

CS 466/666 Spring 2014  
Assignment 4  
Due Noon, June 9, 2014

You are on your honour to present your own work and acknowledge your sources.

1. [14 marks] This question deals with an algorithm to find the second largest of  $n$  values. In class we mentioned/proved that if we scan  $n$  values comparing each with the maximum seen so far we expect the number of “new maxima” to be  $\sum 1/i \cong \ln n + \gamma$ , where  $\gamma = .577\dots$

Take a random sample,  $R$ , of  $r$  of the values;  $r = n/\lg n$  (or so).

By repeatedly pairing the elements that have “won” every comparison, find the maximum of the sample (in  $r-1$  comparisons). Initialize  $m_1$  as the sample maximum and  $m_2$  as the element that was the maximum of half the sample but “lost” to the maximum in the last comparison.

Scan through the remaining elements (i.e. those not in  $R$ ), comparing each to  $m_2$  and if it is larger than  $m_2$  compare it with  $m_1$ . Update  $m_1$  to be the maximum seen so far and  $m_2$  to be the second largest (perhaps excluding some of the values from the  $R$ ).

When finished the scan,  $m_1$  is the maximum of the set; under some condition, you may know  $m_2$  is the second largest value in the set. If you cannot guarantee  $m_2$  is the second largest, find the maximum of  $m_2$  and the random sample elements that “lost” directly to the original  $m_1$ .

- a. [2 marks] What is the maximum number of comparisons this method could use? (Off by  $O(1)$  is OK, but no more. Briefly justify your claim.)
- b. [4 marks] What condition, detectable in this algorithm, would guarantee that  $m_2$  is the second largest after finishing the scan of all elements? What is the probability that this condition will hold? (Briefly justify this claim)
- c. [2 marks] If the condition above is not satisfied, how many more comparisons are required?
- d. [4 marks] What is the expected number of times  $m_2$  is replaced (i.e. the number of times a second comparison is done) in scanning the  $n - n/\lg n$  non sample elements? (Give the expected number of times including the constant factor in the lead term, and justify the claim)
- e. [4 marks] Given the probability of the condition (of part b.) being satisfied and the expected number of comparisons used in that case, and the probability of it not being satisfied and the number of comparisons used in the latter case, what is the expected number of comparisons used by the method. (Justify this claim.)