3 HCF and LCM

LOOKING BACK

1. Factors of 14 are 1, 2, 7, and 14. $140 = 14 \times 10$, then state whether the following statements are true or false:
   a. 2 is a factor of 140.
   b. 10 is not a factor of 14.
   c. 7 is the only common factor of 14 and 140.
   d. 140 is a multiple of 10.
2. $120 = 30 \times 4$ and $120 = 10 \times 12$, then state whether the following statements are true or false:
   a. 30 is a factor of 120.
   b. 10 is a factor of 30.
   c. Only factors of 120 are: 1, 4, 10, 12, and 30.
   d. 4 is a factor of 30.

PRIME FACTORISATION

C-1.2

Consider a number, say, 30. This is a composite number. We can write 30 as $30 = 2 \times 15$. But 15 is also a composite number which can be written as $15 = 3 \times 5$.

Therefore, $30 = 2 \times 3 \times 5$.

Again, we can write 30 as $30 = 5 \times 6$. As 6 is a composite number and $6 = 2 \times 3$.

So, $30 = 5 \times 2 \times 3$.

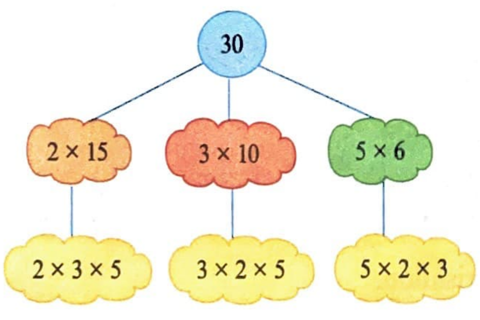
Yet again, we can write 30 as $30 = 3 \times 10$. But $10 = 2 \times 5$.

Therefore, $30 = 3 \times 2 \times 5$.

When we write $30 = 2 \times 3 \times 5$, none of the numbers 2, 3, or 5 are composite numbers. 2, 3, and 5 are the *prime factors* of 30.

$30 = 2 \times 3 \times 5$ *is the* ***prime factorisation*** *of* 30.

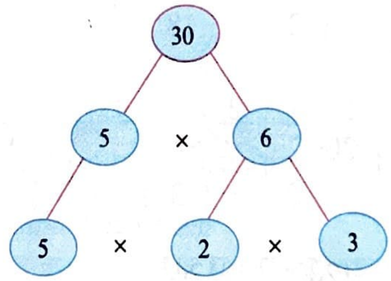
Now we can represent this as:



Prime factorisation *is the expression of a given number as a product of its prime factors.*

The prime factorisation of a number can be represented as a ***factor tree***.

Factor tree for 30



$30 = 5 \times 2 \times 3$

Fundamental Theorem of Arithmetic

Every composite number can be factorised into primes in only one way, except for the order of prime factors. This is known as the ***fundamental theorem of arithmetic*** or the ***unique factorisation theorem*** or ***prime factorisation property***.

There can be many factorisations of a number but there can be ***only one*** prime factorisation. Thus, prime factorisation of a number is unique.
For example,
$30 = 2 \times 3 \times 5$ is the prime factorisation of 30 which is unique.

Remember

- 1 and composite numbers are not included in the prime factorisation of any number.

Steps for Finding Prime Factorisation

Example 1: Express 72 as a product of prime factors and draw its factor tree.

Solution: Steps to be followed:

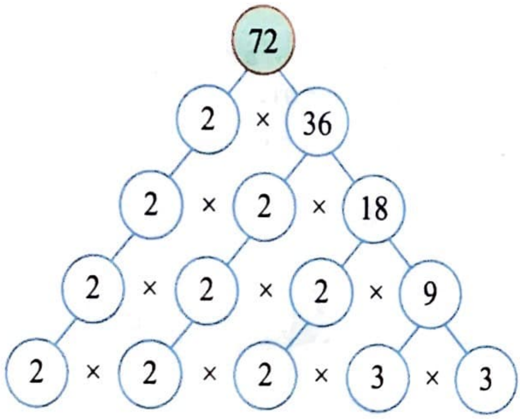
1. Start from the smallest prime number which completely divides 72.
2. Continue dividing the quotient obtained until you get a quotient which is no more divisible by the smallest prime number chosen.
3. Then try to divide this quotient by the next higher prime number that will divide it exactly.
4. Carry on this process until the quotient is also a prime number.

2	72
2	36
2	18
3	9
	3

The product of all the dividing prime factors will give the prime factorisation of 72.

$\therefore \quad 72 = 2 \times 2 \times 2 \times 3 \times 3$

Factor tree for 72 is

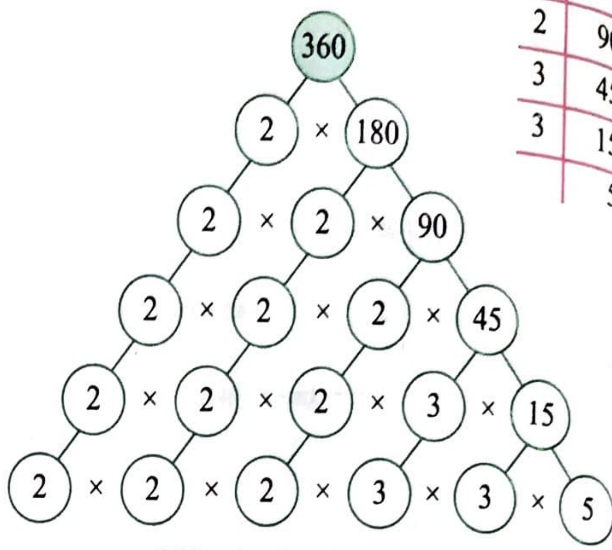


$72 = 2 \times 2 \times 2 \times 3 \times 3$

Example 2: Find the prime factorisation of 360 and draw a factor tree for the same.

Solution: $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$ is the prime factorisation of 360.

2	360
2	180
2	90
3	45
3	15
	5



$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

Remember

- Prime factorisation is the type of factorisation where a number is expressed as a product of its prime factors.
- The prime factorisation property or the fundamental theorem of arithmetic states that every composite number has only one prime factorisation.

Try This!

Find the prime factorisation of: a. 252 b. 891

Exercise 3 A

1. Draw the factor tree and write down the prime factorisation for each of the following numbers.
   a. 20 b. 36 c. 216 d. 272 e. 84 f. 305
2. Express the following as a product of prime factors.
   a. 150 b. 725 c. 230 d. 732 e. 256 f. 67
3. Identify which of the following is the correct prime factorisation.
   a. $48 = 2 \times 2 \times 3 \times 4$ 2×3 b. $168 = 7 \times 2 \times 3 \times 2 \times 2$ ✓ c. $350 = 2 \times 25 \times 7$ 5×5
   d. $28 = 2 \times 14$ 2×7 e. $182 = 2 \times 3 \times 3 \times 3 \times 3$ H.W f. $512 = 2 \times 2 \times 2 \times 8 \times 8$ 2×2×2×2
4. Write the smallest 4-digit number and find its prime factorisation.
5. Write the largest 4-digit number and find its prime factorisation.
6. Draw a factor tree for the predecessor of 100.

COMMON FACTORS

C-1.2

Consider two numbers, say, 6 and 9.

The factors of 6 are 1, 2, 3, and 6.
The factors of 9 are 1, 3, and 9.

We can see that 3 is a factor of 6 and a factor of 9. Hence, we say that 3 is a common factor of 6 and 9.

Let us consider two other numbers 30 and 40.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.
The factors of 40 are 1, 2, 4, 5, 8, 10, 20, and 40.
We find that 1, 2, 5, and 10 are factors of both 30 and 40.

Hence, 1, 2, 5, and 10 are the common factors of 30 and 40.

Example 3: Find the common factors of 54 and 72.

Solution: The factors of 54 are:
1, 2, 3, 6, 9, 18, 27, and 54

The factors of 72 are:
1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

Thus, the common factors are 1, 2, 3, 6, 9, and 18.

Try This!

Find the common factors of 90 and 225.

Highest Common Factor or HCF

Consider the following example. Three groups of athletes from three different schools are taking part in the inaugural march past of an inter-school sports function. The number of athletes from each school are 72, 64, and 48, respectively.

Each school is required to march as a separate group, one behind the other. The number of athletes in each row must be the same for all the three schools and all the rows must be complete. So what would be the maximum number of students in each row for the march past?

Number of students from the first school = 72

Number of students from the second school = 64

Number of students from the third school = 48

Consider the first school. If 72 students have to form rows and columns for the march past, they have to form rectangular blocks. The possibilities are 1 × 72, 2 × 36, 3 × 24, 4 × 18, 6 × 12, and 8 × 9.

Let us consider the second school. Here, the possibilities are 1 × 64, 2 × 32, 4 × 16, and 8 × 8.

For the third school, the possibilities are 1 × 48, 2 × 24, 3 × 16, 4 × 12, and 6 × 8.

School	No. of students	Possibilities	
1st	72	1 × 72	4 × 18
		2 × 36	6 × 12
		3 × 24	8 × 9
2nd	64	1 × 64	4 × 16
		2 × 32	8 × 8
3rd	48	1 × 48	4 × 12
		2 × 24	6 × 8
		3 × 16	

But each row must have the same number of students.

Possible formations

One in a row	Two in a row	Four in a row	Eight in a row
1 × 72	2 × 36	4 × 18	8 × 9
1 × 64	2 × 32	4 × 16	8 × 8
1 × 48	2 × 24	4 × 12	8 × 6

From the possible formations, we can see that 8 is the highest possible number of athletes in a row such that all three school teams can march in rectangular blocks and also have the maximum possible number of students in a row. The first school will have 9 rows with 8 athletes in each row. The second school will have 8 rows of 8 athletes and the third school will have 6 rows of 8 athletes.

We can see that 8 is the highest factor common to—72, 64, and 48. So, 8 is said to be the **highest common factor** (HCF) of 72, 64, and 48. Note that 8 is the greatest number that divides these 3 numbers.

Finding HCF by Listing Factors

Let us find the highest common factor of the numbers 72, 64, and 48 by listing their factors.

Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72

Factors of 64: 1, 2, 4, 8, 16, 32, and 64

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48

The common factors of 72, 64, and 48 are 1, 2, 4, and 8. The highest of these common factors is 8.

Hence, the HCF of 72, 64, and 48 is 8.

Example 4: Find the HCF of 18, 27, and 45 by listing factors.

Solution: The factors of 18 are 1, 2, 3, 6, 9, and 18.
The factors of 27 are 1, 3, 9, and 27.
The factors of 45 are 1, 3, 5, 9, 15, and 45.
The common factors are 1, 3, and 9.
The highest common factor (HCF) is **9**.

Remember

- The HCF or the highest common factor of two or more numbers is the greatest number (factor) that exactly divides all the given numbers.
- HCF is also called Greatest Common Divisor (GCD).

Exercise 3 B

18/07/2024

1. Find the common factors of the following pairs of numbers.
   a. 12, 72 b. 25, 50 c. 66, 64 d. 5, 12 e. 75, 125 f. 36, 45 g. 24, 35 h. 72, 63
2. Find the HCF of the following pairs of numbers by listing the factors.
   a. 24, 72 b. 45, 81 c. 18, 21 d. 52, 78 e. 69, 39 f. 108, 72 g. 45, 64 h. 27, 63

METHODS OF FINDING HCF

C-1.2

Let us discuss some more methods of finding the HCF of two or more given numbers.

Finding HCF by Prime Factorisation

To find the HCF of two or more numbers using prime factorisation.

Let us now find the HCF of 72, 64, and 48.

Step 1: Find the prime factorisation of each of the numbers.

2	72
2	36
2	18
3	9
	3

2	64
2	32
2	16
2	8
2	4
	2

2	48
2	24
2	12
2	6
	3

$72 = 2 \times 2 \times 2 \times 3 \times 3$
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$48 = 2 \times 2 \times 2 \times 2 \times 3$

Step 2: Circle the common factors in the prime factorisation of all the three given numbers.

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$
$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Step 3: Multiply the common factors to get the HCF.

Thus, the HCF of 72, 64, and 48 $= 2 \times 2 \times 2 =$ **8**.

Example 5: Find the HCF of 98 and 112 using the prime factorisation method.

Solution:

2	98
7	49
	7

2	112
2	56
2	28
2	14
	7

$98 = 2 \times 7 \times 7$

$112 = 2 \times 2 \times 2 \times 2 \times 7$

The highest common factor of 98 and 112 $= 2 \times 7 = 14$

Finding HCF using Division by Common Factors

Divide all the numbers by any factor common to all of the given numbers. If there are still any common factors, again divide the quotients by them and keep dividing until there is no common factor for all the given numbers. The product of these common factors will give the highest common factor (HCF) of these numbers.

Example 6: Find the HCF of 20, 28, and 36.

Solution:

2	20, 28, 36
2	10, 14, 18
	5, 7, 9

HCF of 20, 28, and 36 = 2 × 2 = 4

Finding HCF by Long Division Method

For Two Numbers

Step 1: If two numbers are given, divide the greater number by the smaller number. If the remainder is zero, the smaller number is the HCF of the two numbers.

Example 7: Find the HCF of 32 and 8.

Solution:

```
8)32(4
 -32
   0
```

Thus, 8 is the HCF of 32 and 8.

Step 2: If the remainder is not zero in Step 1, take the remainder as the new divisor and the divisor of Step 1 as the new dividend, then divide (see example 8).

If the remainder is zero, the divisor of this step is the required HCF. If not, then proceed to Step 3.

Step 3: Take the remainder of Step 2 as the new divisor and divisor of Step 2 as the new dividend, then divide. If the remainder is 0, the divisor of Step 3 is the HCF. If not then proceed in the same manner till the remainder is zero.

The last divisor thus obtained is the HCF.

Example 8: Find the HCF of 289 and 391.

Solution: By long division method we have

```
289)391(1
   -289
    102)289(2
       -204
         85)102(1
           - 85
             17)85(5
               -85
                 0
```

Since, the last divisor is 17, the HCF of 289 and 391 is 17.

For Three Numbers

Step 1: Choose any two of the three given numbers and find their HCF.

Step 2: Find the HCF of the HCF obtained in Step 1 and the third number.

Step 3: The HCF obtained in Step 2 is the required HCF of the three given numbers.

Example 9: Find the HCF of 106, 159, and 371.

Solution: Let us first find the HCF of 106 and 159. You could start with a different pair of numbers as well.

```
106)159(1
   -106
     53)106(2
       -106
          0
```

∴ The HCF of 106 and 159 is 53.

Now, we shall find the HCF of 53 and 371.

```
53)371(7
  -371
     0
```

∴ The HCF of 53 and 371 is 53.

Hence, the HCF of 106, 159, and 371 is 53.

Try This!

1. Find the HCF using prime factorisation: **a.** 10, 25, and 90 **b.** 39, 65, and 130
2. Find the HCF using long division method: **a.** 20 and 185 **b.** 44 and 484

Exercise 3 C

1. Find the HCF of the following numbers by prime factorisation.
   a. 12, 16 b. 54, 72 c. 30, 96 d. 128, 32
   e. 240, 135 f. 85, 136 g. 242, 88 h. 333, 185
2. Find the HCF of the following groups of numbers using the method of division by common factors.
   a. 12, 18, 24 b. 28, 35, 49 c. 32, 64, 96, 128 d. 70, 105, 175
   e. 24, 45, 63 f. 84, 96, 120 g. 325, 525 h. 64, 96, 144
3. Find the HCF of the following groups of numbers using the long division method.
   a. 144, 198 b. 234, 572 c. 144, 180, 384 d. 45, 60, 330
   e. 70, 140, 420 f. 184, 230, 276 g. 136, 170, 255 h. 1624, 1276, 522

WORD PROBLEMS

C-1.2

Example 10: Find the greatest number that will divide 79, 117, and 59 leaving the remainders 7, 9, and 11, respectively.

Solution: As the word greatest is used, we have to find the highest common factor that divides 79 leaving a remainder of 7, 117 leaving a remainder of 9, and 59 leaving a remainder of 11. First subtract the respective remainders from the corresponding numbers and then find the HCF. Hence, the numbers which are divisible by the highest common factor are:

$79 - 7 = 72$, $117 - 9 = 108$, and $59 - 11 = 48$.

2	72, 108, 48
2	36, 54, 24
3	18, 27, 12
	6, 9, 4

The HCF of 72, 108, and 48 $= 2 \times 2 \times 3 = 12$.

Hence, 12 is the greatest number that will divide 79, 117, and 59 leaving remainders 7, 9, and 11, respectively.

Example 11: 135 *rosogullas*, 90 *laddoos*, and 75 pieces of *burfees* have been made in a sweet shop. The shopkeeper packs each variety separately in boxes. If he wants to pack all the above sweets in boxes such that each box contains an equal number of sweets, what is the greatest number of sweets he can pack in each box?

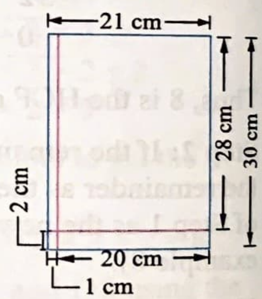
Solution: We have to find the greatest number of sweets he can pack in a box. That means we have to find out the largest number which can divide 135, 90, and 75 exactly, i.e., the HCF of the three numbers.

The HCF of 135, 90, and 75 $= 5 \times 3 = 15$.

Hence, he has to pack 15 sweets in each box.

Example 12: From a rectangular sheet of paper 30 cm by 21 cm, Lisa cut out equal square pieces of paper. She had to cut off 2 cm from the length and 1 cm from the breadth of the rectangular sheet to get these squares. What is the longest possible length of the side of these squares if the sides are in full cm?

Solution: Length of the sheet = 30 cm
Breadth of the sheet = 21 cm
After cutting 2 cm length and 1 cm breadth,
length = 28 cm,
and breadth = 20 cm.



This 28 cm × 20 cm is to be cut into squares.
Hence, the longest side of the square is the HCF of 28 and 20.

HCF of 28 and 20 $= 2 \times 2 = 4$

2	28, 20
2	14, 10
	7, 5

∴ The sides of the square pieces of paper are 4 cm each.

Example 13: A room is 5 m 60 cm long and 2 m 40 cm broad. What are the dimensions of the largest square tiles that can be fixed on the floor, so that the tiles need not be cut to cover the floor entirely?

Solution: The length of the room = 5 m 60 cm

= 5 m + 60 cm

= 500 cm + 60 cm

= 560 cm

The breadth of the room = 2 m 40 cm

= 2 m + 40 cm

= 200 cm + 40 cm

= 240 cm

We have to find the dimensions of the largest square tiles that can be used to cover the floor entirely so that the tiles need not be cut. Which means we have to find out the HCF of the length and the breadth of the room, i.e., the HCF of 560 cm and 240 cm.

2	560, 240
2	280, 120
2	140, 60
2	70, 30
5	35, 15
	7, 3

The HCF of 560 and 240 = $2 \times 2 \times 2 \times 2 \times 5 = 80$

Thus, the side of the square tiles must be 80 cm.

Remember

- Though HCF is mentioned as the largest, greatest, highest, or maximum number or value, remember that it is a factor, and hence the HCF of a group of numbers is smaller than or equal to the smallest number of the group.
- In a group of numbers, if the numbers are multiples of the smallest number, then the smallest number is the HCF of the group of numbers.

Exercise 3 D

1. Find the greatest number which divides 34, 60, and 85 leaving remainders of 7, 6, and 4, respectively.
2. Find the greatest number which divides 129, 161, and 224 leaving remainders of 5, 6, and 7, respectively.
3. Find the greatest number which divides 189, 223, and 347 leaving remainders of 9, 3, and 7, respectively.
4. Find the greatest number which will divide 264 and 168 leaving a remainder of 8 in each case.
   (***Hint:*** First subtract the common remainder 8 from both the numbers and then find the highest common divisor.)
5. Find the greatest number that will divide 264 and 336 and leave a remainder of 12 in each case.

Application-Based Questions

24/06/2024

H.C.F

6. Swami packed 288 oranges and 624 apples in boxes. He packed oranges and apples in separate boxes. If he packed an equal number of fruits in each box, find the maximum number of fruits he put in each box. H.C.F

7. Santosh, a physical education teacher, arranged three groups of 140, 91, and 63 students for a march past. If he arranged the same number of students in each row, find the number of students he arranged in each row. H.C.F
8. Jenny, the florist, had 72 roses, 27 gladioli, and 54 marigolds to be used to make bouquets. She has to make identical bouquets having all three varieties of flowers. What is the maximum number of identical bouquets that Jenny can make if she uses all the flowers? H.C.F

9. A room is 7 m 20 cm long and 5 m 20 cm broad. If square tiles have to be laid in such a way that an exact number of tiles fit in and no tile has to be cut, what is the greatest length of the side of the square tiles? H.C.F
10. The length, breadth, and height of a room are 7 m 20 cm, 5 m 60 cm, and 4 m, respectively. What is the greatest length of the tape that can be used to measure all three dimensions, the tape being used an exact number of times in each case?

COMMON MULTIPLES

C-1.2

Consider two numbers say 6 and 8. The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, and so on.

The multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, and so on.

Notice both the lists of multiples. You will find that 24, 48, 72, and so on, are multiples which are common to both 6 and 8. So, they are the common multiples of 6 and 8.

Example 14: List three common multiples of 8 and 12.

Solution: Multiples of 8 are:

8, 16, 24, 32, 40, 48, 56, 64, 72, 80, ...

Multiples of 12 are:

12, 24, 36, 48, 60, 72, 84, ...

Thus, three common multiples of 8 and 12 are 24, 48, and 72.

Lowest Common Multiple or LCM

Let us consider the following example to understand LCM.

Sachin, Raja, and Sanjay decide to jog around the circular track of their school playground. They take 4 minutes, 6 minutes, and 8 minutes, respectively, to complete one full round of the circular track. All of them start at the same time from the same point on the track and jog in the same direction for an hour. After how many minutes will all three of them meet again at the starting point?

We need to find the common time where they meet earliest or at a minimum time spent.

We can say that Sachin will reach the starting point after every 4 minutes, that is, after the 4th, 8th, 12th, 16th minute, and so on.

Raja will be reaching the starting point after every 6 minutes, that is, after the 6th, 12th, 18th, 24th minute, and so on.

Sanjay will be reaching the starting point after every 8 minutes, that is, after the 8th, 16th, 24th, 32nd minute, and so on.

For Sachin, it is multiples of 4,
i.e., 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 48, 52, 56, 60, ...

For Raja, it is multiples of 6,
i.e., 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...

For Sanjay, it is multiples of 8,
i.e., 8, 16, 24, 32, 40, 48, 56, ...

Common multiples of 4, 6, and 8 are 24, 48, and so on, and smallest of the common multiples, i.e., **lowest common multiple (LCM)** is 24.

$\therefore$ They will meet again after 24 minutes.

Try This!

Find the LCM of 20, 30, and 40.

Remember

- The lowest common multiple (LCM) of two or more numbers is the lowest (smallest or least) of their common multiples.
- LCM of a group of numbers is either equal to the largest number of the group or is greater than it.

Finding LCM by Prime Factorisation

Step 1: Find the prime factorisation of the given numbers.

Step 2: Find the common factors.

Step 3: Multiply the common factors and the other factors also, i.e., common and uncommon factors are to be multiplied.

Example 15: Find the LCM of 84 and 96.

Solution:

2	84
2	42
3	21
	7

2	96
2	48
2	24
2	12
2	6
	3

$84 = 2 \times 2 \times 3 \times 7$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

The common factors of 84 and 96 are 2, 2, and 3.

The common factors are 2, 2, and 3. The factors which are not common to both the numbers are 7, 2, 2, and 2.

LCM is the product of the common factors and the factors which are not common.

That is, $\underbrace{2 \times 2 \times 3}_{\text{common factors}} \times \underbrace{2 \times 2 \times 2 \times 7}_{\text{other factors}} = 672$

This means that 672 is a multiple of 84 and 96, and it is the smallest multiple common to both these numbers. So, 672 is the LCM of 84 and 96.

The common factors are multiplied only once, eg. 2, 2, 3 are common factors of 84 and 96, so they will be multiplied once only.

Finding LCM by Common Division

Step 1: Write the given numbers in a row separated by commas.

Step 2: Divide these numbers by the least prime number which divides at least one of the given numbers.

Step 3: Write the quotients and the numbers that are not divisible by the prime number in the second row. Now repeat Steps **2** and **3** with this row and continue till the numbers in a row are all 1.

Step 4: The LCM is found by multiplying all the prime divisors and quotients other than 1.

It is not necessary to start the division with the least prime number; you can start the division with any prime number which divides at least one of the numbers. But it is convenient to start with the least prime number first.

Example 16: Find the LCM of 88 and 64.

Solution: The LCM is the product of all the divisors and the quotients if any.

2	88, 64
2	44, 32
2	22, 16
2	11, 8
2	11, 4
2	11, 2
	11, 1

Common divisors
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Quotient = 11.

The LCM of 88 and 96 is
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 = 704.$

Try This!

1. Find the LCM of the following pairs of numbers by prime factorisation.
 a. 36 and 72 b. 27 and 30
2. Find the LCM of 32, 39, and 52 by common division method.

Exercise 3 E

1. List the first five multiples of the following numbers.
 a 6,12,18,24,30 b. 7,14,21,28,35 c. 9,18,27,36,45 d. 10,20,30,40,50 e. 16, 32,64, 80, 96
2. List the multiples of each pair and find the LCM.
 a 4, 20 b. 16, 64 c. 15, 40 d. 8, 9
3. Find the LCM using prime factorisation.
 a. 72, 90 b. 16, 30, 42 c. 105, 175, 140 d. 28, 15, 42
 e. 105, 135 f. 82, 102 g. 92, 128 h. 62, 94
4. Find the LCM of the following numbers by the common division method.
 a. 40, 80, 120, 160 b. 30, 48, 120 c. 72, 108, 144 d. 84, 144, 96
 e. 39, 65, 78 f. 51, 102, 34 g. 70, 140, 350 h. 63, 147, 84

WORD PROBLEMS

C-1.2

Example 17: Find the least number, which when divided by 20, 30, and 40, leaves a remainder of 7 in each case.

Solution: The least number that is divisible by 20, 30, and 40 is the least common multiple of these three numbers.

2	20, 30, 40
2	10, 15, 20
5	5, 15, 10
2	1, 3, 2
3	1, 3, 1
	1, 1, 1

LCM of 20, 30, 40 is
$2 \times 2 \times 2 \times 5 \times 3 = 120.$

Thus, 120 is a number which is exactly divisible by 20, 30, and 40. We need a number that leaves a remainder of 7 in each case. This means that the required number is 7 more than 120.

So, the least number divisible by 20, 30, and 40, leaving a remainder of 7 for each is $120 + 7 = 127$.

Example 18: Find the least number, which when divided by 36, 24, and 48, leaves remainders 33, 21, and 45, respectively.

Solution: The least number divisible by 36, 24, and 48 is the LCM of these numbers.

2	36, 24, 48
2	18, 12, 24
2	9, 6, 12
3	9, 3, 6
	3, 1, 2

The LCM of 36, 24, and 48 is $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$.

According to the question, when we divide the number by 36, the remainder is 33; when we divide the number by 24, the remainder is 21; when we divide the number by 48, the remainder is 45.

Observe: $36 - 33 = 3$, $24 - 21 = 3$, and $48 - 45 = 3$.

The LCM 144 is completely divisible by 36, 24, and 48.

But if we take away 3 from the LCM the divisor 36 will be short of 3 and will leave a remainder of 33.

Similarly, the divisor 24 will leave a remainder of 21, and the divisor 48 will leave 45 as the remainder. Hence, the required number is $144 - 3 = 141$.

Example 19: One kilogram box of *burfees* has 16 pieces. One kilogram box of *rosogullas* has 12 pieces. If we want to buy an equal number of both, and if we can buy only in full boxs, what is the least number of each sweet we would have to buy? How many kilograms of each do we need to buy?

Solution:

Given: 16 *burfees* in one box,
12 *rosogullas* in one box

Since we have to get the least number of each of them, we need to find the LCM of 16 and 12.

2	16, 12
2	8, 6
2	4, 3
2	2, 3
3	1, 3
	1, 1

LCM of 16 and 12 is $2 \times 2 \times 2 \times 2 \times 3 = 48$.

So, we need to buy 48 *burfees*, i.e., $(48 \div 16 =)$ 3 kg and 48 *rosogullas*, i.e., $(48 \div 12 =)$ 4 kg.

Example 20: Find the smallest 3-digit number which is exactly divisible by 4, 5, and 6.

Solution: The LCM of 4, 5, and 6 is exactly divisible by 4, 5, and 6.

$\text{LCM} = 2 \times 2 \times 3 \times 5 = 60$

2	4, 5, 6
2	2, 5, 3
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

But we need the smallest 3-digit number which is a multiple of 60 as 4, 5, and 6 exactly divide only multiples of 60, which are:

120, 180, 240, ...

$\therefore$ The smallest 3-digit number which is exactly divisible by 4, 5, and 6 is 120.

Example 21: The neon signboard of a jewellery shop has three coloured features; the logo of the shop in red, the name of the shop in green, and the picture of some jewellery in yellow. The logo comes on every 20 seconds, the name every 24 seconds, and the picture of the jewellery shines every 12 seconds. Lights on all these three features were switched on together. After how much time will all these three light up together?

Solution: Here we have to find the LCM of 20, 24, and 12 to calculate the time after which the three features light up together.

2	20
2	10
5	5
	1

2	24
2	12
2	6
3	3
	1

2	12
2	6
3	3
	1

$20 = 2 \times 2 \times 5$

$24 = 2 \times 2 \times 2 \times 3$

$12 = 2 \times 2 \times 3$

Thus, $2 \times 2 \times 2 \times 3 \times 5 = 120$, which is the LCM of the three numbers. So, all three of the lights will light up together again after 120 seconds, or $(120 \div 60 =)$ 2 minutes.

Example 22: Find a number close to 5000 that is divisible by 33, 55, and 25.

Solution: To find a number close to 5000 that is divisible by 33, 55, and 25, we first have to find the LCM of the three numbers.

$\therefore \text{LCM} = 11 \times 3 \times 5 \times 5 = 825$

11	33, 55, 25
5	3, 5, 25
3	3, 1, 5
5	1, 1, 5
	1, 1, 1

Dividing 5000 by 825 gives us a quotient, approximately equal to 6.

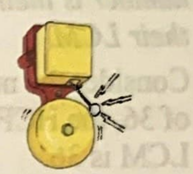
Thus, the required number is $825 \times 6 = 4950$, which is divisible by all three numbers.

Exercise 3 F

1. The LCM of a pair of numbers is 4 and their sum is 6. What are the numbers?
2. The LCM of three different numbers is 4. What are the numbers?
3. The LCM of two numbers is 12 and their sum is 10. What are the numbers?
4. Find the smallest number that is divisible by 3, 4, 5, 6, 10, and 15.
5. Find the least number that is divisible by 30, 50, 60, and 90.
6. Find the least number which when divided by 25, 45, and 60 leaves a remainder of 20 for each.

Application-Based Questions

7. Find the smallest number which when divided by 18, 12, and 24 leaves a remainder of 16, 10, and 22, respectively.
8. The school bell rings every 40 minutes and the clock tower of the city centre rings every 60 minutes. At 8 a.m. on a day both the bells sounded together. At what time will both of them make their sounds together again?



9. A toy train completes one round of a circular track in 120 seconds. Another one completes a round in 180 seconds. Both the trains start together from a station and run in opposite directions. After how many minutes will both the trains meet for the first time at the station from where they started?

10. Lali wants to buy equal number of oranges and apples. The oranges are packed in packets of 6 and the apples are packed in packets of 10. What is the least number of apples she can buy? How many packets of each fruit should she buy?
11. Find a number between 800 and 900 which is divisible by 22, 33, and 66.

PROPERTIES OF HCF AND LCM C-1.2

1. The HCF of 6 and 10 is 2. So, 2 is a factor of both 6 and 10. Also, 2 is the smallest amongst 2, 6, and 10. Thus, *the HCF of given numbers is either smaller than all the given numbers or equal to the smallest number.*
2. The LCM of 6 and 10 is 30. 30 is a multiple of both 6 and 10. Also, 30 > 10 and 6, i.e., it is the greatest amongst 6, 10, and 30. Thus, *the LCM of given numbers is either greater than all the given numbers or equal to the biggest number.*
3. *Two numbers that have only 1 as a common factor are known as* ***co-prime numbers***. Consider two numbers 35 and 39.

Now, $35 = 1 \times 5 \times 7$

and $39 = 1 \times 3 \times 13$.

Common factor = 1

$\therefore$ 35 and 39 are co-prime numbers.

HCF of 35 and 39 = 1.

Thus, *HCF of co-prime numbers is 1.*

4. Again consider 35 and 39 which are co-prime.

LCM of 35 and 39 $= 3 \times 5 \times 7 \times 13$

$= 35 \times 39$

Thus, *the LCM of co-prime numbers = the product of the co-primes.*

3	35, 39
5	35, 13
7	7, 13
13	1, 13
	1, 1

5. HCF of 6 and 10 = 2
LCM of 6 and 10 = 30
Also, $30 = 2 \times 15 = 2 \times 3 \times 5$,
i.e., 2 is a factor of 30, or HCF is a factor of LCM.
Thus, *the HCF of given numbers is a factor of their LCM.* In other words, *the LCM of given numbers is a multiple of their HCF.*
6. The numbers 2 and 3 are prime numbers.
HCF of 2 and 3 is 1. Thus, *the HCF of two or more prime numbers is 1.*
7. *Given any two numbers, if the first number is a factor of the second number then the first number is their HCF and the second number is their LCM.*
Consider the numbers 6 and 36. As 6 is a factor of 36, the HCF of the two numbers is 6 and their LCM is 36.

Relationship Between HCF and LCM

Complete the table and find the relationship between HCF and LCM.

S. No.	Numbers		HCF	LCM	HCF × LCM	Product of the numbers
1	6	10	2	30	60	60
2	25	80	5	400		2000
3	12	30	6			
4	18	27				
5	45	27				
6	48	60				

What do you observe?

If a and b are two numbers, then

$$a \times b = \text{HCF} \times \text{LCM}.$$

Hence, $a = \dfrac{\text{HCF} \times \text{LCM}}{b}$ and $b = \dfrac{\text{HCF} \times \text{LCM}}{a}$.

Or, $\text{HCF} = \dfrac{a \times b}{\text{LCM}}$ and $\text{LCM} = \dfrac{a \times b}{\text{HCF}}$.

So, we can say that

The product of two given numbers = HCF × LCM of the numbers
∴ One number = $\dfrac{\text{HCF} \times \text{LCM}}{\text{Other number}}$

Example 23: Two numbers have their HCF as 8 and LCM as 96. One of the numbers is 24. Find the other number.

Solution: HCF = 8, LCM = 96, One number = 24

The other number = $\dfrac{\text{LCM} \times \text{HCF}}{\text{Given number}} = \dfrac{96 \times 8}{24} = 32$

Example 24: The product of two numbers is 5040 and their HCF is 12. What is their LCM?

Solution: We know that the product of two numbers is equal to the product of their HCF and LCM. Given two numbers a and b,

$$a \times b = \text{HCF} \times \text{LCM}.$$

Or, $\text{LCM} = \dfrac{a \times b}{\text{HCF}}$

We have HCF = 12, and $a \times b = 5040$.

Thus, $\text{LCM} = \dfrac{a \times b}{\text{HCF}} = \dfrac{5040}{12} = 420$

Therefore, LCM of the two numbers is 420.

Exercise 3 G

1. Identify the numbers which are co-primes to the first number in the row.
   a. 63, 27, 93, 43, 23, 33, 53
   b. 85, 70, 17, 51, 16, 21, 34
2. Find the pairs of numbers which are co-prime and then find their HCF and LCM.
   a. 11, 19 b. 63, 36 c. 93, 30 d. 93, 32 e. 32, 42 f. 50, 39 g. 72, 32 h. 86, 75
3. The HCF of two numbers is 12 and their product is 4320. What is their LCM? If one of the numbers is 60, what is the other number?
4. The HCF and the LCM of two numbers is 15 and 450, respectively. If one number is 75, what is the other number?
5. The HCF of two numbers is 5 and their LCM is 595. If one of the numbers is 85, find the other number.
6. The LCM of two numbers is 819. If the two numbers are 63 and 117, find the HCF.
7. The product of HCF and LCM of two numbers is 119. Find the two numbers if none of them is one.

Chapter Check-Up

SELF ASSESSMENT

Multiple Choice Questions

1. Which of the following pairs is not co-prime?
   a. 6, 10 b. 7, 12 c. 1, 5 d. 21, 23
2. LCM of two numbers is 210. Then which of the following is not the HCF of the numbers?
   a. 35 b. 70 c. 90 d. 105

Practice Time

3. Find the common factors of each of the following pairs of numbers.
   a. 9, 12 b. 18, 36 c. 56, 48 d. 40, 12
4. Find the HCF of the following numbers by listing the factors.
   a. 12, 15 b. 24, 32 c. 25, 75 d. 21, 9
   e. 34, 60 f. 25, 40 g. 56, 64
5. Find HCF by prime factorisation.
   a. 18, 30 b. 96, 72, 60 c. 120, 144, 96
6. Find the HCF of the following groups of numbers using the common division method.
   a. 18, 36, 27 b. 128, 84, 154 c. 320, 420, 180
7. Find the HCF of the following groups of numbers using the long division method:
   a. 50, 65 b. 45, 60, 75 c. 192, 240, 1024
8. Identify the pairs of numbers which are co-primes and find their HCF and LCM.
   a. 30, 25 b. 40, 27 c. 6, 7 d. 120, 49
9. Circle the numbers which are co-prime to the first number in the row.
   a. 40, 60, 33, 72, 81, 37 b. 47, 94, 86, 43, 188, 230
10. List multiples of each pair and find the LCM.
    a. 6, 7 b. 8, 12 c. 9, 15 d. 5, 35
11. Find the LCM using prime factorisation.
    a. 75, 150, 200 b. 36, 24 c. 300, 1050
12. Find the greatest number which can divide 36, 81, and 108.
13. Find the greatest number which can divide 89, 53, and 77 and leave a remainder of 5 in each case.
14. Find the least number which when divided by 3, 4, 5, 6, 10, and 12 leaves a remainder of 2 in each case.
15. Four bells ring at intervals of 3, 7, 12, and 14 minutes, respectively. All four rang together at 12 noon. When will they ring together again?

Case-Study Based Question

16. Preeti baked 256 butter cream cookies, 448 cashew nut cookies and 736 choco nut cookies to sell at her stall in a fair. She wants to pack the cookies in packets so that each packet has the same number of cookies of the same kind.
    a. If she wants to pack the largest number of cookies possible in each pack, how many packets will she need to pack? Also, find the number of packets of each kind of cookies.
    b. If 16 of the choco nut cookies are broken, then how many can she pack in each box? Also, find the number of packets she will need to pack.

Creative Thinking

When an army commander wanted to transport his battalion of soldiers, he considered 30-seater, 40-seater, or 50-seater buses. In all three cases, he found that 10 seats were left vacant. If he managed to transport his battalion in 45 same type of buses leaving 10 seats vacant, what is the number of soldiers in his battalion?

Everyday Maths

1. Shobha has two pieces of ribbon. One piece is 720 cm long and the other piece is 900 cm long. She wants to cut both these ribbons into strips of equal length that are as long as possible. How long should each strip be?
2. Jason noticed that his house number is exactly divisible by 7. His friend Sheshan pointed out to him that it is also exactly divisible by 18. If his house number is a 4-digit number, find the smallest number that could be his house number.

   (*Hint:* Find the smallest 4-digit common multiple of 7 and 18.)

Cross-Curricular Connect

C-10.1

(Science) Measurements of distances have been an important part of everyday life since ancient times. In ancient times people used *Angul* (finger), *Muthi* (fist), foot length, pace or steps, cubit, and yard for measuring lengths. Even today in some towns of India, flower sellers use their forearm for measuring the length of a garland. For the sake of uniformity, scientists all over the world accepted standard units of measurements known as the **International System of Units** (SI units). The SI unit of length is metre (m) and 1 m = 100 cm.

The floor of Manpreet's room is 8 m 96 cm long and 6 m 72 cm wide. He wants to use square tiles in his room. Find the minimum number of square tiles of the same size needed to cover the entire floor.

Being Indian

Being physically fit and active is important for everyone, especially students, as being fit ensures a healthy body and mind, which helps build self-esteem and confidence.

Regular physical activity has many benefits, such as, it improves concentration, helps in building a strong and healthy body, improves sleep quality, and many more.

a. Arun, Shaji, and Mohit cycle every day along the circular ring road. They take 8, 12, and 16 minutes, respectively, to take a complete round. If all of them start together at the same time from the same place, after how much time will all three of them meet at the same starting place again?

b. Do you also exercise regularly to keep yourself healthy?

Higher Order Thinking Skills

1. The HCF and LCM of two numbers is 4 and 288. What are the two numbers?
2. Counting the number of rose plants she bought for her rose farm, Ayushi found that she can plant these roses exactly in rows of 12 or 15. Find the number of rose plants she had if the number was between 100 and 150.

Mental Maths

PA

1. If the LCM and the HCF of two numbers is 9 and 3, respectively, what are the numbers?
2. The LCM of 5, 7, 8, and 9 is also the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 18, 20, 21, 24, 28, 30, 35, 36, 40, 42, 45, 56, 60, 63, 70, 72, 84, 90, 105, 120, 126, 140, 168, 180, 210, 252, 280, 315, 360, 420, 504, 630, 840, 1260 and 2520. Explain the reason.
3. When a group of students were formed into groups of 5 or 6, one student was left out on both occasions. When they were grouped as groups of 7, nobody was left out. What is the least number of students in the group?

Journal

Does the prime factorisation of a number differ depending upon which factors one chooses first? Discuss with your partner.

Maths Lab Activity

Aim: To understand the method of long division for finding HCF by using ribbons of different lengths.

Materials Required: Two ribbons of different colours and of different lengths.

Method: To find the HCF of two or more numbers means to find the largest number that can exactly divide the given numbers without any remainder. In the case of ribbons it is the length of the longest piece that can be cut from both without any waste of the ribbons.

Let 'a' and 'b' be two ribbons in two different colours and two different lengths. 'b' being smaller, let us see how many lengths of 'b' will fit into 'a'.

In this case we see that 'b' fits in two times and a length of 'c' is left. This means that 'b' is not the longest piece that can divide 'a' and 'b' exactly.

Now with the length 'c' try and see how many pieces of 'c' can be cut from 'b'. If 'c' can exactly divide 'b', then 'c' can divide 'a' also exactly.

In this case, 'b' is two times the length of 'c' and a length of 'd' is left out. This means that a length of 'c' cannot be cut from 'a' or 'b' exactly.

If length of 'd' can be cut exactly from 'c', then 'd' is the longest piece that can be cut exactly from 'c', 'b', and 'a'. In this case, 'd' is one time the length of 'c' and a length of 'e' is left out.

But, we see that 'e' is exactly two times the length of 'd'. Thus 'e' is the longest piece that can be cut exactly from 'd', 'c', 'b' and 'a'.

Thus, the length 'e' becomes the HCF of 'a' and 'b'.

Symbolic representation of the above

b) a (quotient
p
Remainder → c) b (quotient
q
Remainder → d) c (quotient
r
Remainder → e) d (quotient
d
0

Hence, 'e' is the highest common factor of 'a' and 'b'.