2

Playing With Numbers

18/06/2024

LOOKING BACK

DIAGNOSTIC ASSESSMENT

5 × 8 = 40 and 32 × 3 = 96

State whether the following statements are true or false.

1. 8 is a factor of 40.
2. 40 is a multiple of 5.
3. 96 is a factor of 32.
4. 3 is a factor of 96.
5. 32 is a multiple of 96.
6. 5 is a multiple of 40.

ORDER OF OPERATIONS

C-1.3

While solving a mathematical expression involving various operations, like addition, subtraction, multiplication, division, etc., it is important to know about the order of operations.
To avoid getting different answers for the same mathematical expression, let us learn the order of operations and simplification of different brackets.

We will recollect and learn more about factors and multiples, prime and composite numbers, even and odd numbers, and divisibility tests.

Consider a mathematical expression **36 – 6 ÷ 3.**

If we subtract 6 from 36 and then divide the result by 3, we get

$$36 - 6 = 30 \quad \text{then} \quad 30 \div 3 = 10.$$

However, if we first divide 6 by 3 and then subtract the result from 36, we get

$$6 \div 3 = 2 \quad \text{then} \quad 36 - 2 = 34.$$

Let us take another example **48 + 6 ÷ 3.**
If we add 48 and 6 first and then divide the result by 3, we get

$$48 + 6 = 54 \quad \text{then} \quad 54 \div 3 = 18.$$

If we divide 6 by 3 and then add 48, we get

$$6 \div 3 = 2 \quad \text{then} \quad 48 + 2 = 50.$$

In both these examples, we find that the answers differ depending upon the order in which the operations are performed.

In order to avoid the ambiguity in the answers obtained before, the operations in a mathematical expression are performed in the following order—*division, multiplication, addition,* and *subtraction.*

This order is expressed in short as **'O D M A S'** where O = of, D = Division, M = Multiplication, A = Addition, S = Subtraction.

Therefore, in the expression 36 – 6 ÷ 3, as per the convention, the operation of division should be performed before subtraction.

Hence, 6 ÷ 3 = 2 and then 36 – 2 = 34.

In the second expression 48 + 6 ÷ 3, again division should be performed first.

Hence, 6 ÷ 3 = 2 and then 48 + 2 = 50.

In a series of operations, we use the convention 'ODMAS'. 'O' stands for 'of'. It takes predominance even over division. So the operation 'of' is performed even before division or multiplication.

Example 1: Simplify: $7 \times 3 + 60 \div 10 - 4$

Solution: $7 \times 3 + 60 \div 10 - 4$

$= 7 \times 3 + 6 - 4$ (division; $60 \div 10 = 6$)

$= 21 + 6 - 4$ (multiplication; $7 \times 3 = 21$)

(21 and 6 are to be added and 4 is to be subtracted)

$= 27 - 4$ (addition; $21 + 6 = 27$)

$= 23$ (subtraction; $27 - 4 = 23$)

Thus, $7 \times 3 + 60 \div 10 - 4 = 23$

Example 2: Simplify:

$800 + 400 \div 4 - 10 \text{ of } 70 - 3 \times 60$

Solution: $800 + 400 \div 4 - 10 \text{ of } 70 - 3 \times 60$

O $= 800 + 400 \div 4 - 700 - 3 \times 60$ (10 of 70 means 10×70)

D $= 800 + 100 - 700 - 3 \times 60$ $(400 \div 4 = 100)$

M $= 800 + 100 - 700 - 180$ $(3 \times 60 = 180)$

A $= 900 - 700 - 180$ $(800 + 100 = 900)$

S $= 200 - 180$ $(900 - 700 = 200)$

$= 20$ $(200 - 180 = 20)$

Try This!

Simplify the following.

a. $6 \times 4 + 30 \div 15 - 20$

b. $200 + 100 \text{ of } 2 - 1000 \div 2 + 2 \times 100$

Simplification of Brackets

Lata, a school student, bought 27 sweets and ate 9 of them. She distributed the remaining sweets equally among three of her friends. How many sweets did she give to each of them?

How do you express this problem as a mathematical expression?

It is clear that we have to subtract the 9 sweets that Lata ate from the 27 sweets she had before dividing them among her three friends. So, we know that we have to perform the operation of subtraction before division in this case.

We put $27 - 9$ in brackets as $(27 - 9)$ and then divide by 3. Thus, we write $(27 - 9) \div 3$.

This means that we have to first perform the operation inside the brackets and then proceed to perform any other operation.

Example 3: Evaluate: $30 + (20 - 10)$

Solution: Here we have to first perform the operation of subtraction as it is inside the brackets.

So, $30 + (20 - 10) = 30 + 10 = 40$

Example 4: Rohan bought a book for ₹55 and a pen for ₹25. He gave ₹100 to the shopkeeper. What did he receive as balance?

Solution: In this case, we have to first add the cost of the book and that of the pen, and then subtract the sum from ₹100.

The balance received (we put brackets for the sum) $= 100 - (55 + 25)$.

First, perform the operation inside the brackets.

So, $55 + 25 = 80$

Then subtract this from 100.

So, $100 - 80 = 20$

Thus, Rohan received ₹20 as balance.

Example 5: Simplify: $20 - (10 + 2 - 3)$

Solution: $20 - (10 + 2 - 3) = 20 - (12 - 3)$

$= 20 - 9 = 11$

Example 6: Simplify: $3 \times (5 - 4)$

Solution: First perform the operation inside the brackets.

$\therefore 3 \times (5 - 4) = 3 \times 1 = 3$

Note: The symbol $\therefore$ is used to refer to 'therefore.'

Try This!

Simplify: $15 + (33 - 6)$

Exercise 2 A

1. Write a mathematical expression for each of the following using brackets.

   a. 7 is multiplied to the difference of 17 and 5. 7×(17-5)

   b. Subtract 9 from the sum of 27 and 8. (27+8)-9

   c. Add 27 to the difference of 5 and 3. 27+(5-3)

   d. 36 is divided by the difference of 9 and 5. (9-5)÷36

   e. Divide the sum of 12 and 6 by the difference of 6 and 3. (12+6)÷(6-3)

   f. Add 7 multiplied by 6 to the difference of 4 and 3. (7×6)+(4-3)

2. Simplify the following:

   a. $18 - (3 + 5)$= 10

   b. $40 \times 10 \div 5 + 20$= 80

   c. $25 \div 5 + 30 - 35$=0

   d. $32 + 96 \div (7 + 9)$=38

   e. $24 + 33 \div (34 - 23)$= 27

   f. $80 \div (15 + 8 - 3) + 4$ =8

KINDS OF BRACKETS

C-1.3

The use of brackets takes us to a new order of operations, that is **B O D M A S.** This means that the operation inside the brackets comes before O D M A S. Even inside the brackets we follow the same order, O D M A S.

There are different kinds of brackets.

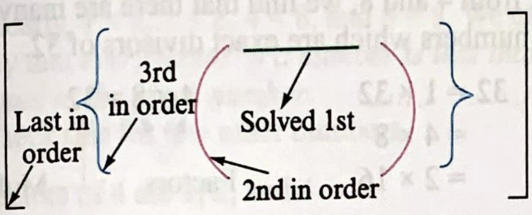
[ ] are known as box brackets, or square brackets, or big brackets.

{ } are known as curly brackets or braces.

( ) are known as simple brackets, or round brackets, or parentheses, or small brackets (as mentioned in the previous section).

— known as line brackets or vinculum, are not commonly used.

Very often we use more than one set of brackets. In such cases, the expressions within the brackets are simplified in the order —, ( ), { }, and [ ].



Example 7: Simplify: $48 \div (16 - \overline{4+4})$

Solution: $48 \div (16 - \overline{4+4})$

First remove the innermost bracket.

$= 48 \div (16 - 8)$ $(4 + 4 = 8)$

Then remove the simple brackets.

$= 48 \div 8$ $(16 - 8 = 8)$

Perform the operation of division.

$= 6$

Example 8: Simplify:

$100 - [20 + \{50 - (40 - 10)\}]$

Solution: $100 - [20 + \{50 - (40 - 10)\}]$

First remove the innermost brackets.

$= 100 - [20 + \{50 - 30\}]$ $(40 - 10 = 30)$

Now remove the curly brackets.

$= 100 - [20 + 20]$ $(50 - 30 = 20)$

Next remove the box brackets.

$= 100 - 40$ $(20 + 20 = 40)$

$= 60$

Example 9: Simplify: $18 \div 3[8 - 3(4 - 2)] + 1$

Solution: $18 \div 3\,[8 - 3\,(4 - 2)] + 1$

$= 18 \div 3\,[8 - 3\,(2)] + 1$ $(4 - 2 = 2)$

Here, 3(2) means 3×2. Now, which operation would you perform first? Division $(18 \div 3)$ or Multiplication (3×2)? **Think!**

When brackets are used in place of '×' symbol, the brackets take predominance over both multiplication and division.

$= 18 \div 3\,[8 - 6] + 1$ $(3 \times 2 = 6)$

$= 18 \div 3\,[2] + 1$ $(8 - 6 = 2)$

$= 18 \div 6 + 1$ $(3 \times 2 = 6)$

$= 3 + 1$ $(18 \div 6 = 3)$

$= 4$

Example 10: Simplify:

$15 + 9 \div 3 - [5 \times 3 - \{5 - (7 - 4)\}]$

Solution: $15 + 9 \div 3 - [5 \times 3 - \{5 - (7 - 4)\}]$

Start with the innermost bracket, i.e., $(7 - 4)$

$= 15 + 9 \div 3 - [5 \times 3 - \{5 - 3\}]$ $(5 - 3)$

$= 15 + 9 \div 3 - [5 \times 3 - 2]$ (5×3)

$= 15 + 9 \div 3 - [15 - 2]$ $(15 - 2)$

$= 15 + 9 \div 3 - 13$ $(9 \div 3)$

$= 15 + 3 - 13$ $(15 + 3)$

$= 18 - 13$ $(18 - 13)$

$= 5$

Try This!

Find the answer in both the cases and try to understand the difference in the answers of:

$(15 + 9) \div 3$ and $15 + 9 \div 3$

=10

Remember

Order of Operations

- 1^{st} operation: ***B*** Brackets
  The innermost brackets are removed first according to the rule of removing brackets.
- 2^{nd} operation: ***O*** of
  Multiply the two numbers having 'of' between them.
- 3^{rd} operation: ***D*** Division
- 4^{th} operation: ***M*** Multiplication
- 5^{th} operation: ***A*** Addition
- 6^{th} operation: ***S*** Subtraction

Exercise 2 B

1. Simplify.
   a. $70 + 2 \times 5 + 3 \text{ of } 10 - 60 \div 6$
   b. $7 + [12 - \{8 + 3 - (9 \text{ of } 6 + 1 - 13 \times 4)\}]$
   c. $5 + [14 + 5 - \{6\,(5 + 1 - 4)\}]$
   d. $100 \times 10 + [400 \div \{100 - (50 - 30)\}]$
   e. $20 - 2(5 - 4) \times \{3 - (5 - 3)\}$
   f. $45 + 3\{34 - \overline{18 - 14}\} \div 3\,[17 + 3 \times 4 - (2 \times 7)]$

MULTIPLES AND FACTORS

C-1.2

Colour all the numbers in Table 2.1 that are exactly divisible by 2, i.e., 2, 4, 6, 8, 10, 12, ... up to 100. These coloured numbers are all multiples of 2.

Table 2.1

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiples

When a number *a* is multiplied by another number *b*, the product is the multiple of both the numbers *a* and *b*. For example,

$$3 \times 4 = 12$$

So, 12 is a multiple of 3, and 12 is also a multiple of 4.

Also, $12 \times 1 = 12$ and $2 \times 6 = 12$.

Hence, 12 is a multiple of 1, 2, 3, 4, 6, and 12.

Example 11: Find all the multiples of 8 between 60 and 100.

Solution:

$8 \times 7 = 56$ $\quad$ $8 \times 11 = 88$
$8 \times 8 = 64$ $\quad$ $8 \times 12 = 96$
$8 \times 9 = 72$ $\quad$ $8 \times 13 = 104$
$8 \times 10 = 80$

These are all multiples of 8. But 56 and 104 are not between 60 and 100. Hence, the required multiples of 8 are 64, 72, 80, 88, and 96.

Factors

We know that, $4 \times 3 = 12$.
Here, 4 and 3 are factors of 12.

A factor of a number is an exact divisor of that number.

For example, 4 exactly divides 32. So, 4 is a factor of 32, and 32 is a multiple of 4. We know that 8 also divides 32 exactly. So, 8 is also a factor of 32, and 32 is a multiple of 8.

Apart from 4 and 8, we find that there are many other numbers which are exact divisors of 32.

$$32 = 1 \times 32 = 4 \times 8 = 2 \times 16$$

$\underbrace{4 \times 8}_{\text{Factors}} = \underbrace{32}_{\text{Multiple}}$

Therefore, 1, 2, 4, 8, 16, and 32 are all factors of 32. Hence, 32 is a multiple of all these numbers.

Let us find the factors of 8.

1) 8 (8
-8
0

Quotient = 8
Remainder = 0
$1 \times 8 = 8$

2) 8 (4
-8
0

Quotient = 4
Remainder = 0
$2 \times 4 = 8$

3) 8 (2
-6
2

Quotient = 2
Remainder = 2
$3 \times 2 \neq 8$

8) 8 (1
-8
0

Quotient = 1
Remainder = 0
$8 \times 1 = 8$

So, $1 \times 8 = 8$; $\quad 2 \times 4 = 8$; $\quad 8 \times 1 = 8$.

$\therefore$ 1, 2, 4, and 8 are exact divisors of 8.

So, 1, 2, 4, and 8 are factors of 8.

Example 12: Find the factors of 36.

Solution:

$1 \times 36 = 36$. 1 and 36 are factors of 36.
$2 \times 18 = 36$. 2 and 18 are factors of 36.
$3 \times 12 = 36$. 3 and 12 are factors of 36.
$4 \times 9 = 36$. 4 and 9 are factors of 36.
$6 \times 6 = 36$. 6 is a factor of 36.
Hence 1, 2, 3, 4, 6, 9, 12, 18, and 36 are all factors of 36.

Facts about Multiples and Factors

a. $1 \times 2 = 2$, $1 \times 3 = 3$.
So, *1 is a factor of every number.*

b. We know that,
$2 \times 1 = 2$ $\quad 3 \times 1 = 3 \quad 4 \times 1 = 4$
and it is true for all numbers. So, it can be said that *every number is a factor of itself.*

c. $1 \times 8 = 8 \quad 2 \times 4 = 8$
$4 \times 2 = 8 \quad 8 \times 1 = 8$
Clearly, $1 < 8$, $2 < 8$, $4 < 8$, and $8 = 8$. We can say that *every factor of a number is less than or equal to the given number.*
Check this for few other numbers.

d. Factors of 4 are 1, 2, and 4.
Factors of 8 are 1, 2, 4, and 8.
Factors of 16 are 1, 2, 4, 8, and 16.
Factors of 64 are 1, 2, 4, 8, 16, 32, and 64.
The number of factors are 3, 4, 5, and 7 respectively, i.e., the number of factors are countable. Thus, *the number of factors of a given number are always finite*, i.e., they can be counted.

e. $7 \times 1 = 7 \quad 7 \times 2 = 14 \quad 7 \times 3 = 21$
$7 \times 4 = 28 \quad 7 \times 5 = 35$
$7 = 7$, $14 > 7$, $21 > 7$, $28 > 7$, $35 > 7$.
7, 14, 21, 28, 35 are multiples of 7 and all these are either equal to or greater than 7.
Thus, *every multiple of a number is equal to or greater than the given number.*

f. Multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, ... This list does not end. So, it can be said that the *number of multiples of a number of a given number is infinite.*

g. 2 is a multiple of 2.
32 is a multiple of 32.
Thus, *every number is a multiple of itself.*

Remember

- 1 is a factor of every number.
- Every number is a factor of itself.
- Every factor of a number is less than or equal to the given number.
- The number of factors of a given number are always finite.
- Every number is a multiple of itself.
- The number of multiples of a given number is infinite.
- Every multiple of a number is equal to or greater than the given number.

Try This!

1. Write down the first five multiples of 7.
2. Write down the first five multiples of 10.
3. Write all the factors of 15.

Perfect Number

Factors of 6 are 1, 2, 3, and 6.

Now, the sum of the factors of 6

$= 1 + 2 + 3 + 6 = 12 = 2$ times 6

Factors of 28 are 1, 2, 4, 7, 14, and 28.

Now, the sum of the factors of $28 = 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2$ times 28.

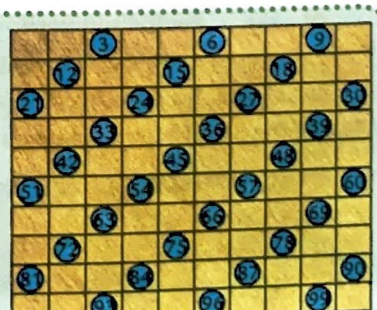
Numbers like 6 and 28 are called ***perfect numbers.***

*A number is called a **perfect number** if the sum of all its factors is equal to twice the number.*

Maths and Art

21 CENTURY SKILLS Creativity

Take two different coloured sheets and make a 100 squares grid on both. On the first sheet, write the numbers from 1 to 100. Out of the second sheet cut out circles after leaving every 2 squares. Overlay the second sheet on top of the first sheet and see all the multiples of 3.
Now use different coloured sheets and find multiples of 4 and 5.



Exercise 2 C

20/06/2024

1. Look at the table and answer the following.

   a. List the multiples of 2 from 1 to 40.

   b. What are the multiples of 3 between 20 and 40?

   c. What are the multiples of 4 between 70 and 90?

   d. How many even numbers are there between 11 and 31? =10

   e. List the even numbers between 51 and 81.

   f. How many multiples of 6 are there between 1 and 50? =8(=6, 12, 18, 24, 30, 36, 42 & 48)

   g. How many multiples of 5 are there between 11 and 51? =8

   h. How many odd numbers are there between 10 and 40? =15

   i. List all the odd numbers between 50 and 90.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. List all the factors of the following numbers.

   a. 12 b. 25 c. 16 d. 24 e. 18 f. 77 g. 288 h. 250

3. Fill in the blanks.

   a. If a number a is exactly divisible by b, then b is a Factor of a and $a \div b$ will have 0 as the remainder.

   b. A number is a Factor and a Multiple of itself.

4. Write all the multiples of the following numbers between 55 and 105.

   a. 10 b. 15 c. 20 d. 25 e. 30 f. 40

5. Write all the multiples of the following numbers between 200 and 300.

   a. 55 b. 63 c. 82 d. 91 e. 43 f. 25

6. Write all the multiples of 2:

   a. starting from 102 up to 130 b. starting from 340 up to 360

Higher Order Thinking Skills

Critical thinking

1. A scout master wanted his 72 cadets to take part in a parade, marching in a rectangular formation. He wanted that the columns as well as the rows must have more than 6 students. If each row has fewer students than the number of the columns, how many rows will he be able to make in the formation he wishes?
2. 8 cakes have to be divided equally among 9 children. Sheela tried many ways of cutting the cakes into equal pieces. What is the smallest number of pieces she has to make of each cake so that she can easily give equal amount of cake to each child without wasting any part of the cake? How many pieces will each child get?
3. There are 36 square tables. They can be arranged in multiple ways. If the tables are to be arranged in a rectangular shape, how many such arrangements are possible?

   Hint: Two of the possible arrangements are 1 × 36 (1 row of 36 tables) and 2 × 18 (2 rows with 18 tables in each row).

TYPES OF NUMBERS

C-1.2

Prime and Composite Numbers

Write the factors of the numbers given in Table 2.2.

We find that:

a. 1 is the only number with just one factor.

Table 2.2

Number	Factors	Number	Factors
1	1	11	
2	1, 2	12	
3	1, 3	13	
4	1, 2, 4	14	
5		15	
6		16	1, 2, 4, 8, 16
7		17	
8	1, 2, 4, 8	18	
9		19	
10		20	

b. There are many numbers with exactly two factors, 1 and the number itself. For example, 2, 3, 5, and 7. These are called ***prime numbers***.

c. There are many numbers with more than two factors, for example, 8 and 16. These are called ***composite numbers***.

Let us learn more about prime numbers through the exercise in table 2.3. Follow the steps given here and cross out composite numbers.

Table 2.3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1: Number 1 is neither prime nor composite. Cross it out.

Step 2: Number 2 is a prime number. Do not cross it out. But all other multiples of 2 are composite, so cross them out. In other words, we are crossing out all composite numbers which are multiples of 2.

Step 3: Number 3 is a prime number. So do not cross it out. But cross out all multiples of 3 as they are all composite numbers.

Step 4: Number 4 and its multiples have already been crossed out when we crossed out the multiples of 2, and the number 4 itself is a multiple of 2. Number 5 is a prime number, so do not cross it out. Cross out all the other multiples of 5, not already crossed out.

Step 5: Number 6 and all its multiples are crossed out when we crossed out the multiples of 2 and 3. Number 7 is a prime number. Do not cross it out. Cross out all the multiples of 7, not already crossed out.

Step 6: The multiples of 8, 9, and 10 have already been crossed out when we crossed out the multiples of 2, 3, and 5. Number 1 and all the composite numbers up to 100 have been crossed out. So the numbers left uncrossed in the table are all prime numbers. This method of finding the prime numbers is known as ***Eratosthenes' Sieve*** named after the Greek mathematician Eratosthenes (274 BC – 194 BC), who derived it.

Sieve of Eratosthenes

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Remember

- Numbers with only two factors, 1 and the number itself, are known as prime numbers. Examples are 2, 3, 5, 7, 11, and 13.
- Numbers with more than two factors are called composite numbers. Examples are 4, 6, 8, 9, 10, and 12.
- Number 2 is the smallest and the only even prime number.
- Number 1 is neither prime nor composite.

Co-prime Numbers

*Two numbers that have only 1 as their common factor are known as **co-prime numbers**.*

Thus, all prime numbers are co-prime numbers. However, composite numbers can also be co-prime numbers.

Example 13: Are 35 and 39 co-prime numbers?

Solution: $35 = 7 \times 5 \times 1$

$39 = 3 \times 13 \times 1$

Though both 35 and 39 are composite numbers, the only common factor is 1. Therefore, 35 and 39 are co-prime numbers.

Twin Primes

Let us again consider table 2.3. The prime numbers 17 and 19 have got only one number, 18, in between them. In other words, these two primes are close to each other with a gap of only one number. Such pairs of primes are called ***twin primes***. 17 is called the twin prime of 19 and vice versa.

Prime Triplet

*A set of three consecutive prime numbers, differi... by 2, is called a **prime triplet**.* The only prime tr... is (3, 5, 7).

Even and Odd Numbers

The multiples of 2 are called even numbers or we can say that *the numbers which are exactly divisi... by 2 are called **even numbers***. For example, 2, 4, ... 8, 10, 12, and so on.

*The numbers that are not divisible by 2 are known as **odd numbers**.* For example, 1, 3, 5, 7, 9, 11, ar... so on.

Mental Maths PA

1. What is the twin-prime number of 61 if it has a twin-prime number?
2. How many prime numbers are there between 1 and 50?
3. List all the prime numbers between 1 and 100.

Exercise 2 D

24/06/2024

1. How many prime numbers exist between:
   a. 1 and 10? =4 b. 1 and 20?=8 c. 20 and 40? =4 d. 40 and 80? =10 e. 80 and 100?=
2. Fill in the boxes with the words *even* or *odd*.
   a. The sum of two even numbers is always even.
   b. The sum of two odd numbers is always even.
   c. The sum of an even and an odd number is always odd.
   d. The product of two even numbers is always even.
   e. The product of two odd numbers is always odd.
   f. The product of an even and an odd number is always even.
3. Find out the prime numbers from the following numbers.
   a. 141 b. 181 c. 86 d. 83 e. 81 f. 67 g. 93 h. 61 i. 43 j. 97
   k. 91 l. 30 m. 163 n. 187 o. 184 p. 131 q. 169 r. 119 s. 134 t. 177
4. Which of the following pair of numbers are co-prime numbers?
   a. 52 and 81 b. 294 and 256 c. 88 and 187 d. 675 and 392
5. Write the prime numbers from 1 to 100 whose ones place digit is:
   (*Note:* The ones place is also called as units place.)
   a. 1-11 31, 41, 61, 71 b. 3-13, 23, 43, 53, 73 c. 5 d. 7 H.W
6. Which are the pairs of twin primes from 1 to 100?
   (2,3) (3,5) (5,7) (11,13) (17,19) (29,31) (41,43) (59,61) (71,73)

TESTS FOR DIVISIBILITY BY 2, 3, 4, & 5

C-1.2

An understanding of the tests of divisibility is very helpful in finding the factors of a given number. Factors are useful for dividing a collection of things into equal groups. For example, 24 pastries can be packed into boxes containing 1, 2, 3, 4, 6, 8 , or 12 pastries in each box. Here, 1, 2, 3, 4, 6, 8, and 12 are factors of 24.

Divisibility by 2

Consider the number 38. We know that 10, 20, 30, 40, etc., are all divisible by 2. As 10 is a multiple of 2, all the multiples of 10 are also divisible by 2. So in 38, 30 is divisible by 2.
We have to check whether 8, the ones place digit in 38, is divisible by 2. Since 8 is divisible by 2, 38 is also divisible by 2.

Consider the numbers 1132, 12134, 11546, 121128, and 2123240. It is difficult to say whether they are divisible by 2, just by looking at them. Now the digits at the ones place are 2, 4, 6, 8, and 0, respectively. When these numbers are divided by 2, the remainder is 0. Thus, *a number is divisible by 2 if the digit at the ones place is 0, 2, 4, 6, or 8.*

Divisibility by 3

Let us see what happens when we divide a number by 3.

Observe the remainder and the sum of the digits of a number in each of the following cases.

Dividend	Divisor	Quotient	Remainder	Sum of the digits
1	3	0	1	1
10	3	3	1	1
11	3	3	2	2
12	3	4	0	3
24	3	8	0	6
100	3	33	1	1
126	3	42	0	9
171	3	57	0	9
101	3	33	2	2
487	3	162	1	19

We find that wherever the remainder is 0, 3 exactly divides the number and the sum of the digits of that number is a multiple of 3. Thus, *if the sum of the digits of a number is divisible by 3, then the number is divisible by 3.*

Consider the number 813. The sum of the digits is $8 + 1 + 3 = 12$. 12 is divisible by 3. Hence, 813 is also divisible by 3.

Example 14: Is 721 divisible by 3?

Solution: The sum of the digits is $7 + 2 + 1 = 10$. 10 is not divisible by 3. Hence, 721 is not divisible by 3.

Divisibility by 4

Consider the number 324. We know that 100, 200, 300, 400, etc., are all divisible by 4 as $25 \times 4 = 100$. In 324, 300 is divisible by 4. So we have to check whether 24 is divisible by 4. It is divisible by 4 as $6 \times 4 = 24$. So, the number 324 is divisible by 4.

A number is divisible by 4 if the number formed by the last two digits of the number is divisible by 4.

Example 15: Which of these numbers is divisible by 4?

a. 736 b. 970

Solution:

a. In 736, 700 is divisible by 4 as all multiples of 100 are divisible by 4. The number formed by the last two digits, 36, is also divisible by 4. Hence, 736 is divisible by 4.
b. In 970, 900 is divisible by 4, but 70 is not divisible by 4. Hence, 970 is not divisible by 4.

Divisibility by 5

We know that any multiple of 5 always has its units digit as either 5 or 0. Examples are 25, 100, 130, etc., which are all divisible by 5.

A number is divisible by 5 if the units digit of the number is either 5 or 0.

Example 16: Which of these numbers are divisible by 5?

a. 3457 b. 275 c. 8130

Solution:

a. In 3457, the units digit is 7. Hence, the number is not divisible by 5.
b. In 275, the units digit is 5. Hence, 275 is divisible by 5.
c. In 8130, the units digit is 0. Hence, 8130 is divisible by 5.

Exercise 2 E 26|06|2024

1. Which of the following numbers are divisible by 2 (multiples of 2)?
   a. 362 b. 731 c. 895 d. 8246 e. 217 f. 812 g. 6022 h. 5818
2. Which of the following numbers are divisible by 3 (multiples of 3)?
   a. 231 b. 343 c. 861 d. 783 e. 61 f. 81 HW
3. Which of the following numbers are divisible by 4 (multiples of 4)?
   a. 132 b. 72 c. 382 d. 1236 e. 7141 f. 8612 HW
4. Which of the following numbers are divisible by 5 (multiples of 5)?
   a. 375 b. 25 c. 83 d. 70004 e. 6105 f. 2100 g. 8325 h. 5005 HW

TESTS FOR DIVISIBILITY BY 6 & 8

Divisibility by 6

C-1.2

A number divisible by 6 must be an even number as 6 is an even number. We have $6 = 2 \times 3$. So, a number divisible by 6 must be divisible by both 2 and 3, i.e., it should satisfy the divisibility rules of both 2 and 3.

Let's consider a few examples:

a. 351 is not an even number. Hence, 351 is not divisible by 2 and thus it cannot be divisible by 6.

b. 422 is an even number. But the sum of its digits is 8, which is not divisible by 3. So, 422 is not divisible by 3 and hence, is not divisible by 2 and thus it cannot be divisible by 6.

c. 792 is an even number. Hence, it is divisible by 2. The sum of its digits is equal to 18, which is divisible by 3. So, 792 is divisible by 2 and 3. Therefore, it is divisible by 6.

Divisibility by 8

We know that 1000, 2000, 3000, etc., are all multiples of 8 as $8 \times 125 = 1000$.

Consider the number 7128. Since 1000 is divisible by 8. 7000 is also divisible by 8. We must check if 128 is divisible by 8. Since, 128 (8×16) is divisible by 8, 7128 is also divisible by 8.

A number is divisible by 8 if the number formed by the last three digits of the number is divisible by 8.

Example 17: Which of the following numbers are divisible by 8?

a. 34672 b. 384132

Solution:

a. In 34672, the number formed by the last three digits is 672 (8×84) which is divisible by 8. Hence, 34672 is divisible by 8.

b. In 384132, the number formed by the last three digits is 132 which is not divisible by 8. Hence, 384132 is not divisible by 8.

Exercise 2 F 26|06|2024

HW

1. From the following, find the numbers divisible by 6.
   a. 672 b. 813 c. 7312 d. 1236 e. 4314
   f. 689 g. 263 h. 164 i. 8135 j. 7236
2. From the following, find the numbers divisible by 8.
   a. 328 b. 4728 c. 8256 d. 9096 e. 6324
   f. 8004 g. 5369 h. 6072 i. 4568 j. 4821

TESTS FOR DIVISIBILITY BY 9, 10, & 11

C-1.2

Divisibility by 9

Let us see what happens when a number is divided by 9. Observe the remainder and the sum of the digits of the number.

Dividend	Divisor	Quotient	Remainder	Sum of the digits
1	9	0	1	1
9	9	1	0	9
17	9	1	8	8
27	9	3	0	9
81	9	9	0	9
162	9	18	0	9
372	9	41	3	12
891	9	99	0	18

We find that wherever the remainder is 0, 9 exactly divides the number. The sum of the digits of such numbers is 9 or a multiple of 9.

For example, the sum of the digits of 3980439 is 36 and 36 is divisible by 9. So, 3980439 is divisible by 9.

A number is divisible by 9 if the sum of the digits of a number is divisible by 9.

Example 18: Which of the following numbers are divisible by 9?

a. 3457 b. 8154 c. 3465

Solution:

a. In 3457, the sum of the digits $= 3 + 4 + 5 + 7 = 19$ and this number is not divisible by 9. Hence, 3457 is not divisible by 9.

b. In 8154, the sum of the digits $= 8 + 1 + 5 + 4 = 18$ which is divisible by 9. So, 8154 is divisible by 9.

c. The sum of the digits $= 3 + 4 + 6 + 5 = 18$ is divisible by 9. Hence, 3465 is divisible by 9.

Divisibility by 10

A number is divisible by 10 if its units digit is 0.

For example, 20, 30, 90, etc., are all divisible by 10 as their units digit is 0. A number such as 43 is not divisible by 10.

Example 19: Which of the following numbers are divisible by 10?

a. 1347 b. 780 c. 8003

Solution:

a. In 1347, the ones digit is 7 and not zero. Hence, 1347 is not divisible by 10.

b. In 780, the ones digit is zero. Hence, 780 is divisible by 10.

c. In 8003, the ones digit is not zero. Hence, 8003 is not divisible by 10.

Remember

- A number is divisible by 9 if the sum of the digits of the number is divisible by 9.
- A number is divisible by 10 if its units digit is 0.

Divisibility by 11

Number	Sum of the digits (at odd places from right)	Sum of the digits (at even places from right)	Difference	Divisible by 11
121	1 + 1 = 2	2	2 - 2 = 0	Yes
1331	1 + 3 = 4	3 + 1 = 4	4 - 4 = 0	Yes
123456	6 + 4 + 2 = 12	5 + 3 + 1 = 9	12 - 9 = 3	No
3456788	8 + 7 + 5 + 3 = 23	8 + 6 + 4 = 18	23 - 18 = 5	No

From the table we can see that, *a number is divisible by 11 if the difference of the sums of the alternate digits is either 0 or divisible by 11.*

Consider the number 7139.

$7139 \div 11 = 649$. Here the sum of the alternate digits 9 and 1 is equal to the sum of the alternate digits 3 and 7. Hence, the difference between the sums is 0.

$81345 \div 11 = 7395$ and the remainder is 0. The sum of the alternate digits $5 + 3 + 8 = 16$. The sum of the other set of alternate digits $4 + 1 = 5$. The difference between these two sums $16 - 5 = 11$, which is divisible by 11.

In 3456788, the sum of the alternate digits $= 8 + 7 + 5 + 3 = 23$ and the sum of the other set of alternate digits $= 8 + 6 + 4 = 18$. Their difference $23 - 18 = 5$, which is not divisible by 11. Hence, the number 3456788 is not divisible by 11.

Example 20: Which of the following numbers are divisible by 11?

a. 3467 **b.** 53064 **c.** 81895

Solution:

a. In 3467, the difference of the sum of alternate digits $7 + 4$ and $6 + 3$ is 2. It is neither zero nor a multiple of 11. Hence, 3467 is not divisible by 11.

b. In 53064, the difference of the sum of alternate digits $4 + 0 + 5$ and $6 + 3$ is zero. Hence, the number 53064 is divisible by 11.

c. In 81895, the difference of the sum of the alternate digits $5 + 8 + 8$ and $9 + 1$ is $21 - 10 = 11$. This is divisible by 11. Hence, 81895 is divisible by 11.

Some General Properties of Divisibility

1. *If a number is divisible by another number, then it is divisible by every factor of that number.* For example, 48 is divisible by 12. So, 48 is also divisible by 1, 2, 3, 4, 6, and 12.
2. *If a number is divisible by two or more co-prime numbers, it must be divisible by their product.* For example, 56 is divisible by 4 and 7 which are co-prime numbers, and it is also divisible by their product $(4 \times 7 =)$ 28.
3. *If a number is a factor of each of the two given numbers, then it must be a factor of their sum.* For example, 6 is a factor of 24 and 48, and 6 is also a factor of $(24 + 48 =)$ 72.
   We can also say *if two given numbers are divisible by a number, then their sum is also divisible by that number.*

> If a number is a factor of the sum of two numbers, then it is not necessarily a factor of each of the numbers. For example, $25 + 14 = 39$ which is divisible by 13 or 13 is a factor of 39. But, 13 is not a factor of either 25 or 14.

4. *If a number is a factor of two numbers then it is a factor of their difference also.* For example, 7 is a factor of 56 and 35, and $56 - 35 = 21$; 7 is a factor of 21 also.

Exercise 2 G

27/06/2024

1. From the following, find the numbers divisible by 9.
   a. 8163 b. 7214 c. 8353 d. 6345 e. 1584 f. 3617 g. 6273 h. 8001
2. Identify the numbers divisible by 10.
   a. 29 b. 430 c. 89 d. 77 e. 120 f. 33 g. 17908 h. 3640
3. Identify the numbers divisible by 11.
   a. 71412 b. 376277 c. 6116 d. 86124 e. 643214
   f. 20438 g. 48295 h. 14909 i. 97526 j. 563761
4. You are given three digits 7, 2, and 9. H.W
   a. How many three-digit numbers can be formed without repeating any of them? 6times
   b. How many of the numbers formed in a. are divisible by 3? Why?
   c. How many of the numbers formed in a. are divisible by 9? Why?
   d. How many of the numbers formed in a. are divisible by 6? Why?

Application-Based Questions

5. $72a384$ is a number in which one of the digits is 'a'. If the number is exactly divisible by 9, what is the numerical value of 'a'?
6. $46a7b2$ is a 6-digit number of in which a and b are two missing digits. This number is divisible by 9. Find the least possible value of $a + b$. Also, state the maximum possible value of $a + b$.
7. There are 150 students waiting for some boats to cross a river. The students have to be divided into equal groups to get into different boats. In how many ways can you group the students such that each group has more than 5 students and less than 50 students?

Chapter Check-Up

SELF ASSESSMENT

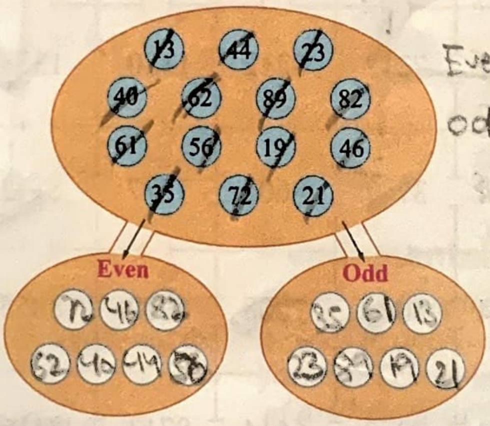
Multiple Choice Questions

1. Which of the following numbers is a multiple of 15?
   a. 115 b. 135 c. 70 d. 85
2. Which of the following numbers is divisible by 11?
   a. 2248 b. 3648 c. 1561 d. 2695

Practice Time

3. Simplify the following.
   a. 6 of 20 ÷ 5 – 10 =14
   b. 85 – 20 ÷ 4 + 10 of 2 = 65
   c. 37 + 26 ÷ 2 + 2 of 25 – 80 ÷ 2 = 60
   d. 45 + (35 – 12) – 5 = 73
   e. 20 ÷ (2 of 3 + 8 – 4) + 7 = 9
   f. 80 – {20 – (5 + 2 – 4 ÷ 2)} = 64
   g. 90 + {10 + 15 of 3 – (20 + 30 – 45 ÷ 5)} = 94
   h. 8 – {3 – 2 + (6 – 4 + 1)} = 2
4. What are the multiples of 5 between 20 and 50?
5. Write all the factors of 72.= 1, 2, 3, 4, 8, 9, 72, 36, 24, 18, 12
6. Is 28 a perfect number? Explain. Give one more example of a perfect number.
7. List the twin primes between 1 and 50. (2,3) (3,5) (5,7) (11,13) (17,19) (29,31) (41,43)
8. Which of the following pairs of numbers are co-prime numbers?
   a. 441 and 288 b. 91 and 27 c. 2310 and 2431
9. Separate the odd and even numbers from the following.

927
972
297
279
729
792



Even:- 72, 46, 82, 62, 40, 44, 56
odd:- 35, 61, 13, 23, 89, 19, 21

10. Identify the numbers which are divisible:
    a. by 3.
       i. 319 ii. 528 iii. 116 iv. 4173
    b. by 4.
       i. 343 ii. 454 iii. 14468 iv. 1376
    c. by 5.
       i. 6556 ii. 34515 iii. 8106 iv. 7800
11. Identify the numbers which are divisible:
    a. by 6.
       i. 348 ii. 294 iii. 232 iv. 94
    b. by 8.
       i. 458 ii. 1472 iii. 6132 iv. 1104
    c. by 9.
       i. 333 ii. 90 iii. 469 iv. 2149
    d. by 10.
       i. 31 ii. 50 iii. 1250 iv. 505
    e. by 11.
       i. 121 ii. 894036 iii. 693 iv. 40982

Maths Lab Activity

Tests of Divisibility

Objective: To enable the children to recognise the multiples of numbers by applying the relevant tests of divisibility

Materials Required: A maze of numbers and three factors 5, 6, and 8, a pencil, and an eraser.

Preparation: Students work in pairs.

Steps:

1. Note that the number at the starting point is 480. The options to move to the next box are 2008 or 3421.
2. Check if 2008 is a multiple of any one of the numbers 5, 6, or 8. (Remember that the number to which we can move has to be a multiple of any one of 5, 6, or 8.) By applying the tests of divisibility, we find that 2008 is not a multiple of 5 or 8 but is a multiple of 6. Also, 3421 is not a multiple of any of these three numbers. Hence, an arrow is drawn to the box containing 2008.
3. From 2008, the next two options are 1209 or 1264. 1209 is not a multiple of any of the three given factors. Hence, move on to 1264, as 1264 is a multiple of 8.
4. Keep on moving till you reach the end.

Start

480	3421	2225	3342	456	7473	2818
2008	1264	7212	8212	1380	1338	7626
1209	9124	3747	8602	7777	8811	5124
2030	4104	5004	7008	2032	8855	6416
4128	2103	7052	6614	8713	3116	2089
2136	565	3054	9019	2016	3318	9102
9971	6046	7806	9088	7015	3313	1096
8884	6713	1557	7209	4118	617	960

End