allows for multiple universes in LQG. Ekpyrotic universe created by brane collisions means "Big Bang" was really a "Big Bounce". Number of dimensions? Lots of theoretical cosmological models that match lambda-CDM.

A brief history of Quantum Gravity

- 1916: Einstein points out that quantum effects must modify GR
- 1927: Oskar Klein suggests quantum gravity should modify concepts of spacetime
- 1930s: Rosenfeld applies Pauli method to linearized Einstein field equations
- 1934: Blokhintsev and Gal'perin discuss the spin-two quantum of the gravitational field, naming it the graviton
- 1936: Bronstein realizes need for background independence, issues with Riemannian geometry
- 1938: Heisenburg points out issues with the dimensionality of the gravitational coupling constant for a quantum theory of gravity
- 1949: Peter Bergmann and group begin the canonical approach to quantum gravity. Bryce Dewitt applies Schwinger's covariant quantization method to the gravitational field for his thesis.
- 1952: Gupta develops the covariant approach to quantum gravity.

Before I get to my research, let me give you some background that will help you understand the question I'm looking to answer (and also explain the two papers we're looking at)

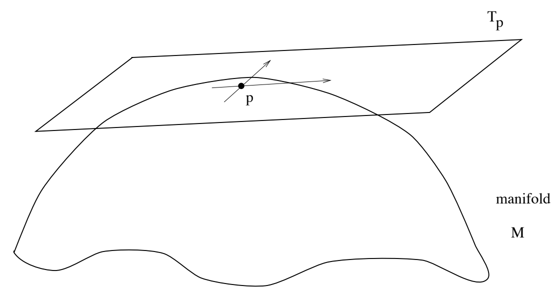
Background independence: QFT is formulated using a fixed background. GR requires the background be dynamical, resulting from solutions of the EFE

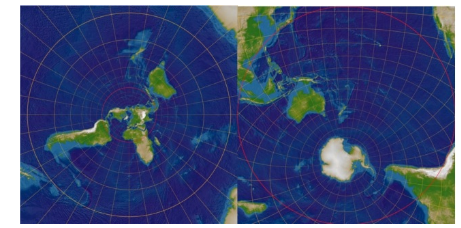
A brief history of Quantum Gravity

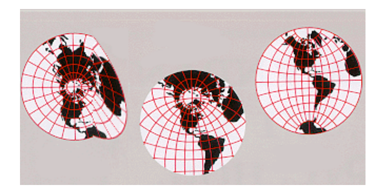
- 1957: Charles Misner introduces the "Feynman quantization of general relativity", or sum over histories approach. His paper outlines the three approaches, canonical, covariant, and sum over history in modern terms
- 1961: Arnowit, Deser, and Misner complete the ADM formulation of GR
- 1962: Using ADM, Peres writes the Hamilton-Jacobi formulation of GR
- 1964: Feynman starts loop corrections to GR, but notices unitarity was lost for naive Feynman rules. DeWitt adds correction terms involving loops of fictitious fermionic particles, solving the problem.
- 1966: Faddeev and Popov independently come up with DeWitt's "Faddeev-Popov" ghosts.
- 1967: DeWitt's seminal paper, completed in 1966 but not published until 1967 due to publishing charges, develops canonical quantum gravity and the Wheeler-DeWitt equation. DeWitt and Feynman complete the Feynman rules for GR.
- 1969: Charles Misner starts quantum cosmology by truncating the Wheeler-DeWitt equation to a finite number of degrees of freedom

The Wheeler-DeWitt equation is equation 5.20 (derived from 5.5)

Quick intro to GR







GR is defined on Pseudo-Riemannian manifolds.

Pseudo-Riemannian manifolds: a generalization of a Riemannian manifold where the metric tensor is not positive definite, such as the metric for GR

Riemannian manifold: a smooth manifold M equipped with an inner product (metric) on the tangent space TpM at each point p that varies smoothly.

Smooth manifold: A topological manifold with a globally defined differential structure. Also known as a differentiable manifold. A non-differentiable manifold example would be an atlas of charts for the globe that has a kink in the Tropic of Cancer, chart = coordinate system

Differential structure: complicated, enables one to take derivatives. =1 for D=1,2,3, is greater than 1 for D>=4

Manifold: A topological space that near each point resembles Euclidean space

Topological manifold: A locally Euclidean Hausdorff space

Hausdorff space: a topological space where distinct points have disjoint neighborhoods

Topological space: A set X of points along with a set of neighborhoods for each point (collection t of subsets of X) that satisfy axioms:

1. The empty set and X are in t
2. The intersection of any collection of sets in t is also in t
3. The union of any pair of sets in t is also in t

Quick intro to GR

$$ds^2 = e^{2\lambda}dt^2 - e^{2(\nu-\lambda)}\left(dr^2 + dz^2\right) - r^2 e^{-2\lambda}d\phi^2$$

Line Element

Metric

$$g_{\mu\nu} = \begin{pmatrix} e^{2\lambda} & 0 & 0 & 0 \\ 0 & -e^{2(\nu-\lambda)} & 0 & 0 \\ 0 & 0 & -e^{2(\nu-\lambda)} & 0 \\ 0 & 0 & 0 & -r^2e^{-2\lambda} \end{pmatrix}$$

Connection

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right)$$

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$$

$$R = R^{\mu}_{\mu} = g^{\mu\nu}R_{\mu\nu}$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

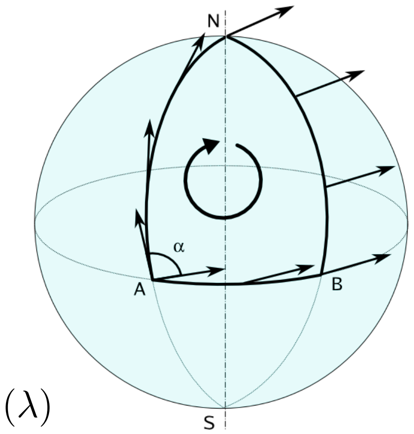
Curvature

Matter

Matter tells space how to curve, space tells matter how to move

Connection is torsion free and metric compatible = covariant derivative of metric is zero everywhere

Parallel Transport



$\partial \mu T_{\mu\nu} = ??$

Not a tensor

Tensor

$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda$

Parallel Transport $\frac{D}{d\lambda} = \frac{dx^\mu}{d\lambda}\nabla_\mu = 0 \quad \text{along} \quad x^\mu(\lambda)$

That is, the directional covariant derivative is equal to zero along the curve x^μ parameterized by λ. For a vector this can be simply written as:

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda = 0$$

Geodesic equation

Parallel Transport of the tangent vector

$$\frac{D}{d\lambda}\frac{dx^\mu}{d\lambda} = \frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0$$

Parallel transport is trivial in flat space, but not in curved space

A partial derivative is not a tensor. A covariant derivative is. We like tensors in GR, because they're valid everywhere.

Geodesic equation: A geodesic is a curve along which the tangent vector is parallel transported

Theorema Egregium -> curvature of a surface can be determined entirely by measuring distances along paths on the surface (intrinsic curvature). Math: the Gaussian curvature of a surface is invariant under local isometry

Intrinsic curvature: A torus has zero intrinsic curvature, but non-zero extrinsic curvature. A line only has extrinsic curvature (ie. it only has curvature given an embedding), whereas surfaces can have curvature independent of embedding.

Gaussian curvature: Although defined in terms of extrinsic curvature, it is an intrinsic curvature. It is the product K=k_1*k_2 of the two principle curvatures k_1 and k_2

Principle curvatures (of a point on a surface): The two eigenvalues of the shape operator at that point. Informally, the max and min of the two curvatures defined by the normal curvatures of the intersection between the normal plane of the point and the surface (in 3D)

Shape Operator: The differential of the Gauss map f

Gauss Map: A map of a surface R^n to the S^(n-1) unit sphere. The Jacobian determinant of the Gauss map is the Gaussian curvature.

Isometry: a diffeomorphic map that pulls back the metric tensor on the second manifold to the metric tensor on the first

Pull-back: f:M->N is a smooth map between smooth manifolds M and N. Then f* is the map of space of 1-forms on N to the 1-forms on M

1-form: linear space of the cotangent bundle. A k-form is the k-th exterior power of the cotangent bundle

Exterior power: the k-th wedge product: x_1^x_2^...^x_k

Wedge, or Exterior Product: For A as a degree p k-form and B as a degree q k-form, A^B=(-1)^{pq}B^A

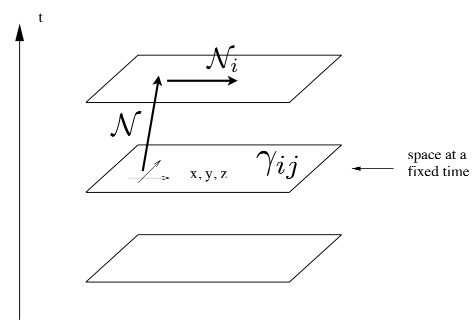
(A^B)_{/mu_1.../mu_(p+q)}=(p+q)!/(p!q!)A_{[/mu_1.../mu...p}B_{/mu_{p+1}.../mu_{p+q}] where antisymmetrization [/mu,/nu] = 1/n!(alternating sum over permutation of indices) so if A and B are 1-forms then A^B = 2A_{[/mu}B_{/nu]} = A_/mu B_/nu - A_/nu B_/mu

Cotangent bundle: The dual of the tangent bundle

Tangent bundle: The disjoint union of tangent spaces of a manifold M. The dimension of TM is twice that of M. The TM of a circle is a cylinder, consisting of all the tangent lines to that point joined together in a smooth and non-overlapping manner.

Tangent space: Every point x of a differentiable manifold M has attached a real vector space of the possible directions at which one can tangentially pass through x. Each tangent space of an n-dimensional manifold is an n-dimensional vector space

ADM 3+1 formalism



$$g_{\mu\nu} = \begin{pmatrix} \mathcal{N} & \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 \\ 0 & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & 0 & \gamma_{22} & \gamma_{23} \\ 0 & 0 & 0 & \gamma_{33} \end{pmatrix}$$

$$\mathcal{N}_i =^{(4)} g_{0i}$$

$$\mathcal{N} = \left(-^{(4)}g_{00}\right)^{-\frac{1}{2}}$$

ADM foliates spacetime into spacelike hypersurfaces labelled by a time coordinate and 3 spatial coordinates

Quantum gravity Hamilton-Jacobi equations

$$H + \frac{\partial S}{\partial t} = 0$$

$$S_H = \frac{1}{2\kappa} \int \sqrt{-g} R d^4 x$$

$$\hat{g}_{ij}(t, x^k) \to g_{ij}(t, x^k)$$

$$\hat{\pi}_{ij}(t, x^k) \to -i \frac{\delta}{\delta g_{ij}(t, x^k)}$$

Plus commutators gives the Wheeler–DeWitt

Wheeler-DeWitt equation

$$\left[-G_{ijkl}\frac{\delta^2}{\delta\gamma_{ij}\delta\gamma_{kl}} -^3 R(\gamma)\gamma^{1/2} + 2\Lambda\gamma^{1/2}\right]\Psi[\gamma_{ij}] = 0$$

$$G_{ijkl} = \frac{1}{2}\gamma^{-1/2}\left(\gamma_{ik}\gamma_{jl} + \gamma_{il}\gamma_{jk} - \gamma_{ij}\gamma_{kl}\right)$$

Gijkl is the metric on the superspace and 3R is the scalar curvature

The superspace is the space of all three geometries ... a 5+1 space (-,+,+,+,+,+) described in DeWitt's 1967 paper

Recall gamma_ij is the 3-metric of the spacelike hypersurfaces

Wheeler-DeWitt gives ground state and wave functions of excited states of matter field for the universe

Semiclassical Universe from First Principles

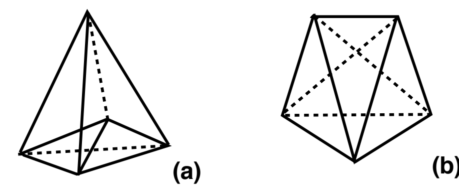
- Ambjorn, Jurkiewicz, and Loll get quantum cosmology without the Wheeler-DeWitt equation using the sum-over-histories approach of Causal Dynamical Triangulations
- They "observe" a bounce, or state of vanishing spatial extension which tunnels to a universe of finite linear extension
- What is Causal Dynamical Triangulations?

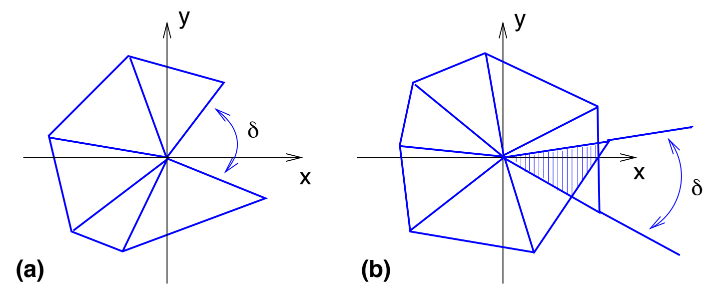
Ekpyrotic models also observe a bounce, producing a flat big bang universe from the covariant approach (superstring theory)

Causal Dynamical Triangulations

- Causal, or Lorentzian, signature for the metric (which avoids problems of degenerate quantum geometries which occur with Euclidean metrics)
- Non-perturbative path integral without a fixed background. Spacetime is determined dynamically as part of the theory
- Spacetime is approximated by tiling flat simplicial building blocks (Regge calculus) without coordinates (simplicial manifold)
  1. Any face of a simplex in T is a simplex in T
  2. Two simplices in T are either disjoint or share a common face
- Well-defined Wick rotation (analytic continuation)

Causal Dynamical Triangulations





Name	Dim	0-faces	1-faces	2-faces	3-faces	4-faces	Causal Structure
Vertex	0	1					
Edge	1	2	1				{1,1}
Triangle	2	3	3	1			{2,1} {1,2}
Tetrahedron	3	4	6	4	1		{3,1} {2,2} {1,3}
Pentatope	4	5	10	10	5	1	{4,1} {3,2} {2,3} {1,4}

Table 1: Types and causal structures of simplices

Pentatopes
Curvature entirely on vertices (3d) or hinges (4d)

Causal Dynamical Triangulations

$$G(g_i, g_f; t_i, t_f) = \int \mathcal{D}[g] e^{iS_{EH}}$$

$$S_H = \frac{1}{2\kappa} \int \sqrt{-g}(R - 2\Lambda) d^4x$$

$$S_R = \frac{1}{8\pi G} \sum_h V_h \delta_h - \frac{\Lambda}{8\pi G} \sum_d V_d$$

Gravitational path integral
Einstein-Hilbert action
Discrete E-H action, Vh volume of hinge, Vd volume of top level d-simplex, delta h deficit angle

CDT

$$S = \frac{1}{8\pi G} \sum_{space\text{-}like\ h} Vol(h) \frac{1}{i} \left(2\pi - \sum_{d\text{-}simplices} \Theta \right) - \frac{1}{8\pi G} \sum_{time\text{-}like\ h} Vol(h) \left(2\pi - \sum_{d\text{-}simplices} \Theta \right) - \frac{\Lambda}{8\pi G} \sum_{time\text{-}like}$$

$$\alpha \to -\alpha, \alpha > \frac{7}{12}$$

$$\int \mathcal{D}[g] e^{iS} \to \sum_T \frac{1}{C(T)} e^{iS(T)}$$

$$Z = \sum_T exp \left((k_0 + 6\Delta) N_0 - k_4 N_4 - \Delta (2N_4^{(4,1)} + N^{(3,2)}) \right)$$

$$k_0 = \frac{\sqrt{3}}{8G}, k_4 = \frac{\Lambda\sqrt{5}}{768\pi G} + \frac{\sqrt{3}}{8G} \left(\frac{5}{2\pi} cos^{-1}(1/4) - 1 \right)$$

Spacelike and timelike hinges

Wick rotation

Replace gravitational path integral by sum over distinct triangulations where C(T) is the order of the automorphism group (equivalent triangulations) of the triangulation

Substitution of the Wick action into the sum yields the partition function

Metropolis algorithm makes “ergodic move” at a probability proportional to the change in the action

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