Graph-Tool

August 19, 2018

1 Network Optimization

Optimizing a network's virtual backbone using Graph Theory

1.1 Import Tools

1.1.1 Import Graph Tool Modules

Specifically graph_tool main, draw and collection modules

```
In [1]: import graph_tool.all
        import graph_tool as gt
        import graph_tool.draw
        import graph_tool.collection
```

```
/home/achal/.virtualenvs/network/lib/python3.5/importlib/_bootstrap.py:222: RuntimeWarning: nump
  return f(*args, **kwds)
```

1.1.2 Import Other Required Modules

These Modules include Matplotlib

```
In [2]: import matplotlib as mpl
```

```
In [3]: %matplotlib inline
```

```
In [4]: import _pickle as pickle
```

```
In [5]: from tqdm import tqdm_notebook as tqdm
```

1.2 Algorithms

1.2.1 Exact Exponential Algorithms

Minimum Dominating Set

A bitmask approach to compute the minimum dominating set of a graph

```
In [43]: def min_dominating_set(graph):
             vertices = list(graph.vertices())
             vertex_neighbours_dict = {}

             for vertex in graph.vertices():
                 vertex_neighbours = set(vertex.out_neighbors())
                 vertex_neighbours.add(vertex)
                 vertex_neighbours_dict[vertex] = vertex_neighbours

             min_dominating_set = set(vertices)

             for i in tqdm(range(2**(len(vertices)))):
                 dominated = set()
                 dominating = set()
                 for j in range(len(vertices)):
                     if not i&(1<<j):
                         continue
                     dominating.add(vertices[j])
                     if len(dominating) >= len(min_dominating_set):
                         break
                     dominated = dominated.union(vertex_neighbours_dict[vertices[j]])
                     if len(dominated) == len(vertices):
                         min_dominating_set = dominating
                         break

             dominated_property_map = graph.new_vertex_property("bool")
             dominated_property_map.set_value(0)
             for vertex in min_dominating_set:
                 dominated_property_map[vertex] = 1

             return dominated_property_map

In [44]: def min_connected_dominating_set(graph):
             vertices = list(graph.vertices())
             vertex_neighbours_dict = {}

             for vertex in graph.vertices():
                 vertex_neighbours = set(vertex.out_neighbors())
                 vertex_neighbours.add(vertex)
                 vertex_neighbours_dict[vertex] = vertex_neighbours

             min_dominating_set = set(vertices)

             for i in tqdm(range(2**(len(vertices)))):
                 dominated = set()
                 dominating = set()
                 for j in range(len(vertices)):
                     if not i&(1<<j):
```

```
                    continue
                if j>0 and not vertices[j] in dominated:
                    continue

                dominating.add(vertices[j])

                if len(dominating) >= len(min_dominating_set):
                    break
                dominated = dominated.union(vertex_neighbours_dict[vertices[j]])
                if len(dominated) == len(vertices):
                    min_dominating_set = dominating
                    break

        dominated_property_map = graph.new_vertex_property("bool")
        dominated_property_map.set_value(0)
        for vertex in min_dominating_set:
            dominated_property_map[vertex] = 1

        return dominated_property_map
```

1.2.2 Heuristic Approximate Algorithms

Minimum Dominating Set

A Greedy approach for computing the minimum dominating set of an undirected graph

```
In [8]: def min_dominating_set(graph):
        #     import pdb; pdb.set_trace()
            dominated_set = set()
            dominating_set = set()
            vertex_neighbours_dict = {}
        #     total_vertices = len(graph.vertices())

            for vertex in graph.vertices():
                vertex_neighbours = set(vertex.out_neighbors())
                vertex_neighbours.add(vertex)
                vertex_neighbours_dict[vertex] = vertex_neighbours

        #     print(vertex_neighbours_dict)

            def get_max_potential_vertex():
                max_potential = 0
                max_p_vertex = None
                for vertex, neighbours in vertex_neighbours_dict.items():
                    if len(neighbours) > max_potential:
                        max_potential = len(neighbours)
                        max_p_vertex = vertex
                return max_p_vertex
```

```
        while vertex_neighbours_dict:
            current_vertex = get_max_potential_vertex()
#           print("The current vertex is", current_vertex)
#           print("The current dominated set is", dominated_set)
#           print("The neighbours not dominated yet for the current vertex is", vertex_nei
            current_vertex_neighbours = vertex_neighbours_dict.pop(current_vertex, None)
#           print("Recheck", current_vertex_neighbours)
            dominated_set = dominated_set.union(current_vertex_neighbours)
            dominating_set.add(current_vertex)
            to_delete = set()
            for vertex, neighbours in vertex_neighbours_dict.items():
                neighbours = neighbours.difference(current_vertex_neighbours)
                vertex_neighbours_dict[vertex] = neighbours
                if not neighbours:
                    to_delete.add(vertex)
            for vertex in to_delete:
                vertex_neighbours_dict.pop(vertex)


        dominated_property_map = graph.new_vertex_property("bool")
        dominated_property_map.set_value(0)
        for vertex in dominating_set:
            dominated_property_map[vertex] = 1

        return dominated_property_map
```

Minimum Connected Dominating Set

A Greedy approach to compute the minimum connected dominating set of a graph

```
In [9]: def min_connected_dominating_set(graph):
            dominated_set = set()
            dominating_set = set()
            vertex_neighbours_dict = {}
            possible_vertex_options = set()

            for vertex in graph.vertices():
                vertex_neighbours = set(vertex.out_neighbors())
                vertex_neighbours.add(vertex)
                vertex_neighbours_dict[vertex] = vertex_neighbours


            def get_max_potential_vertex(iterable):
                max_potential = 0
                max_p_vertex = None
                for vertex in iterable:
                    neighbours = vertex_neighbours_dict.get(vertex, [])
```

```
            if len(neighbours) > max_potential:
                max_potential = len(neighbours)
                max_p_vertex = vertex
        return max_p_vertex

    first_vertex = get_max_potential_vertex(vertex_neighbours_dict.keys())
    possible_vertex_options.add(first_vertex)



    while possible_vertex_options:
        current_vertex = get_max_potential_vertex(possible_vertex_options)
        if not current_vertex:
            break
        possible_vertex_options.remove(current_vertex)
        to_delete = set()
        current_vertex_neighbours = vertex_neighbours_dict.pop(current_vertex, set())
        possible_vertex_options = possible_vertex_options.union(current_vertex_neighbour
        dominated_set = dominated_set.union(current_vertex_neighbours)
        dominating_set.add(current_vertex)
        for vertex, neighbours in vertex_neighbours_dict.items():
            neighbours = neighbours.difference(current_vertex_neighbours)
            vertex_neighbours_dict[vertex] = neighbours
            if not neighbours:
                to_delete.add(vertex)
        for vertex in to_delete:
            vertex_neighbours_dict.pop(vertex)

    dominated_property_map = graph.new_vertex_property("bool")
    dominated_property_map.set_value(0)
    for vertex in dominating_set:
        dominated_property_map[vertex] = 1

    return dominated_property_map
```

1.3 Sample Graphs

1.3.1 Library Collections

List of available graphs

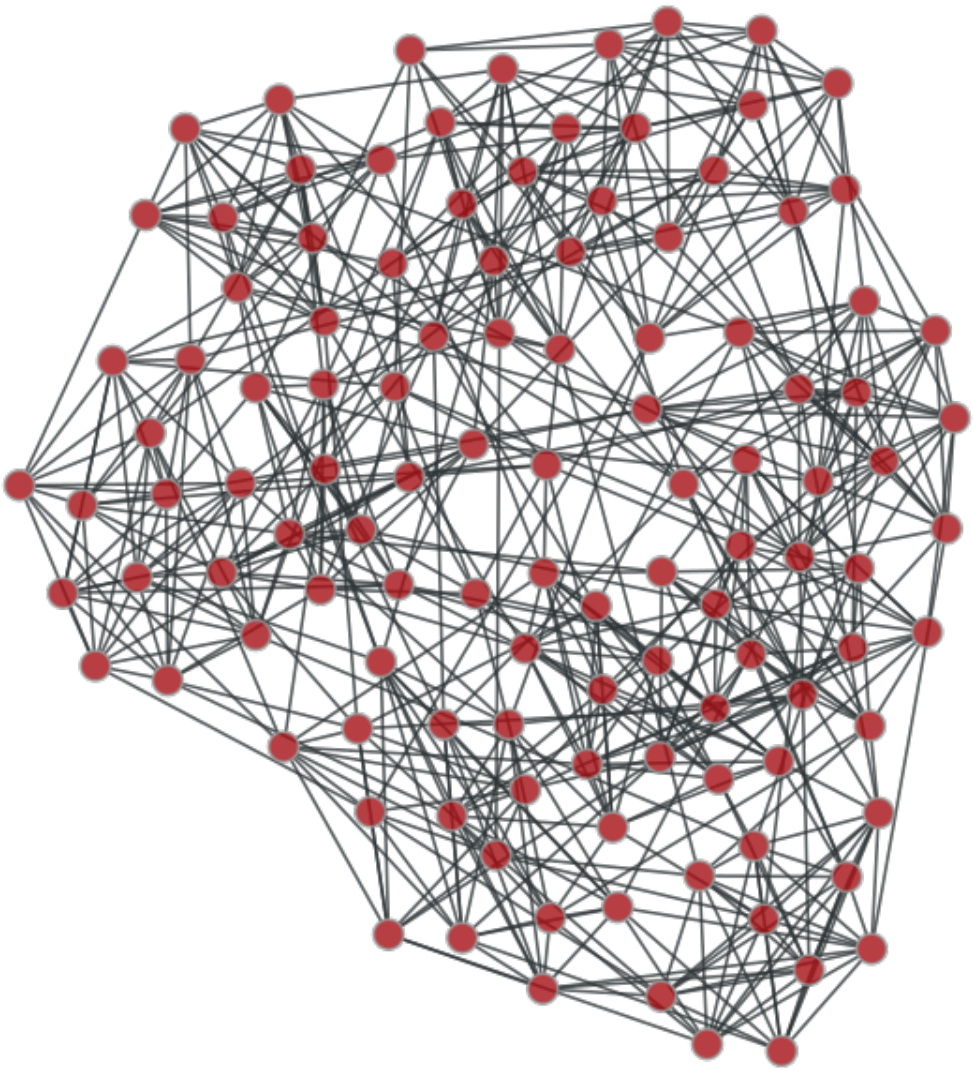
```
In [10]: list(gt.collection.data.keys())
```

```
Out[10]: ['as-22july06',
          'celegansneural',
          'polblogs',
          'football',
```

```
 'cond-mat-2003',
 'polbooks',
 'karate',
 'dolphins',
 'cond-mat-2005',
 'power',
 'astro-ph',
 'netscience',
 'hep-th',
 'serengeti-foodweb',
 'pgp-strong-2009',
 'cond-mat',
 'email-Enron',
 'lesmis',
 'adjnoun']
```

Visualise Graph

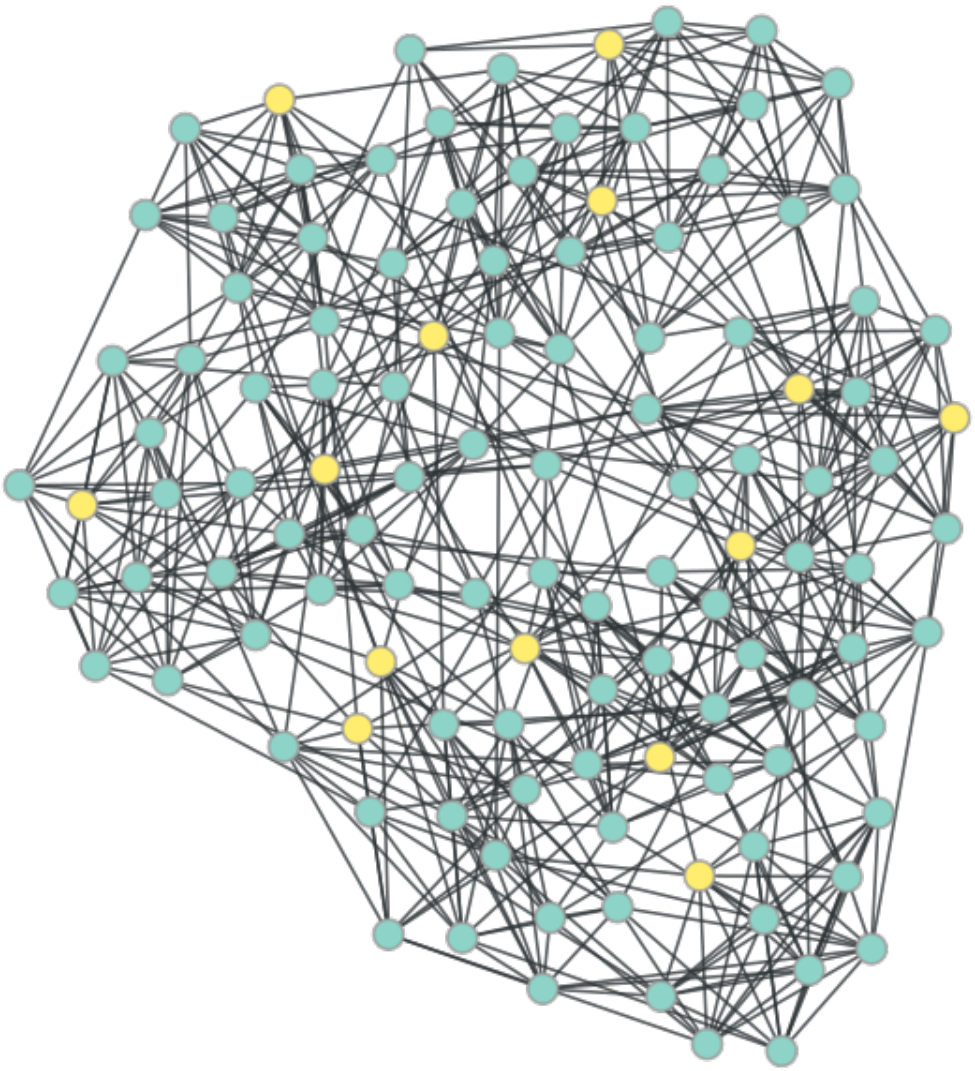
```
In [11]: # select graph from library collection
         graph = gt.collection.data["football"]

In [12]: # uniform position
         pos = gt.draw.sfdp_layout(graph)

In [13]: gt.draw.graph_draw(graph, pos=pos)
```



Out[13]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

Minimum Dominating Set

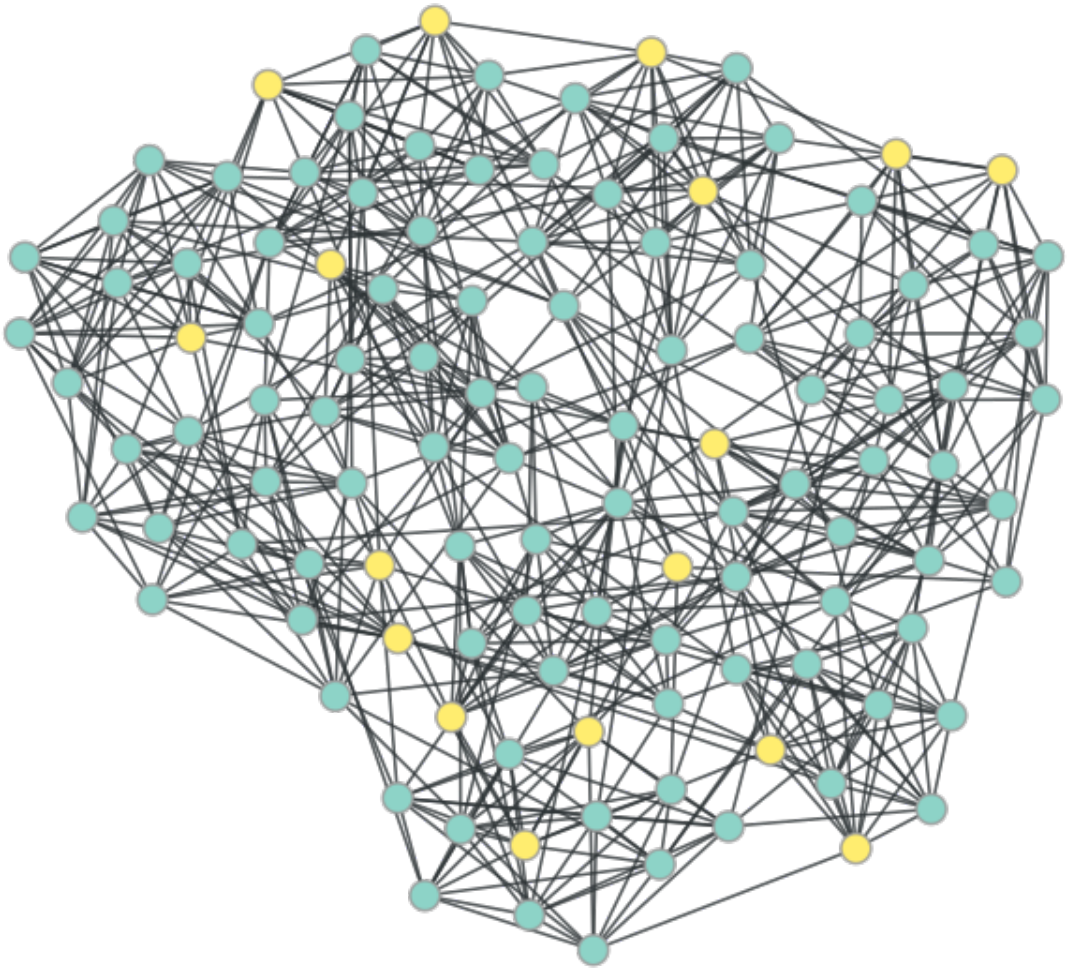
In [14]: gt.draw.graph_draw(graph, vertex_fill_color=min_dominating_set(graph), pos=pos)



Out[14]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

Minimum Connected Dominating Set

In [15]: gt.draw.graph_draw(graph, vertex_fill_color=min_connected_dominating_set(graph))



```
Out[15]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0
```

1.3.2 Custom Graphs

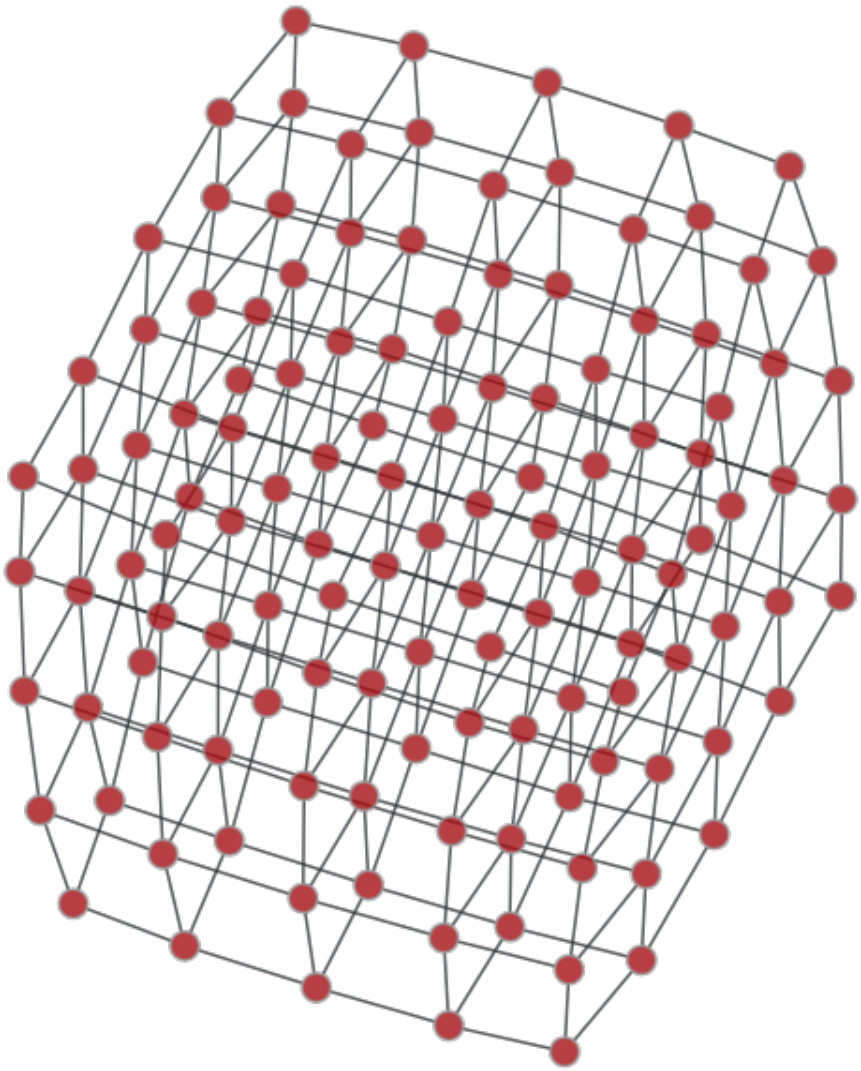
Lattice Graph

Visualize Graph

```
In [16]: graph = gt.generation.lattice([5,5,5], periodic=False)
```

```
In [17]: pos = gt.draw.sfdp_layout(graph)
```

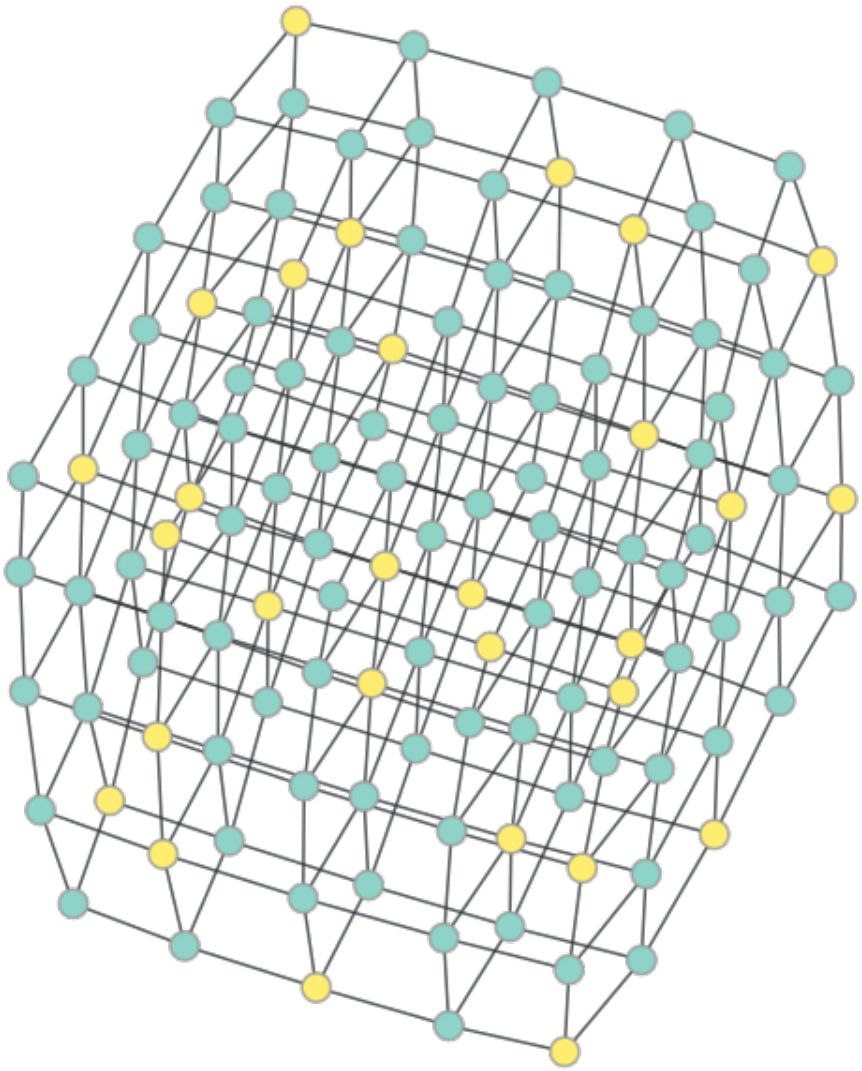
```
In [18]: gt.draw.graph_draw(graph, pos=pos)
```



Out[18]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

Minimum Dominating Set

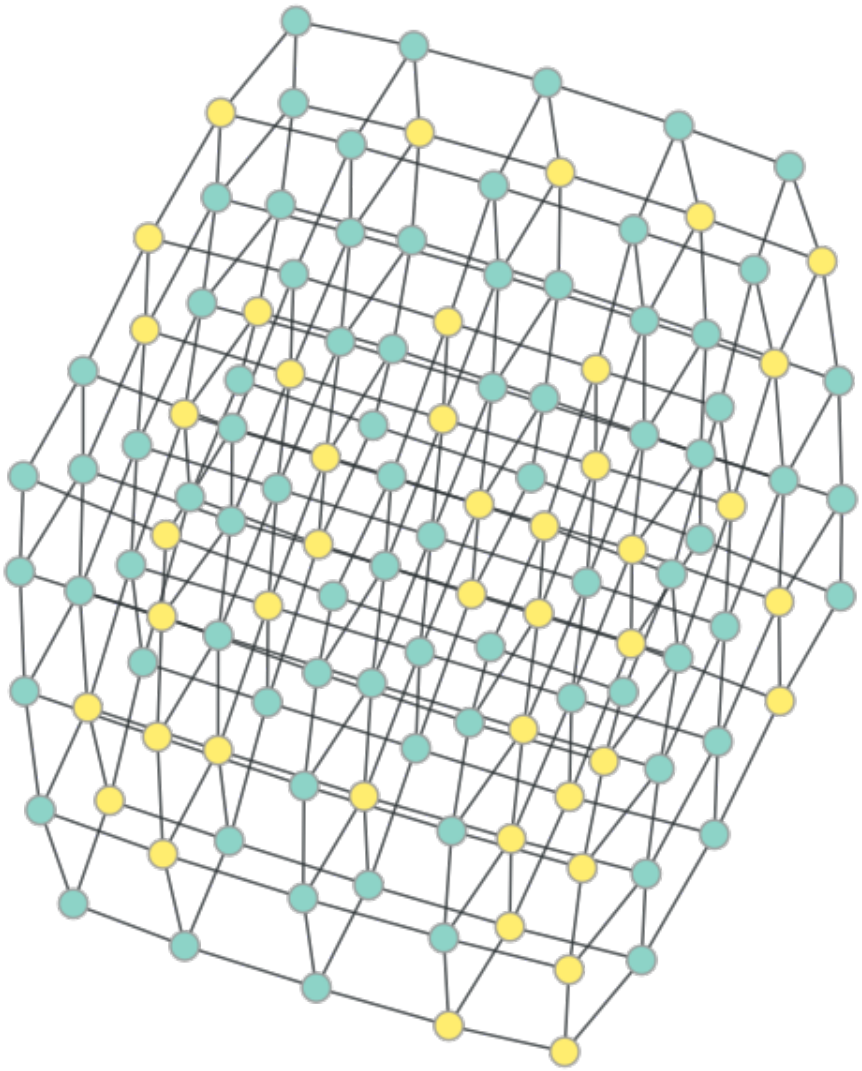
In [19]:
```
gt.draw.graph_draw(graph, pos=pos, vertex_fill_color=min_dominating_set(graph) )
```



Out[19]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

Minimum Connected Dominating Set

In [20]: gt.draw.graph_draw(graph, pos=pos, vertex_fill_color=min_connected_dominating_set(graph



```
Out[20]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0
```

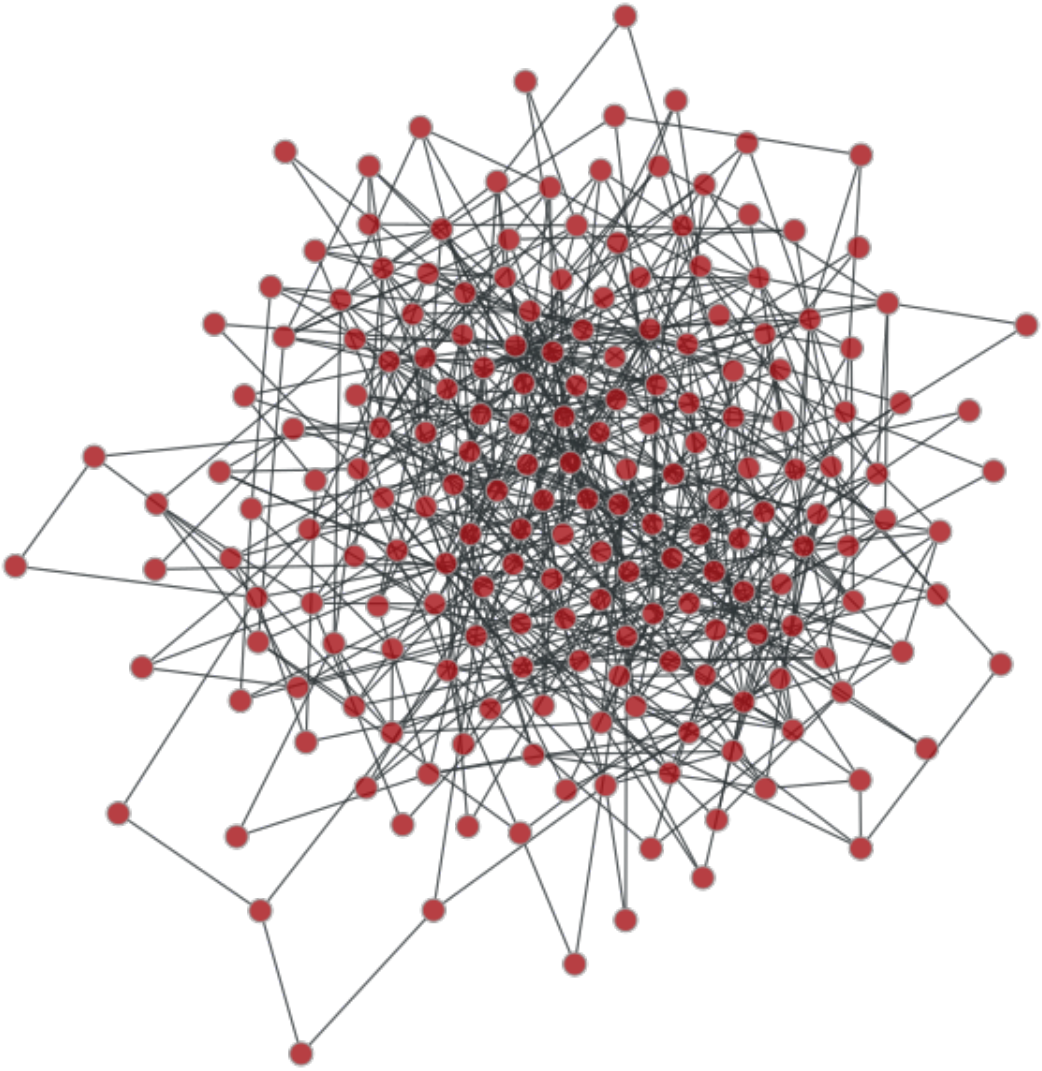
Random Graph

```
In [21]: # limit 10
         from random import random as random_number
         def random_degree_sampler():
             return int((random_number()*10) +2)
```

Visualize Graph

```
In [22]: graph = gt.generation.random_graph(N=200, deg_sampler=random_degree_sampler, directed=F

In [23]: pos = gt.draw.sfdp_layout(graph)
```

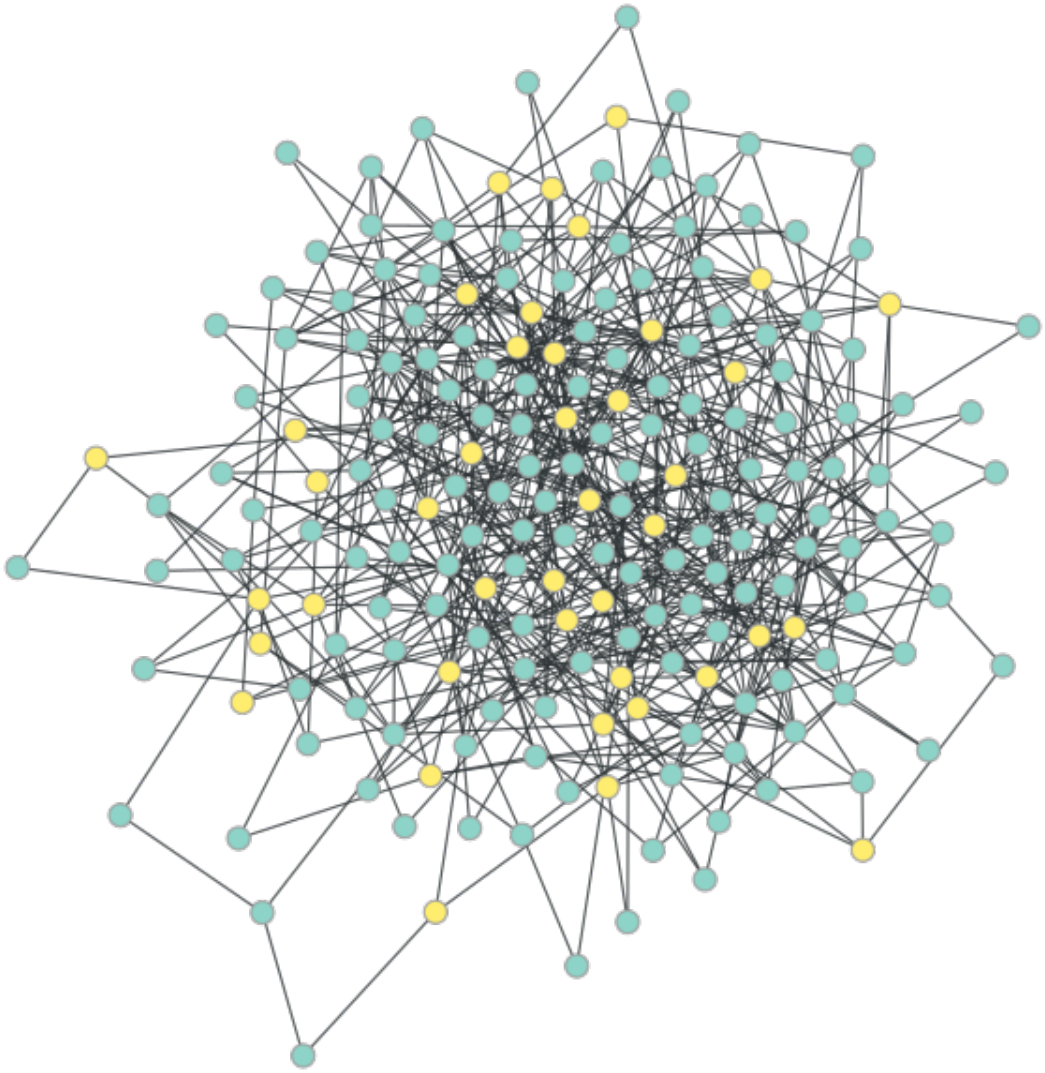
In [24]:
```
gt.draw.graph_draw(graph, pos=pos)
```



Out[24]:
```
<PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0
```

Minimum Dominating Set

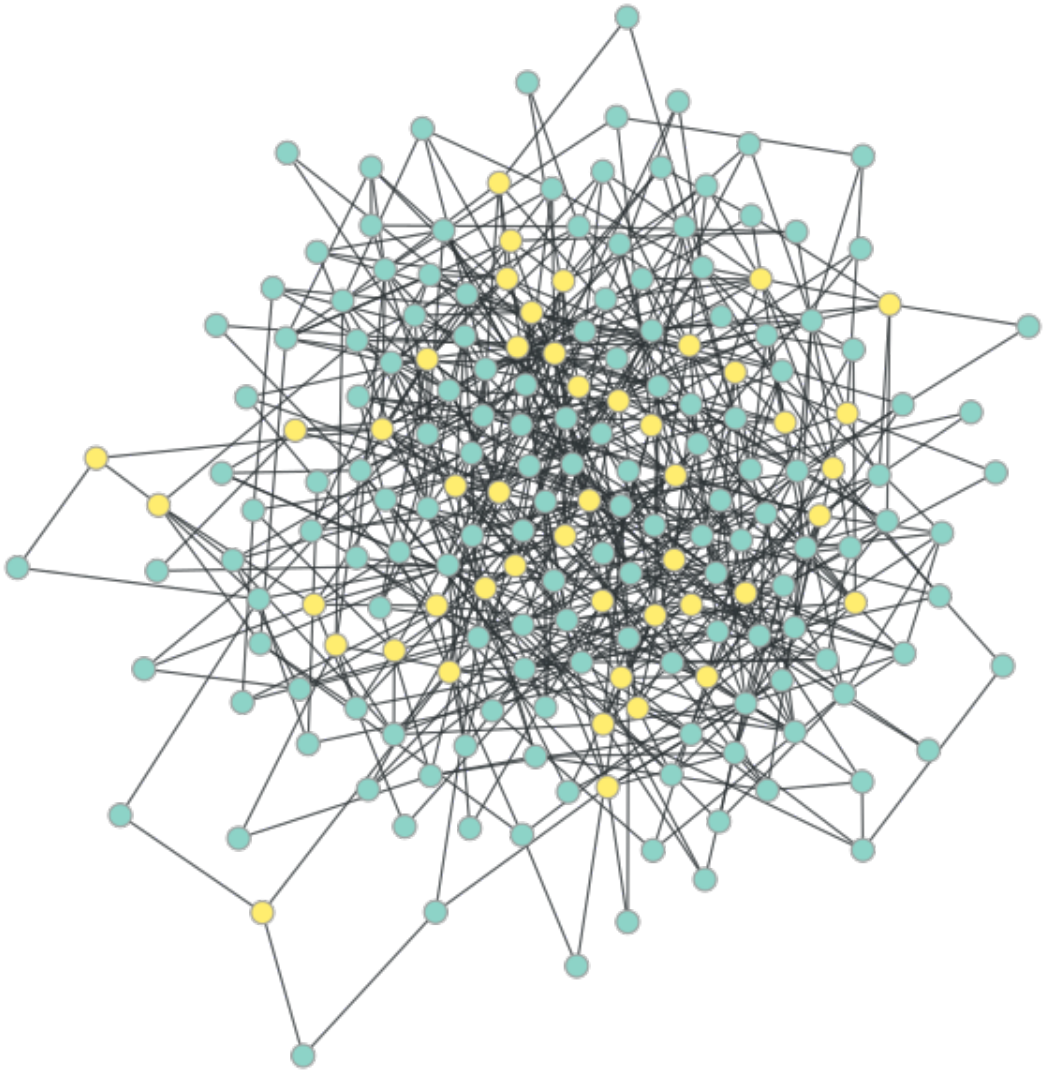
In [25]:
```
gt.draw.graph_draw(graph, pos=pos, vertex_fill_color=min_dominating_set(graph))
```



Out[25]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

Minimum Connected Dominating Set

In [26]: gt.draw.graph_draw(graph, pos=pos, vertex_fill_color=min_connected_dominating_set(graph



```
Out[26]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0
```

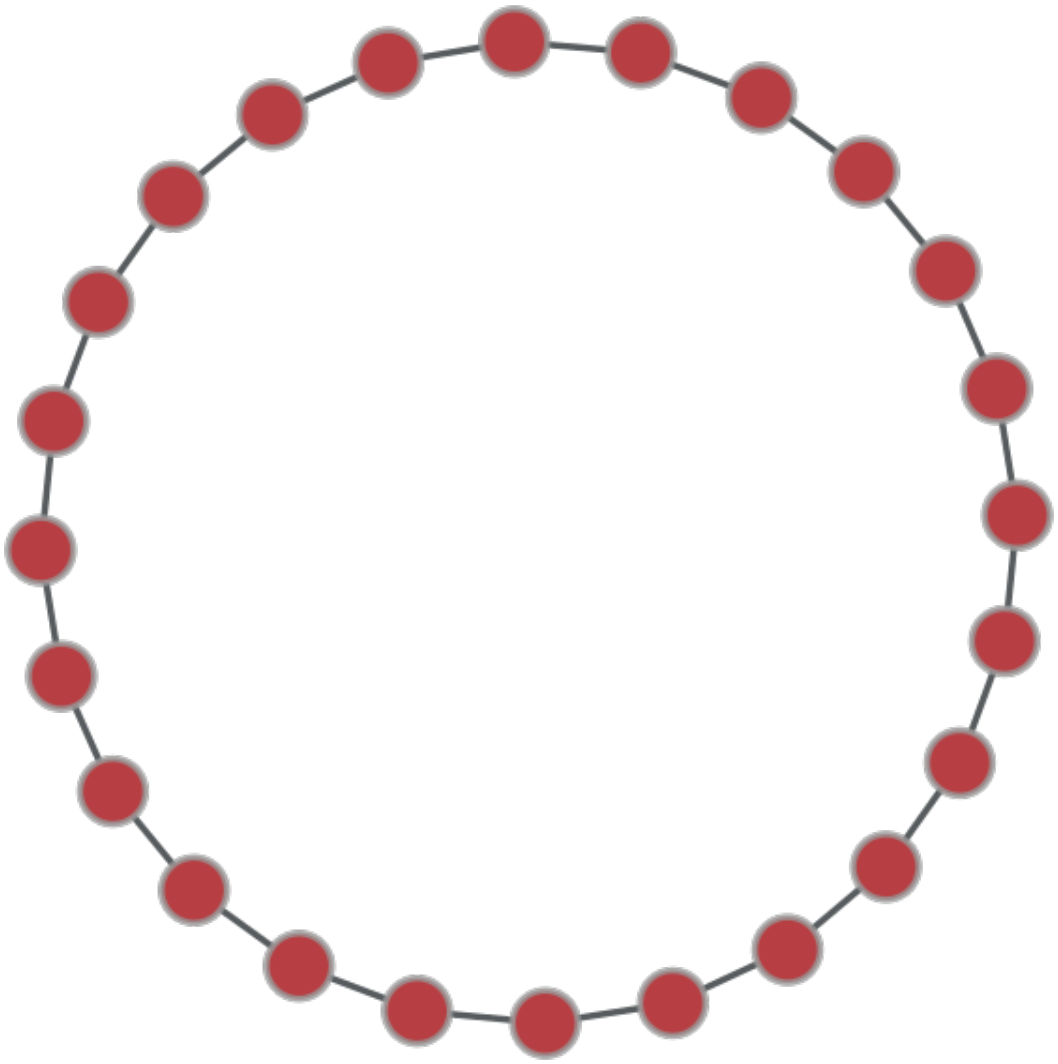
Circular Graph

Visualize Graph

```
In [27]: graph = gt.generation.circular_graph(N=24, directed=False, k=1)
```

```
In [28]: pos = gt.draw.sfdp_layout(graph)
```

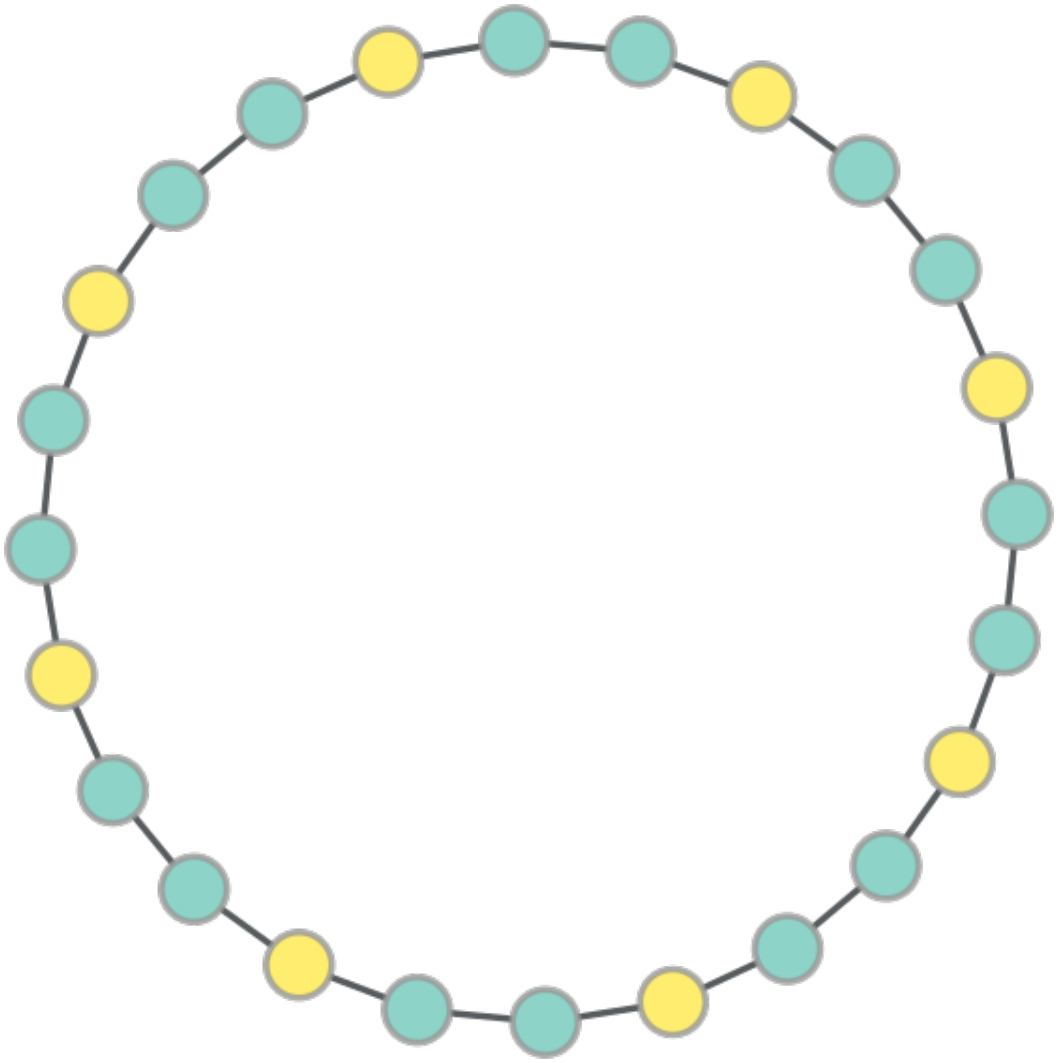
```
In [29]: gt.draw.graph_draw(graph, pos=pos)
```



Out[29]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

Minimum Dominating Set

In [30]: gt.draw.graph_draw(graph, pos=pos, vertex_fill_color=min_dominating_set(graph))

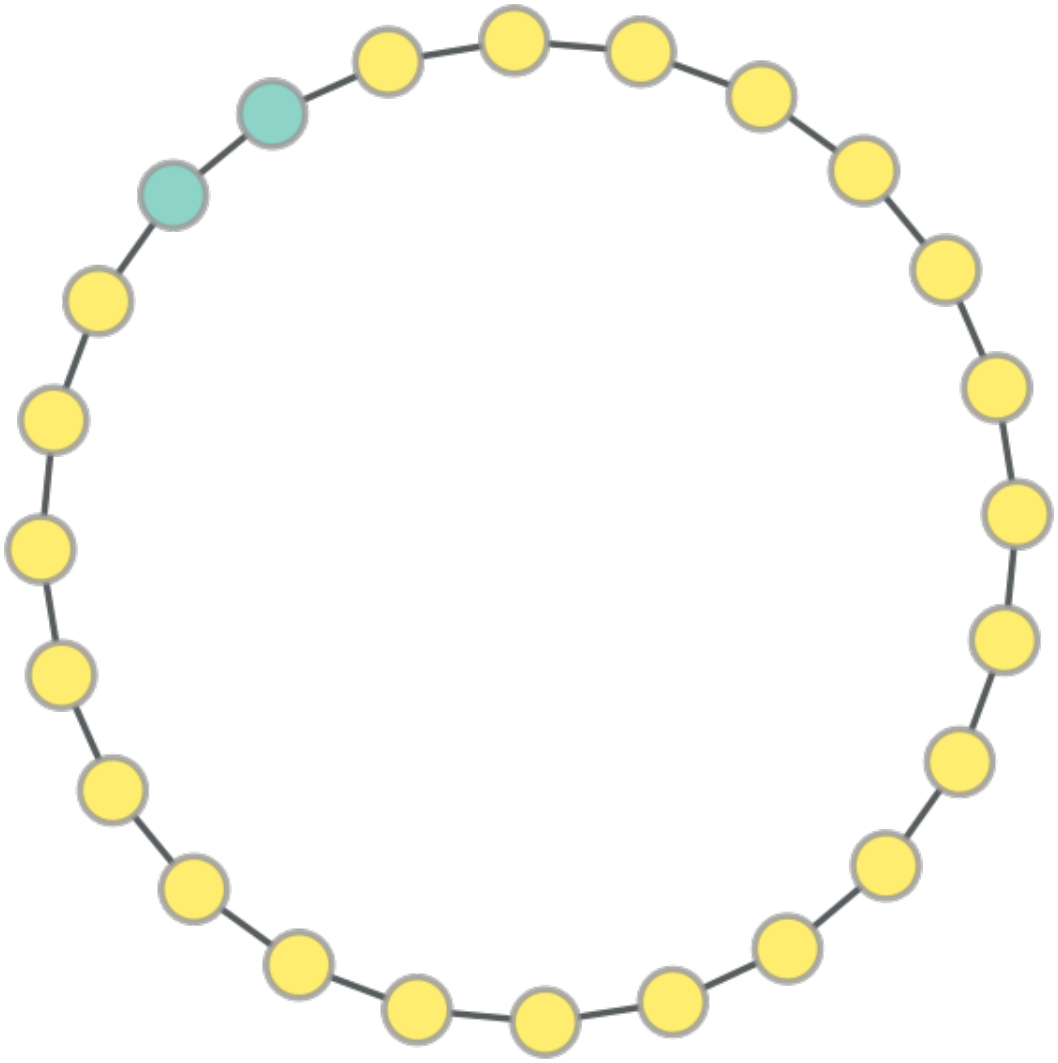


```
Out[30]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0
```

```
In [31]: # gt.draw.GraphWidget(graph, pos=pos, vertex_fill_color=min_dominating_set(graph))
```

Minimum Connected Dominating Set

```
In [32]: gt.draw.graph_draw(graph, pos=pos, vertex_fill_color=min_connected_dominating_set(graph
```



```
Out[32]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0
```

1.3.3 Counter Example Graphs

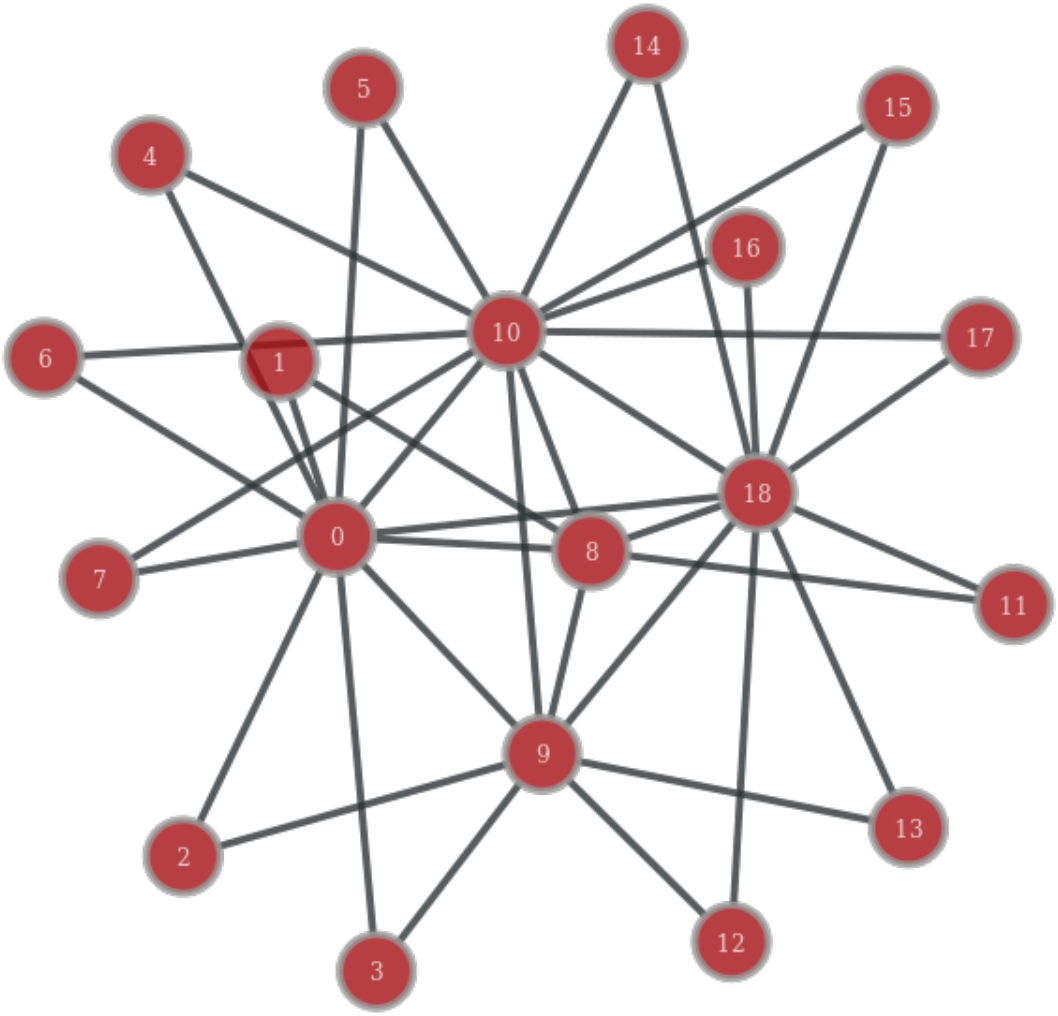
Greedy Heuristic Algorithms

```
In [33]: # with open('counter_example_greedy_heuristic.pickle', 'wb') as f:
         #     pickle.dump(save_counter_example_graph_1, f)
```

```
In [34]: with open('counter_example_greedy_heuristic.pickle', 'rb') as f:
             graph = pickle.load(f)
```

```
In [35]: pos = gt.draw.sfdp_layout(graph)
```

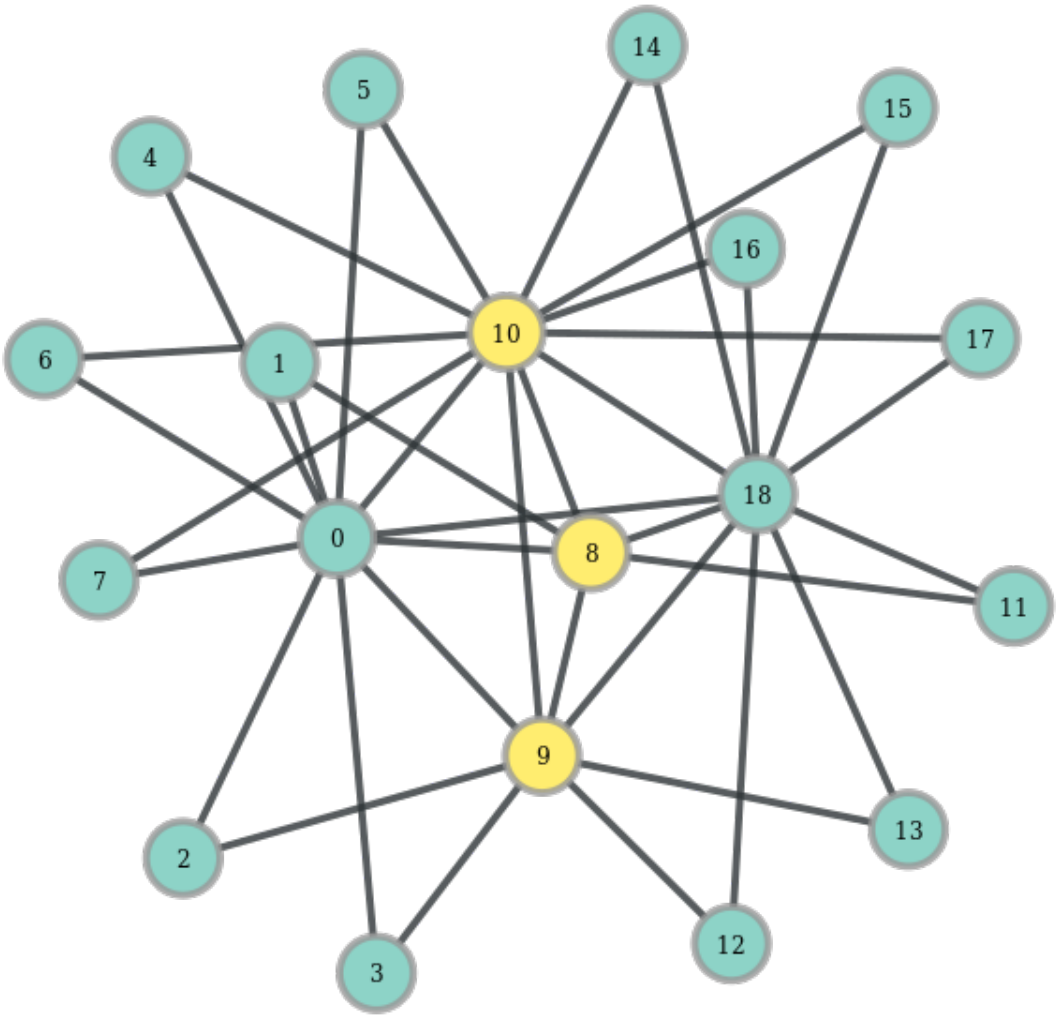
In [36]: gt.draw.graph_draw(graph, vertex_text=graph.vertex_index, pos=pos)



Out[36]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

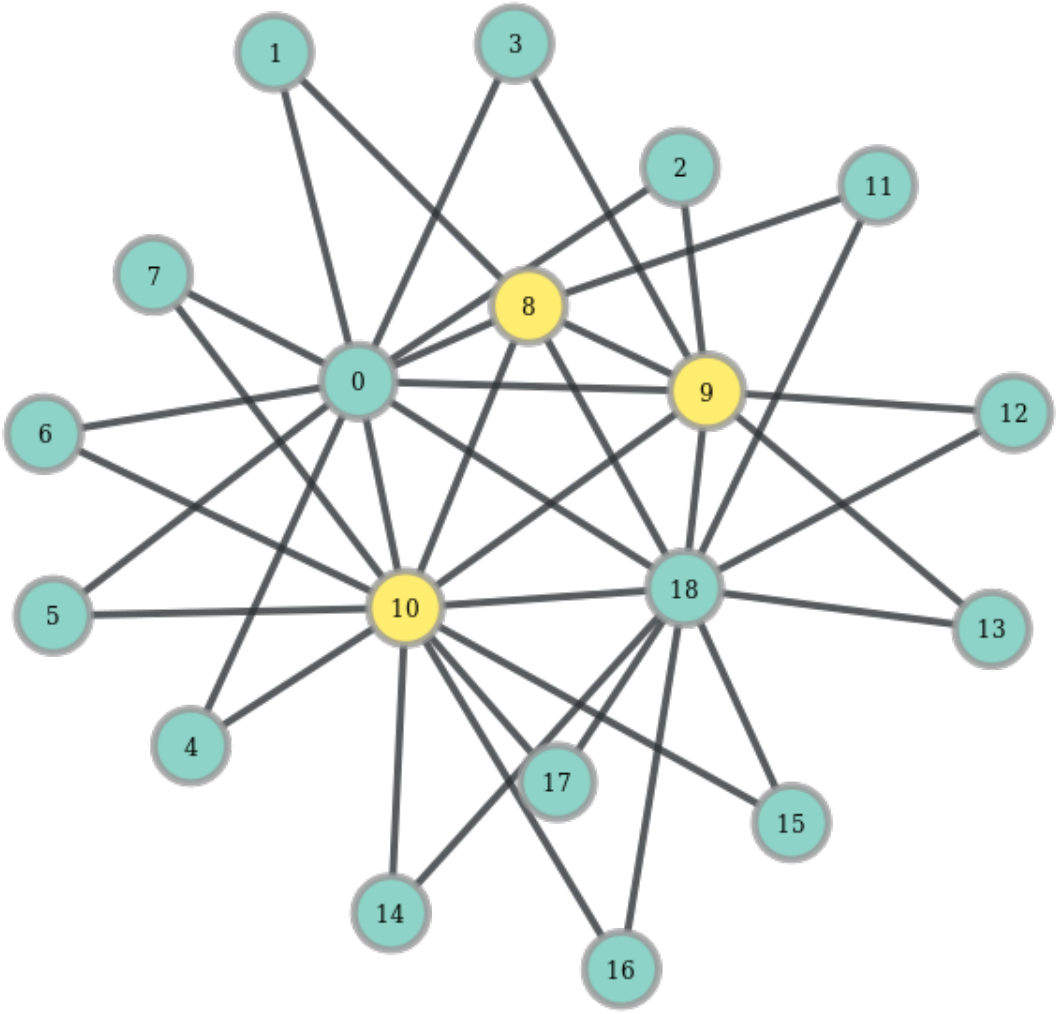
Approximate Algorithm Results

In [37]: gt.draw.graph_draw(graph, vertex_fill_color=min_dominating_set(graph), vertex_text=grap



Out[37]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

In [40]: gt.draw.graph_draw(graph,vertex_fill_color=min_connected_dominating_set(graph), vertex_

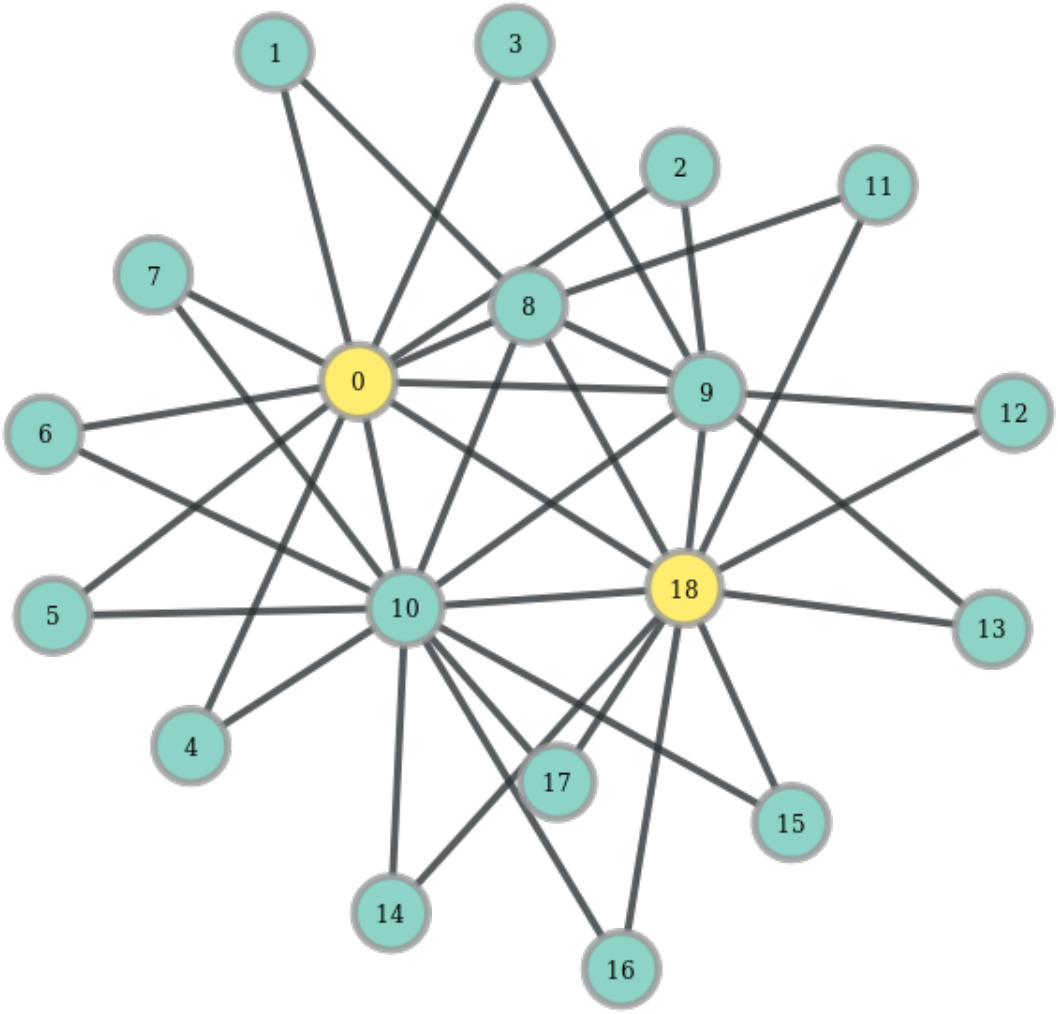


Out[40]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

Exact Algorithm Results

In [47]: gt.draw.graph_draw(graph, vertex_fill_color=min_dominating_set(graph), vertex_text=grap

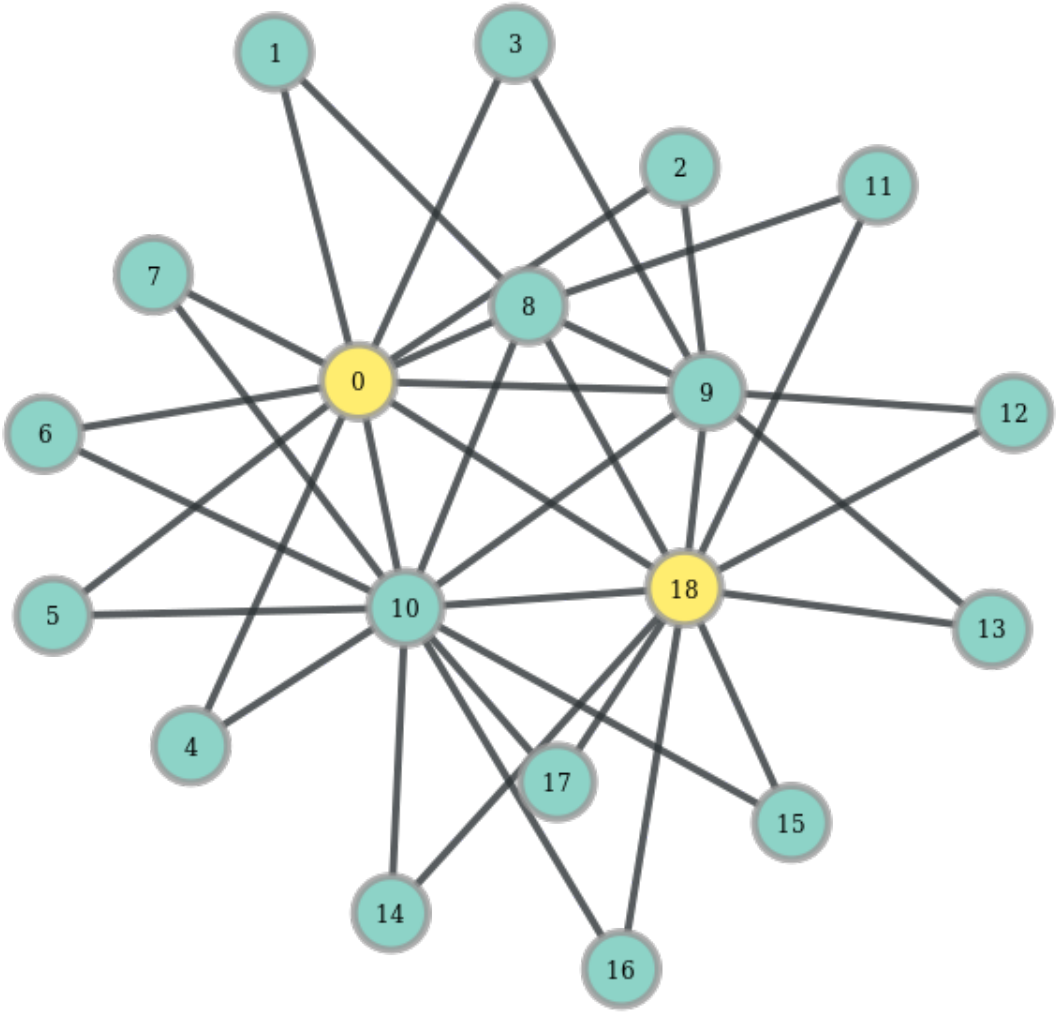
HBox(children=(IntProgress(value=0, max=524288), HTML(value='')))



Out[47]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

In [48]: gt.draw.graph_draw(graph, vertex_fill_color=min_connected_dominating_set(graph), vertex

HBox(children=(IntProgress(value=0, max=524288), HTML(value='')))



Out[48]: <PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph 0

In [39]: gt.draw.interactive_window(graph, pos=pos,vertex_fill_color=min_connected_dominating_se

Out[39]: (<PropertyMap object with key type 'Vertex' and value type 'vector<double>', for Graph
<PropertyMap object with key type 'Vertex' and value type 'bool', for Graph 0x7f5e34f4