Ejemplo:

$$\max Z = -x_1 + x_2 + 2x_3$$

s.a

$$x_1 + 4x_2 - x_3 \leq 20$$
$$-2x_1 + 4x_2 + 2x_3 \leq 60$$
$$2x_1 + 3x_2 + x_3 \leq 50$$
$$x_1, x_2, x_3 \geq 0$$

FORMA NORMAL DE MÁXIMO (FNM)

$$\max Z = -x_1 + x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6$$

s.a:

$$x_1 + 4x_2 - x_3 + x_4 = 20$$
$$-2x_1 + 4x_2 + 2x_3 + x_5 = 60$$
$$2x_1 + 3x_2 + x_3 + x_6 = 50$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

$$n - m = 6 - 3 = 3 > 0$$
$$\binom{6}{3} = \frac{6!}{3! \cdot 3!} = 20$$

a) Base posible inicial

$$I = \{4, 5, 6\} \rightarrow V.B \qquad B = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$J = \{1, 2, 3\} \rightarrow V.N.B$$

$$B^{-1} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\bar{\bar{X}}^B = \begin{vmatrix} \bar{\bar{x}}_4 \\ \bar{\bar{x}}_5 \\ \bar{\bar{x}}_6 \end{vmatrix} = \begin{vmatrix} 20 \\ 60 \\ 50 \end{vmatrix} \geq 0$$

$$\bar{\bar{X}}^B = B^{-1} b = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 20 \\ 60 \\ 50 \end{vmatrix} = \begin{vmatrix} 20 \\ 60 \\ 50 \end{vmatrix}$$

b) calcular; $\bar{\bar{Z}}$, Y, Z^R, $C^R - Z^R$

$$\bar{\bar{Z}} = C^B \bar{\bar{X}}^B = [0 \quad 0 \quad 0] \begin{vmatrix} 20 \\ 60 \\ 50 \end{vmatrix} = 0$$

$$Y = B^{-1} R$$

$$Y = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 4 & -1 \\ -2 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 4 & -1 \\ -2 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$Z^R = C^B Y = [0 \quad 0 \quad 0] \begin{vmatrix} 1 & 4 & -1 \\ -2 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = [0 \quad 0 \quad 0]$$

$$C^R - Z^R = [-1 \quad 1 \quad 2] - [0 \quad 0 \quad 0] = [-1 \quad 1 \quad 2]$$

DISPONIENDO EN EL TABLERO SIMPLEX:

	x_1	x_2	x_3	x_4	x_5	x_6	θ
$-\bar{\bar{Z}} = 0$	-1	1	2	0	0	0	
$\bar{\bar{x}}_4 = 20$	1	4	-1	1	0	0	
$\bar{\bar{x}}_5 = 60$	-2	4	2	0	1	0	60/2
$\bar{\bar{x}}_6 = 50$	2	3	1	0	0	1	50/1

pivot

x_3: VE
x_5: VS

PRIMERA ITERACION:

F_1	0	-1	1	2	0	0	0	
F_2	20	1	4	-1	1	0	0	
F_3	60	-2	4	2	0	1	0	$\times\ 1/2$
F_4	50	2	3	1	0	0	1	

0	-1	1	2	0	0	0	
20	1	4	-1	1	0	0	
30	-1	2	1	0	1/2	0	$\times$ (-2) (1) (-1)
50	2	3	1	0	0	1	

-60	1	-3	0	0	-1	0
50	0	6	0	1	1/2	0
30	-1	2	1	0	1/2	0
20	3	1	0	0	-1/2	1

	x_1	x_2	x_3	x_4	x_5	x_6
$-\bar{\bar{Z}} = -60$	1	-3	0	0	-1	0
$\bar{\bar{X}}_4 = 50$	0	6	0	1	1/2	0
$\bar{\bar{X}}_3 = 30$	-1	2	1	0	1/2	0
$\bar{\bar{X}}_6 = 20$	3 pivot	1	0	0	-1/2	1

X_1: VE
X_6: VS

SEGUNDA ITERACION:

-60	1	-3	0	0	-1	0	
50	0	6	0	1	1/2	0	
30	-1	2	1	0	1/2	0	
20	3	1	0	0	-1/2	1	$\times\ 1/3$

-60	1	-3	0	0	-1	0	
50	0	6	0	1	1/2	0	
30	-1	2	1	0	1/2	0	
20/3	1	1/3	0	0	-1/6	1/3	$\times$ (-1) (1)

-200/3	0	-10/3	0	0	-5/6	-1/3
50	0	6	0	1	1/2	0
110/3	0	7/3	1	0	1/3	1/3

$20/3 \quad 1 \quad 1/3 \quad 0 \quad 0 \quad -1/6 \quad 1/3$

	x_1	x_2	x_3	x_4	x_5	x_6
$-\bar{\bar{Z}} = -200/3$	0	$-10/3$	0	0	$-5/6$	$-1/3$
$\bar{\bar{X}}_4 = 50$	0	6	0	1	$1/2$	0
$\bar{\bar{X}}_3 = 110/3$	0	$7/3$	1	0	$1/3$	$1/3$
$\bar{\bar{X}}_1 = 20/3$	1	$1/3$	0	0	$-1/6$	$1/3$

$$Z^{opt} = 200/3$$

$$x_1^* = 20/3 \,, \quad x_2^* = 0 \,, \quad x_3^* = 110/3$$

Ejemplo ②

$$\text{Min } Z = 16x_1 + 12x_2$$

s. a:

$$4x_1 + 4x_2 \geq 2$$
$$8x_1 + 4x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

FNM

$$\text{Max } R = -16x_1 - 12x_2$$

s.a.

$$4x_1 + 4x_2 - x_3 = 2$$
$$8x_1 + 4x_2 - x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

$n - m = 4 - 2 = 2 > 0$

SOLUCION: MÉTODO DE DOS FASES

I. PRIMERA FASE.

a) Formular el problema artificial:

$$\text{Max } W = -\sum u_i = -u_1 - u_2$$

s. a

$$4x_1 + 4x_2 - x_3 + 0x_4 + u_1 + 0u_2 = 2$$
$$8x_1 + 4x_2 + 0x_3 - x_4 + 0u_1 + u_2 = 3$$
$$x_1, x_2, x_3, x_4, u_1, u_2 \geq 0$$

b) Seleccionar una base inicial

$$I = \{1^u, 2^u\} \;; \quad J = \{1, 2, 3, 4\}$$

$$B = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} ; \text{ por cuanto, } \bar{\bar{X}}^B = \begin{vmatrix} \bar{\bar{u}}_1 \\ \bar{\bar{u}}_2 \end{vmatrix} = \begin{vmatrix} 2 \\ 3 \end{vmatrix} \geq 0$$

c) calcular las matrices: Y, Z^R, $C^R - Z^R$, $\bar{\bar{Z}}$