Growth Effects of Flat-Rate Taxes

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Recent estimates of the potential growth effects of tax reform vary wildly, ranging from zero to eight percentage points. Using an endogenous growth model, we assess which model features and parameter values are important for determining the quantitative impact of tax reform. We find that the critical parameters are factor shares, depreciation rates, the elasticity of intertemporal substitution, and the elasticity of labor supply. The elasticities of substitution in production, on the other hand, are relatively unimportant. The quantitative estimates in several recent papers are compared with each other and with some of the evidence from U.S. experience. We find that Robert Lucas's conclusion, that tax reform would have little or no effect on the U.S. growth rate, is theoretically robust and consistent with the evidence.

Growth models with endogenously determined rates of technical change provide a useful framework for studying the effects of fiscal policy on the long-run growth rate. Recent papers by Jones and Manuelli (1990), King and Rebelo (1990), Lucas (1990), Rebelo (1991), Yuen (1991), Kim (1992), Gomme (1993), Jones, Manuelli, and Rossi (1993), Pecorino (1993, 1994), and others have used endogenous growth models to look at both the positive and normative effects of

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taxation. Unfortunately, although all these authors use a similar basic framework and calibrate their models to U.S. data, their quantitative conclusions differ wildly.

For example, Lucas (1990) calculates that eliminating the capital tax and raising the labor tax in a revenue-neutral way would have a trivial effect on the U.S. growth rate, changing it by only 0.03 of a percentage point; Jones et al. (1993) conclude that eliminating all distorting taxes could raise it by as much as eight percentage points! King and Rebelo (1990) and Kim (1992) conclude that tax reform would raise the growth rate by modest, but nontrivial, amounts.

The goals of the present paper are to trace the sources of these conflicting results by examining the features of the preferences, technology, and tax policy that are critical for conclusions about the long-run growth effects of taxation. Our main finding is that Lucas's conclusion is robust. Specifically, if human capital's share is large in all sectors, if the sector producing human capital is lightly taxed, and if long-run labor supply is fairly inelastic, then taxing returns in the sectors producing consumption goods and physical capital does not have large growth effects.

We also compare the predictions of the various models with some of the evidence offered by U.S. experience. Income tax revenue as a percentage of gross national product increased permanently from about 2 percent to about 15 percent in 1942. This large rise in income tax rates produced no noticeable effect on the average growth rate of the economy. In our view, this is evidence against models that predict large growth effects from taxation. In order for these models to be consistent with the evidence that we discuss, the growth effects of an income tax increase would have to have been exactly offset by the effects of changes in public expenditures, other taxes, and other determinants of the growth rate.

We look at flat-rate taxes levied on the income from physical and human capital, the proceeds of which are rebated in lump-sum form, and we compare steady-state growth rates. As shown in Rebelo (1991), if all income is taxed at a common rate τ, then, compared with the values in an untaxed economy, the steady-state input ratios and factor shares in all sectors are unchanged, and the interest rate is reduced by the factor $1 - \tau$. The change in the growth rate is then equal to the change in the interest rate multiplied by the elasticity of intertemporal substitution. These effects are independent of the properties of the production functions (beyond linear homogeneity) and preferences (beyond a constant rate of time preference and constant elasticity of intertemporal substitution).

If income from different sources is taxed at different rates, however, the situation is substantially more complicated. For determining growth effects, the important features of the economy include two types of parameters. The first group consists of those, such as factor shares and depreciation rates, that can be calibrated in a straightforward way on the basis of quantities that, at least in principle, are observable in an economy following a balanced growth path. Gathering the necessary data may or may not be easy, and particular societies may or may not choose to gather them, but the required data are generated by the economy. The second group consists of parameters, such as elasticities of substitution in production and the elasticity of labor supply, that cannot be calibrated on the basis of observations from an economy growing along a balanced path. Price variation is needed to estimate these elasticities, and relative prices do not vary along the balanced growth paths of the models we consider. One of our goals here is to assess the sensitivity of growth effects to the values of these hard-to-observe parameters.

First we consider the technology parameters. We assume that the preferences and technologies are of the constant elasticity of substitution (CES) variety and compare economies that have different elasticities of substitution but, in the absence of taxes, have identical interest rates, factor shares, and factor price ratios. That is, we compare economies that are observationally equivalent in the untaxed steady state (or when both factors are taxed at the same rate) but respond differently when factors are taxed asymmetrically. Under the assumption of inelastic labor supply, we show that while the quantitative responses of the steady-state interest rate and growth rate to fiscal reform are quite sensitive to the factor share parameters in the input-producing sectors, they are very insensitive to the substitution elasticities. Surprisingly, steady-state revenues are also quite insensitive to the elasticity parameters.

Elastic labor supply is then considered. First we show that if leisure time is quality adjusted in the same way work time is, then the analysis of the preceding sections remains unchanged: only a reinterpretation of the results is needed. We then show that if leisure time is measured in "raw hours," the effect of taxation on the interest rate depends on the elasticity of labor supply, and we derive a quantitative relationship for the interest rate effect in terms of that elasticity. Although the currently available empirical estimates of the relevant elasticity are very mixed, there seems to be little evidence suggesting that it is large.

We then proceed to a numerical comparison of some of the tax models mentioned earlier. We find that the very small growth effect found by Lucas is due to the technology he uses for the sector producing human capital, and the very large effect found by Jones et al. is due to their assumption about the elasticity of labor supply. We show that the calculated growth effects of taxation are also very sensitive to assumptions about the rate of depreciation and the tax treatment of depreciation, and we argue that some commonly used assumptions overstate the potential growth effects of tax reform.

The rest of the paper is organized as follows. The basic model is described in Section I, the technology parameters are studied in Section II, and elastic labor supply is discussed in Section III. Numerical comparisons are carried out in Section IV, and conclusions are drawn in Section V. Derivations are contained in Appendix A.

I. The Basic Model

Time is continuous, all markets are perfectly competitive, and there is an infinitely lived representative household. At each date t, existing stocks of physical and human capital, k_t and h_t, are used as inputs into production. These stocks depreciate at the rates δ_1 and δ_2, respectively. The household owns the stocks, which it supplies, inelastically, to firms. The economy has three types of firms, which produce new physical capital, I_{1t}, new human capital, I_{2t}, and consumption goods, c_t. The household uses its income to buy goods of all three types. Let q_{it} and p_{it}, $i = 1, 2$, be the net-of-tax returns and purchase prices for the two factors, measured in terms of contemporaneous consumption goods. The revenue from all taxes is rebated to the household in lump-sum form, T_t, at each date t.

The household's problem, given (k_0, h_0) and $\{p_{1t}, p_{2t}, q_{1t}, q_{2t}, T_t, t \geq 0\}$, is to choose $\{c_t, I_{1t}, I_{2t}, k_t, h_t, t \geq 0\}$ to solve

$$\max \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} dt$$

subject to

$$\dot{k}_t = I_{1t} - \delta_1 k_t, \tag{1a}$$

$$\dot{h}_t = I_{2t} - \delta_2 h_t, \tag{1b}$$

and

$$p_{1t} I_{1t} + p_{2t} I_{2t} + c_t - q_{1t} k_t - q_{2t} h_t - T_t \leq 0, \tag{1c}$$

where $\rho, \sigma > 0$.

To characterize a competitive equilibrium, the conditions for utility maximization must be combined with those for profit maximization, budget balance for the government, and market clearing. Only balanced growth paths will be considered.[1] Along any such path, con-

[1] As shown in Bond, Wang, and Yip (1992), the model here is globally asymptotically stable, so balanced growth paths represent the long run of the economy. See Faig (1991) and Caballé and Santos (1993) for other analyses of stability, and see Mulligan and Sala-i-Martin (1993) for a discussion of transitional dynamics.

sumption and both kinds of capital grow at a common, constant rate, g, and the interest rate, r, and the sectoral allocation of factors are constant.

Let G, H, and F be constant returns to scale production functions for the sectors producing physical capital, human capital, and consumption goods; let z_j, $j = 1, 2, 3$, be the ratio of human to physical capital employed in each sector; and let τ_{ij} be the flat-rate tax—constant over time—on income earned by factor $i = 1, 2$, employed in sector $j = 1, 2, 3$. If income is taxed gross of depreciation, then the balanced growth path satisfies

$$r = \rho + \sigma g, \tag{2a}$$

$$(1 - \tau_{11})G_1(1, z_1) - \delta_1 = r, \tag{2b}$$

$$(1 - \tau_{22})H_2(1, z_2) - \delta_2 = r, \tag{2c}$$

$$\frac{(1 - \tau_{21})G_2(1, z_1)}{(1 - \tau_{11})G_1(1, z_1)} = \frac{(1 - \tau_{22})H_2(1, z_2)}{(1 - \tau_{12})H_1(1, z_2)}, \tag{2d}$$

and

$$\frac{(1 - \tau_{21})G_2(1, z_1)}{(1 - \tau_{11})G_1(1, z_1)} = \frac{(1 - \tau_{23})F_2(1, z_3)}{(1 - \tau_{13})F_1(1, z_3)}, \tag{2e}$$

plus market-clearing conditions for the three outputs and resource constraints for the two factors. Equation (2a) relates the consumption growth rate to the interest rate; (2b) and (2c) equate the real rates of return on each factor, net of taxes and depreciation, to the interest rate; and (2d) and (2e) follow from the equality of factor returns in all sectors.

It is important for our purpose that the system is block-recursive: (2b)–(2d) can be solved for (r, z_1, z_2). Then (2a) determines g, (2e) determines z_3, and the five market-clearing conditions determine the remaining variables. Moreover, since (2b)–(2d) involve only the technologies, depreciation rates, and tax rates in the input-producing sectors, only those factors affect the interest rate. The preference parameters ρ and σ affect only the way the interest rate is translated into a growth rate, the relationship in (2a). The technology and tax rates in the consumption goods sector affect only the input ratio in that sector, the relationship in (2e).

We begin with two special cases that are very simple to analyze and for which the intuition is clear. They are also useful for illustrating the consequences of depreciation deductions. If the tax rates on income earned in the input-producing sectors vary only by factor, $\tau_{ij} = \tau_i$, $i, j = 1, 2$, *or* only by sector, $\tau_{ij} = \tau_j$, $i, j = 1, 2$, then (2b)–(2d) take the simpler form

$$(1 - \tau_1)G_1(1, z_1) - \delta_1 = r, \tag{3a}$$

$$(1 - \tau_2)H_2(1, z_2) - \delta_2 = r, \tag{3b}$$

and

$$\frac{G_2(1, z_1)}{G_1(1, z_1)} = \frac{H_2(1, z_2)}{H_1(1, z_2)}. \tag{3c}$$

In addition, assume that the two depreciation rates are equal: $\delta_1 = \delta_2$. Alternatively, if income is taxed *net* of depreciation and tax rates vary only by factor, then the steady-state conditions are

$$(1 - \tau_1)[G_1(1, z_1) - \delta_1] = r, \tag{4a}$$

$$(1 - \tau_2)[H_2(1, z_2) - \delta_2] = r, \tag{4b}$$

and

$$\frac{G_2(1, z_1)}{G_1(1, z_1)} = \frac{H_2(1, z_2)}{H_1(1, z_2)}, \tag{4c}$$

where τ_i, $i = 1, 2$, is the tax rate on income earned by factor i. We are primarily interested in how tax policy affects the interest rate, since the change in the growth rate is proportional to it: $\Delta g = \Delta r/\sigma$.

In the first special case, income from both factors is taxed at a common rate τ. Then clearly the rate of return, gross or net depending on the tax treatment of depreciation, is reduced by the factor $1 - \tau$, and the input ratios are unaffected. These conclusions hold for any constant returns to scale production functions. The intuition here is that a uniform tax rate is like a neutral downward shift in the (gross or net) production function, with an offsetting lump-sum subsidy. Moreover, it is clear that taxes levied on factors employed in the consumption goods sector have no effect on the growth rate. (In fact, in this setting such a tax is equivalent to a consumption tax and so is completely nondistorting; see Rebelo [1991].)

In the second special case, human capital is the only input into its own production, the assumption used in Lucas (1990). Then $H(k_2, h_2) = Bh_2$, and it follows immediately from (3b) or (4b) that the interest rate is completely determined by the coefficient B, appropriately adjusted for taxes and depreciation: $r = (1 - \tau_2)B - \delta_2$ or $r = (1 - \tau_2)(B - \delta_2)$. Hence taxes in the other sectors have no effect on the interest rate or growth rate. The intuition here is that the human capital sector offers an investment that yields a rate of return B, regardless of what else is happening in the economy. But along the balanced path *all* investments must yield the *same* rate of return, so all must yield B.

In the general case, however, when tax rates vary by factor or sector or both, the effect on the interest rate depends on properties of the production functions in both input-producing sectors. An obvious conjecture is that the degree of substitutability between the two factors is crucial. If they are highly substitutable, one might expect that the economy adjusts by substituting away from the more heavily taxed factor and that other variables, including the interest rate, adjust very little. Thus one might expect a large impact on input ratios and factor shares and a relatively small impact on the interest rate. The reverse might be expected if the substitution possibilities are poor.

In the next section we examine this conjecture more carefully. We find that the conclusions about input ratios and factor shares are correct, but the one about the interest rate is not. The response of the interest rate, and hence of the growth rate, is very insensitive to the elasticities of substitution in the production technologies.

II. The Unimportance of Substitution Elasticities

To study the effects of taxing factors or sectors at different rates, we shall fix values for (z_1, z_2, r, q) and compare economies with CES technologies for producing physical and human capital that, when untaxed, have the specified input ratios, interest rate, and rental ratio $q = q_2/q_1$.

Fix (z_1, z_2, r, q). Assume that tax rates vary only by factor or only by sector, so that (3a)–(3c) hold. The latter assumption is not too bad for the United States, where in the education sector physical capital and the (substantial) fraction of the labor input representing students' time are both untaxed, and in the goods sector incomes from physical and human capital are taxed at similar rates. The discussion in the rest of this section will emphasize this interpretation. It is worth noting, however, that the steady state for such an economy is identical to the one that would prevail if tax rates varied only by factor and capital income was taxed more heavily. Also, for simplicity assume that both depreciation rates are zero: $\delta_1 = \delta_2 = 0$. There is no reason to think that the substitution elasticities interact with the depreciation rates in an important way, so this assumption seems innocuous.

For each pair of elasticity parameters $\eta, \gamma \geq 0$, there are unique CES functions, call them $G(\cdot, \cdot; \eta)$ and $H(\cdot, \cdot; \gamma)$, with scale and weight parameters that depend on (z_1, z_2, q, r), such that (3a)–(3c) hold. Those functions are

$$G(k_1, h_1; \eta) = \begin{cases} r[w^{1-\alpha}k_1^{\alpha} + (1-w)^{1-\alpha}(qh_1)^{\alpha}]^{1/\alpha}, & \eta \neq 1 \\ r\left(\dfrac{k_1}{w}\right)^{w}\left(\dfrac{qh_1}{1-w}\right)^{1-w}, & \eta = 1 \end{cases} \tag{5a}$$

and

$$H(k_2, h_2; \gamma) = \begin{cases} \left(\frac{r}{q}\right)[v^{1-\beta}k_2^\beta + (1-v)^{1-\beta}(qh_2)^\beta]^{1/\beta}, & \gamma \neq 1 \\ \left(\frac{r}{q}\right)\left(\frac{k_2}{v}\right)^v \left(\frac{qh_2}{1-v}\right)^{1-v}, & \gamma = 1, \end{cases} \tag{5b}$$

where $w = 1/(1 + qz_1)$ and $v = 1/(1 + qz_2)$ are the factor shares for capital in the two sectors, and where $\alpha = 1 - (1/\eta)$, and $\beta = 1 - (1/\gamma)$. All the economies in the two-dimensional family indexed by (η, γ) have identical steady states. Although they have different elasticities of substitution, those elasticities cannot be identified by observing an economy growing along a balanced path.

If income earned in different sectors is taxed at different rates, however, these economies respond differently. Describe a tax policy by the retention rates $m = (m_1, m_2)$, where $m_i = (1 - \tau_i)$, $i = 1, 2$. Let $R(m, \eta, \gamma)$, $Q(m, \eta, \gamma)$, and $Z_i(m, \eta, \gamma)$, $i = 1, 2$, denote the (net-of-tax) interest rate, (gross-of-tax) rental ratio, and input ratios along the balanced growth path in the economy with tax policy m and elasticity parameters (η, γ). Then (3a)–(3c) imply that

$$m_1 G_1(1, Z_1) = m_2 H_2(1, Z_2) = R \tag{6a}$$

and

$$\frac{G_2(1, Z_1)}{G_1(1, Z_1)} = \frac{H_2(1, Z_2)}{H_1(1, Z_2)} = Q, \quad \text{all } m, \eta, \gamma. \tag{6b}$$

We are interested in two questions about solutions to (6). The first is whether, for fixed (η, γ), the interest rate R, rental ratio Q, and input ratios Z_1 and Z_2 are sensitive to the tax policy m. The second is whether, for fixed tax policy m, the solution (R, Q, Z_1, Z_2) is sensitive to the elasticities (η, γ).

We begin with some basic properties of the solution. Fix (η, γ). Clearly, multiplying both retention rates by a common factor $\lambda > 0$ changes the interest rate $R(m)$ by λ and has no effect on the rental ratio $Q(m)$ or input ratios $Z_1(m)$ and $Z_2(m)$. That is, Q, Z_1, and Z_2 are homogeneous of degree zero in the pair $m = (m_1, m_2)$, and R is homogeneous of degree one.

It is also easy to show (arguing by contradiction) that $R(m)$ lies between $m_1 r$ and $m_2 r$. That is, the interest rate lies between the bounds defined by the economies in which both tax rates are set at the higher level and both at the lower level. In addition, the relative rental rate rises for the factor produced in the more heavily taxed sector: $m_2 < m_1$ implies $Q > q$, and conversely. The intuition behind this is clear: the relative cost of production is raised for the factor produced in the more heavily taxed sector, and the rental rate for that factor must rise to cover its higher cost. Finally, the input ratios in both industries shift away from the more heavily taxed factor: $m_2 < m_1$ implies $Z_i < z_i$, $i = 1, 2$, and conversely. This is an immediate consequence of cost minimization.

More detailed information requires solving (6) explicitly. First, from (6b) we obtain

$$\frac{Z_1(m, \eta, \gamma)}{z_1} = \left[\frac{Q(m, \eta, \gamma)}{q}\right]^{-\eta} \tag{7a}$$

and

$$\frac{Z_2(m, \eta, \gamma)}{z_2} = \left[\frac{Q(m, \eta, \gamma)}{q}\right]^{-\gamma}, \quad \text{all } m, \eta, \gamma. \tag{7b}$$

These are the familiar expressions that, for CES technologies, relate changes in the input ratios to changes in the factor cost ratio: the elasticities are η and γ. Substituting from these expressions into (6a), we can solve for the rental rate and interest rate.

A. *Identical Technologies:* G = H

We begin with the special case in which physical and human capital are produced with the same technology, $\eta = \gamma$ and $w = v$. Then

$$\frac{Q(m, \eta, \eta)}{q} = Q(m, \eta, \eta) = \left(\frac{m_2}{m_1}\right)^{-1} \tag{8a}$$

and

$$\frac{R(m, \eta, \eta)}{r} = \begin{cases} [wm_1^{\eta-1} + (1 - w)m_2^{\eta-1}]^{1/(\eta-1)}, & \eta \neq 1 \\ m_1^w m_2^{1-w}, & \eta = 1, \text{all } m. \end{cases} \tag{8b}$$

If the sectors producing physical and human capital are taxed at different rates, then production costs for the two capital goods will differ by the tax differential. The change in the rental rates must exactly offset that cost differential, independent of η and w. It then follows immediately from (7) that the change in the input ratio is also independent of w, although it increases with η.

The change in the interest rate is "produced" with a CES function that has the retention rates m_1 and m_2 as inputs and the factor shares w and $1 - w$ as weights. Thus if human capital's share is relatively large (as it is in the U.S. economy), then the weight on the retention rate in the human capital sector, m_2, is correspondingly large, and the tax rate on income earned in the sector producing physical capital, m_1, has relatively little impact on the interest rate and growth rate.

Although the magnitude of the interest rate response is sensitive to the share parameter w, for empirically plausible ranges for the parameters it is likely to be insensitive to the elasticity parameter η. Two results show this. First, when $m_2/m_1 \approx 1$, the response of the interest rate to changes in tax policy, to a first-order approximation, is independent of η. To see this, differentiate (8b) with respect to the retention rates and find that

$$\left.\frac{d\ln(R/r)}{d\ln(m_1)}\right|_{m_1=m_2} = w,$$

$$\left.\frac{d\ln(R/r)}{d\ln(m_2)}\right|_{m_1=m_2} = 1 - w, \quad \text{all } \eta.$$

This conclusion is a direct consequence of the fact that, for fixed (r, z), all members of the family of production functions defined in (5) have identical marginal rates of substitution when $Z = z$ and that the latter holds if $m_1 = m_2$.

More generally, if m_2/m_1 is not close to one, differentiate (8b) with respect to the elasticity parameter to obtain

$$\left.\frac{d\ln(R/r)}{d\eta}\right|_{\eta=1} = \frac{w(1-w)[\ln(m_2/m_1)]^2}{2}, \quad \text{all } m.$$

Since $w \in (0, 1)$, the term $w(1 - w)/2$ is no greater than .125. If m_2/m_1 lies between ½ and two, then the squared term is no greater than .48, so the entire expression is less than .060. Therefore, in the neighborhood of the Cobb-Douglas case, the interest and growth effects of tax policies, within the empirically relevant policy range, are very insensitive to the elasticity parameter.

In this case, then, all our questions have very sharp answers. The rental ratio Q/q is unit-elastic with respect to m_2/m_1, regardless of η and w. The response of the input ratio Z/z is very sensitive to the elasticity η and independent of w. The change in the interest rate, R/r, is much more sensitive to the retention rate in the human capital sector, m_2, than to the retention rate in the physical capital sector, m_1 (under the assumption that human capital's share, $1 - w$, is large relative to physical capital's share, w), and the factor share w is critical in determining the size of the response. And if the tax policy treats income almost symmetrically ($m_1 \approx m_2$) or if the production function is close to Cobb-Douglas ($\eta \approx 1$), then the change in the interest rate is very insensitive to the production elasticity. Thus while η is critical for determining the response of the input ratio and factor shares, it is not important for determining the growth effect.

B. Different Technologies: G $\neq$ H

Next consider the case in which the production technologies differ but both elasticities of substitution are unity, $w \neq v$, $\eta = \gamma = 1$. In this case we find that

$$\frac{Q(m, 1, 1)}{q} = \left(\frac{m_2}{m_1}\right)^{-1/(1-w+v)} \tag{9a}$$

and

$$\frac{R(m, 1, 1)}{r} = m_1^{v/(1-w+v)} m_2^{(1-w)/(1-w+v)}, \quad \text{all } m. \tag{9b}$$

Since v, $1 - w \in (0, 1)$, the elasticity of Q/q with respect to m_2/m_1 exceeds ½ in absolute value. If $v < w$, then the elasticity exceeds one in absolute value.

The change in the interest rate is "produced" from the retention rates m_1 and m_2 with a Cobb-Douglas technology, where the weights are the factor shares v and $1 - w$, normalized to sum to unity. Here the weights v and $1 - w$ are the "alien" factor shares: the share of *physical* capital in the *human* capital sector and the share of *human* capital in the *physical* capital sector. Therefore, if v is substantially smaller than $1 - w$ (as it is in the United States), then the conclusion above still holds: the tax on the human capital sector is relatively much more important in determining the interest rate and growth rate. If $v = 0$, as in Lucas (1990), then $R/r = m_2$, and the change in the interest rate depends *only* on the tax in the human capital sector.

Finally, in the general case in which the two production functions differ from each other and both elasticities differ from unity, we find that

$$\frac{R(m, \eta, \gamma)}{r} = m_1\left\{w + (1 - w)\left[\frac{Q(m, \eta, \gamma)}{q}\right]^{-(\eta-1)}\right\}^{1/(\eta-1)} \tag{10a}$$

$$= m_2\left\{v\left[\frac{Q(m, \eta, \gamma)}{q}\right]^{\gamma-1} + (1 - v)\right\}^{1/(\gamma-1)}, \quad \eta, \gamma \neq 1, \text{all } m. \tag{10b}$$

With these expressions, it is straightforward (but tedious) to show that the conclusions reached for the Cobb-Douglas case hold more generally: the elasticity of the rental ratio Q/q with respect to m_2/m_1

is negative and exceeds ½ in absolute value; and v and $1 - w$ are critical for the relative importance of m_1 and m_2 in determining R/r.

For small differences in the two tax rates, that is, for m_2/m_1 close to unity, this is particularly clear. Note that $m_2/m_1 = 1$ implies $Q/q = 1$, and evaluate the derivatives of (10) to find that

$$\left.\frac{d\ln(Q/q)}{d\ln(m_2/m_1)}\right|_{m2/m1=1} = -\frac{1}{1-w+v},$$

$$\left.\frac{d\ln(R/r)}{d\ln(m_1)}\right|_{m2/m1=1} = \frac{v}{1-w+v},$$

$$\left.\frac{d\ln(R/r)}{d\ln(m_2)}\right|_{m2/m1=1} = \frac{1-w}{1-w+v}, \quad \text{all } \eta, \gamma.$$

Notice, too, that these derivatives do not depend on η and γ. Therefore, the conclusions for the case in which $G = H$ also hold more generally: to a first-order approximation, the response of the rental ratio and the interest rate are independent of the substitution elasticities.

More generally, if m_2/m_1 is not close to one, differentiate (10) with respect to η to obtain

$$\left.\frac{d\ln(Q/q)}{d\eta}\right|_{\eta=\gamma=1} = \frac{w(1-w)[\ln(m_2/m_1)]^2}{2(1-w+v)^3},$$

$$\left.\frac{d\ln(R/r)}{d\eta}\right|_{\eta=\gamma=1} = \frac{vw(1-w)[\ln(m_2/m_1)]^2}{2(1-w+v)^3}, \quad \text{all } m.$$

The derivatives with respect to γ are similar. The argument above holds here as well: if m_2/m_1 lies between ½ and two, then this term is very small. Therefore, in a neighborhood of the Cobb-Douglas case, the effects of tax policy on the interest rate and growth rate are very insensitive to the elasticity parameters. That is, the solution in (9) is a very good approximation if the production elasticities are close to one.

To summarize, then, suppose that w is small relative to $1 - w$, that $0 < v < w$, and that both elasticities are near unity. Then the elasticity of the rental ratio Q/q with respect to m_2/m_1 is a little greater than one in absolute value and is insensitive to the elasticity parameters. The change in the interest rate, R/r, is much more sensitive to m_2 than to m_1, and the factor shares v and $1 - w$ are critical in determining the response. Moreover, the interest rate response is very insensitive to the production elasticities, so conclusions about the growth effects of taxation are insensitive to η and γ. The changes in the input ratios, Z_1/z_1 and Z_2/z_2, however, are sensitive to the elasticities.

Some of the results above hold if m_2/m_1 is close to one or if both substitution elasticities are close to one. To get some idea of what "close" means, we can simply compute the ratios in (7) and (10) as functions of the technology parameters (η, γ, w, v) and the policy parameters (m_1, m_2). Note that Q/q and R/rm_1, as well as the input ratios, depend on the policy parameters only through the ratio m_2/m_1. Also note that factor share and input ratios are very closely related:

$$\frac{S_i(m, \eta, \gamma)}{s_i} = \frac{Z_i(m, \eta, \gamma) Q(m, \eta, \gamma)}{z_i q}, \quad i = 1, 2,$$

where $s_1 = (1 - w)/w$ and $s_2 = (1 - v)/v$ are the factor share ratios in the untaxed economy, and the S_i's are the ratios in the taxed economy. Since factor shares are more readily observed, we plot them instead.

Figure 1 displays the ratios R/rm_1, Q/q, S_1/s_1, and S_2/s_2 as functions of m_2/m_1. The factor shares $w = .4$ and $v = .2$ are used throughout, and the elasticity parameters take values of .5, 1.0, and 2. Each panel has six curves, drawn for six different elasticity pairs (η, γ), as indicated.

The results are quite striking. The effects of a given tax policy m_2/m_1 on the interest rate ratio R/rm_1 and rental ratio Q/q are very insensitive to the elasticity parameters. The effect on the factor share ratio in each industry is very sensitive to that industry's own elasticity, however. Experiments with other parameter values indicate that these conclusions are robust.

Next consider the share parameters. Figure 2 shows the response of the interest rate for Cobb-Douglas technologies with different pairs of share parameters (w, v), where $w = .2$ and .4, and $v = .05$, .2, and .3, as indicated. The shares, unlike the elasticities, are quantitatively important in determining the interest rate.

The share of tax revenue in total income in the steady state is also quite insensitive to the elasticity parameters. Figure 3 plots the ratio of revenue to total income, including all income generated in the sector producing human capital, as a function of m_2, for $m_1 = .65$ and for share parameters $w = .40$ and $v = .20$. The four curves are drawn for different elasticity pairs (η, γ), where each parameter takes values of .5 or 2.0. The elasticity parameters have very little effect on the revenue ratio. In addition, the figure looks very similar if income is defined narrowly, excluding output in the sector producing human capital.

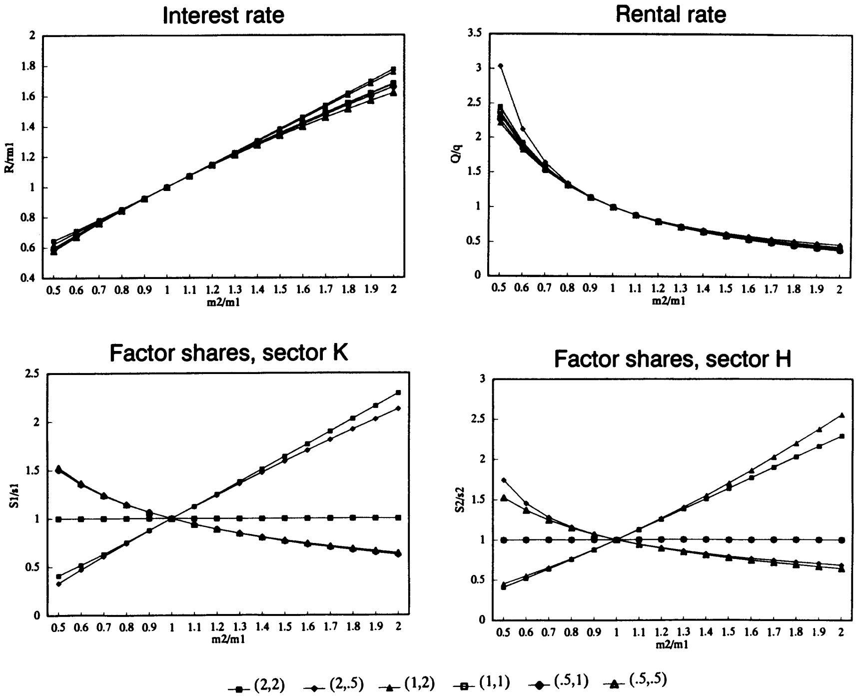

FIG. 1.—Factor shares $(w, v) = (.40, .20)$, various elasticity pairs (η, γ)

C. *Transitional Dynamics*

To a first-order approximation, transitional dynamics in the neighborhood of the untaxed steady state are also independent of the elasticity parameters. To see this, observe that the dynamics depend on the functions G and H and their derivatives, evaluated near the steady state. But by construction, all members of a family of technologies defined by (5a) and (5b) have identical steady-state marginal products. That is, for fixed (z_1, z_2, r, q), the derivatives $G_i(1, z_1; \eta)$ and $H_i(1, z_2; \gamma)$, $i = 1, 2$, are independent of η and γ. It then follows that for any (k_i, h_i) with k_i/h_i near z_i, the level of output is also approximately independent of the elasticity parameter. For example, choose (k_1, h_1) such that $k_1/h_1 \approx z_1$, and let (k_1^s, h_1^s) be any nearby point satisfying $h_1^s/k_1^s = z_1$. Then

$$
\begin{aligned}
&G(k_1, h_1; \eta)\\
&\quad\approx G(k_1^s, h_1^s; \eta) + (k_1 - k_1^s)G_1(1, z_1; \eta) + (h_1 - h_1^s)G_2(1, z_1; \eta)\\
&\quad= k_1 G_1(1, z_1; \eta) + h_1 G_2(1, z_1; \eta),
\end{aligned}
$$

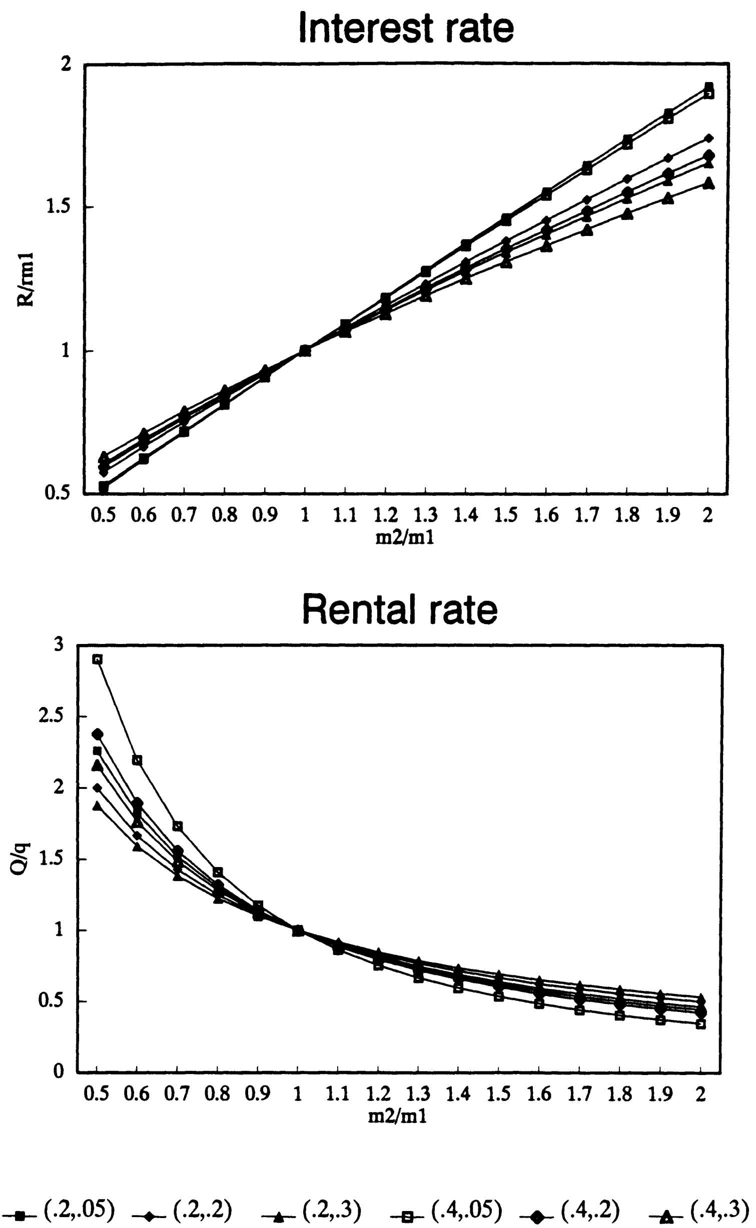

FIG. 2.—Cobb-Douglas technologies, various factor shares (w, v)

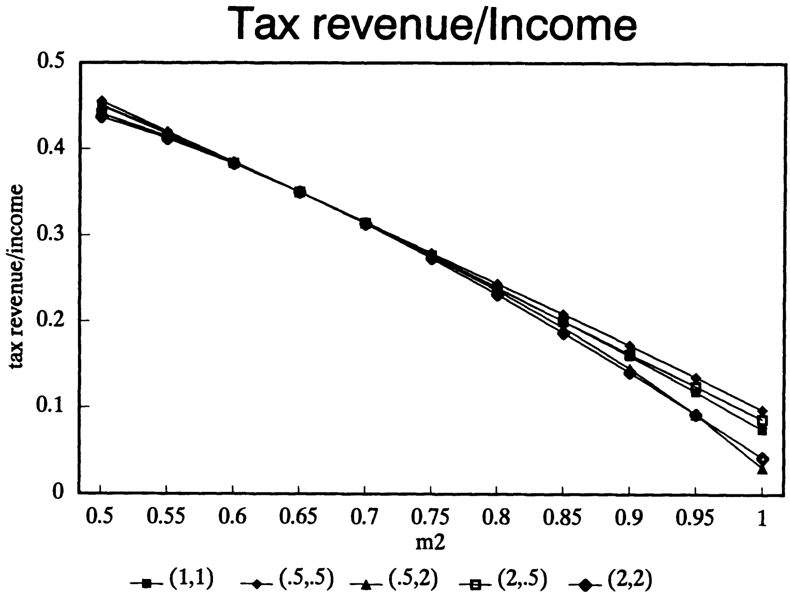

FIG. 3.—Factor shares $(w, v) = (.40, .20)$, various elasticity pairs (η, γ)

where the second line uses a first-order Taylor series approximation and the third uses Euler's theorem for homogeneous functions. As we have already noted, the derivatives in the third line are independent of η. An analogous argument holds for the human capital sector.

To predict accurately the impact of a given tax policy on the long-run interest rate and growth rate, it is important to know the share parameters w and v. The elasticity parameters η and γ, however, are of minor importance. The policy's impact on input ratios and factor shares, on the other hand, is sensitive to η and γ but not to w and v. Since w and v are observable in the untaxed steady state but η and γ are not, it may be easier to predict the impact of fiscal reform on the interest rate than its impact on factor ratios and factor shares.

III. Elastic Labor Supply

In this section we shall briefly discuss two ways of incorporating elastic labor supply into the basic model. Let l_4 denote the proportion of time devoted to leisure.

First, suppose that leisure time is quality adjusted in the same way work time is,[2] and suppose that instantaneous preferences have the

[2] See Becker (1981, chaps. 1, 2) for an interpretation of these preferences in terms of home production.

form

$$u(c, l_4 h) = \frac{V(c, l_4 h)^{1-\sigma}}{1 - \sigma}, \quad \sigma > 0,$$

where V: $\mathbb{R}_+ \times [0, 1] \to \mathbb{R}_+$ is strictly increasing, strictly quasi-concave, continuously differentiable, and homogeneous of degree one. The new control variable l_{4t} adds a new first-order condition, which determines the steady-state mix of consumption and leisure. If taxes are levied by sector, gross of depreciation, the new condition is

$$\frac{V_2(\hat{c}, l_4 z)}{V_1(\hat{c}, l_4 z)} = (1 - \tau_3) F_2(1, z_3), \tag{2f}$$

where $\hat{c} = c/k$ and $z = h/k$. Equations (2a)–(2e) are unaltered, however, so the interest rate and growth rate are unaffected. A consumption tax now distorts the labor-leisure mix but still has no effect on the growth rate.

Alternatively, suppose that utility depends on pure leisure time, unadjusted for the level of human capital, and suppose that instantaneous preferences have the form

$$u(c, l_4) = \frac{[cv(l_4)]^{1-\sigma}}{1 - \sigma}, \quad \sigma > 0,$$

where v: $[0, 1] \to \mathbb{R}_+$ is strictly increasing, strictly concave, and twice continuously differentiable, and concavity of u requires that $-\sigma v''(l_4) v(l_4) > (1 - 2\sigma)[v'(l_4)]^2$.

If income is taxed gross of depreciation and tax rates vary only by sector, then (2c) is altered and a new condition determining the labor-leisure trade-off is added. The new conditions are

$$(1 - \tau_2) H_2(1, z_2)(1 - l_4) - \delta_2 = r \tag{2c'}$$

and

$$\frac{\hat{c} v'(l_4)}{v(l_4)} = (1 - \tau_3) z F_2(1, z_3). \tag{2f'}$$

These changes imply that the system of equations is no longer block-recursive: the interest rate and factor intensities can no longer be determined in isolation. Hence it is more difficult to explore analytically the properties of the steady state. Nevertheless, it can still be shown that a consumption tax has no effect on the steady-state growth rate. In this case, though, the conclusion holds when the tax revenue is *not* rebated to households.

Moreover, an argument exactly analogous to the one in Section II can be used to study the sensitivity of growth effects to the elasticity of labor supply. Simply substitute $(1 - l_4)m_2$ for m_2 in (6a) and notice that the rest of the analysis is unchanged, except that l_4 is unknown. In particular, (10b) holds with $(1 - l_4)m_2$ replacing m_2. Then differentiate as before to find that

$$\left.\frac{d\ln(R/r)}{d\ln(m_1)}\right|_{m_2/m_1=1} = \frac{1}{1+x}\left[x + \frac{d\ln(1-l_4)}{d\ln(m_1)}\right],$$

$$\left.\frac{d\ln(R/r)}{d\ln(m_2)}\right|_{m_2/m_1=1} = \frac{1}{1+x}\left[1 + \frac{d\ln(1-l_4)}{d\ln(m_2)}\right],$$

where $x = v/(1 - w)$. Recall that $v/(1 - w)$ is the ratio of the alien factor shares. In Lucas's (1990) model, $v = 0$, so $x = 0$. As the first expression shows, in this case a tax on the sector producing physical capital affects the growth rate *only* if the supply of labor is elastic.

Notice that the importance of elastic labor supply in producing growth effects is greater the smaller are v and w In the next section we shall see that the large growth effects found by Jones et al. (1993) result from low values for v and w, together with a large labor supply elasticity.

IV. Quantitative Comparisons

In this section we shall use the framework developed above to compare the results in Lucas (1990), King and Rebelo (1990), Kim (1992), and Jones et al. (1993) on the potential effect of tax reform on the long-run growth rate of the U.S. economy. As noted above, the quantitative conclusions in these papers differ dramatically. We shall see below that these sharp differences in the conclusions arise from differences in the assumptions about the share parameter in the sector producing human capital, the depreciation rate for human capital, and the elasticity of labor supply.

Two features appear in some of the models we are comparing that do not fit into the framework used here. The first feature is a production function for human capital that displays diminishing point-in-time returns (see Heckman [1976] and Rosen [1976] for a further discussion). Technologies of this sort dampen the impact of changes in the rate of return on incentives to invest. The second feature is a utility function that has pure leisure (unadjusted for quality) as an argument. As we saw in the last section, elastic labor supply of this form magnifies the impact of changes in tax policy since it provides a second avenue, in addition to adjustments to consumption, by which consumers can respond to changes in the rate of return on investment. We shall see below that diminishing point-in-time returns do not seem to be quantitatively important in any of the analyses, but elastic labor supply does have a quantitatively significant effect in the Jones et al. model.

In this section we allow tax rates to vary by both factor and sector, and we assume that income is taxed gross of depreciation. First we simplify each of the models to conform to our setup and calibrate it using the author's parameter values. We then carry out the tax experiment(s) performed by him to see whether altering the specification has changed the model's responses significantly. Then, for each calibration, we perform the exercise of eliminating all income taxes. Cobb-Douglas production functions are used throughout, so $\eta = \gamma = 1$.

A. *The Lucas Model*

First consider the model in section 4 of Lucas (1990). It includes diminishing point-in-time returns in human capital accumulation, elastic labor supply, and an elasticity of substitution of .60 (rather than unity) in the goods production technology. In addition, Lucas ignores depreciation, which can be interpreted as an assumption that the production functions are defined net of depreciation and returns are taxed net of depreciation. Lucas's baseline parameters are displayed in table 1. The key assumptions are that human capital is produced using human capital only and that the human capital sector is untaxed.

TABLE 1

Description of Lucas (1990) Model

Additional features	Diminishing point-in-time returns to human capital Elastic labor supply CES production function with .6 elasticity of substitution Taxes levied net of depreciation	
Baseline parameters	$\delta_k = 0$, $\delta_h = 0$, $w = .24$, $v = .00$, $\tau = (.26, .40, .00, .00)$, $\sigma = 2$, $\rho = .0340$	
Benchmark indicators	$g = .0150$, $r = .0640$	
	Change in the Growth Rate	
	Original Model	Modified Version
Experiments:		
Reduce capital tax to 0% and raise labor tax to 46%	−.0003	.0000
Eliminate all taxes		.0000

The experiment Lucas runs is cutting the capital tax to zero while raising the labor tax to .46. As noted above, if labor supply is inelastic, this change has no effect on the growth rate. The nonzero effect Lucas obtains comes from the fact that labor supply in his model is (very slightly) elastic.

B. *The King and Rebelo Model*

Next consider the model in section 3 of King and Rebelo (1990). Except for diminishing point-in-time returns in human capital accumulation, their model is identical to ours: labor supply is inelastic, technologies are Cobb-Douglas, and returns are taxed gross of depreciation. Their baseline parameters are displayed in table 2.

King and Rebelo look at two experiments, increasing all tax rates by .10 and raising the tax rate on physical capital only by .10. Each experiment is performed for two sets of parameter values. In each case, our modified version of their model gives results very similar to the original results. Thus introducing diminishing point-in-time returns in the sector producing human capital does not make much difference quantitatively.

Eliminating all taxes in the modified model raises the growth rate by .0330 for King and Rebelo's baseline parameters, or when a lower value (v = .05 instead of .33) is used for capital's share in the sector producing human capital. If a lower depreciation rate for human

TABLE 2

Description of King and Rebelo (1990) Model

Additional features	Diminishing point-in-time returns to human capital	
Baseline parameters	δ_k = .1, δ_h = .1, w = .33, v = .33, τ = (.20, .20, .20, .20), σ = 1, ρ = .0120	
Benchmark indicators	g = .0200, r = .0320	
	Change in the Growth Rate	
	Original Model	Modified Version
Experiments:		
Increase all taxes by .10	−.0152	−.0167
Raise capital tax by .10	−.0052	−.0058
Raise capital tax by .10 with v = .05	−.0011	−.0012
Increase all taxes by .10 with δ_h = .012	−.0067	−.0071
Eliminate all taxes		.0330
Eliminate all taxes with v = .05		.0330
Eliminate all taxes with δ_h = .012		.0143

TABLE 3

DESCRIPTION OF KIM (1992) MODEL

Additional features	Inflation tax Partial deduction of depreciation on physical capital	
Baseline parameters	$\delta_k = .05$, $\delta_h = .01$, $w = .34$, $v = .34$, $\tau = (.34, .17, .34, .17)$, $\sigma = 1.94$, $\rho = .01$	
Benchmark indicators	$g = .0150$, $r = .0391$	
	CHANGE IN THE GROWTH RATE	
	Original Model	Modified Version
Experiment:		
Eliminate all taxes	.0085	.0091

capital is used, $\delta_h = .012$ instead of .100, eliminating all taxes raises the growth rate by .0143. Although this is still a significant effect, it is much more modest than the one calculated for King and Rebelo's baseline parameters.

C. *The Kim Model*

Kim (1992) begins with a much more detailed description of the U.S. tax system but is able to aggregate to obtain effective net tax rates on the two factors. After the aggregation, his model is identical to ours except for the presence of an inflation tax and the partial deductibility of depreciation on physical capital. His baseline parameters, which are displayed in table 3, are similar to King and Rebelo's except that the depreciation rates are substantially lower and σ is substantially higher. Kim's experiment is to eliminate all taxes. This change raises the growth rate by .0085 in his model and by .0091 in our version of it.

D. *The Jones et al. Model*

Finally, consider model 2 in Jones et al. (1993). They assume that labor supply is elastic, and their production function for human capital has human capital and market goods as inputs. This is important because of the tax treatment of various factors: human capital employed directly in the sector producing human capital is not taxed, but all factors employed in producing the market goods used by that sector are taxed. In other respects, their model conforms with our setup: the technologies are Cobb-Douglas and taxes are levied gross of depreciation. Modifying our structure to incorporate their tax structure is not hard, so we shall do so.

TABLE 4

Description of Jones et al. (1993) Model

Additional features	Elastic labor supply Market goods used in producing human capital	
Baseline parameters	$\delta_k = .1$, $\delta_h = .1$, $w = .36$, $\psi = .48$, $\tau = (.21, .31, .00, .00)$, $\sigma = 1.5$, $\rho = .02$, $1 - \bar{l}_4 = .29$, $l_2 = .12$	
Benchmark indicators	$g = .0200$, $r = .0500$	
	Change in the Growth Rate	
	Original Model	Modified Version
Experiments:		
Eliminate all taxes	.0350	.0211
Eliminate all taxes with $\sigma = 1.1$	.0830	.0333

The sector producing human capital uses market goods, I_{2t}, and human capital as inputs. Let l_{2t} be the proportion of time allocated as direct labor input. The constraints (1b) and (1c) for the consumer's problem become

$$\dot{h}_t = \Phi I_{2t}^{\psi}(l_{2t}h_t)^{1-\psi} - \delta_2 h_t \tag{1b'}$$

and

$$I_{1t} + I_{2t} + c_t - q_{1t}k_t - q_{2t}(1 - l_{2t} - \bar{l}_4)h_t - T_t \leq 0, \tag{1c'}$$

where $\bar{l}_4$ is the (fixed) proportion of time allocated to leisure and $1 - l_2 - \bar{l}_4$ the proportion allocated to goods production, and prices are unity because all market goods are produced with the same technology. The baseline parameter values used by Jones et al. are displayed in table 4. Notice that $v = \psi w = .17$ is the (implicit) share of physical capital in the sector producing human capital.

The experiment Jones et al. run is to eliminate all taxes. They do this for a number of alternative values for the parameter σ (cf. their table 3). Comparisons with our version of their model are reported in table 4 for $\sigma = 1.5$ and $\sigma = 1.1$. In both cases their growth effects are much larger than ours: increases of .035 and .083 compared with .021 and .033. The reason is the very high elasticity of labor supply in their model. Their model predicts that for $\sigma = 1.5$ the total supply of labor to nonleisure activities, $1 - l_4$, increases by 14 percent, and for $\sigma = 1.1$ by an astounding 48 percent!

E. *Comparing the Experiments*

The seven experiments reducing all tax rates to zero produce growth effects between 0.0 and 3.3 percentage points. The low value is an

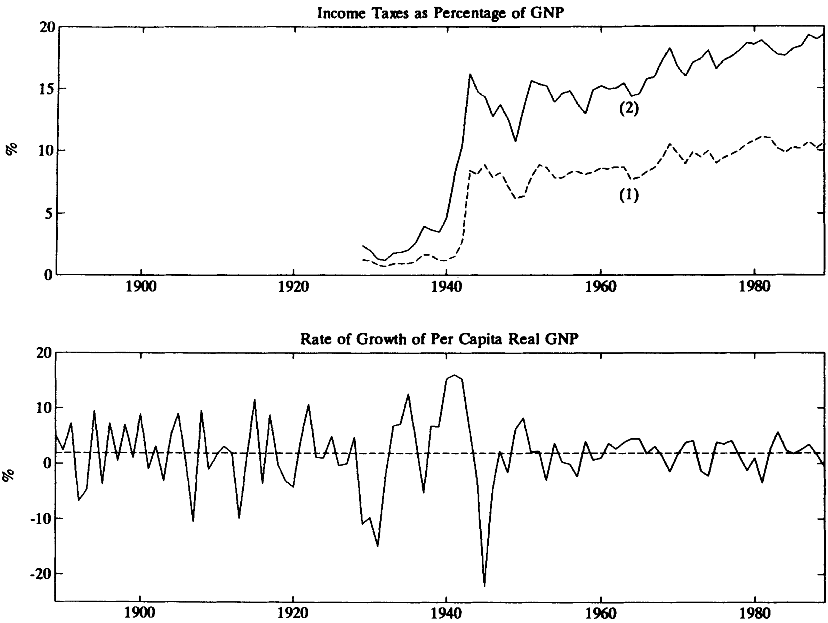

FIG. 4.—U.S. income taxes and growth rates, 1889–1989

immediate consequence when (untaxed) human capital is the only input in the production of human capital. The high value occurs when the elasticity of intertemporal substitution and the depreciation rates are high. Moreover, elastic labor supply can raise that figure even higher. Can anything be said about which range of values is most plausible?

U.S. experience provides an interesting tax reform experiment. Before the Sixteenth Amendment was approved in 1913, the U.S. Constitution severely restricted the ability of the federal government to levy taxes on income. Even after approval of the amendment, income tax revenues were a negligible fraction of GNP until World War II. That fraction increased dramatically in the early 1940s, from 2 percent to 15 percent of GNP.

Figure 4 shows income tax revenue as a fraction of GNP and the growth rate of per capita real GNP for the period 1889–1989.[3] In line 1, revenue consists of revenue from federal, state, and local individual income taxes. In line 2, it includes, in addition, revenue from social security and retirement taxes and from the federal corporate profits tax.

[3] The tax data are taken from the *Survey of Current Business*. The GNP and population data are taken from *One Hundred Years of Economic Statistics* (published by the *Economist*) for 1889–1928 and the *Economic Report of the President* for 1929–91.

While there are other aspects of government policy that changed after World War II, we would expect, on the basis of some of the models above, that such a dramatic increase in income taxation would generate a noticeable negative effect on the growth rate. It does not. The growth rate of per capita real GNP, while it displays substantial variation both before and after 1942, displays no clear break in its average value.

We performed three statistical tests, all of which confirm the visual impression that the average growth rate is the same before and after 1942. The first was a t-test for the difference in means (allowing for different variances). The average growth rate is 2.31 before 1942 and 1.22 after, and the t-value for the difference is .92. For the second, we estimated the mean rate of growth for the two periods by regressing the logarithm of per capita real GNP on a constant and a time trend, and we performed a Chow test. The least-squares growth rates are 1.37 and 1.61 in the two subperiods, and the $F_{1,99}$-value is 3.57. The final test was nonparametric. The median growth rates are 2.10 and 2.11 in the two subperiods, and the p-values are .38 and .46.[4] In no case could we reject, at the 5 percent level, the hypothesis that the average growth rate was the same in the two subperiods.

There are some obvious caveats with our simple comparison of the growth rates in two periods with low and high income tax rates. We have not controlled for numerous factors that may have varied across the two periods, including the level of other taxes and the level and composition of public expenditures.

Nevertheless, we view this evidence as suggesting that the growth effects implied by some of the calibrations above are implausibly large. Among the endogenous growth models, the U.S. experience over the last century seems to accord best with Lucas's calibration or, more generally, with any in which human capital's share is large in both sectors and the sector producing human capital is lightly taxed. (It also accords well with any model in which growth is driven by exogenous technical change.) In addition, several modifications of the other models substantially dampen the growth effects they produce.

First, the assumption that income is taxed gross of depreciation overstates the impact of tax reduction. To see this, suppose that all income is taxed at the same rate, so factor proportions in both industries are independent of the tax rate, and let r be the interest rate when there are no taxes. Then it follows from (3) and (4) that raising the tax rate by $\Delta\tau$ reduces the (after-tax) interest rate by $\Delta\tau(r + \delta)$

[4] The second test was motivated by Watson's (1992) Monte Carlo study, which shows that the least-squares estimate of the growth rate is more robust than the geometric mean to differences in the serial correlation properties of the series. The third test is described in Gibbons (1985, pp. 131–40).

if income is taxed gross of depreciation, but by only $\Delta\tau r$ if income is taxed net of depreciation. If the interest and depreciation rates are about equal, the former effect is about twice as large as the latter. In the United States, depreciation of physical capital is at least partly deductible, so assuming that income is taxed gross of depreciation exaggerates the potential effects of tax reform.

Second, a depreciation rate of 10 percent for physical capital is too high. Calculations based on the capital consumption allowance and estimates of the aggregate capital stock produce an average depreciation rate of a little over 6 percent (see App. B).

Third, a depreciation rate of 10 percent for human capital is probably also too high. Estimates of depreciation at the individual level range from 0.2 percent (Heckman 1976), to 1.2 percent (Mincer 1974), to 3–4 percent (Haley 1976). If working lifetimes are about 40 years, then, when population growth is ignored, about 2.5 percent of the workforce retires each year. Retirees have more experience but less education than younger workers, and average wages peak well before retirement for most workers. If retirees embody 2.5–4.0 percent of the total stock of human capital, then summing individual depreciation and retirement effects gives a range of 2.7–8.0 percent. The latter figure involves substantial double counting, however: if individual depreciation rates are high, then retirees account for a correspondingly smaller proportion of the total stock of human capital.

Fourth, there is little evidence of the very strong labor supply effects postulated by Jones et al. Average weekly hours per employed person have fallen very steadily and dramatically over the last century, from 53.5 hours in 1889 to 34.6 hours in 1989. Labor force participation has risen over the same period, with the proportion of the population that is employed growing from 35.0 percent in 1889 to 47.4 percent in 1989. The trend in hours slightly outweighs the trend in employment, with weekly hours per head of population falling from 18.7 to 16.4.[5]

If the King and Rebelo model is recalibrated with $\delta_k = \delta_h = .06$, eliminating all taxes raises the long-run growth rate by 2.5 percentage points rather than 3.3. If depreciation on physical capital is taken to be fully deductible, the growth effect is further reduced, to 1.8 percentage points. And as tables 2 and 3 show, even smaller growth effects are obtained if a smaller value is used for the depreciation

[5] The employment/population ratio (in millions) is 21.6/61.8 = .350 for 1889 and 117.3/247.3 = .474 for 1989. For the earlier year, the figure for population is taken from Maddison (1982, table B2), and the figures for employment and average weekly hours are taken from Kendrick (1961, tables A-VII, A-IX). For the later year, all data are taken from the 1993 *Economic Report of the President* (tables B-29, B-30, and B-42).

rate on human capital, δ_h, or for the elasticity of intertemporal substitution, $1/\sigma$.

V. Conclusions

The U.S. economy over the last century conforms very well to the description of a balanced growth path, with stable values for the capital/output ratio, capital's share in income, leisure's share in total time, and the real interest rate. Hence those data can have very little information about some of the elasticities that are relevant here. One goal of the present paper has been to study how sensitive quantitative conclusions about growth effects are to these badly estimated parameters.

On the technology side, we found that the elasticities of substitution in production are not critical for growth or revenue effects. Thus assuming Cobb-Douglas production functions in all sectors is harmless: within a wide range, conclusions about growth and revenue effects are very insensitive to this assumption. (Elasticities may be important for welfare conclusions, however, since they are critical in determining the size of the distortion in input ratios resulting from asymmetric taxation factor incomes; see, e.g., Lucas [1990] and Davies and Whalley [1991].)

By contrast, share parameters are quite important for growth effects. Excellent information about factor shares in the goods-producing sector is readily available, but a better estimate of capital's share in the sector producing human capital—a parameter about which information should be available—would be very useful.

On the preference side, two elasticities are important. Differences in estimates of the elasticity of intertemporal substitution can easily account for differences by a factor of two or three in estimates of growth effects. Similarly, differences in estimates of the long-run elasticity of labor supply can account for differences of several percentage points in beliefs about interest rate effects. For the reasons discussed above, it seems unlikely that aggregate U.S. time series can be used to improve our estimates of either parameter. Other data sources (cross-section or cross-country) are needed.

The depreciation rates for both types of capital, the tax treatment of depreciation, and the tax treatment of inputs in the sector producing human capital are also critical for determining growth effects. Among these, the depreciation rate for human capital is the most problematic. Although depreciation rates for individual human capital have been estimated from age-earnings profiles, those estimates are inappropriate in the current context, where the largest source of depreciation comes from the fact that lifetimes are finite. An overlapping generations model would allow a more satisfactory treatment of this issue, but at the cost of raising a new and equally difficult problem: how human capital is transmitted from one generation to the next.

The results above show that Lucas's (1990) result is robust: if human capital's share is large in all sectors and if the sector producing human capital is lightly taxed, then taxing returns in the physical capital sector will have very modest growth effects. We should emphasize, however, that even if the growth effects of tax reform are small, the welfare effects may be large, as shown in Lucas (1990) and Davies and Whalley (1991). In particular, capital taxation can lead to a fairly large bias in the composition of the capital stock between its physical and human components, a bias that can have serious welfare consequences. The analysis here suggests that studying these consequences in models in which growth is exogenous (or absent) may be harmless. There is, as yet, no theoretical presumption or empirical evidence of substantial growth effects from factor taxation.

Appendix A

A. *Balanced Growth Conditions*

Let λ_1, λ_2, and ν be the multipliers for the constraints (1a)–(1c) in the household's problem. Then the conditions for a maximum are

$$c_t^{-\sigma} = \nu_t, \tag{A1}$$

$$\lambda_{it} = \nu_t p_{it}, \quad i = 1, 2, \tag{A2}$$

and

$$\dot{\lambda}_{it} = (\rho + \delta_i)\lambda_{it} - \nu_t q_{it}, \quad i = 1, 2. \tag{A3}$$

Along the balanced growth path, consumption grows at a constant rate g, so $\dot{c}_t/c_t = g$. (Both kinds of capital also grow at this rate.) Since factor prices are constant, it then follows from (A1) and (A2) that

$$\frac{\dot{\nu}_t}{\nu_t} = \frac{\dot{\lambda}_{1t}}{\lambda_{1t}} = \frac{\dot{\lambda}_{2t}}{\lambda_{2t}} = -\sigma g. \tag{A4}$$

Factors are paid their marginal products, and the net-of-tax return on factor i must be equal in all sectors. Hence

$$\begin{aligned} q_{it} &= (1 - \tau_{i1})p_{1t}G_i(\theta_{1t}k_t, l_{1t}h_t) + \omega_i\tau_{i1}p_{it}\delta_i \\ &= (1 - \tau_{i2})p_{2t}H_i(\theta_{2t}k_t, l_{2t}h_t) + \omega_i\tau_{i2}p_{it}\delta_i, \quad i = 1, 2, \end{aligned} \tag{A5}$$

where θ_{it} and l_{it}, $i = 1, 2, 3$, are the proportions of the physical and human capital stocks employed in the three sectors, and $\omega_i \in [0, 1]$ is the extent to which depreciation on factor i is tax deductible. (An analogous condition also holds for the consumption goods sector.) Substitute from (A2), (A4), and (A5) into (A3) and set $\omega_1 = \omega_2 = 0$ to get (2a)–(2c). Equations (2d) and (2e) follow directly from (A5). To derive (4a) and (4b), use a similar argument with $\omega_1 = \omega_2 = 1$.

In addition to (2a)–(2e), the equilibrium conditions are market clearing in the three output markets and resource constraints for the two factors of production:

$$g + \delta_1 = \theta_1 G(1, z_1), \tag{A6}$$

$$g + \delta_2 = \left(\frac{\theta_2}{z}\right) H(1, z_2), \tag{A7}$$

$$\hat{c} = \theta_3 F(1, z_3), \tag{A8}$$

$$\sum_{i=1}^{3} \theta_i = 1, \tag{A9}$$

and

$$\sum_{i=1}^{3} z_i \theta_i = z, \tag{A10}$$

where $\hat{c} = c/k$ and $z = h/k$.

The transversality condition holds if $\rho > (1 - \sigma)g$, exactly the condition needed to ensure that total utility is bounded if consumption grows at the rate g. It will be assumed throughout that this is the case.

B. *A Family of CES Functions*

Any CES function can be written as

$$G(k, h) = \begin{cases} A[\theta k^\alpha + (1 - \theta)h^\alpha]^{1/\alpha}, & \alpha \neq 0 \\ A k^\theta h^{1-\theta}, & \alpha = 0, \end{cases}$$

where $A > 0$, $0 \leq \theta \leq 1$, and $\alpha \leq 1$. Fix (z_1, r, q) and $\alpha \leq 1$. We require (3a) to hold and the ratio of the marginal products to equal the rental ratio q:

$$G_1(1, z; \alpha) = \theta A[\theta + (1 - \theta)z^\alpha]^{(1-\alpha)/\alpha} = r,$$

$$\frac{G_1(1, z; \alpha)}{G_2(1, z; \alpha)} = z^{1-\alpha}\frac{\theta}{1 - \theta} = q.$$

Solve for A and θ as functions of α and (z_1, r, q) to obtain (5a).

C. *Tax Revenue*

Assume that tax rates vary only by factor. Since factors earn the same returns in all sectors, all three production functions are homogeneous of degree one, and $G_1/G_2 = Q$, we find that

$$\frac{\text{tax revenue}}{\text{income}} = \frac{\tau_1 k_t p_1 G_1 + \tau_2 h_t p_1 G_2}{k_t p_1 G_1 + h_t p_1 G_2} = \frac{\tau_1 Q + \tau_2 Z}{Q + Z}.$$

To express Z in terms of Q, first use (2a) and (A7) to find that

$$\Theta_2 = \frac{Z[(R - \rho)/\sigma]}{H(1, Z_2)}.$$

Assume that consumption goods and physical capital are produced with the same technology, $F = G$. Then $Z_1 = Z_3$, so (A9) and (A10) imply that

$$Z = Z_1 + (Z_2 - Z_1)\Theta_2.$$

Combining these two facts, we find that

$$Z = \frac{Z_1 H(1, Z_2)}{H(1, Z_2) - (Z_2 - Z_1)[(R - \rho)/\sigma]}.$$

Then, using (7) and (8b) or (9b), we can write Z_1, Z_2, and R in terms of Q/q. The latter can be computed by using (8a) or (9a).

D. *Elastic Labor Supply*

If leisure time is quality adjusted, we must modify the objective function for the household's decision problem and put $(1 - l_{4t})h_t$ in place of h_t in the budget constraint (1c). In addition, we must modify the government's budget constraint and the resource constraint for human capital. Equation (A1) is replaced by a pair of conditions and (A3) is slightly altered. The new conditions are

$$V(c_t, l_{4t}h_t)^{-\sigma} V_1(c_t, l_{4t}h_t) = \nu_t,$$

$$V(c_t, l_{4t}h_t)^{-\sigma} V_2(c_t, l_{4t}h_t) = \nu_t q_{2t},$$

$$\dot{\lambda}_{2t} = (\rho + \delta_2)\lambda_{2t} - V^{-\sigma}(c_t, l_{4t}h_t) V_2(c_t, l_{4t}h_t) l_{4t} - \nu_t q_{it}(1 - l_{4t}).$$

Use the first two together with (A5) to derive (2f). Use the second and third to obtain (A3), and derive (2a)–(2e) as before.

If leisure is not quality adjusted, then the new conditions for the household's maximum are

$$v(l_{4t})^{1-\sigma} c_t^{-\sigma} = \nu_t,$$

$$c_t^{1-\sigma} v(l_{4t})^{-\sigma} v'(l_{4t}) = \nu_t q_{2t} h_t,$$

$$\dot{\lambda}_{2t} = (\rho + \delta_2)\lambda_{2t} - \nu_t q_{it}(1 - l_{4t}).$$

Use the first two together with (A5) to derive (2f′). Substitute from (A2), (A4), and (A5) into the third to get (2c′).

Consider a consumption tax, the proceeds of which are thrown away.

Among the steady-state conditions are

$$\frac{\hat{c}v'(l_4)}{v(l_4)} = (1 - \tau_3)zF_2(1, z_3),$$

$$\hat{c} = (1 - \tau_3)\theta_3 F(1, z_3).$$

Moreover, $\hat{c}$ and τ_3 do not appear in any of the other steady-state conditions. Substituting to eliminate $\hat{c}$ and $1 - \tau_3$, we obtain one equation that can be combined with the other steady-state conditions to solve for the capital allocation θ_i, $i = 1, 2, 3$; the time allocation l_i, $i = 1, 2, 3, 4$; the aggregate factor ratio h/k; and the growth rate g. The remaining equation determines $\hat{c}$, which depends on τ_3. As in the case in which leisure is exogenous, the only effect of an increase in τ_3 is a reduction in consumption.

E. The Jones et al. Model

With (1b′) and (1c′) as constraints in the household's problem, the conditions for a maximum are (A1) and

$$\lambda_{1t} = \nu_t,$$

$$\lambda_{2t}\Phi\psi\mu_t^{1-\psi} = \nu_t,$$

$$\lambda_{2t}\Phi(1 - \psi)\mu_t^{-\psi} = \nu_t q_{2t},$$

$$\dot{\lambda}_{1t} = (\rho + \delta_1)\lambda_{1t} - \nu_t q_{1t},$$

$$\dot{\lambda}_{2t} = (\rho + \delta_2)\lambda_{2t} - \lambda_{2t} l_{2t}\Phi(1 - \psi)\mu_t^{-\psi} - \nu_t q_{2t}(1 - l_{2t} - \bar{l}_4),$$

where $\mu = l_2 h/I_2$ is the input ratio in the human capital sector. Using (A4) and (A5), we find that the conditions for a steady state include

$$r = \rho + \sigma g,$$

$$(1 - \tau_{11})Awz_1^{1-w} - \delta_1 = r,$$

$$(1 - \tau_{21})A(1 - w)z_1^{-w} = \frac{1 - \psi}{\psi\mu},$$

$$(1 - l_2 - \bar{l}_4)(1 - \psi)\Phi\mu^{-\psi} - \delta_2 = r,$$

$$g + \delta_2 = l_2\Phi\mu^{-\psi}.$$

Appendix B

The depreciation rate is equal to the ratio (capital consumption allowance/output)/(capital/output). From the 1991 *Economic Report of the President* we find that for 1989, CCA/Y = 554/5,201 = .107. Christiano (1988, pp. 260–62) calculates the capital/output ratio to be 2.65, of which 10 percent is consumer durables, 33 percent producer structures and equipment, 33 percent government and private residential, and 24 percent government nonresidential. Since the capital consumption allowance figures exclude consumer

TABLE B1

Year	K/Y	CCA/Y	(CCA/Y)/(K/Y)
1950	1.44	.0885	.0613
1955	1.44	.0887	.0620
1960	1.52	.0939	.0620
1965	1.46	.0880	.0600
1970	1.52	.0951	.0620
1975	1.61	.1065	.0660

durables and government, the relevant capital/output ratio for our purposes is $K/Y = 2.7 \times [1 - .10 - (.025 \times .33) - .24] = 1.73$, where we have used the information in Young and Musgrave (1980, tables 1.A.2, 1.A.6) to estimate the proportion of residential capital owned by the government. Combining these two figures, we obtain a depreciation rate of $.107/1.73 = .062$.

This figure agrees well with calculations based on the capital stock estimates in Young and Musgrave. Table B1 uses their figures (table 1.A.2) for the capital stock, excluding government capital, consumer durables, and inventories, and figures from the 1991 *Economic Report of the President* (tables B-1, B-3, and B-16) for the other series.

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