CS 360 Fall 2013
Assignment 1.
Due Wednesday September 25 at noon.

For all problems you are expected to justify your answers, by showing your work or stating arguments, as is appropriate.

1. For each of the following statements, prove the statement if it is always holds true (for every choice of sets x, Y, Z), and provide a counter example if not:
   (a) If every member if Y is an infinite set, then so is $\bigcap Y$.
   (b) If $Y \subseteq Z$ then $\bigcap Y \subseteq \bigcap Z$.
2. Let $Y = \{B \subseteq \mathbf{N} : (\mathbf{N} \setminus B) \text{ is a finite set}\}$. Describe the set $\bigcap Y$ explicitly (recall that $\bigcap Y = \{z : \forall x \in Y, : \ z \in x\}$. That is, we assume that all members of Y are sets and take their intersection).
3. Is the following statement true? Prove your claim.

   For any non-empty language, $L \subseteq \{0, 1\}^*$,

   $$\epsilon \in L \text{ if and only if } L \subseteq LL$$

   (where ϵ denotes the empty string).
4. Define a language $L_{times} \subseteq \{a, b, c\}^*$ by *structural induction* as $I(A, P)$ where :
   - $A = \{abaca\}$
   - $P = \{F_1, F_2\}$ where, for every i, j, k, $F_1(a^i ba^j ca^k) = a^{i+1} ba^j ca^{k+j}$ and $F_2(a^i ba^j ca^k) = a^i ba^{j+1} ca^{k+i}$.

   In other words, a string is in L_{times} if and only if the above rules imply that it has to be there.

   For each of the following strings, determine if it is in L_{times} or not:

   (i) *ababab*. (ii) *aaabaacaaaaaa*. (iii) *aabacaaaa*.

   Prove each statement you make (to prove that a string belongs to the language, provide a sequence of strings, such that each string there is either *abaca* or its membership in the language is implied by applying the functions F_1, F_2, to previous strings in the sequence. To prove that a string is not in L_{times}, prove that some property of strings holds for all members of L_{times} but fails for that string).

5. You are sitting in a restaurant with two glasses in front of you, each filled to 3/4 of its capacity. The first glass contains wine and the second contains water. Out of boredom, you start playing the following game: You pour liquid from one glass to the other, and mix the liquid in that other glass. You then repeat your pouring game over and over again (without spilling anything). Prove, that as long as you follow the above steps, there will always be more wine than water in the first glass. (**Hint**: *use structural induction on the set of all obtainable states of the glasses*).

6. Consider the regular expressions, $r_1 = \lambda + (0+1)^*1$ and $r_2 = (0^*1)^*$. Prove that $L(r_1) = L(r_2)$. Recall, that you should show two sided inclusion between these two languages.

7. For each of the languages listed below, give a regular expression that represents the language. Briefly justify your answer. The languages are all over alphabet $\Sigma = \{a, b\}$.

   (a) The language of all strings with two consecutive a's, but no three consecutive a's. (e.g. aa, $aabaa$, $babaa$ are in the language but $abab$, $aaaab$ are not).

   (b) The set of strings whose first and last letters are different.