CS360 Fall 2013 -Assignment 4

Due Wednesday, Nov 6th, 11:59am

1. Let us call a language *Reg-pumpable* if it satisfies the conclusion of the pumping lemma for regular languages. Namely, L is Reg-pumpable if there exist some n such that for every $w \in L$, if $|w| > n$ then the is a decomposition $w = xyz$ so that for any $i \geq 0$, $xy^iz \in L$.
   Similarly, say that a language L is *CF-pumpable* if it satisfies the conclusion of the CFL pumping lemma.

   (a) [**5 points**] Prove that every language that is Reg-pumpable is also CF-pumpable.

   (b) [**5 points**] Prove that the converse of the above claim is false. Namely, find a language that is CF-pumpable but not Reg-pumpable. Prove your claims.

   (c) [**5 points**] A language L over some alphabet, Σ, is a *Length language* if, for any word, $w \in L$, every w' of the same length is also in L (namely, $\forall w, w',\ |w| = |w'|$ implies $w \in L$ if and only if $w' \in L$). Prove that every CF-pumpable length language is also Reg-pumpable.

   (d) [**10 points**] Prove that $L_{cube} = \{a^{n^2} : n \geq 0\}$ is not a context-free language.

2. For each of the following languages determine whether it is a regular language or not, and whether it is a CFL or not:

   (a) [**5 points**] $L_{sq} = \{a^n b^m :\ n \equiv m (mod\ 3)\}$.

   (b) [**10 points**] $L_p = \{w \in \{a, b\}^* :\ |w|\ is\ a\ prime\ number\ \}$.

   (c) [**5 points**] $L_{odd\_eq} = \{a^j b^k c^\ell : j\ is\ odd\ \texttt{or}\ k = \ell\}$.

3. Define a grammar, $G = (V, T, S, P)$, by setting $V = \{S\}$, $T = \{a, b\}$ and $P = \{S \mapsto ab|aSb|aSSb\}$.

   (a) [**10 points**] Prove that $L(G) = I\left(\{ab\}, \{\frac{w}{awb}, \frac{w_1, w_2}{aw_1w_2b}\}\right)$. That is, $L(G)$ equals the language that is defined by structural induction as the closure of the core set, $\{ab\}$, under the operations, $\frac{w}{awb}$ and $\frac{w_1, w_2}{aw_1w_2b}$.

   (b) [**5 points**] Prove that, for every $w \in L(G)$, $n_a(w) = n_b(w)$.

   (c) [**5 points**] Prove that for every $w \in L(G)$, if w' is a proper initial segment of w (that is, there exist some word $w'' \neq \epsilon$, such that $w = w'w''$) then $n_a(w') > n_b(w')$.

   (d) [**5 points**] Is $L(G)$ a regular language? Prove your claim.

4. For a language L over some finite alphabet, Σ define a new language $F_0(L) = \{0^{|w|} : w \in L\}$. That is, the set of all strings of 0's that have the same length as some word in L.

   (a) [**10 points**] Prove that for every regular language L the language $F_0(L)$ is also regular.

   (b) [**10 points**] Prove that for every context free language L the language $F_0(L)$ is also context free.

   (c) [**10 points**] Find a language L such that L is not a context free language but $F_0(L)$ is regular. Prove your claims.