APPENDIX 8A. SOME USEFUL LATIN SQUARES AND HOW TO USE THEM TO CONSTRUCT GRAECO–LATIN AND HYPER-GRAECO–LATIN SQUARE DESIGNS

Before running a Latin square or similar design be sure to randomize the design. For example, randomly permute first the rows and then the columns, and finally randomly assign the treatments to the letters.

3×3

$$\begin{array}{ccc} A & B & C \\ B & C & A \\ C & A & B \end{array} \qquad \begin{array}{ccc} A & B & C \\ C & A & B \\ B & C & A \end{array}$$

To form the 3×3 Graeco–Latin square superimpose the two designs. Thus, using Greek letter equivalents for the second 3×3 Latin square, we have

$$\begin{array}{ccc} A\alpha & B\beta & C\gamma \\ B\gamma & C\alpha & A\beta \\ C\beta & A\gamma & B\alpha \end{array}$$

4×4

$$\begin{array}{cccc} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{array} \qquad \begin{array}{cccc} A & B & C & D \\ D & C & B & A \\ B & A & D & C \\ C & D & A & B \end{array} \qquad \begin{array}{cccc} A & B & C & D \\ C & D & A & B \\ D & C & B & A \\ B & A & D & C \end{array}$$

These three 4×4 Latin squares may be superimposed to form a hyper-Graeco–Latin square. Superimposing any pair gives a Graeco–Latin square design.

5×5

$$\begin{array}{ccccc} A & B & C & D & E \\ B & C & D & E & A \\ C & D & E & A & B \\ D & E & A & B & C \\ E & A & B & C & D \end{array} \qquad \begin{array}{ccccc} A & B & C & D & E \\ C & D & E & A & B \\ E & A & B & C & D \\ B & C & D & E & A \\ D & E & A & B & C \end{array} \qquad \begin{array}{ccccc} A & B & C & D & E \\ D & E & A & B & C \\ B & C & D & E & A \\ E & A & B & C & D \\ C & D & E & A & B \end{array} \qquad \begin{array}{ccccc} A & B & C & D & E \\ E & A & B & C & D \\ D & E & A & B & C \\ C & D & E & A & B \\ B & C & D & E & A \end{array}$$

These four 5×5 Latin squares may be superimposed to form a hyper-Graeco–Latin square. Also, superimposing any three gives a hyper-Graeco–Latin square design. Similarly, superimposing any pair gives a Graeco–Latin square design.

6 × 6

A	*B*	*C*	*D*	*E*	*F*
B	*A*	*F*	*E*	*C*	*D*
C	*F*	*B*	*A*	*D*	*E*
D	*C*	*E*	*B*	*F*	*A*
E	*D*	*A*	*F*	*B*	*C*
F	*E*	*D*	*C*	*A*	*B*

No 6 × 6 Graeco–Latin square exists.

7 × 7

A	*B*	*C*	*D*	*E*	*F*	*G*		*A*	*B*	*C*	*D*	*E*	*F*	*G*
B	*C*	*D*	*E*	*F*	*G*	*A*		*C*	*D*	*E*	*F*	*G*	*A*	*B*
C	*D*	*E*	*F*	*G*	*A*	*B*		*E*	*F*	*G*	*A*	*B*	*C*	*D*
D	*E*	*F*	*G*	*A*	*B*	*C*		*G*	*A*	*B*	*C*	*D*	*E*	*F*
E	*F*	*G*	*A*	*B*	*C*	*D*		*B*	*C*	*D*	*E*	*F*	*G*	*A*
F	*G*	*A*	*B*	*C*	*D*	*E*		*D*	*E*	*F*	*G*	*A*	*B*	*C*
G	*A*	*B*	*C*	*D*	*E*	*F*		*F*	*G*	*A*	*B*	*C*	*D*	*E*

These two 7 × 7 Latin squares can be superimposed to give a Graeco–Latin square design.

8 × 8

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*		*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
B	*A*	*D*	*C*	*F*	*E*	*H*	*G*		*E*	*F*	*G*	*H*	*A*	*B*	*C*	*D*
C	*D*	*A*	*B*	*G*	*H*	*E*	*F*		*B*	*A*	*D*	*C*	*F*	*E*	*H*	*G*
D	*C*	*B*	*A*	*H*	*G*	*F*	*E*		*F*	*E*	*H*	*G*	*B*	*A*	*D*	*C*
E	*F*	*G*	*H*	*A*	*B*	*C*	*D*		*G*	*H*	*E*	*F*	*C*	*D*	*A*	*B*
F	*E*	*H*	*G*	*B*	*A*	*D*	*C*		*C*	*D*	*A*	*B*	*G*	*H*	*E*	*F*
G	*H*	*E*	*F*	*C*	*D*	*A*	*B*		*H*	*G*	*F*	*E*	*D*	*C*	*B*	*A*
H	*G*	*F*	*E*	*D*	*C*	*B*	*A*		*D*	*C*	*B*	*A*	*H*	*G*	*F*	*E*

These two 8 × 8 Latin squares can be superimposed to give a Graeco–Latin square design.

9 × 9

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*		*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
B	*C*	*A*	*E*	*F*	*D*	*H*	*I*	*G*		*G*	*H*	*I*	*A*	*B*	*C*	*D*	*E*	*F*
C	*A*	*B*	*F*	*D*	*E*	*I*	*G*	*H*		*D*	*E*	*F*	*G*	*H*	*I*	*A*	*B*	*C*
D	*C*	*F*	*G*	*H*	*I*	*A*	*B*	*C*		*B*	*C*	*A*	*E*	*F*	*D*	*H*	*I*	*G*
E	*F*	*D*	*H*	*I*	*G*	*B*	*C*	*A*		*H*	*I*	*G*	*B*	*C*	*A*	*E*	*F*	*D*
F	*D*	*E*	*I*	*G*	*H*	*C*	*A*	*B*		*E*	*F*	*D*	*H*	*I*	*G*	*B*	*C*	*A*
G	*H*	*I*	*A*	*B*	*C*	*D*	*E*	*F*		*C*	*A*	*B*	*F*	*D*	*E*	*I*	*G*	*H*
H	*I*	*G*	*B*	*C*	*A*	*E*	*F*	*D*		*I*	*G*	*H*	*C*	*A*	*B*	*F*	*D*	*E*
I	*G*	*H*	*C*	*A*	*B*	*F*	*D*	*E*		*F*	*D*	*E*	*I*	*G*	*H*	*C*	*A*	*B*

These two 9 × 9 Latin squares can be superimposed to give a Graeco–Latin square design.

For other Latin squares see Fisher and Yates (1963). The designs above are from that source and are reproduced here with permission.