Lecture 12
Topic 9: Factorial treatment structures (Part I)

Factorial experiments

Allow the investigation of the effects of a number of variables on some response …

…in a highly *efficient* manner, and

…in a way that also allows the systematic characterization of relationships *among* factors.

Terminology

Factors Individual classes of treatments
Designated by upper-case letters (A, B, C, etc.)
e.g. Temperature, Gender, etc.

Levels: The different categories *within* each factor
Designated by lower-case letters with subscripts (a_1, a_2, a_3, etc.)
e.g. (0, 10, and 20º C), (♀ and ♂), etc.

A 2x3 factorial experiment
A 3x6x4 factorial experiment
Etc.

Factorial experiments and experimental designs

Experimental designs are characterized by *the method of randomization*.

Factorial experiments are characterized by a certain *treatment structure*, namely:

> The full set of treatments is composed of *all possible combinations of the levels of all factors* under investigation.

Factor A:	Temperature	Levels: 0, 10, and 20º C
Factor B:	Gender	Levels: ♀ and ♂

A factorial treatment structure will have 3x2 = 6 total treatment combinations:

0º C ♀ 10º C ♀ 20º C ♀ 0º C ♂ 10º C ♂ 20º C ♂

A factorial treatment structure may occur within *any* experimental design.

Examples of a 4 x 2 factorial experiment within three different experimental designs:

Since Factor A has 4 levels (1, 2, 3, 4) and Factor B has 2 levels (1, 2), there are eight different treatment combinations: (11, 12, 13, 14, 21, 22, 23, 24).

CRD with 3 replications

24 **23 13 23** ***24*** **14 13 23 11** ***24*** **12 14 22 13 12 21 21 11 22 12 11 22 21 14**

RCBD with 3 blocks

13 12 21 23 11 ***24*** **14 22**	**12 11** ***24*** **23 13 22 21 14**	***24*** **14 22 21 11 13 23 12**

8 x 8 Latin Square

24	**11**	**22**	**12**	**13**	**14**	**23**	**21**
21	**23**	**13**	**14**	**22**	**12**	**11**	***24***
12	**14**	***24***	**11**	**23**	**21**	**22**	**13**
13	**22**	**21**	***24***	**11**	**23**	**14**	**12**
23	**12**	**11**	**13**	**21**	**22**	***24***	**14**
14	***24***	**23**	**22**	**12**	**13**	**21**	**11**
11	**21**	**12**	**23**	**14**	***24***	**13**	**22**
22	**13**	**14**	**21**	***24***	**11**	**12**	**23**

Example of a 2x2 factorial experiment organized as a CRD

The two factors are Nitrogen levels (N_0 and N_1) and Phosphorous levels (P_0 and P_1) applied to a crop. The response variable is yield (lbs/acre).

Factor		**A = N level**			
	Level	**$a_1 = N_0$**	**$a_2 = N_1$**	**Means**	**a_2-a_1**
B = P level	**$b_1 = P_0$**	40.9	47.8	44.4	6.9 (*se* A,b_1)
	$b_2 = P_1$	42.4	50.2	46.3	7.8 (*se* A,b_2)
	Means	41.6	49.0	45.3	7.4 (*me* A)
	b_2-b_1	1.5 (*se* B,a_1)	2.4 (*se* B,a_2)	1.9 (*me* B)	

Main effect (*me*): The average difference between means of two levels of a factor.

Simple effect (*se*): The difference between means of two levels of a factor, *at one specified level of the other factor*.

Interactions and terminology

Many ways to say the same thing:

When the effect of one factor depends on the level of another factor, the two factors are said to exhibit an interaction (i.e. the factors are not independent).

If the simple effects of one factor are the same across all levels of another factor, the two factors are said not to exhibit an interaction (i.e. the factors are independent).

Interaction is a common and fundamental scientific idea.

The primary objective of a factorial experiment is to study the interactions among factors.

The Interaction SS measures the departure of group means from their expected values based on a purely additive model.

Synergism The combination of the levels of two factors *enhances* each others' effect.
Interference The combination of the levels of two factors *inhibits* each others' effect.

Both synergism and interference increase the Interaction SS.

		Factor B			A Means	τ_A
Factor A	Rep	b_1	b_2	b_3		
a_1	1	5.0	12.0	3.0	6.3	-2.2
	2	7.0	9.0	2.0		
	Mean	6.0	10.5	2.5		
a_2	1	9.0	14.0	8.0	10.7	2.2
	2	10.0	17.0	6.0		
	Mean	9.5	15.5	7.0		
B Means		7.8	13.0	4.8	**8.5**	
τ_B		-0.8	4.5	-3.8		

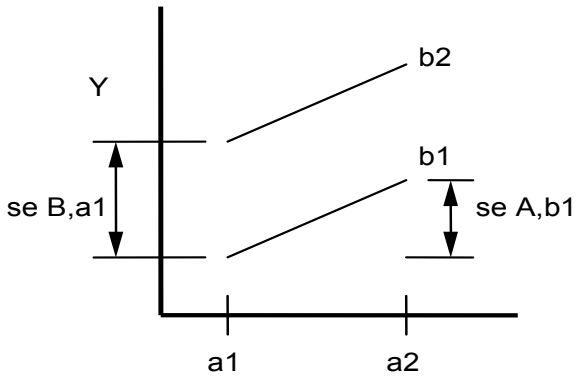
$SS_A = 56.3$
$SS_B = 139.5$
$SS_{A:B} = 1.2$

Where do these come from?

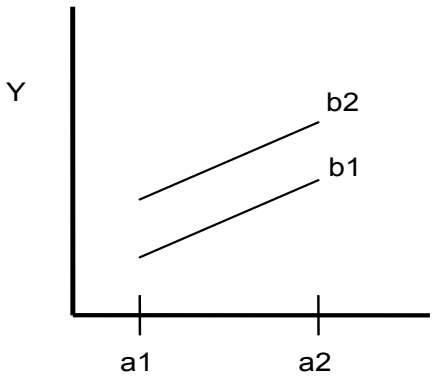
What happens if we have only 1 rep per A*B combination?

The differences between the simple effects of two factors (**first-order interactions** or **two-way interactions**), can be visualized in *interaction plots*:

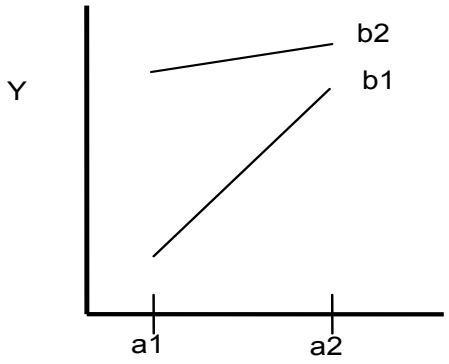
a. High me B, no interaction



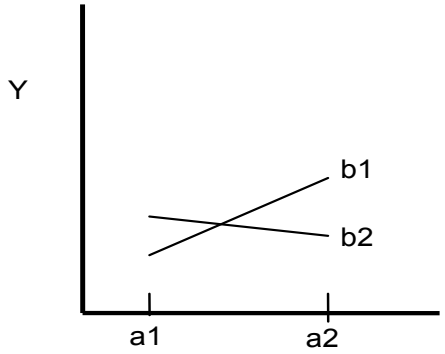
b. Low me B, no interaction



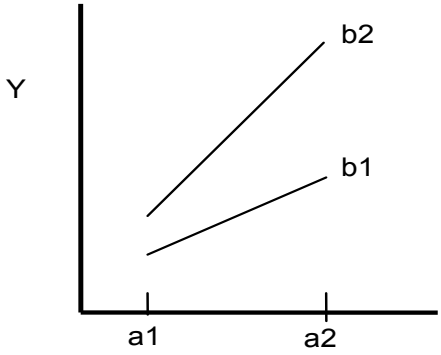
c. An interaction may be a difference in *magnitude* of the response



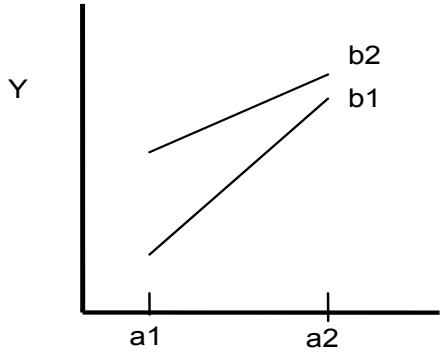
d. It may also be a difference in *direction* of response



e. Synergism



f. Interference



Reasons for carrying out factorial experiments

1. *To investigate interactions*
2. *To establish the dependence or independence of factors of interest*
3. *To offer recommendations that must apply over a wide range of conditions*

Some disadvantages of factorial experiments

1. *Size*
   To investigate 5 factors (3 levels each) in a factorial experiment requires, at minimum, **243** experimental units.
2. *Interpretation*
   Higher order interactions (three-way, four-way, etc.) are very difficult to interpret.

Distinguishing factors from experimental units

1. A factorial experiment in which leaf discs are grown in 10 different tissue culture media (all possible combinations of 5 different types of sugars and 2 different pH levels).
2. A CRD in which each sugar solution is prepared twice, in two separate batches.

	Sugar Type				
	1	2	3	4	5
pH_1	*	*	*	*	*
pH_2	*	*	*	*	*

	Sugar Type				
	1	2	3	4	5
Batch 1	*	*	*	*	*
Batch 2	*	*	*	*	*

The two pH classes are **common** across the entire study

The two Batch "classes" are not. Batch 1 for Treatment 1 has no closer relation to Batch 1 for Treatment 2 than it does to Batch 2 for Treatment 2.

Do the classes in question have a consistent meaning across the experiment, or are they simply ID's?

Distinguishing factors from blocks

1. A factorial experiment in which leaf discs are grown in 10 different tissue culture media (all possible combinations of 5 different types of sugars and 2 different pH levels).

2. An RCBD in which two sets of media are prepared by two different technicians.

	Sugar Type				
	1	2	3	4	5
pH_1	*	*	*	*	*
pH_2	*	*	*	*	*

	Sugar Type				
	1	2	3	4	5
Tech1	*	*	*	*	*
Tech2	*	*	*	*	*

Both experiments require two-way ANOVAs, but they differ in their *objectives*.

Are the effects of the classes in question of interest, or do they simply offer a means of partitioning variation from the error?

The two-way factorial analysis (Model I ANOVA)

The linear model:

$$Y_{ijk} = \mu + \tau_{Ai} + \tau_{Bj} + (\tau_A\tau_B)_{ij} + \varepsilon_{ijk}$$

τ_{Ai} is the fixed effect of factor A (i = 1,...,a)
τ_{Bj} is the fixed effect of factor B, (j = 1,...,b)
$(\tau_A\tau_B)_{ij}$ is the *interaction* of factor A level i with factor B level j
ε_{ijk} is the error associated with replication k of the factor combination ij (k = 1,..,r)

In dot notation:

$$Y_{ijk} = \overline{Y}_{...} + (\overline{Y}_{i..} - \overline{Y}_{...}) + (\overline{Y}_{.j.} - \overline{Y}_{...}) + (\overline{Y}_{ij.} - \overline{Y}_{i..} - \overline{Y}_{.j.} + \overline{Y}_{...}) + (Y_{ijk} - \overline{Y}_{ij.})$$

factor A effect | factor B effect | interaction effect (A*B) | experimental error

The null hypotheses: $\tau_{Ai} = 0$ $\tau_{Bj} = 0$ $(\tau_A\tau_B)_{ij} = 0$

$$TSS = SSA + SSB + SS(AB) + SSE$$

The ANOVA

The Treatment SS is partitioned into three orthogonal components:

$$\overbrace{SSA + SSB + SS(AB)}^{SST}$$

A 3x4 factorial has: 12 treatment combinations
11 df for treatment
2 df for Factor A
3 df for Factor B
6 df for the A*B Interaction

This partitioning is valid even when the overall F test among treatments is not significant!

Source	df	SS	MS	F
Factor A	a - 1	SSA	MSA	MSA/MSE
Factor B	b - 1	SSB	MSB	MSB/MSE
AxB	(a - 1)(b - 1)	SSAB	MSAB	MSAB/MSE
Error	ab(r - 1)	SSE	MSE	
Total	rab - 1	TSS		

The significance of the interaction F test determines what kind of subsequent analysis is appropriate:

No significant interaction: Subsequent analysis (mean comparisons, contrasts, etc.) are performed on the *main effects*.

Significant interaction: Subsequent analysis (mean comparisons, contrasts, etc.) are performed on the *simple effects*.

Example of a 2 x 3 factorial experiment within an RCBD

Data: The number of quack-grass shoots per square foot after spraying with maleic hydrazide. Treatments are maleic hydrazide applications rates (**R**: 0, 4, and 8 lbs/acre) and delay in cultivation after spraying (**D**: 3 and 10 days).

D	R	Block 1	Block 2	Block 3	Block 4	Means
3	0	15.7	14.6	16.5	14.7	15.38
	4	9.8	14.6	11.9	12.4	12.18
	8	7.9	10.3	9.7	9.6	9.38
10	0	18.0	17.4	15.1	14.4	16.23
	4	13.6	10.6	11.8	13.3	12.33
	8	8.8	8.2	11.3	11.2	9.88
Means		12.30	12.62	12.72	12.60	12.56

Randomization?

```
#The ANOVA
quack_mod<-lm(Number ~ D + R + D*R + Block, quack_dat)  ← WHAT'S MISSING??
anova(quack_mod)
```

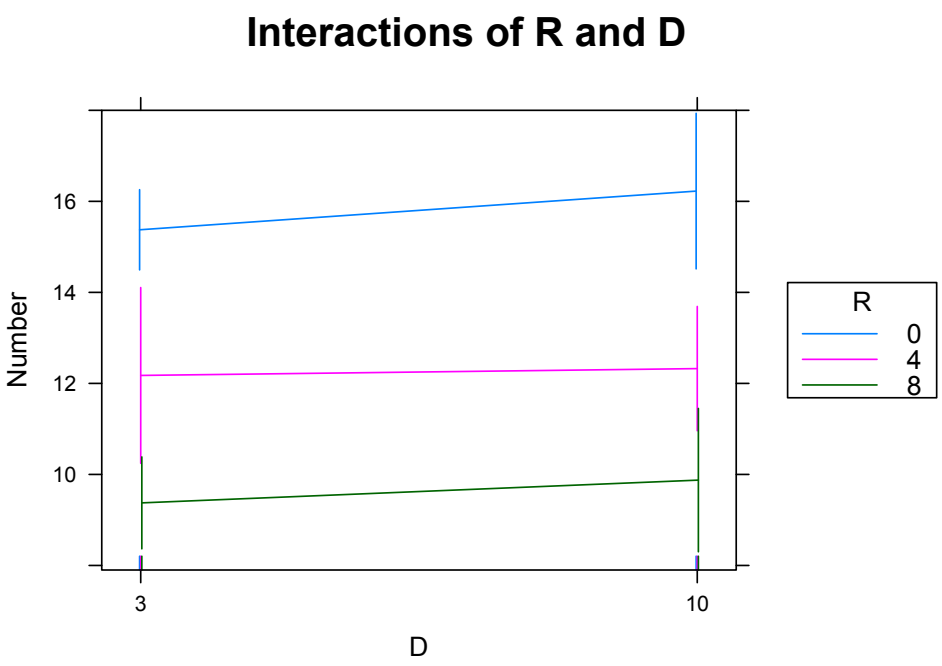
The output

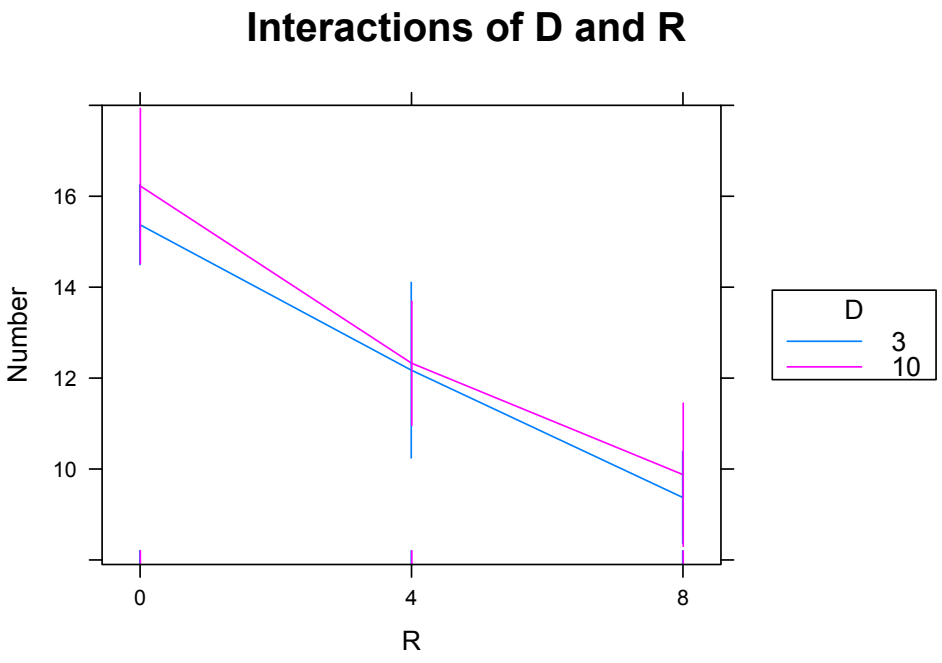
```
Analysis of Variance Table

          Df  Sum Sq Mean Sq F value    Pr(>F)
D          1   1.500   1.500  0.5713    0.4614
R          2 153.663  76.832 29.2630 6.643e-06 ***
Block      3   0.582   0.194  0.0738    0.9731
D:R        2   0.490   0.245  0.0933    0.9114 NS
Residuals 15  39.383   2.626
```

Note that the 15 error df =
Block*D (3 df) + Block*R (6 df) + Block*D*R (6 df)

Essentially parallel lines in the interaction plots reflect the absence of an interaction:





If no interaction is present, you proceed by analyzing the *main effects*.

```r
#Analyzing the main effects
library(agricolae)
HSD.test(quack_mod, "D")
HSD.test(quack_mod, "R")
```

Tukey means separations

D, means

```
alpha: 0.05 ; Df Error: 15
Critical Value of Studentized Range: 3.014325
Honestly Significant Difference: 1.409971

Means with the same letter are not significantly different.
```

Groups, Treatments and means

a	10	12.81
a	3	12.31

R, means

```
alpha: 0.05 ; Df Error: 15
Critical Value of Studentized Range: 3.673378
Honestly Significant Difference: 2.104414

Means with the same letter are not significantly different.
```

Groups, Treatments and means

a	0	15.8
b	4	12.25
c	8	9.625

```
#Performing a trend analysis on the factor R
# Contrast 'Linear'    -1,0,1
# Contrast 'Quadratic' 1,-2,1
contrastmatrix<-cbind(c(-1,0,1),c(1,-2,1))
contrasts(quack_dat$R)<-contrastmatrix

quack_Rcontrast_mod<-aov(Number ~ D + R + D*R + Block, quack_dat)
summary(quack_Rcontrast_mod, split = list(R = list("Linear" = 1,
                                                   "Quadratic" = 2)))
```

Contrasts (trend analysis of R)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
D	1	1.50	1.50	0.571	0.461	
R	2	153.66	76.83	29.263	6.64e-06	***
R: Linear	**1**	**152.52**	152.52	58.092	1.56e-06	***
R: Quadratic	**1**	**1.14**	1.14	0.435	0.520	
Block	3	0.58	0.19	0.074	0.973	
D:R	2	0.49	0.25	0.093	0.911	
D:R: Linear	**1**	**0.12**	0.12	0.047	0.832	
D:R: Quadratic	**1**	**0.37**	0.37	0.140	0.714	
Residuals	15	39.38	2.63			

Notice that this trend analysis using orthogonal contrasts partitioned not only the SS for the factor R but also the SS of the interaction D*R. Recall the original ANOVA table:

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
D	1	1.500	1.500	0.5713	0.4614	
R	**2**	**153.663**	76.832	29.2630	6.643e-06	***
Block	3	0.582	0.194	0.0738	0.9731	
D:R	**2**	**0.490**	0.245	0.0933	0.9114	
Residuals	15	39.383	2.626			

In this case, no significant interaction components were found "hiding" inside the overall non-significant interaction.

But it is possible to find significant interaction components within an overall non-significant interaction!

SO: When is it worth partitioning the Interaction SS?