CS360 Fall 2013-Assignment 3

Due Wednesday, October 23th, 11:59am

1. (a) [**10 points**] Prove that for every pair of regular expressions, r_1, r_2, there exist a regular expression r, such that $L(r) = L(r_1) \setminus L(r_2)$. (For sets A, B, we define $A \setminus B$ as $\{x : x \in A \text{ and } x \notin B\}$).

   (b) [**20 points**] Find a regular expression r over $\Sigma = \{a, b\}$, such that $L(r) = L((ab + b)^\star + (ba + b)^\star) \setminus L((b^\star ab^\star)$. Explain in detail how you constructed r and why it does the job.

2. (a) [**10 points**] Prove that for every finite set K of regular languages, $\cap K$ is also a regular language.

   (b) [**Bonus 5 points**] Does the previous claim remain valid if we allow K to be infinite? Prove your claim.

3. Given a language L, define a language

$$P(L) = \{w \in L : \text{ for every } x, y, z, \text{ such that } w = xyz, \text{ if } y \neq \epsilon \text{ then } \exists i \text{ such that } xy^iz \notin L\}$$

   (a) [**10 points**] Prove that for every regular language L, the language $P(L)$ is also regular.

   (b) [**10 points**] Find language L for which $P(L)$ is not a regular language.

4. For each of the following languages, determine whether they are regular or not. If they are, provide an appropriate regular expression or an ϵ-NFA, or show how their existence follows from closure properties that we have shown in class. If they are not regular, prove why.

   (a) [**10 points**] $L_1 = \{x \in \{a, b\}^\star$: every occurrence of aaa in x is immediately followed by a longer sequence of b's $\}$.

   (b) [**10 points**] $L_3 = \{x \in \{a, b\}^\star : n_a(x) - n_b(x) = 2(mod5)\}$, (where, for a letter $\sigma \in \Sigma$, and a word $w \in \Sigma^*$, $n_\sigma(w)$ denotes the number of occurrences of σ in w).

   (c) [**10 points**] $L_4 = \{a^k b^l c^m : l + k = m(mod3)\}$.

   (d) [**10 points**] $L_5 = \{(o1)^k (10)^k : k \in N\}$.

5. [**Bonus 10 points**] Let f be any function from the natural numbers to the natural numbers. Let L be a regular language over some alphabet Σ such that, for every $n \in N$,

$$\{w \in \Sigma^\star : f(n) \leq |w| \leq f(n) + n\} \subseteq L$$

   (that is, every word of length between $f(n)$ and $f(n) + n$ belongs to L). Prove that $\Sigma^\star \setminus L$ is finite.