

Quantum Gravity

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PHY 245C

Why Quantum Gravity?

- Problems it solves (or should solve)
 - Black Hole Entropy
 - Information loss paradox
 - Cosmological Constant
 - Closed Timelike Curves
 - Wormholes, Global Hyperbolicity

Black Hole Entropy

- Drop a box of gas of defined entropy into the Black Hole, and you just have a (bigger) black hole [Carlip, <http://www.physics.ucdavis.edu/Text/Carlip.html#thermo>]
- Since Entropy is supposed to increase, where did it go?
- Beckenstein: $S = (c/4\hbar)kA$
 - A is surface area of event horizon
 - k is Boltzmann's constant
 - Then Black Hole somehow has a temperature
- Hawking: $T = \hbar K / (4\pi^2 kc)$
 - K is surface gravity
 - The smaller the black hole, the hotter

Information Loss paradox

1. Why do classical black holes have finite entropy equal to $\frac{1}{4}$ event horizon area?
2. How does information escape from an evaporating black hole?
3. In essence (modulo a great many technical arguments), the answer to both of these questions is that the finite mass black hole, representing a finite number of energy states N , therefore possesses a discrete energy spectrum. In general, discrete spectra are quantum-mechanically non-degenerate, so knowledge of the precise energy and other (commuting) conserved charges determines the quantum state. But General Relativity charges are generically given by boundary terms; thus, the entire state of the black hole resides in the boundary (asymptotic region), available to all observers. [summarized from <http://adamgetchell.blogspot.com/2007/11/f-regge-calculus-and-other-interludes.html>]

Approaches

- Perturbative Quantum Theory -> String Theory -> Non-perturbative string theory -> M-Theory
- Background independent -> Geometrodynamics -> Non-perturbative, Hamiltonian approach -> Loop Quantum Gravity
- Regge Calculus -> Causal Dynamical Triangulations

Loop Quantum Gravity

- Background independence = spacetime metric is not external structure on which matter fields and gravitational perturbations propagate. Metric is dynamical entity which becomes fluctuating quantum operator. Necessary feature of successful quantum gravity theory.
- Requires non-perturbative function. For example, consider the perturbed metric:

$g = g_0 + h$ where h is the perturbation (graviton)

Then g_0 is a specific background, and we lose background independence

- Thus, LQG discards perturbation theory, Fock spaces, background metrics, and so on.

LQG program

- Construct a QFT on a differential manifold M (instead of a background spacetime M with a particular g_0)
- Show for any given g_0 “background metric” the theory contains semiclassical sector which looks like ordinary QFT on (M, g_0)

Background Independence

- [L. Smolin, hep-th/0507235]
- General Relativity is mostly relational:
 - Dimension
 - Topology
 - Differential structure
 - Signature
 - Metric and fields
- A spacetime can be denoted by (M, g_{ab}, f) where M is the first four properties, g_{ab} is the metric, and f are the fields. Then M is fixed for a particular model, and constitutes a background.
- However, GR can be said to be relative because:
 - An equivalence relationship, the diffeomorphism ϕ , is defined as a smooth invertible map from a manifold to itself by taking point p to the point $\phi \cdot p$, dragging the fields along by $(\phi \cdot f)(p) = f(\phi^{-1} \cdot p)$
 - Diffeomorphisms of a manifold form a group $\text{Diff}(M)$
 - GR postulates that physical spacetimes correspond to an equivalence class of manifolds, metrics, and fields under all actions of $\text{Diff}(M)$, denoted $\{M, g_{ab}, f\}$

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- Dimension and topology are coded in $\{M, g_{ab}, f\}$
- Everything else is a system of relationships between events
- Events are not points of a manifold, but are identified only by coincidences between values of fields (preserved by diffeomorphisms)
- Relationships are:
 1. Causal order (which events precede which given by lightcone structure)
 2. Measure (spacetime volume of sets defined by causal order)

It can be shown that information in $\{M, g_{ab}, f\}$ is completely characterized by the causal structure and measure.

[Malament, arXiv:gr-qc/0506065]

Hence, GR is relational apart from the specification of topology, differential structure, and dimensionality

Causal set theory

- A causal set is a partially ordered set such that the intersection of the past and future of any pair of events is a finite set. Elements of the causal set are physical events, and partial ordering describes causality
- 1. A history of the universe consists only of a causal set.
- 2. Quantum dynamics is defined by assigning to each history a complex number which is its quantum amplitude

Causal sets are motivated by the expectation of discrete spacetime at the Planck scale. In particular

- A causal set C approximates a classical spacetime $\{M, g_{ab}, f\}$ if to each event e in C there is an event e in $\{M, g_{ab}, f\}$:
- 1. Causality is preserved
- 2. On average, there is 1 event e from C per Planck volume of $\{M, g_{ab}, f\}$

Inverse problem for causal sets

- Given $\{M, g_{ab}, f\}$ it is easy to define C
- But no C approximates a low dimensional manifold
- No characterization expressed in terms of relations in C allow selection of those causal sets that approximate spacetimes (but see Causal Dynamical Triangulations)

Hence, causal sets have so far failed to lead to a good physical theory of quantum gravity

However, loop quantum gravity, which relates causal relations to local changes in the relations of quantum geometry, may address this

Results from LQG

- GR and SUGRA for $n > 2+1$ can be rewritten as a gauge theory such that the configuration space is the space of a connection field A_a on a spatial manifold Σ .
- Metric info is contained in conjugate momenta.
- Gauge symmetry includes diffeomorphisms of a spacetime manifold $\Sigma \times \mathbb{R}$
- Dynamics take simple form of a constrained topological field theory containing a term defined from BF (background field, http://en.wikipedia.org/wiki/BF_theory) theory plus another term generating quadratic constraint

More results from LQG

If T is a classical gravitational theory described by the diffeomorphism equivalence class $\{M, A_a, f\}$ then:

- Quantization of T results in a unique Hilbert space, H , such that
 1. Wilson loops are represented by operators that create normalizable states
(http://en.wikipedia.org/wiki/Wilson_loops)
 2. Operator algebra with electric field is represented faithfully
 3. Diffeomorphisms of Σ are unitary without anomaly

A way to easily represent LQG is via a graph of node relationships, where the edges are representations of the group/algebra A describing properties shared by the nodes, and the nodes are invariants of A

LQG benefits, drawbacks

- Non-perturbatively quantizes 3-space with quantized area and volume operators
- But: No semiclassical limit recovering GR shown
- But-but: A generalized Kodama state has been argued, via the Immirzi parameter, provides a good semiclassical limit which is CPT invariant, normalizable, chiral, and describes parity violation in quantum gravity. [http://en.wikipedia.org/wiki/Kodama_state]
- Calculates entropy of black holes
- But: requires a special value of the Immirzi parameter
- Big Bang replaced with Big Bounce
- Not a unified theory of all interactions (such as string theory)
- But: the algebra that one uses to quantize in LQG is particular to 3+1 dimension, so no extra dimensions needed
- Cannot predict matter content, and in fact, no experimental predictions from LQG which aren't also made by the Standard Model or GR
- But: allows for smaller structure than quarks, all the way down to the Planck scale
- Not mathematically unique
- But: No reason for fundamental theories to be mathematically unique (and neither is string theory!)

String theory benefits, drawbacks

- String theory predicts dimensionality (ten), particle content, and SUSY
- But: must be decompactified from $3+1$ to 10 dimensions, and there are countably infinite numbers of compactifications from 10 dimensions to 4. This collection of different flux vacua are called the landscape, so in this sense string theory is also not unique
- Anthropic principle: Considered harmful or beneficial, depending on who you are talking to.
 - Can a theory requiring the anthropic principle really be called fundamental? [L. Smolin, hep-th/0407213]
 - What happens to stars if you allow the constants of nature to vary? [Fred Adams, arXiv:0807.3697v1, referred from Cosmic Variance, July 24th, 2008]
- Background Independence achieved in M-theory
- But: No-one knows exactly what M-theory really is

Results

- Extra-dimensions (String)
- AdS-CFT (String)
- Holography (String)
- Supersymmetry (String)
- Non-commutative geometry/K-theory
- Emergent spacetime (CDT)

Bugs/Features

- Stringscape -> Anthropic Principle

Observational Difficulties

- Gravitons are difficult to detect (LIGO)
- LIGO needs 10^{37} to detect
- We detect gravitons about 10^{-35} as good as we detect neutrinos
- If we take a detector the mass of the Earth, squash it into a flat surface of maximal area, and run it for the lifetime of the universe we'll detect 4 gravitons
[Lecture by Freeman Dyson, summarized here:
<http://adamgetchell.blogspot.com/2008/11/can-ligo-detect-graviton.html>]

(Large) Extra dimensions

- Can already rule out anything larger than a few millimeters
- What about the Planck scale?

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