

Model : $\hat{\text{income}} = b_0$

Loss Function : $L(b_0) = \sum_{i=1}^n (\hat{y}_i - y_i)^2 + b_0^2$

To find optimal value for b_0 , we need to minimize the loss function

$$\frac{dL(b_0)}{db_0} = 0 \Rightarrow \frac{d(\sum_{i=1}^n (\hat{y}_i - y_i)^2 + b_0^2)}{db_0} = 0$$

Since $\hat{y}_i = \hat{\text{income}} = b_0$, the equation is rewritten as:

$$\frac{d(\sum_{i=1}^n (-y_i + b_0)^2 + b_0^2)}{db_0} = 0$$

$$\frac{d(\sum_{i=1}^n (-y_i + b_0)^2)}{db_0} + \frac{d(b_0^2)}{db_0} = 0$$

$$\sum_{i=1}^n 2(b_0 - y_i)(1) + 2b_0 = 0$$

$$2 \sum_{i=1}^n (b_0 - y_i) + 2b_0 = 0$$

$$2 \sum_{i=1}^n b_0 - 2 \sum_{i=1}^n y_i + 2b_0 = 0$$

$$\sum_{i=1}^n b_0 + b_0 = \sum_{i=1}^n y_i$$

$$nb_0 + b_0 = \sum_{i=1}^n y_i$$

$$b_0 = \frac{\sum_{i=1}^n y_i}{(n+1)}$$

$$b_0 = \text{optimal prediction} = \sum_{i=1}^n y_i / (n+1)$$

$$= (20000 + 35000 + 40000) / 4$$

$$= 23750$$