Background Independent Quantum Gravity

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Why Quantum Gravity?

General Relativity and Quantum Mechanics are Fundamentally Incompatible

- GR violates the Uncertainty Principle
- QM violates Causality

Why Quantum Gravity?

General Relativity and Quantum Field Theory give different answers

- Black Hole Information Paradox
- Wormholes, Time Machines, and Warp Drives
- Origins and structure of the universe

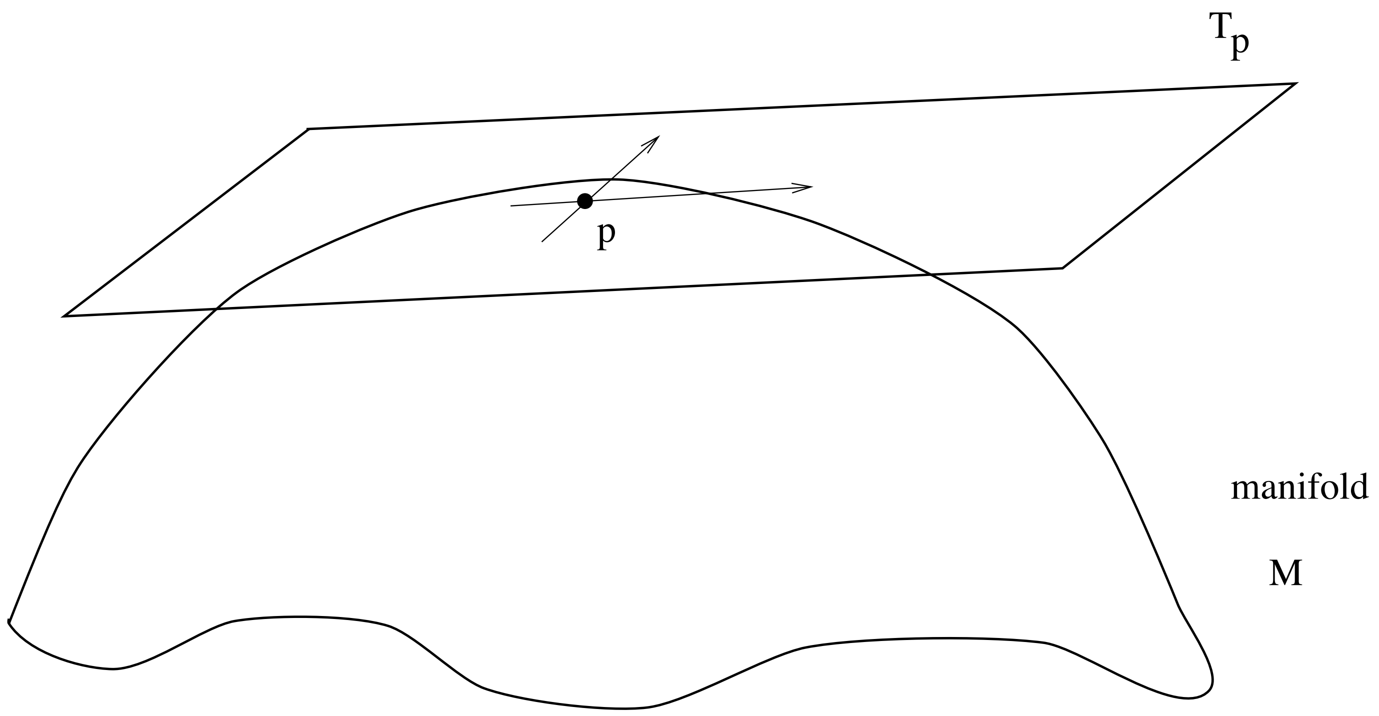
A brief history of Quantum Gravity

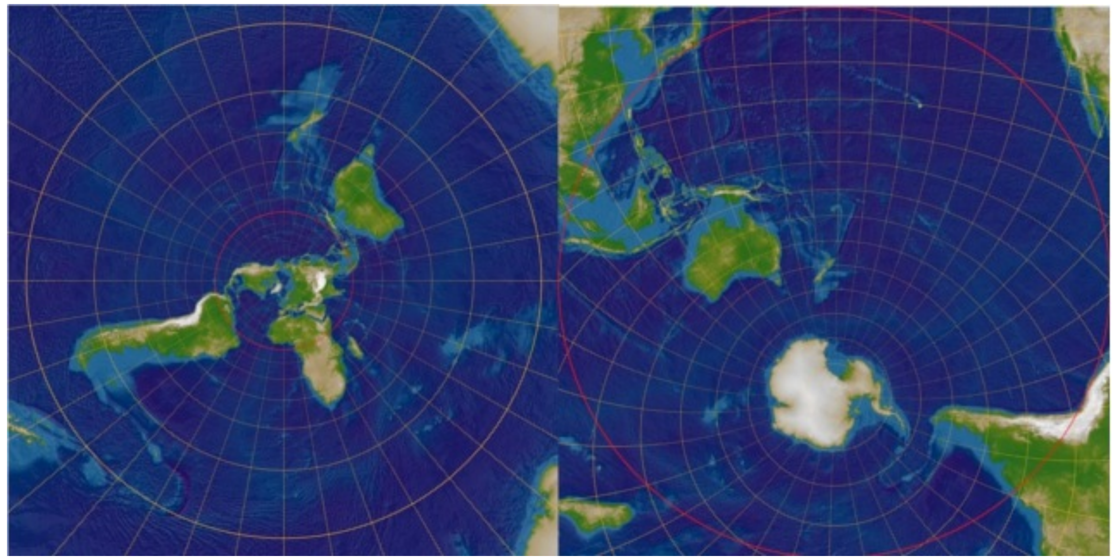
- 1916: Einstein points out that quantum effects must modify GR
- 1927: Oskar Klein suggests quantum gravity should modify concepts of spacetime
- 1930s: Rosenfeld applies Pauli method to linearized Einstein field equations
- 1934: Blokhintsev and Gal'perin discuss the spin-two quantum of the gravitational field, naming it the graviton
- 1936: Bronstein realizes need for background independence, issues with Riemannian geometry
- 1938: Heisenburg points out issues with the dimensionality of the gravitational coupling constant for a quantum theory of gravity
- 1949: Peter Bergmann and group begin the canonical approach to quantum gravity. Bryce Dewitt applies Schwinger's covariant quantization method to the gravitational field for his thesis.
- 1952: Gupta develops the covariant approach to quantum gravity.

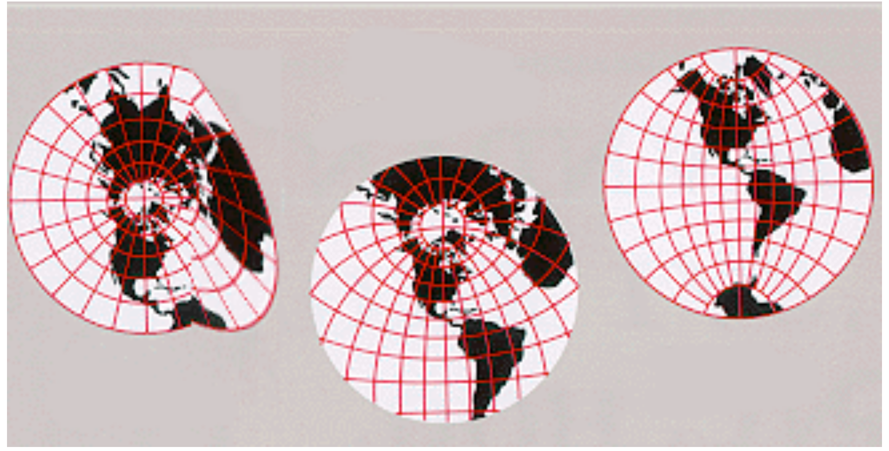
A brief history of Quantum Gravity

- 1957: Charles Misner introduces the "Feynman quantization of general relativity", or sum over histories approach. His paper outlines the three approaches, canonical, covariant, and sum over history in modern terms
- 1961: Arnowit, Deser, and Misner complete the ADM formulation of GR
- 1962: Using ADM, Peres writes the Hamilton-Jacobi formulation of GR
- 1964: Feynman starts loop corrections to GR, but notices unitarity was lost for naive Feynman rules. DeWitt adds correction terms involving loops of fictitious fermionic particles, solving the problem.
- 1966: Faddeev and Popov independently come up with DeWitt's "Faddeev-Popov" ghosts.
- 1967: DeWitt's seminal paper, completed in 1966 but not published until 1967 due to publishing charges, develops canonical quantum gravity and the Wheeler-DeWitt equation. DeWitt and Feynman complete the Feynman rules for GR.
- 1969: Charles Misner starts quantum cosmology by truncating the Wheeler-DeWitt equation to a finite number of degrees of freedom

Quick intro to GR







Quick intro to GR

Line Element

$$ds^2 = e^{2\lambda}dt^2 - e^{2(\nu-\lambda)}\left(dr^2 + dz^2\right) - r^2 e^{-2\lambda}d\phi^2$$

Metric

$$g_{\mu\nu} = \begin{pmatrix} e^{2\lambda} & 0 & 0 & 0 \\ 0 & -e^{2(\nu-\lambda)} & 0 & 0 \\ 0 & 0 & -e^{2(\nu-\lambda)} & 0 \\ 0 & 0 & 0 & -r^2 e^{-2\lambda} \end{pmatrix}$$

Connection

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right)$$

$$R^\rho_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}$$

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$$

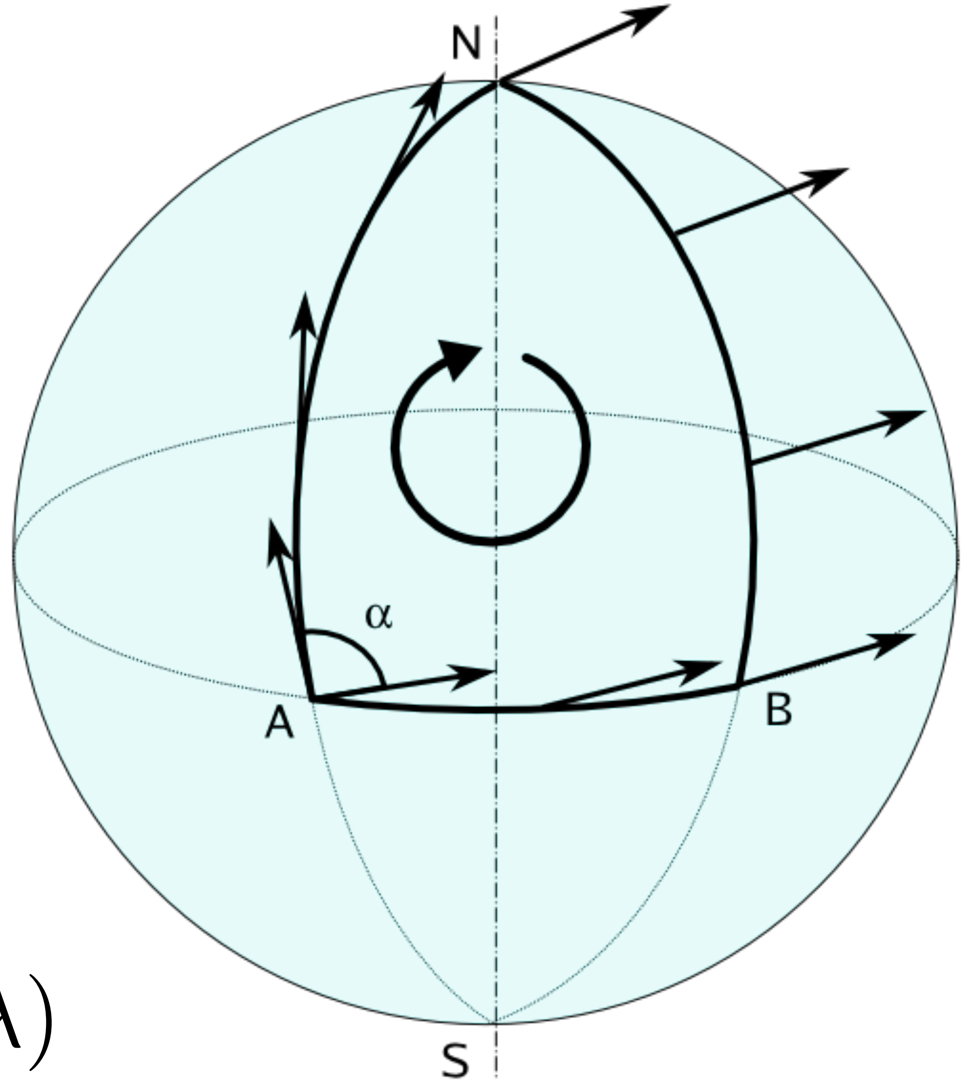
$$R = R^\mu_\mu = g^{\mu\nu}R_{\mu\nu}$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Curvature

Matter

Parallel Transport



$\partial\mu T_{\mu\nu}$ =??

Not a tensor

Tensor

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda$$

Parallel Transport $\frac{D}{d\lambda} = \frac{dx^\mu}{d\lambda}\nabla_\mu = 0 \quad \text{along} \quad x^\mu\left(\lambda\right)$

That is, the directional covariant derivative is equal to zero along the curve x^μ parameterized by λ. For a vector this can be simply written as:

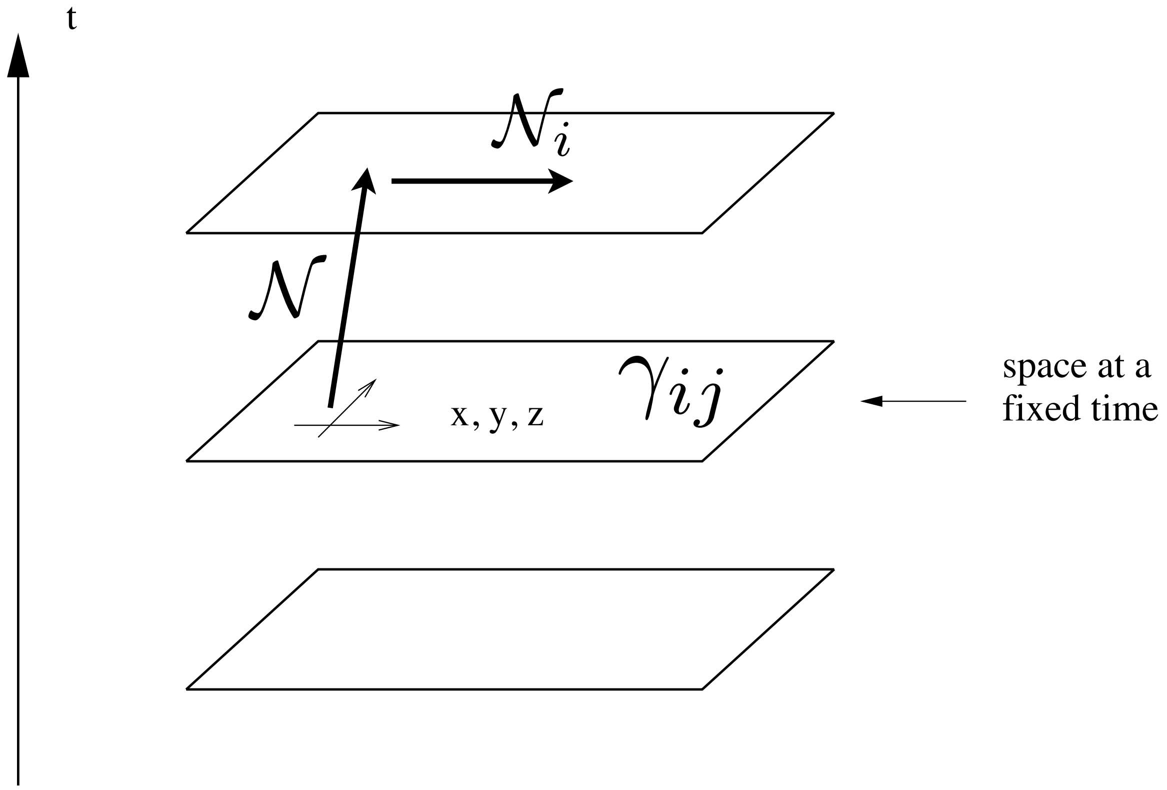
$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda = 0$$

Geodesic equation

Parallel Transport of the tangent vector

$$\frac{D}{d\lambda}\frac{dx^\mu}{d\lambda} = \frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0$$

ADM 3+1 formalism



$$g_{\mu\nu} = \begin{pmatrix} \mathcal{N} & \mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3 \\ 0 & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & 0 & \gamma_{22} & \gamma_{23} \\ 0 & 0 & 0 & \gamma_{33} \end{pmatrix}$$

$$\mathcal{N}_i =^{(4)} g_{0i}$$

$$\mathcal{N} = \left(-^{(4)}g_{00}\right)^{-\frac{1}{2}}$$

Quantum gravity Hamilton-Jacobi equations

$$H + \frac{\partial S}{\partial t} = 0$$

$$S_H = \frac{1}{2\kappa} \int \sqrt{-g} R d^4 x$$

$$\hat{g}_{ij}(t, x^k) \rightarrow g_{ij}(t, x^k)$$

$$\hat{\pi}_{ij}(t, x^k) \rightarrow -i \frac{\delta}{\delta g_{ij}(t, x^k)}$$

Wheeler-DeWitt equation

$$\left[-G_{ijkl}\frac{\delta^2}{\delta\gamma_{ij}\delta\gamma_{kl}} -^3 R(\gamma)\gamma^{1/2} + 2\Lambda\gamma^{1/2}\right]\Psi[\gamma_{ij}] = 0$$

$$G_{ijkl} = \frac{1}{2}\gamma^{-1/2}\left(\gamma_{ik}\gamma_{jl} + \gamma_{il}\gamma_{jk} - \gamma_{ij}\gamma_{kl}\right)$$

Semiclassical Universe from First Principles

- Ambjorn, Jurkiewicz, and Loll get quantum cosmology without the Wheeler-DeWitt equation using the sum-over-histories approach of Causal Dynamical Triangulations
- They "observe" a bounce, or state of vanishing spatial extension which tunnels to a universe of finite linear extension
- What is Causal Dynamical Triangulations?

Causal Dynamical Triangulations

- Causal, or Lorentzian, signature for the metric (which avoids problems of degenerate quantum geometries which occur with Euclidean metrics)

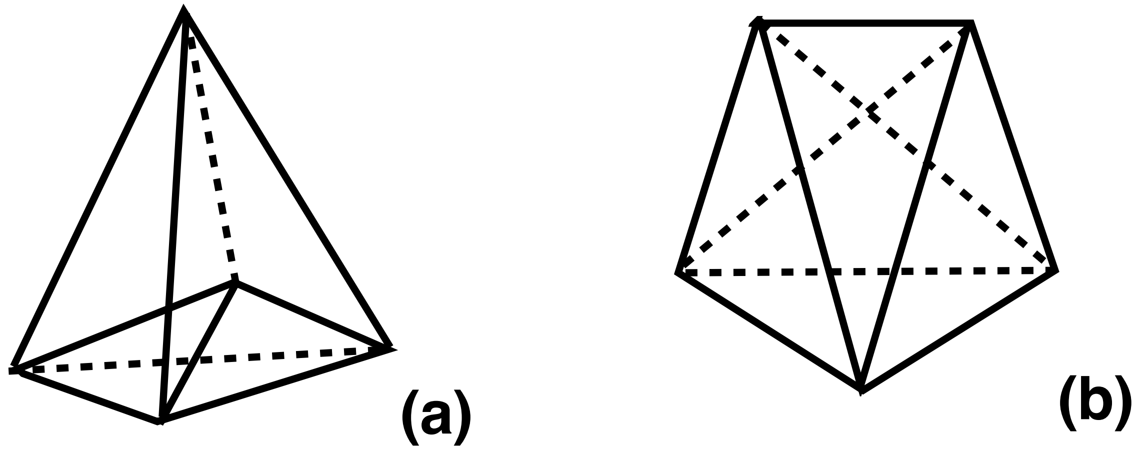
- Non-perturbative path integral without a fixed background. Spacetime is determined dynamically as part of the theory

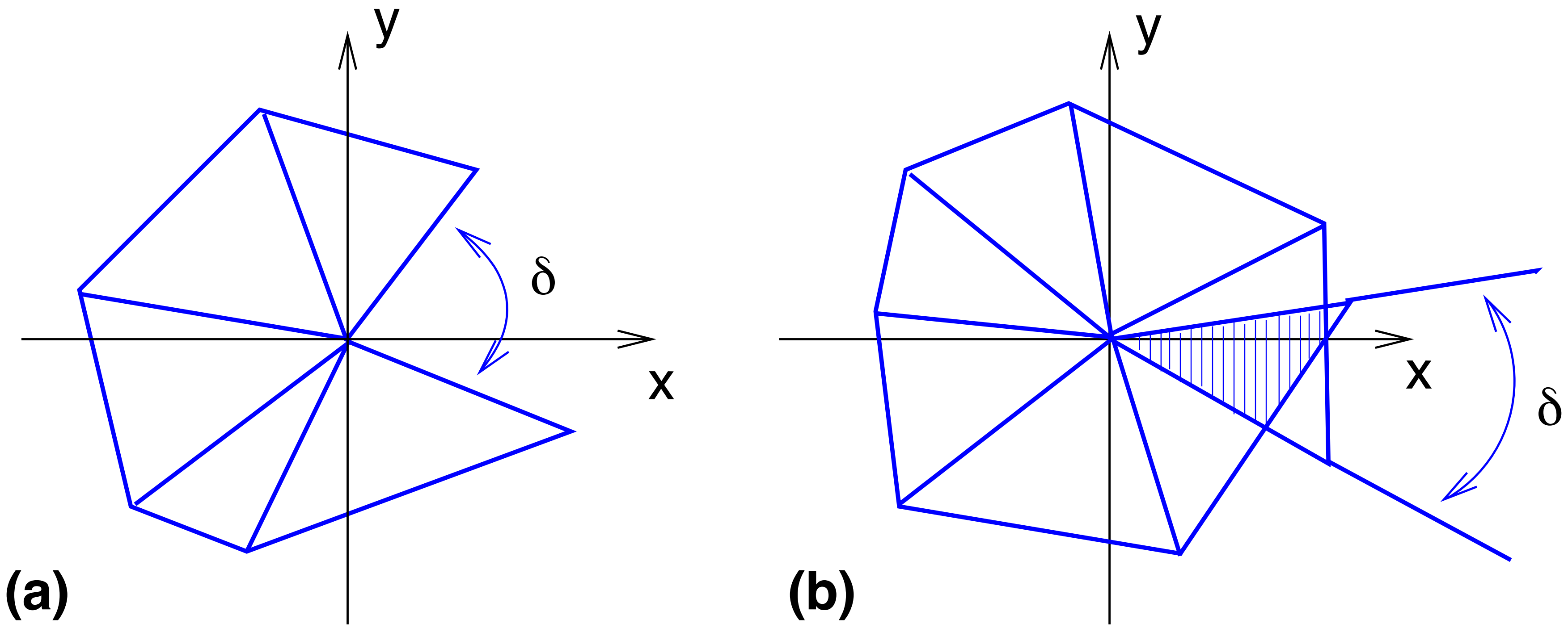
- Spacetime is approximated by tiling flat simplicial building blocks (Regge calculus) without coordinates (simplicial manifold)

  1. Any face of a simplex in T is a simplex in T
  2. Two simplices in T are either disjoint or share a common face

- Well-defined Wick rotation (analytic continuation)

Causal Dynamical Triangulations





Name	Dim	0-faces	1-faces	2-faces	3-faces	4-faces	Causal Structure
Vertex	0	1					
Edge	1	2	1				{1,1}
Triangle	2	3	3	1			{2,1} {1,2}
Tetrahedron	3	4	6	4	1		{3,1} {2,2} {1,3}
Pentatope	4	5	10	10	5	1	{4,1} {3,2} {2,3} {1,4}

Table 1: Types and causal structures of simplices

Causal Dynamical Triangulations

$$G(g_i, g_f; t_i, t_f) = \int \mathcal{D}[g] e^{iS_{EH}}$$

$$S_H = \frac{1}{2\kappa} \int \sqrt{-g}(R - 2\Lambda) d^4x$$

$$S_R = \frac{1}{8\pi G} \sum_h V_h \delta_h - \frac{\Lambda}{8\pi G} \sum_d V_d$$

CDT

$$S = \frac{1}{8\pi G}\sum_{space\text{-}like\ h} Vol(h)\frac{1}{i}\left(2\pi - \sum_{d\text{-}simplices}\Theta\right) - \frac{1}{8\pi G}\sum_{time\text{-}like\ h} Vol(h)\left(2\pi - \sum_{d\text{-}simplices}\Theta\right) - \frac{\Lambda}{8\pi G}\sum_{time\text{-}like}$$

$$\alpha \to -\alpha, \alpha > \frac{7}{12}$$

$$\int \mathcal{D}[g]e^{iS} \to \sum_T \frac{1}{C(T)}e^{iS(T)}$$

$$Z = \sum_T exp\left((k_0 + 6\Delta)N_0 - k_4 N_4 - \Delta(2N_4^{(4,1)} + N^{(3,2)})\right)$$

$$k_0 = \frac{\sqrt{3}}{8G}, k_4 = \frac{\Lambda\sqrt{5}}{768\pi G} + \frac{\sqrt{3}}{8G}\left(\frac{5}{2\pi}cos^{-1}(1/4) - 1\right)$$

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